1. Ded Moroz would like to receive S_2^{-1} roubles at time $T = 2$.

   Assume the framework of Black and Scholes model, S_t is the share price, r is the risk free rate, σ is the volatility.

   How much Ded Moroz should pay now at $t = 0$?

2. The process Y_t is defined by
$$dY_t = W_t^2 dt + 3W_t dW_t,\ Y_0 = 1.$$

   (a) Find dY_t^2;

   (b) Find $\mathbb{E}(Y_t)$ and $\mathbb{V}\mathrm{ar}(Y_t)$.

3. The quarterly y_t is modelled by $ETS(AAA)$ process:
$$\begin{cases} u_t \sim \mathcal{N}(0; 9) \\ s_t = s_{t-4} + 0.1u_t \\ b_t = b_{t-1} + 0.2u_t \\ \ell_t = \ell_{t-1} + b_{t-1} + 0.3u_t \\ y_t = \ell_{t-1} + b_{t-1} + s_{t-4} + u_t \end{cases}$$

   (a) Given that $s_{100} = -1$, $s_{99} = 2$, $s_{98} = -2$, $b_{100} = 0.5$, $\ell_{100} = 4$ find 95% predictive interval for y_{102}.

   (b) In this problem the true values of parameters are exactly specified. Now we consider the real life forecasting problem where parameters are unknown and hence estimated.

   Let's link parameters with noise equation, trend equation, seasonality equation and level equation. How many noise-related parameters are estimated? Trend-related? Seasonality-related? Level-related?

4. Processes (y_t) and (u_t) are stationary and are described by the system
$$\begin{cases} y_t = \beta y_{t-1} + u_t \\ u_t = \alpha u_{t-1} + \nu_t. \end{cases}$$

   The processes (ν_t) is a white noise with variance σ^2. Parameters β and α are less than 1 by absolute value.

   (a) Find the autocovariance function of (y_t).

   (b) Find the first two coefficients δ_1 and δ_2 in the decomposition
$$y_t = \nu_t + \delta_1 \nu_{t-1} + \delta_2 \nu_{t-2} + \dots$$