

Stochastic Processes problems

https://github.com/bdemeshev/stochastic_pro

September 8, 2023

Contents

1	First step analysis	3
2	Solutions	4
3	Sources of wisdom	5

1 First step analysis

- 1.1** Biden and Trump alternately throw a fair dice infinite number of times. Biden throws first. The person who obtain the first 6 wins the game.
- a) What is the probability that Biden will win?
 - b) What is expected number of turns?
 - c) What is variance of turns?
 - d) What is expected number of turns given that Biden won?
 - e) Find the transition matrix of this four state Markov chain.
- 1.2** Elon throws an unfair coin until “head” appears. The probability of “head” is $p \in (0; 1)$. Let N be the total number of throws.
- a) Find $\mathbb{E}(N)$, $\mathbb{E}(\exp(tN))$, $\text{Var}(N)$, $\mathbb{E}(N^3)$.
 - b) What is the probability than N will be even?
- 1.3** Alice and Bob throw a fair coin until the sequence HTT or THT appears. Alice wins if HTT appears first, Bob wins if THT appears first.
- a) Find the probability that Alice wins.
 - b) Find the expected value and variance of the total number of throws.
 - c) Using any open source software find the probability that Alice wins for all possible combinations of three coins sequences for Alice and Bob.
 - d) Now Alice and Bob play the following game. Alice chooses her three coins winning sequence first. Next Bob, knowing the choice of Alice, chooses his three coins winning sequence. Than they throw a fair coin until either of their sequences appears. What is the best strategy for Alice? For Bob? What is the probability that Alice wins this game?
- 1.4** You throw a dice unbounded number of times. If it shows 1, 2 or 3 then the corresponding amount of dollars is added in the pot. If it shows 4 or 5 the game stops and you get the pot. If it shows 6 the game ends and you get nothing. Initially the pot is empty.
- a) What is probability that the game will end by 6?
 - b) What is expected duration of the game?
 - c) What is your expected payoff?
 - d) What is your payoff variance?
 - e) Consider variation-A of the game. Rules are the same, but initially the pot contains 100 dollars. How will the answers to questions (a)-(d) change?
 - f) Consider variation-B of the game. Initially the pot is empty. One rule is changed. If the dice shows 5 the content of the pot is burned and the game continues. How will the answers to questions (a)-(d) change?
- 1.5** Boris Johnson throws a fair coin until 1 appears or until he says “quit”. His payoff is the value of the last throw. Boris optimizes his expected payoff. If many strategies gives the same expected payoff he chooses the strategy that minimizes the expected duration of the game.
- a) What is the optimal strategy and the corresponding expected payoff?

b) What is the expected duration?

c) How the answers to points (a) and (b) will change if Boris should pay 0.3 dollars for each throw?

1.6

1.7

1.8

2 Solutions

3 Sources of wisdom

[Buz+15] Nazar Buzun et al. “Stochastic Analysis in Problems, part 1 (in Russian).” In: *arXiv preprint arXiv:1508.03461* (2015). URL: <https://arxiv.org/abs/1508.03461>.