Привет!

2021-12-~~02~~ 3

→ как выйти на N?

→ хар.-ие ф-ции.

Лекция

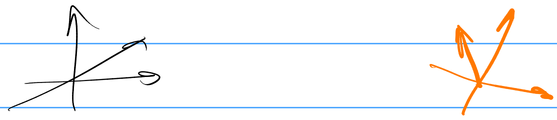
def
$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot \exp\left(-\frac{1}{2\sigma^2}(x-\mu)^2\right)$$

$$\boxed{X \sim N(\mu; \sigma^2)}$$

Откуда?

$$V = \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix}$$ — вектор скор-ей подви-ой част.

[Rot] Закон распр-ия вектора не изм-ся при любом фикс-м повороте / отражении.



$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{pmatrix}$$

$$P(V_1 > 0.3) = P(V_1 > 0.3)$$

[Proj] Проекции вектора V на орт-ые подпр-ва независимы.

[d=2] [ф. плотности распр-ма]

← это можно док-ть из [Rot] и [Proj]

$f(v_1, v_2)$

[Rot] → при повороте $v = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$ ф. плотн остаётся такой же.

$$\underline{f(v_1, v_2)} = h(\underbrace{v_1^2 + v_2^2})$$

[Proj] V_1 и V_2 незав. случ.

$$f(v_1, v_2) = f_{V_1}(v_1) \cdot f_{V_2}(v_2)$$

[Rot] $V_1 \sim V_2$

$$f(v_1, v_2) = f(v_1) \cdot f(v_2)$$

[Rot] отражения $V_1 \sim -V_1$

$$f(v_1) = f(-v_1) = g(v_1^2)$$

$$f(v_1, v_2) = h(v_1^2 + v_2^2) = g(v_1^2) \cdot g(v_2^2) \quad \forall v_1, v_2$$

запахло экспонентой!

$$\underline{h(v_1^2 + 0)} = \underline{g(v_1^2)} \cdot g_0$$

$$\boxed{h(v_1^2 + v_2^2) = k \cdot h(v_1^2) \cdot h(v_2^2)}$$

? ln

...

производ по $V_1^2 = a$

$$h'(v_1^2 + v_2^2) \cdot 1 = \underbrace{k \cdot h'(v_1^2)} \cdot h(v_2^2) \quad \forall v_1, v_2$$

$v_1 = 0$

$$\boxed{h'(v_2^2) = c \cdot h(v_2^2)} \Rightarrow h(v_2^2) = h_0 \cdot e^{cv_2^2}$$

$(e^{3t})' = 3 \cdot e^{3t}$

$(5e^{3t})' = 3 \cdot (5e^{3t})$

$$f(x_1;x_2) = h(v_1^2+v_2^2) = h_0\cdot e^{c(V_1^2+V_2^2)} =$$

$$= \boxed{\sqrt{h_0}\cdot e^{cV_1^2}}\cdot\sqrt{h_0}\cdot e^{cV_2^2}$$

$$a = V_1^2 \qquad \frac{dh}{da} = c\cdot h \qquad \frac{dh}{h} = c\cdot da$$

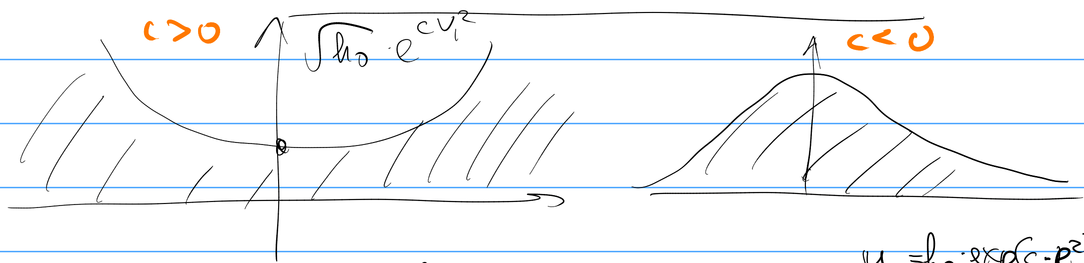
$$\ln h = c\cdot a + const$$

$$h = \underbrace{\boxed{e^{const}}}_{h_0} e^{ca}$$

Шаг 1

$$\iint_{\mathbb{R}^2} f(x_1,x_2)\,dx_1\,dx_2 = 1 \Rightarrow h_0 \text{ через } c$$

$c>0$ или $c<0$?

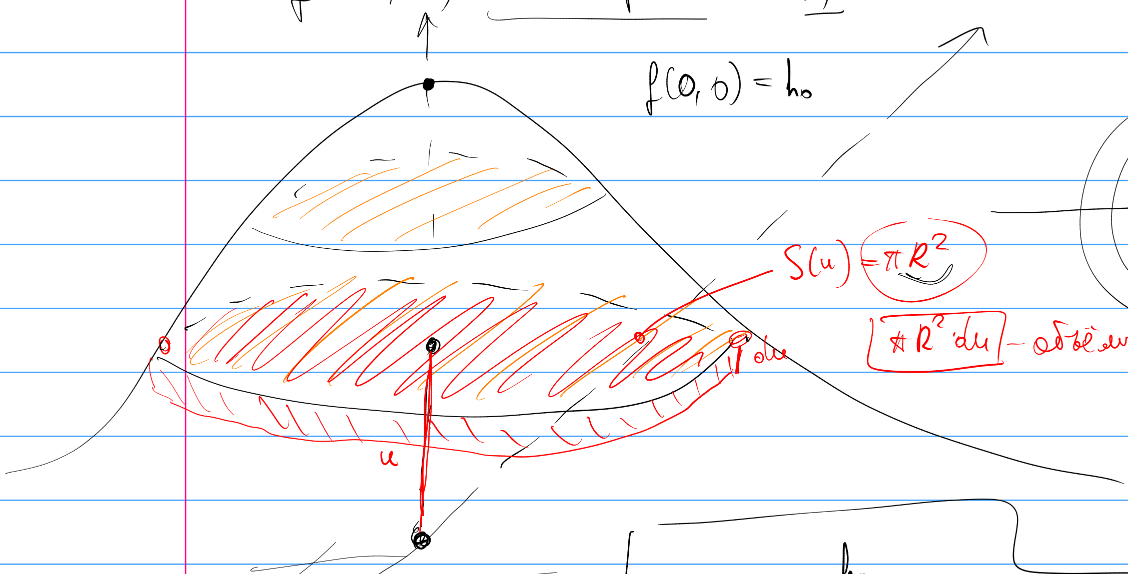


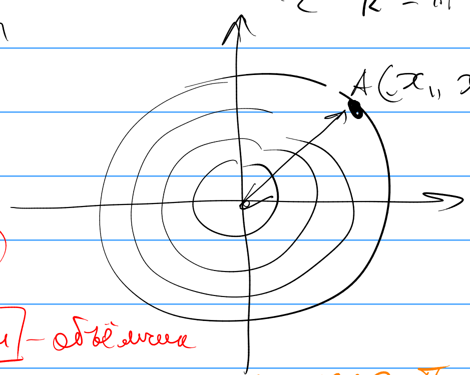
$$f(x_1,x_2) = h_0\cdot\exp(c(x_1^2+x_2^2))$$

$$f(0,0) = h_0$$

$$u = h_0\cdot\exp(c\cdot R^2)$$

$$R^2 = x_1^2+x_2^2$$





$\boxed{\pi R^2\,du}$ — объёмчик

запахло π !

$$\ln u = \ln h_0 + c\cdot R^2$$

$$R^2 = \frac{\ln u - \ln h_0}{c}$$

$$\text{Volume} = 1 = \boxed{\int_0^{h_0} \pi\cdot\frac{\ln u - \ln h_0}{c}\,du}$$

$$\frac{c}{\pi} = \int_0^{h_0} \ln u - \ln h_0\,du$$

$$\frac{c}{\pi} = \int_0^{h_0} \ln u - \ln h_0 \, du$$

$u \in [0; h_0]$

$$\frac{c}{\pi} = \int_0^{h_0} \ln\left(\frac{u}{h_0}\right) du$$

$\frac{u}{h_0} = t \quad u = t \cdot h_0$

$t \in [\;;\;]$

$du = h_0 dt$

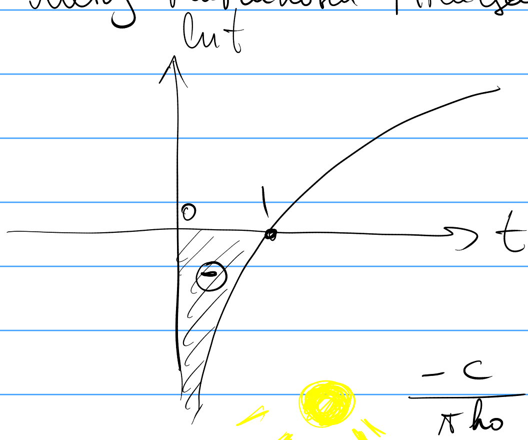
$$\frac{c}{\pi} = \int_0^1 \ln t \cdot h_0 \, dt$$

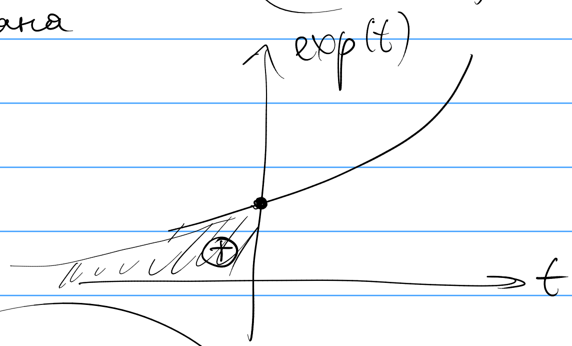
$c<0$

$$\frac{c}{\pi h_0} = \int_0^1 \ln t \, dt \; \ldots$$

$(t \cdot \ln t - t)' = \ln t$

Метод Маркинона Мнацаканяна



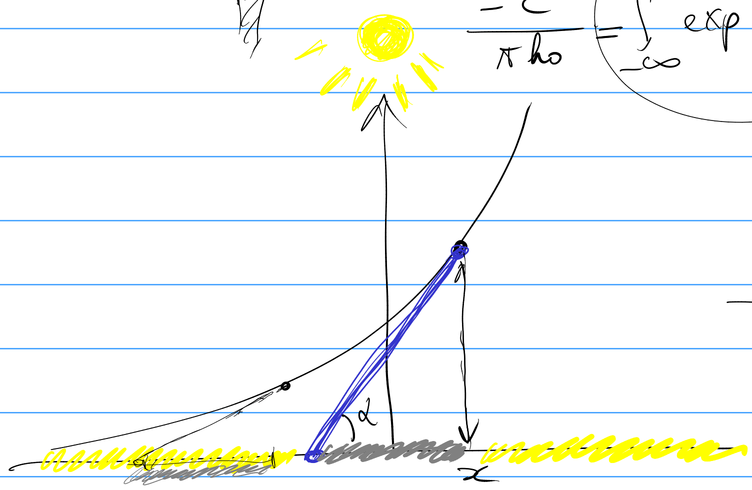


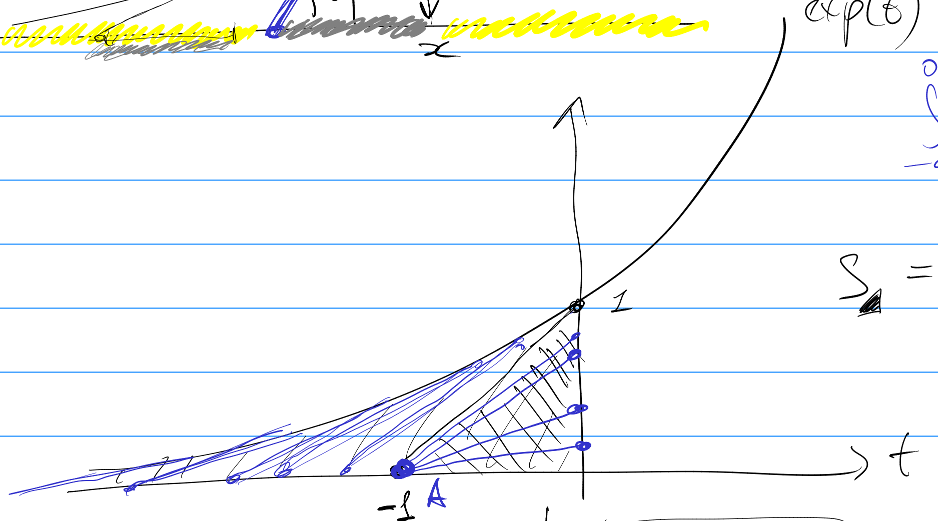
$$\frac{-c}{\pi h_0} = \int_{-\infty}^0 \exp(t)\, dt$$

$(e^x)' = e^x$

$tg\,\alpha = \boxed{e^x = \frac{e^x}{\text{тень}}}$

длина тени? $= 1$





$$\int_{-\infty}^0 \exp(t)\, dt = \frac{1}{2} + \frac{1}{2} = 1$$

$S_\Delta = \frac{1}{2}$

$c = -d$

$$\frac{-c}{\pi h_0} = 1$$

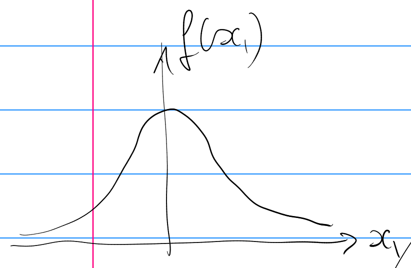
$$\boxed{h_0 = \frac{d}{\pi}}$$

$$\boxed{f(x_1, x_2) = \frac{d}{\pi} \cdot \exp(-d(x_1^2 + x_2^2))}$$

Упр. Найдите α если доп-но из-но, что $Var(V_i) = \sigma^2$.

$$f(x_1, x_2) = \frac{\alpha}{\pi} \cdot \exp\left(-\alpha(x_1^2 + x_2^2)\right)$$

$$= \underbrace{\boxed{\sqrt{\frac{\alpha}{\pi}} \cdot \exp(-\alpha x_1^2)}}_{f(x_1)} \cdot \underbrace{\sqrt{\frac{\alpha}{\pi}} \cdot \exp(-\alpha x_2^2)}_{f(x_2)}$$



$f(x_1)$

$$E(V_1) \overset{?}{=} 0 \leftarrow \text{по аксиоме [Rot]}$$

$$\int_{-\infty}^{\infty} \underbrace{x_1 \cdot \underbrace{f(x_1)}_{\text{четная}}}_{\text{нечетная}} dx_1 = 0$$

$$Var(V_1) = E(V_1^2) = \sigma^2$$

$$\boxed{\int_{-\infty}^{\infty} x_1^2 \cdot f(x_1)\, dx_1 = \sigma^2} \overset{?}{\Rightarrow} \alpha?$$

$$\int_{-\infty}^{\infty} x_1^2 \sqrt{\frac{\alpha}{\pi}} \cdot \exp(-\alpha x_1^2)\, dx_1 = \sigma^2$$

$dx_1 = \frac{da}{\sqrt{\alpha}}$

$a = \sqrt{\alpha} \cdot x_1$

$a^2 = \alpha \cdot x_1^2$

$x_1 \in (-\infty; \infty)$

$a \in (-\infty; \infty)$

$x_1^2 = \frac{a^2}{\alpha}$

$$\int_{-\infty}^{\infty} \frac{a^2}{\alpha} \cdot \sqrt{\frac{\cancel{\alpha}}{\pi}} \cdot \exp(-a^2) \cdot \frac{da}{\cancel{\sqrt{\alpha}}} = \sigma^2$$

$$\int_{-\infty}^{\infty} a^2 \exp(-a^2)\, da = \sigma^2 \cdot \alpha \cdot \sqrt{\pi}$$

$$\int_{-\infty}^{\infty} \underset{u}{a} \cdot \underset{v' = \left(\frac{\exp(-a^2)}{-2}\right)'}{\boxed{a \cdot \exp(-a^2)}} \cdot da = \underset{+\infty}{a} \cdot \underset{\to 0}{\frac{\exp(-a^2)}{-2}} \Bigg|_{a=-\infty}^{a=+\infty} - \int_{-\infty}^{\infty} \underset{u'}{1} \cdot \underset{v}{\frac{1}{-2} \cdot \exp(-a^2)}\, da$$

$$\frac{1}{2}\int_{-\infty}^{\infty}\frac{1}{\sqrt{\pi}}\exp(-a^2)\,da = \sigma^2\cdot d$$

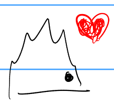
$$\int_{-\infty}^{\infty}\sqrt{\frac{d}{\pi}}\cdot\exp(-dx^2)\,dx = 1 \quad \forall d$$

$$\int_{-\infty}^{\infty}\frac{1}{\sqrt{\pi}}\cdot\exp(-x^2)\,dx = 1$$

$$\frac{1}{2} = \sigma^2\cdot d$$

$$d = \frac{1}{2\sigma^2}$$

$$f(x_1) = \sqrt{\frac{d}{\pi}}\cdot\exp(-dx_1^2) = \frac{1}{\sqrt{2\pi\sigma^2}}\cdot\exp\left(-\frac{1}{2\sigma^2}\cdot x_1^2\right)$$



$[\text{Rot}] + [\text{Proj}] \Longrightarrow$ V_1 и V_2 незав

$f(x_1)$ такая

Аксиомы Гершеля–Максвелла.