Name, group:

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1. Consider the random sample X_1, X_2, ..., X_n from the distribution with the density

$$f(x) = (a+1)x^a, \quad x \in [0;1].$$

   (a) Find the estimator of a using maximum likelihood;

   (b) Find the estimator of a using method of moments.

2. You found a strange coin. You toss it 200 times and it lands 120 times head up and 80 times tail up.

   Test the hypothesis that head and tail have the same probability using significance level $\alpha = 0.05$:

   (a) using Pearson's test;

   (b) using likelihood ratio.

3. Maria measured her weight twice, before and after drinking 100g of smoothie. But the measures differ by 200g. The problem is with scales. Each measurement is the true weight plus random error with expected value zero.

   Provide an unbiased estimate of weighing error variance.

4. You estimate the probability of some event using 1000 observation. Find the maximal possible length of 95% confidence interval for the unknown probability.

5. You have two indepent observations X_1 and X_2. The null hypothesis states that the distribution is uniform on $[0;1]$. The alternative hypothesis states that both random variables have the density $f(x) = 2x$ on $[0;1]$.

   Using Neyman-Person lemma construct the most powerful test for the probability of first type error $\alpha = 0.05$.

6. You found a strange coin. You toss it n times to test the hypothesis that head and tail have the same probability. You use two statistics, LR — likelihood ratio and PT — Pearson's test. Under the null-hypothesis find probability limit of LR/PT as $n \to \infty$.