Pair Bootstrap

$X \leftrightarrow Y$

Bootstrapping Residuals

↳ Wild Bootstrap

$\{x_i, y_i\}$ $\quad i = \overline{1, N}$

$y_i = \beta_1 + \beta_2 x_i + \varepsilon_i$

1) Resample DF with indexes $\{1, \dots, N\}$ with repl.

2) β^*_{OLS}

Fixed-X Sampling or Bootstrapping Residuals

1) $\hat{y}_i = f(x_i, \hat{\beta}_{OLS})$

2) $\hat{\varepsilon}_i = y_i - \hat{y}_i$,

prob. $\hat{\varepsilon}_i = \frac{1}{n}$

3) $\varepsilon^{*b} : \varepsilon_1, \ldots, \varepsilon_n$

$$y_i^{*b} = f(x_i, \hat{\beta}_{OLS}) + \varepsilon_i^{*b}$$

4) $y_i^{*b}, X \Rightarrow \hat{\beta}_{OLS}^{*b}$

5) Repeat N-boot

Wild Bootstrap [Wu]

1) $\hat{y} = f(x_i, \hat{\beta}_{OLS})$

2) $\hat{\varepsilon}_i = y_i - \hat{y}_i$

3) ε_i^{*b}

$$y_i^{*b} = f(x_i, \hat{\beta}_{OLS}) + \frac{t_i^* \cdot \hat{\varepsilon}_i}{\sqrt{1 - h_{ii}}}$$

$$h_{ii} = X (X^T X)^{-1} X^T$$

$t_i^* : a_1, \ldots, a_n$

$$a_i = \frac{\hat{\varepsilon}_i - \bar{\hat{\varepsilon}}_i}{\sqrt{\frac{\sum (\hat{\varepsilon}_i - \bar{\hat{\varepsilon}}_i)^2}{n}}}$$