**Estimation questions**

1. To go to the mountain top I use a gondola lift in the morning. I go back from the top using the same gondola lift in the evening. Cabins are numbered from 1 to $a$.

   I have noticed that the absolute difference of cabin numbers of my two trips was 10.

   (a) Estimate $a$ using maximum likelihood.

   (b) Estimate $a$ using method of moments.

2. Random variables $X_1$, $X_2$, ..., $X_n$ are independent identically distributed with density

$$f(x_i \mid \lambda, a) = \frac{\lambda}{2} \exp(-\lambda|x_i - a|).$$

   Observed values for $n = 3$ are $-3$, 1, 11.

   (a) Estimate $\lambda$ using method of moments for fixed $a = 1$.

   (b) Estimate $\lambda$ and $a$ using maximum likelihood.

3. Random variables $X_1$, ..., $X_n$ are independent and normally distributed $\mathcal{N}(1, 1/b)$.

   (a) Estimate $b$ using maximum likelihood.

   (b) Does the estimator achive the Cramer-Rao lower bound?

   (c) Is the estimator consistent?

   (d) Is the estimator unbiased?

4. Random variables $X_1$, $X_2$, ..., $X_n$ are independent identically distributed with density

$$f(x_i \mid \lambda) = \frac{\lambda}{2} \exp(-\lambda|x_i|).$$

   For $n = 100$ I have 40 negative values with sum equal to $-300$ and 60 positive values with sum equal to 500.

   (a) Test the hypothesis $\lambda = 1$ using LR approach at significance level $\alpha = 0.01$.

   (b) Test the hypothesis $\lambda = 1$ using LM approach at significance level $\alpha = 0.01$.

**Distribution questions**

5. I have three problems in the home assignment. Time spent on each problem is modelled by independend exponentially distributed random variables with rate $\lambda$: $X_1$, $X_2$, $X_3$.

   (a) Find the moment generating function of $X_i$ and hence the moment generating function of $S = X_1 + X_2 + X_3$.

   (b) Find $\mathbb{E}(S^3)$.

   (c) Find the joint density of $R = X_1/(X_1 + X_2 + X_3)$ and $S$.

6. I have $100$ numbers written on small sheets of paper: $x_1$, $x_2$, ..., $x_{100}$. The sum of these numbers is $1$.

   Find the possible values of the sum

$$\frac{x_1}{\sqrt{1-x_1}} + \frac{x_2}{\sqrt{1-x_2}} + \ldots + \frac{x_{100}}{\sqrt{1-x_{100}}}.$$

   Hint: consider a randomly selected number $X$ and apply the Jensen's inequality.