

## Home Assignment 1

1. Let  $\Omega = \mathbb{R}$ . Explicitly find the sigma-algebras  $\mathcal{F}_1 = \sigma(A)$ ,  $\mathcal{F}_2 = \sigma(B)$ ,  $\mathcal{F}_3 = \sigma(A, B)$  where  $A = [-10; 5]$  and  $B = (0; 10)$ .
2. I throw a die once. Let  $X$  be the result of the toss. Count the number of events in sigma-algebras  $\mathcal{F}_1 = \sigma(X)$ ,  $\mathcal{F}_2 = \sigma(\{X > 3\})$ ,  $\mathcal{F}_3 = \sigma(\{X > 3\}, \{X < 5\})$ .
3. Let  $\Omega = \mathbb{R}$ . The sigma-algebra  $\mathcal{F}$  is generated by all the sets of the form  $(-\infty, t]$ ,

$$\mathcal{F} = \sigma(\{(-\infty; t] \mid t \in \mathbb{R}\})$$

Check whether  $A_1 = (0; 10) \in \mathcal{F}$ ,  $A_2 = \{5\} \in \mathcal{F}$ ,  $A_3 = \mathbb{N} \in \mathcal{F}$ .

4. Prove the following statements or provide a counter-example:
  - (a) If  $\mathcal{F}_1$  and  $\mathcal{F}_2$  are sigma-algebras then  $\mathcal{F} = \mathcal{F}_1 \cup \mathcal{F}_2$  is sigma-algebra.
  - (b) If  $X$  and  $Y$  are independent random variables then  $\text{card } \sigma(X, Y) = \text{card } \sigma(X) + \text{card } \sigma(Y)$ .
5. I throw a die infinite number of times. Let  $X_n$  be the result of the  $n$ -th toss. Consider a pack of sigma-algebras:  $\mathcal{F}_n = \sigma(X_1, \dots, X_n)$  and  $\mathcal{H}_n = \sigma(X_n, X_{n+1}, X_{n+2} \dots)$ .

Provide an example of an event such that

- (a)  $A_1 \in \mathcal{F}_{2020}$ ;
- (b)  $A_2 \in \mathcal{H}_{2020}$ ;
- (c)  $A_3 \in \mathcal{F}_{2020}$  and  $A_3 \in \mathcal{H}_{2020}$ ;
- (d)  $A_4 \in \mathcal{F}_n$  for all  $n$ ,  $A_4 \in \mathcal{H}_n$  for all  $n$ ,  $A_4 \neq \emptyset$ ,  $A_4 \neq \Omega$ .

## Home Assignment 2

1.