Today we celebrate 20 January, Penguin Awareness Day.

1. Adélie Penguin would like to receive S_2 roubles at time $T = 3$, where S_t is the share price if and only if $S_2 > 120$. Assume Black-Scholes model is valid, the risk-free rate is $r = 0.1$ and current share price is $S_0 = 100$.

   How much Adélie Penguin should pay now at $t = 0$?

2. Consider stationary $MA(2)$ model, $y_t = 2 + 0.3u_{t-2} + 0.1u_{t-1} + u_t$, where (u_t) is a white noise with $\mathbb{V}\text{ar}(u_t) = 4$.

   You know that $u_{100} = -1$, $u_{99} = 1$.

   (a) Find 95% predictive interval for y_{102}.

   (b) Find the first two values of the autocorrelation function, ρ_1, ρ_2.

   (c) Find the first two values of the partial autocorrelation function, ϕ_{11}, ϕ_{22}.

3. The process y_t is described by a simple $GARCH(1, 1)$ model:

$$\begin{cases} y_t = \sigma_t \nu_t \\ \sigma_t^2 = 1 + 0.2y_{t-1}^2 + 0.3\sigma_{t-1}^2 \\ \nu_t \sim \mathcal{N}(0; 1) \end{cases}$$

   The variables ν_t are independent of past variables y_{t-k}, ν_{t-k}, σ_{t-k} for all $k \geq 1$. The processes y_t, σ_t^2 are stationary.

   Given $\sigma_{100} = 1$ and $\nu_{100} = 0.5$ find 95% predictive interval for y_{100+h} where h tends to infinity.

4. Emperor penguin studies a stochastic analog of the Fibonacci sequence

$$y_t = 10 + y_{t-1} + y_{t-2} + u_t,$$

   where (u_t) is a white noise process. Consider a stationary solution of this equation.

   (a) Find $\mathbb{E}(y_t)$.

   (b) Find dy_t/du_{t-1}.

   Be brave! There are two more exercises!

5. The quarterly y_t is modelled by $ETS(AAA)$ process:

$$\begin{cases} u_t \sim \mathcal{N}(0;9) \\ s_t = s_{t-2} + 0.1u_t \\ b_t = b_{t-1} + 0.2u_t \\ \ell_t = \ell_{t-1} + b_{t-1} + 0.3u_t \\ y_t = \ell_{t-1} + b_{t-1} + s_{t-4} + u_t \end{cases}$$

   (a) Given that $s_{100} = 2$, $s_{99} = -1$, $s_{98} = -1$, $b_{100} = 0.5$, $\ell_{100} = 4$ find 95% predictive interval for y_{102}.

   (b) In this problem particular values of parameters are specified. And how many parameters are estimated in quarterly $ETS(AAA)$ model before real forecasting?

6. Consider the process $y_t = u_1 \sin t + u_2 \cos t$, where (u_t) is a white noise process.

   (a) Is the process (y_t) stationary?

   (b) You know that $y_{100} = 0$ and $y_{99} = -1$. Construct a predictive interval for y_{102} with coverage probability of at least 95%.

   (c) Will the predictive interval for y_{103} be wider or narrower than for y_{102}? You don't need to actually calculate it.

Bonus: How many words «Penguin» have you spotted?