# Иллюстрации к лекциям по курсу 'Теория Вероятностей и Математическая Статистика"

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## Contents

# 1 Использованные пакеты

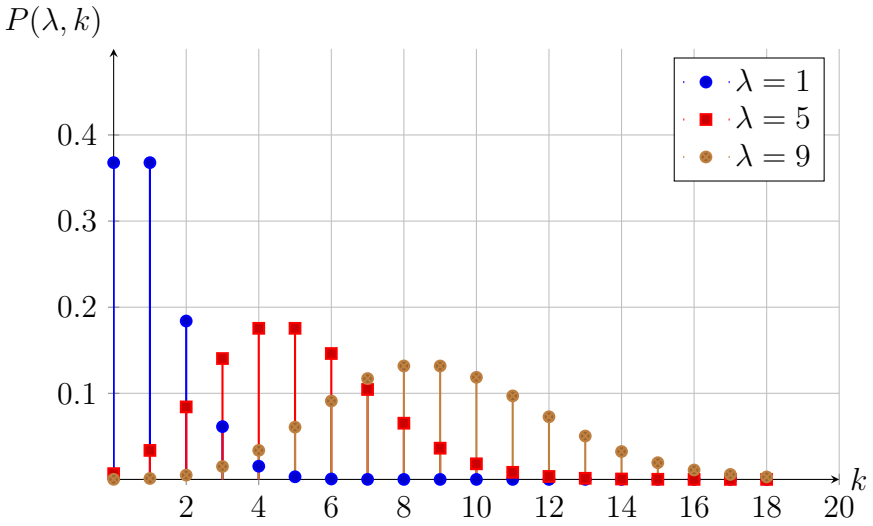
```
\usepackage{pgfplots}
\usepackage{mathtools,amssymb}
\usepackage{tikz}
\usepackage{pgfplots}
\usepackage{listings}
\usepackage{xcolor}
\usepackage{hyperref}
\usepackage[russian,english]{babel}
\usepackage{tkz-euclide}
```

# 2 Дискретные распределения

## 2.1 Распределение Пуассона

### Функция вероятности

$$P(\lambda, k) = \frac{\lambda^k \cdot e^{-\lambda}}{k!}$$

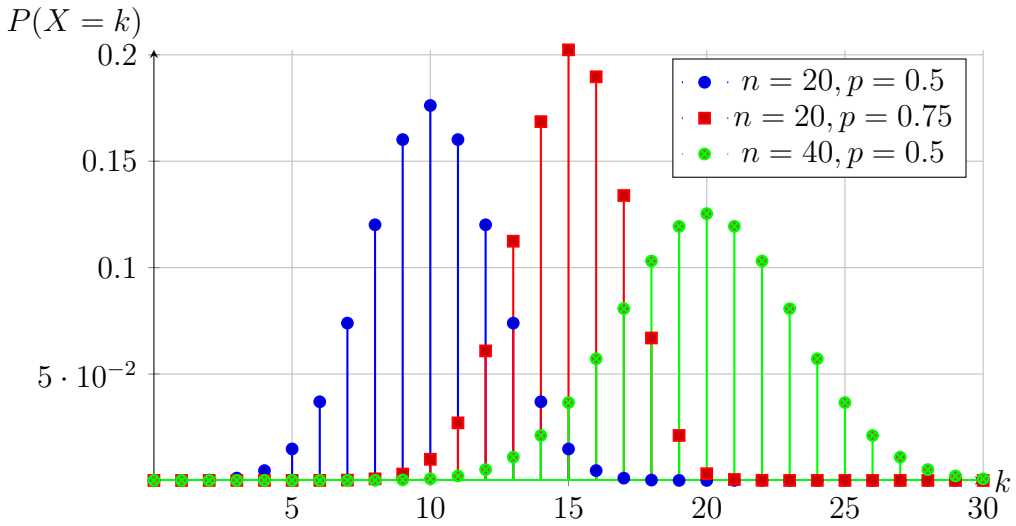



```
\pgfmathdeclarefunction{poiss}{1}{%
  \pgfmathparse{(#1^x)*exp(-#1)/(x!)}}
\begin{center}
\begin{tikzpicture}
\begin{axis}[
               axis x line=center,
               axis y line=center,
               xtick={0,2,...,19},
               ytick={0.1,0.2,...,0.4},
                 domain = 0:18,
                 samples = 19,
                 xlabel={$k$},
                 ylabel={$P(\lambda,k)$},
                 xlabel style={right},
                 ylabel style={above left},
                 ymax=0.5,xmax=20,x post scale=1.4,grid = major]
                 \addplot+[ycomb,blue,thick] {poiss(1)};
                 \addlegendentry{$\lambda = 1$}
                 \addplot+[ycomb,red,thick] {poiss(5)};
                  \addlegendentry{$\lambda = 5$}
                 \addplot+[ycomb,brown,thick] {poiss(9)};
                  \addlegendentry{$\lambda = 9$};
             \end{axis}
\end{tikzpicture}
\end{center}
```

## 2.2 Биномиальное распределение

### Функция вероятности

$$P(X = k) = C_n^k \cdot p^k \cdot (1-p)^{n-k}$$

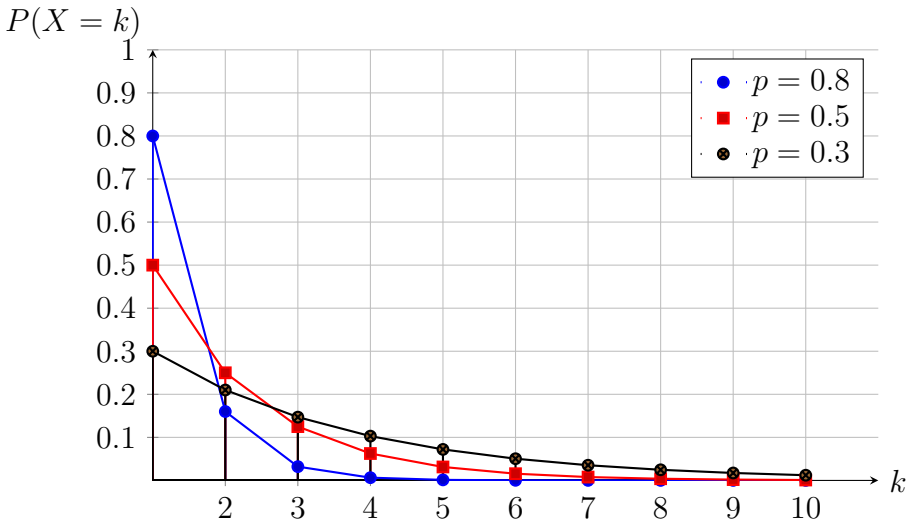



```
\pgfmathdeclarefunction{binom}{2}{\pgfmathparse{
(((#2)!)/(((#2-x)!)*(x!))) * ((#1)^x) * (1-#1)^(#2-x)}%
}
\begin{center}
\begin{tikzpicture}

\begin{axis}[axis x line=center,
               axis y line=center,
                 domain = 0:30,
                 samples = 31,
                 ytick = {0,0.05,0.1,...,0.5},
                 xlabel={$k$},
                 ylabel={$P(X = k)$},
                 xlabel style={right},
                 ylabel style={above left},
                 x post scale=1.6,
                 grid = major]
\addplot+[ycomb,blue,thick]  {binom(0.5, 20)} \closedcycle;
\addlegendentry{$n=20, p=0.5$};
\addplot+[ycomb,red,thick]  {binom(0.75, 20)} \closedcycle;
\addlegendentry{$n=20, p=0.75$};
\addplot+[ycomb,green,thick]  {binom(0.5, 40)} \closedcycle;
\addlegendentry{$n=40, p=0.5$};
\end{axis}
\end{tikzpicture}
\end{center}
```

## 2.3 Геометрическое распределение

### Функция вероятности

$$P(X = k) = p \cdot (1 - p)^{k-1}$$



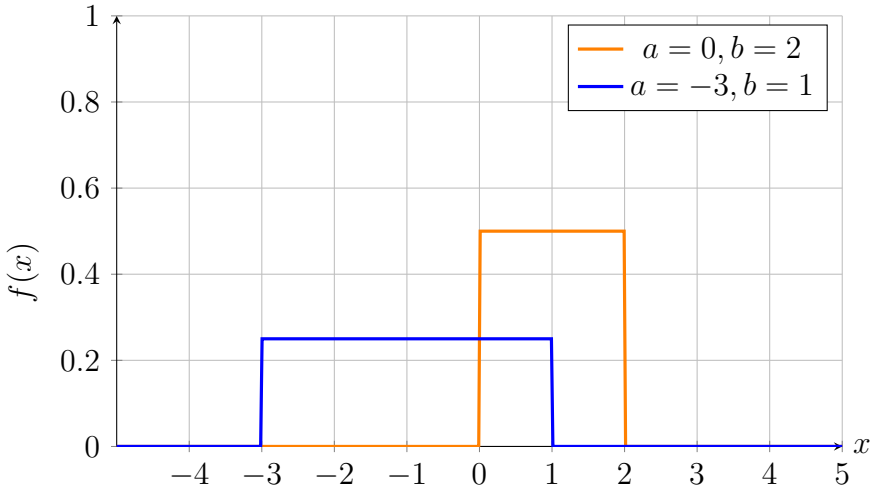

```
\pgfmathdeclarefunction{geom}{1}{\pgfmathparse{( 1 - #1 ) ^ ( x - 1) * #1}}
\begin{center}
\begin{tikzpicture}
\begin{axis}[axis x line=center,
               axis y line=center,
               xtick={1,2,...,10},
               ytick={0.1, 0.2,...,1},
                 domain = 1:10,
                 samples = 10,
                 xlabel={$k$},
                 ylabel={$P(X = k)$},
                 xlabel style={right},
                 ylabel style={above left},
                 ymax=1,
                 xmax=11,
                 x post scale=1.4,
                 grid = major]
\addplot+[ycomb,blue,thick]  {geom(0.8)} \closedcycle;
\addlegendentry{$p = 0.8$};
\addplot+[ycomb,red,thick]  {geom(0.5)} \closedcycle;
\addlegendentry{$p = 0.5$};
\addplot+[ycomb,black,thick]  {geom(0.3)} \closedcycle;
\addlegendentry{$p = 0.3$};
\addplot[blue,thick]  {geom(0.8)};
\addplot[red,thick]  {geom(0.5)};
\addplot[black,thick]  {geom(0.3)};
\end{axis}
\end{tikzpicture}
\end{center}
```

# 3 Непрерывные распределения

## 3.1 Равномерное распределение

### 3.1.1 Функция плотности

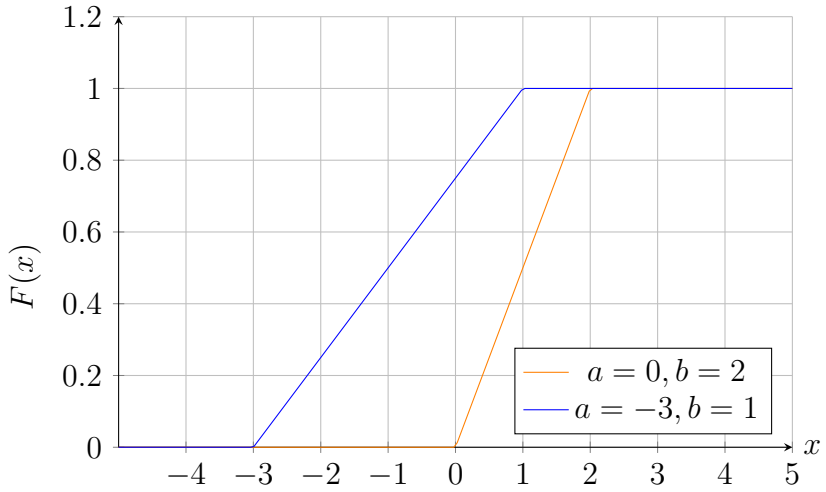
$$f(x) = \begin{cases} \dfrac{1}{b-a} & , x \in [a,b] \\ 0 & , x \notin [a,b] \end{cases}$$




```
\begin{center}
\begin{tikzpicture}[
    declare function={unipdf(\x,\xl,\xu)= (\x>=\xl)*(\x<\xu)*1/(\xu-\xl);}
]

\begin{axis}[axis x line=center,
             axis y line = left,
                 ymin=0,ymax=1,
                 xmin=-5, xmax=5,
                 samples = 500,
                 xlabel={$x$},
                 ylabel={$f(x)$},
                 xlabel style={right},
                 ylabel style={above left},
                 x post scale=1.4,
                 grid = major
]
\addplot [very thick, orange] {unipdf(x,0,2)};
\addlegendentry{$a=0, b=2$};
\addplot [very thick,blue] {unipdf(x,-3,1)};
\addlegendentry{$a=-3, b=1$};
\end{axis}
\end{tikzpicture}
\end{center}
```

### 3.1.2 Функция распределения

$$F(x) = \begin{cases} 1 & , x > b \\ \dfrac{x-a}{b-a} & , x \in [a, b] \\ 0 & , x < a \end{cases}$$

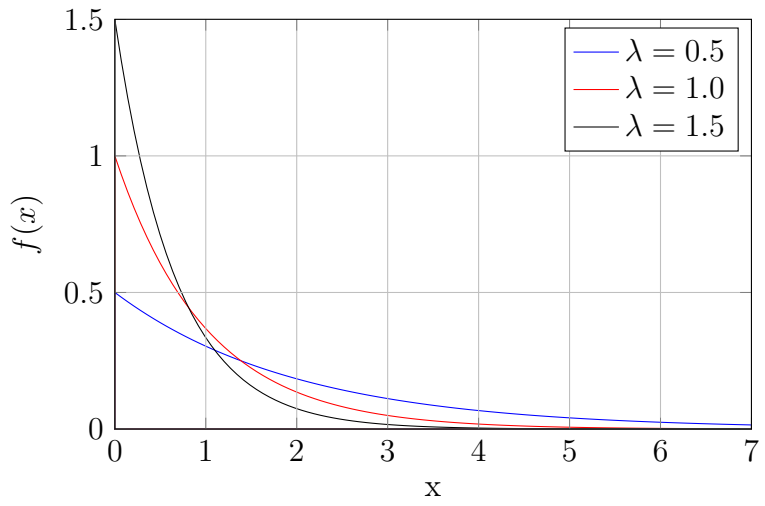



```
\begin{center}
\begin{tikzpicture}[
    declare function={unip(\x,\xl,\xu)= (\x>=\xl)*(\x<\xu)*(\x-\xl)/(\xu-\xl
       ) + (\x>\xu);}
]

\begin{axis}[axis x line=center,
              axis y line=left,
                samples = 200,
                ymax = 1.2,
                xlabel={$x$},
                ylabel={$F(x)$},
                xlabel style={right},
                ylabel style={above left},
                x post scale=1.3,
                grid = major, legend pos=south east
]
\addplot [orange] {unip(x,0,2)};
\addlegendentry{$a=0, b=2$};
\addplot [blue] {unip(x,-3,1)};
\addlegendentry{$a=-3, b=1$};
\end{axis}
\end{tikzpicture}
\end{center}
```

## 3.2 Экспоненциальное распределение

### 3.2.1 Функция плотности

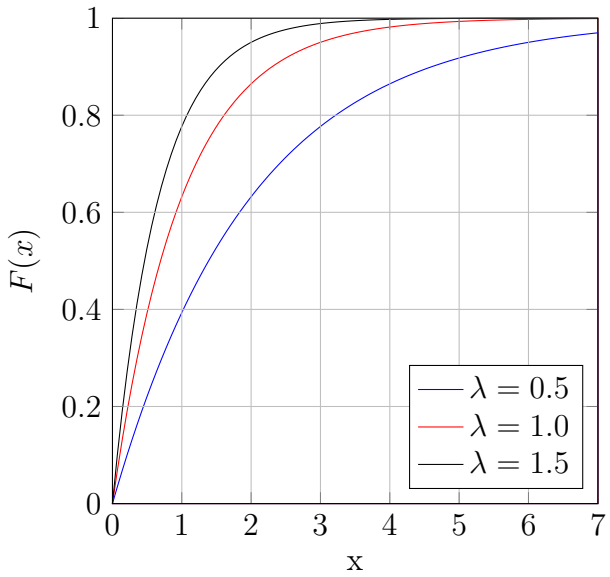
$$f(x;\lambda) = \begin{cases} \lambda e^{-\lambda x} & , x \geq 0 \\ 0 & , x \leq 0 \end{cases}$$




```
\pgfmathdeclarefunction{exp}{1}{\pgfmathparse{exp(-#1 * x)}%}

\begin{tikzpicture}
\begin{axis}
[no markers, domain=0:10,
samples=100, xlabel={x}, enlargelimits=false,
height=7cm, width=7cm, ylabel=$f(x)$,
clip=false, axis on top, grid = major]
   \addplot [domain=1:10, color = blue] {expon(0.5)} \closedcycle;
     \addlegendentry{$ \lambda = 0.5$};
  \addplot [domain=1:10, color = red] {expon(1,0)} \closedcycle;
     \addlegendentry{$ \lambda = 1.0$};
  \addplot [domain=1:10] {expon(1.5)} \closedcycle;
     \addlegendentry{$ \lambda = 1.5$};
\end{axis}
\end{tikzpicture}
```

### 3.2.2 Функция распределения

$$f(x;\lambda) = \begin{cases} 1 - e^{-\lambda x} & , x \geq 0 \\ 0 & , x \leq 0 \end{cases}$$

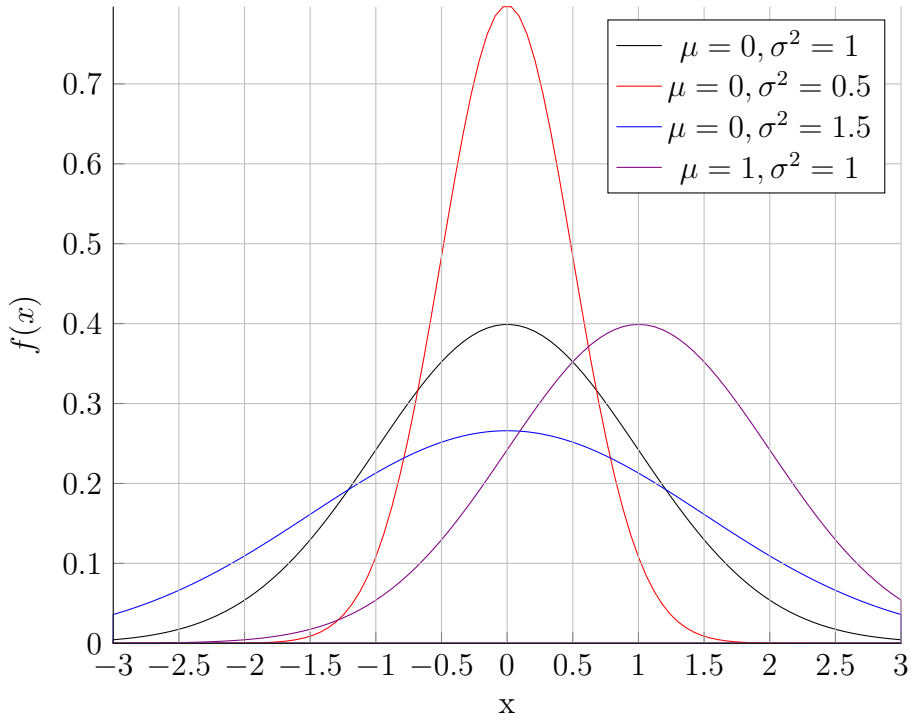



```
\pgfmathdeclarefunction{expondist}{1}{\pgfmathparse{ 1 -  exp(- (#1 * x))}%}

\begin{tikzpicture}
\begin{axis}[no markers,
domain=0:10, samples=100, xlabel={x}, enlargelimits=false, ylabel=$F(x)$,
height=8cm, width=8cm, clip=false, axis on top, legend pos=south east,
grid = major]
  \addplot [domain=0:7, color = blue] {expondist(0.5)} \closedcycle;
    \addlegendentry{$ \lambda = 0.5$};
  \addplot [domain=0:7, color = red] {expondist(1.0)} \closedcycle;
    \addlegendentry{$ \lambda = 1.0$};
  \addplot [domain=0:7] {expondist(1.5)} \closedcycle;
    \addlegendentry{$ \lambda = 1.5$};
\end{axis}
\end{tikzpicture}
```

## 3.3 Нормальное распределение

### Функция плотности

$$f(x, \mu, \sigma) = \frac{1}{\sigma \cdot \sqrt{2\pi}} \cdot e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

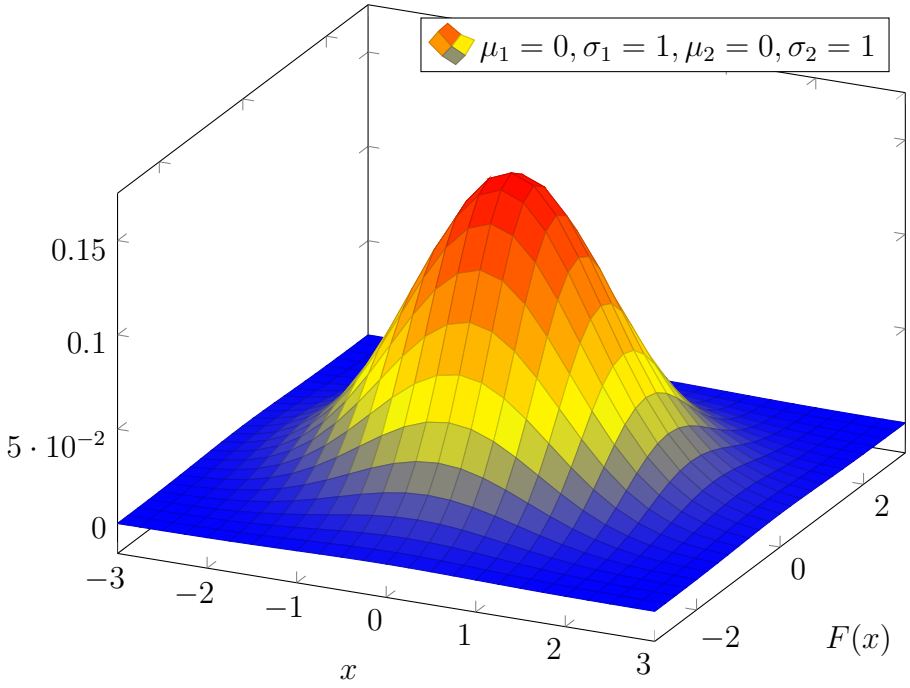



```
\pgfmathdeclarefunction{gauss}{2}{\pgfmathparse{1/(#2*sqrt(2*pi))*exp(-((x
    -#1)^2)/(2*#2^2))}}
\begin{center}
\begin{tikzpicture}
\begin{axis}[no markers, domain=0:10, samples=100,axis lines*=left, xlabel=x
    , ylabel = $f(x)$,
height=10cm, width=12cm, ytick={0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8,
     0.9, 1.0},enlargelimits=false, clip=false, axis on top, grid = major]
\addplot [domain=-3:3] {gauss(0,1)};
\addlegendentry{$\mu = 0, \sigma^2 = 1$};
\addplot [domain=-3:3, color = red] {gauss(0,0.5)};
\addlegendentry{$\mu = 0, \sigma^2 = 0.5$};
\addplot [domain=-3:3, color = blue] {gauss(0,1.5)};
\addlegendentry{$\mu = 0, \sigma^2 = 1.5$};
\addplot [domain=-3:3, color = violet] {gauss(1,1)};
\addlegendentry{$\mu = 1, \sigma^2 = 1$};
\end{axis}
\end{tikzpicture}
\end{center}
```

## 3.4 Многомерное нормальное распределение

### Функция плотности

$$f(x,y) = \frac{1}{2\pi \cdot \sigma_x \cdot \sigma_y} \cdot e^{-\frac{1}{2} \cdot \left( \frac{(x-\mu_x)^2}{\sigma_x^2} + \frac{(y-\mu_y)^2}{\sigma_y^2} \right)}$$

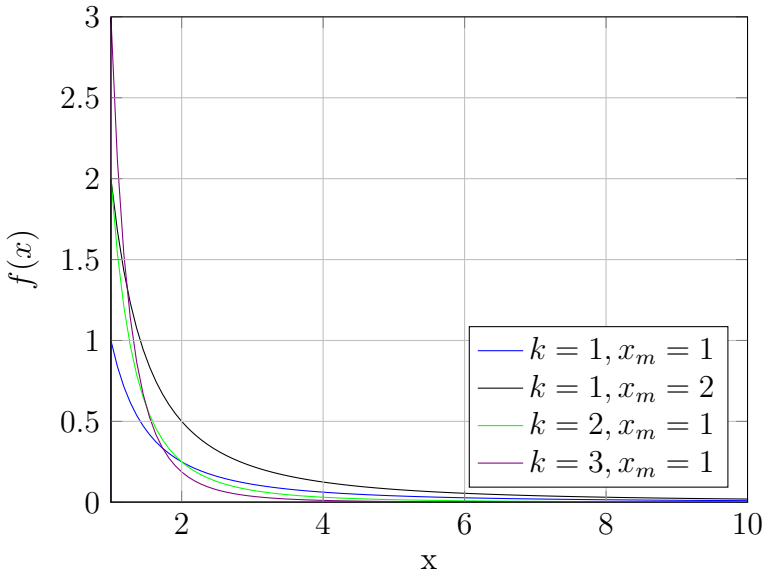



```
\pgfmathdeclarefunction{norm2}{4}{%
  \pgfmathparse{(1/(2*pi*#2*#4))* exp((-0.5)*(((x-#1)/#2)^2+((y-#3)/#4)^2))}
     %
}

\begin{center}
\begin{tikzpicture}
  \begin{axis}[
  height=10cm, width=12cm,
  xlabel=$x$,
  ylabel=$F(x)$]
  \addplot3[surf,domain=-3:3] {norm2(0,1,0,1)};
  \addlegendentry{$\mu_1 = 0, \sigma_1 = 1,\mu_2=0, \sigma_2=1$};
  \end{axis}
\end{tikzpicture}
\end{center}
```

## 3.5 Парето распределение

### 3.5.1 Функция плотности

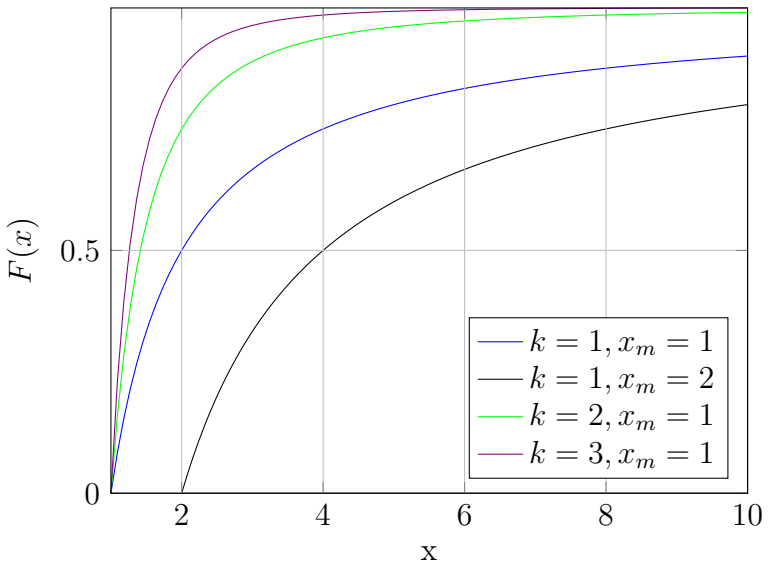
$$f(x) = \frac{k \cdot x_m^k}{x^{k+1}}$$




```
\pgfmathdeclarefunction{pareto}{1}{\pgfmathparse{( #1 * 1 ^ ( #1 ) / ( x ^
    (#1 + 1)))}%
}
\begin{center}
\begin{tikzpicture}

\begin{axis}[no markers, domain=0:10, samples=100, xlabel={x}, ylabel={$f(x)
    $}, enlargelimits=false, legend pos=south east, ytick = {0, 0.5, ..., 3},
height=8cm, width=10cm, clip=false, axis on top,
grid = major]
\addplot [domain=1:10, color = blue] {pareto(1, 1)} \closedcycle;
\addlegendentry{$k = 1, x_m = 1$};
\addplot [domain=1:10, color = black] {pareto(1, 2)} \closedcycle;
\addlegendentry{$k = 1, x_m = 2$};
\addplot [domain=1:10, color = green] {pareto(2, 1)} \closedcycle;
\addlegendentry{$k = 2, x_m = 1$};
\addplot [domain=1:10, color = violet] {pareto(3, 1)} \closedcycle;
\addlegendentry{$k = 3, x_m = 1$};

\end{axis}
\end{tikzpicture}
\end{center}
```

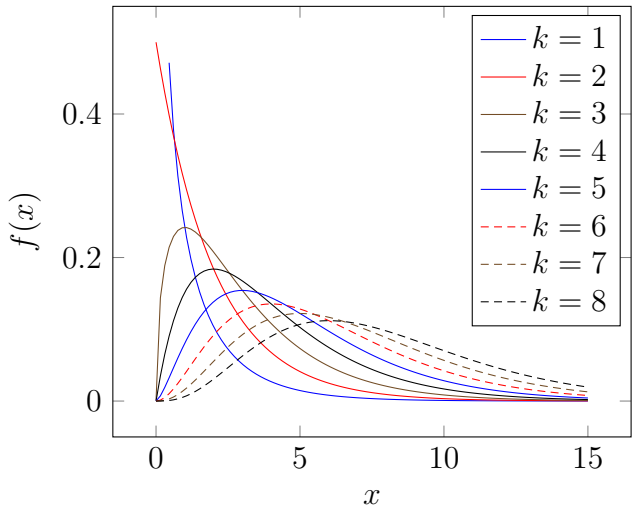
### 3.5.2 Функция распределения

$$F_X(x) = 1 - \left(\frac{x_m}{x}\right)^k$$



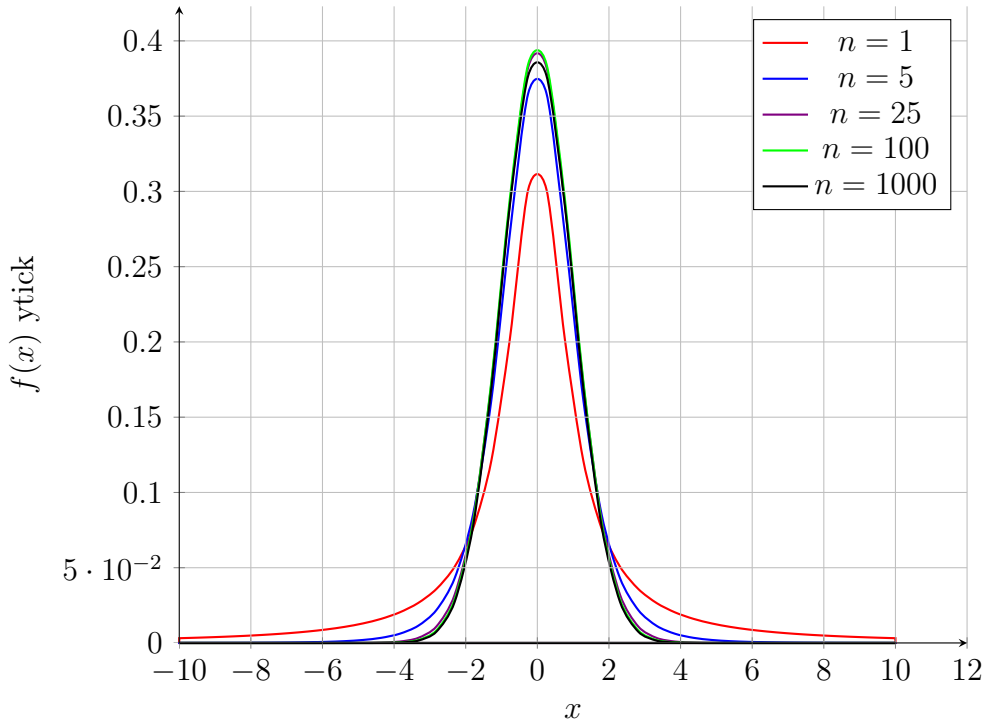

```
\pgfmathdeclarefunction{paretof}{2}{\pgfmathparse{1 - (#2/x)^#1}}

\begin{tikzpicture}
\begin{axis}[no markers, domain=0:10, samples=100, xlabel={x}, ylabel={$P(X
    < x)$}, enlargelimits=false, ytick = {0, 0.5, ..., 3},
height=8cm, width=10cm, clip=false, axis on top, legend pos=south east, ylim
     = 0
grid = major]
\addplot [domain=1:10, color = blue] {paretof(1, 1)} \closedcycle;
\addlegendentry{$k = 1, x_m = 1$};
\addplot [domain=1:10, color = black] {paretof(1, 2)} \closedcycle;
\addlegendentry{$k = 1, x_m = 2$};
\addplot [domain=1:10, color = green] {paretof(2, 1)} \closedcycle;
\addlegendentry{$k = 2, x_m = 1$};
\addplot [domain=1:10, color = violet] {paretof(3, 1)} \closedcycle;
\addlegendentry{$k = 3, x_m = 1$};
\end{axis}
\end{tikzpicture}
```

### 3.6 Хи-квадрат распределение




```
\begin{center}
\begin{tikzpicture}
  \begin{axis}[%
    xlabel = $x$,
    ylabel =  ylabel={$f(x)$},
    samples = 200,
    restrict y to domain = 0:0.5,
    domain = 0.01:15]
    \foreach \k in {1,...,8} {%
      \addplot+[mark={}] gnuplot[raw gnuplot] {%
        isint(x) = (int(x)==x);
        log2 = 0.693147180559945;
        chisq(x,k)=k<=0||!isint(k)?1/0:x<=0?0.0:exp((0.5*k-1.0)*log(x)-0.5*x
           -lgamma(0.5*k)-k*0.5*log2);
        set xrange [1.00000e-5:15.0000];
        set yrange [0.00000:0.500000];
        samples=200;
        plot chisq(x,\k)};
        \addlegendentryexpanded{$k = \k$}}
  \end{axis}
\end{tikzpicture}
\end{center}
```

## 3.7 Распределение Стьюдента




```
\begin{center}
\begin{tikzpicture}[
    declare function={gamma(\z)=
    2.506628274631*sqrt(1/\z)+ 0.20888568*(1/\z)^(1.5)+ 0.00870357*(1/\z)
       ^(2.5)- (174.2106599*(1/\z)^(3.5))/25920- (715.6423511*(1/\z)^(4.5))
       /1244160)*exp((-ln(1/\z)-1)*\z;},
    declare function={student(\x,\n)= gamma((\n+1)/2.)/(sqrt(\n*pi) *gamma(\
       n/2.)) *((1+(\x*\x)/\n)^(-(\n+1)/2.));}
]

\begin{axis}[xlabel = $x$, height=10cm, width=12cm, ylabel=$f(x)$
ytick={0.0, 0.1, 0.2, 0.3, 0.4, 0.5},
enlargelimits=false, clip=false, axis on top,
grid = major
    axis lines=left,
    enlargelimits=upper,
    samples=50
]

\addplot [thick, smooth, domain=-10:10, color = red]{student(x,1)} \
   closedcycle;
\addlegendentry{$n = 1$};
\addplot [thick, smooth, domain=-10:10, color = blue]{student(x,5)} \
   closedcycle;
\addlegendentry{$n = 5$}
\addplot [thick, smooth, domain=-10:10, color = violet]{student(x,25)}
\closedcycle;
\addlegendentry{$n = 25$}
\addplot [thick, smooth, domain=-10:10, color = green]{student(x,100)}
\closedcycle;
\addlegendentry{$n = 100$}
\addplot [thick, smooth, domain=-10:10, color = black]{student(x,1000)}
\closedcycle;
\addlegendentry{$n = 1000$}
```

```
\end{axis}
\end{tikzpicture}
\end{center}
```

# 4 Теоремы

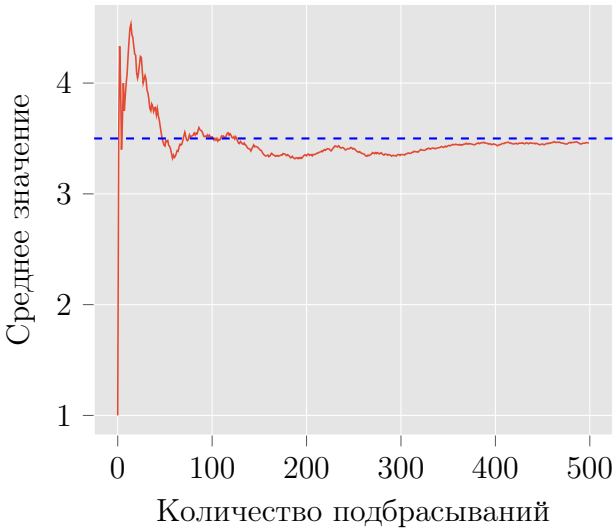
## 4.1 Иллюстрация закона больших чисел

Согласно Закону Больших Чисел, среднее значение конечной выборки из фиксированного распределения близко к математическому ожиданию этого распределения.

$$\frac{x_1 + x_2 + ... + x_n}{n} \to \mathbb{E}(x_i)$$

На данной картинке изображена динамика среднего значения выпавшего кубика в зависимости от количества подбрасываний.

Как можно наблюдать, при достаточно большом количестве подбрасываний среднее значение стремтися к математическому ожиданию.




```
\begin{tikzpicture}

\definecolor{color0}{rgb}{0.886274509803922,0.290196078431373,0.2}

\begin{axis}[
axis background/.style={fill=white!89.8039215686275!black},
axis line style={white},
tick align=outside,
tick pos=left,
x grid style={white},
xlabel={Количество подбрасываний},
xmajorgrids,
xmin=-24.95, xmax=523.95,
xtick style={color=white!33.3333333333333!black},
y grid style={white},
ylabel={Среднее значение},
ymajorgrids,
ymin=0.823333333333333, ymax=4.71,
ytick style={color=white!33.3333333333333!black}
]
\addplot [semithick, color0]
table {%
0 1
1 3.5
2 4.33333333333333
3 4
```

```
4 3.4
5 3.66666666666667
6 4
7 3.75
8 3.88888888888889
9 4
10 4.09090909090909
...
490 3.45010183299389
491 3.45121951219512
492 3.45436105476673
493 3.45748987854251
494 3.45656565656566
495 3.45766129032258
496 3.46277665995976
497 3.46184738955823
498 3.46092184368737
499 3.458
};
\addplot [thick, blue, dashed]
table {%
-24.95 3.5
523.95 3.5
};
\end{axis}

\end{tikzpicture}
```