

Today we celebrate 20 January, Penguin Awareness Day.

1. Adélie Penguin would like to receive  $S_2$  roubles at time  $T = 3$ , where  $S_t$  is the share price if and only if  $S_2 > 120$ . Assume Black-Scholes model is valid, the risk-free rate is  $r = 0.1$  and current share price is  $S_0 = 100$ .

How much Adélie Penguin should pay now at  $t = 0$ ?

2. Consider stationary  $MA(2)$  model,  $y_t = 2 + 0.3u_{t-2} + 0.1u_{t-1} + u_t$ , where  $(u_t)$  is a white noise with  $\text{Var}(u_t) = 4$ .

You know that  $u_{100} = -1$ ,  $u_{99} = 1$ .

- (a) Find 95% predictive interval for  $y_{102}$ .
  - (b) Find the first two values of the autocorrelation function,  $\rho_1, \rho_2$ .
  - (c) Find the first two values of the partial autocorrelation function,  $\phi_{11}, \phi_{22}$ .
3. The process  $y_t$  is described by a simple  $GARCH(1, 1)$  model:

$$\begin{cases} y_t = \sigma_t \nu_t \\ \sigma_t^2 = 1 + 0.2y_{t-1}^2 + 0.3\sigma_{t-1}^2 \\ \nu_t \sim \mathcal{N}(0; 1) \end{cases}$$

The variables  $\nu_t$  are independent of past variables  $y_{t-k}, \nu_{t-k}, \sigma_{t-k}$  for all  $k \geq 1$ . The processes  $y_t, \sigma_t^2$  are stationary.

Given  $\sigma_{100} = 1$  and  $\nu_{100} = 0.5$  find 95% predictive interval for  $y_{100+h}$  where  $h$  tends to infinity.

4. Emperor penguin studies a stochastic analog of the Fibonacci sequence

$$y_t = 10 + y_{t-1} + y_{t-2} + u_t,$$

where  $(u_t)$  is a white noise process. Consider a stationary solution of this equation.

- (a) Find  $\mathbb{E}(y_t)$ .
- (b) Find  $dy_t/du_{t-1}$ .

Be brave! There are two more exercises!

5. The quarterly  $y_t$  is modelled by  $ETS(AAA)$  process:

$$\begin{cases} u_t \sim \mathcal{N}(0; 9) \\ s_t = s_{t-2} + 0.1u_t \\ b_t = b_{t-1} + 0.2u_t \\ \ell_t = \ell_{t-1} + b_{t-1} + 0.3u_t \\ y_t = \ell_{t-1} + b_{t-1} + s_{t-4} + u_t \end{cases}$$

- (a) Given that  $s_{100} = 2$ ,  $s_{99} = -1$ ,  $s_{98} = -1$ ,  $b_{100} = 0.5$ ,  $\ell_{100} = 4$  find 95% predictive interval for  $y_{102}$ .
- (b) In this problem particular values of parameters are specified. And how many parameters are estimated in quarterly  $ETS(AAA)$  model before real forecasting?

6. Consider the process  $y_t = u_1 \sin t + u_2 \cos t$ , where  $(u_t)$  is a white noise process.

- (a) Is the process  $(y_t)$  stationary?
- (b) You know that  $y_{100} = 0$  and  $y_{99} = -1$ . Construct a predictive interval for  $y_{100}$  with coverage probability of at least 95%.
- (c) Will the predictive interval for  $y_{103}$  be wider or narrower than for  $y_{102}$ ? You don't need to actually calculate it.

Bonus: How many words «Penguin» have you spotted?