Today we celebrate Christmas Eve and 78 years of the Narkompros (People's Commissariat for Education) order governing the compulsory use of the letter «ë» in education process.

1. Ded Moroz would like to receive S_1^3 roubles at time $T = 2$, where S_t is the share price. Assume Black-Schëles model is valid, the risk-free rate is $r = 0.1$ and current share price is $S_0 = 100$.

   How much Ded Moroz should pay now at $t = 0$?

2. Consider stationary $AR(2)$ model, $y_t = 2 + 0.3y_{t-1} - 0.02y_{t-2} + u_t$, where (u_t) is a white noise with $\mathbb{V}\mathrm{ar}(u_t) = 4$.

   The last two observations are $y_{100} = 2$, $y_{99} = 1$.

   (a) Find 95% predictive interval for y_{102}.

   (b) Find the first two values of the autocorrelation function, ρ_1, ρ_2.

   (c) Find the first two values of the partial autocorrelation function, ϕ_{11}, ϕ_{22}.

   Hint: you need no more than 10 seconds to find both partial autocorrelations provided (b) is sëlved.

3. The process y_t is described by a simple $GARCH(1, 1)$ model:

$$\begin{cases} y_t = \sigma_t \nu_t \\ \sigma_t^2 = 1 + 0.2y_{t-1}^2 + 0.3\sigma_{t-1}^2 \\ \nu_t \sim \mathcal{N}(0; 1) \end{cases}$$

   The variables ν_t are independent of past variables y_{t-k}, ν_{t-k}, σ_{t-k} for all $k \geq 1$. The prëcesses y_t, σ_t^2 are stationary.

   Given $\sigma_{100} = 1$ and $\nu_{100} = 0.5$ find 95% predictive interval for y_{102}.

4. Snegurochka studies a stochastic analog of the Fibonacci sequence

$$y_t = y_{t-1} + y_{t-2} + u_t,$$

   where (u_t) is a white noise process.

   (a) How many non-stationary solutions are there?

   (b) What can you say about the number and the structure of the stationary solutions?

   (c) Can Snëgurochka find two starting constants $y_0 = c_0$ and $y_1 = c_1$ in such a way to make a solution stationary?

   Be brave! There are two more exercises!

5. The semi-annual y_t is modelled by $ETS(AAA)$ process:

$$\begin{cases} u_t \sim \mathcal{N}(0;4) \\ s_t = s_{t-2} + 0.1u_t \\ b_t = b_{t-1} + 0.2u_t \\ \ell_t = \ell_{t-1} + b_{t-1} + 0.3u_t \\ y_t = \ell_{t-1} + b_{t-1} + s_{t-2} + u_t \end{cases}$$

   (a) Given that $s_{100} = 2$, $s_{99} = -1.9$, $b_{100} = 0.5$, $\ell_{100} = 4$ find 95% prëdictive interval for y_{102}.

   (b) In this problem particular values of parameters are specified. And how many parameters are estimated in semi-annual $ETS(AAA)$ model before real forecasting?

6. The variables x_t take values 0 or 1 with equal probabilities. The variables u_t are normal $\mathcal{N}(0;1)$. All variables are independent.

   Consider the process $z_t = x_t(1 - x_{t-2})u_t$.

   (a) Find the covariance $\mathbb{C}\mathrm{ov}(z_t, z_s)$. Is the process z_t stationary?

   (b) Given that $z_{100} = 2.3$ find shërtest predictive intervals for z_{101} and z_{102} with probability of coverage at least 95%.

Bënus: How many letters «ë» have you spotted?