Home Assignment 1

1. Let $\Omega = \mathbb{R}$. Explicitely find the sigma-algebras $\mathcal{F}_1 = \sigma(A)$, $\mathcal{F}_2 = \sigma(B)$, $\mathcal{F}_3 = \sigma(A, B)$ where $A = [-10; 5]$ and $B = (0; 10)$.

2. I throw a die once. Let X be the result of the toss. Count the number of events in sigma-algebras $\mathcal{F}_1 = \sigma(X)$, $\mathcal{F}_2 = \sigma(\{X > 3\})$, $\mathcal{F}_3 = \sigma(\{X > 3\}, \{X < 5\})$.

3. Let $\Omega = \mathbb{R}$. The sigma-algebra $\mathcal{F}$ is generated by all the sets of the form $(-\infty, t]$,

$$\mathcal{F} = \sigma\left(\{(-\infty; t] \mid t \in \mathbb{R}\}\right)$$

   Check whether $A_1 = (0; 10) \in \mathcal{F}$, $A_2 = \{5\} \in \mathcal{F}$, $A_3 = \mathbb{N} \in \mathcal{F}$.

4. Prove the following statements or provide a counter-example:

   (a) If $\mathcal{F}_1$ and $\mathcal{F}_2$ are sigma-algebras then $\mathcal{F} = \mathcal{F}_1 \cup \mathcal{F}_2$ is sigma-algebra.

   (b) If X and Y are independent random variables then $\operatorname{card}\sigma(X, Y) = \operatorname{card}\sigma(X) + \operatorname{card}\sigma(Y)$.

5. I throw a die infinite number of times. Let X_n be the result of the n-th toss. Consider a pack of sigma-algebras: $\mathcal{F}_n = \sigma(X_1, \ldots, X_n)$ and $\mathcal{H}_n = \sigma(X_n, X_{n+1}, X_{n+2}\ \ldots)$.

   Provide and example of an event such that

   (a) $A_1 \in \mathcal{F}_{2020}$;

   (b) $A_2 \in \mathcal{H}_{2020}$;

   (c) $A_3 \in \mathcal{F}_{2020}$ and $A_3 \in \mathcal{H}_{2020}$;

   (d) $A_4 \in \mathcal{F}_n$ for all n, $A_4 \in \mathcal{H}_n$ for all n, $A_4 \neq \emptyset$, $A_4 \neq \Omega$.

Deadline: 25 September 2020, 21:00 MSK.

Home Assignment 2

1.