Home Assignment 1

1. Consider the Markov chain with the transition matrix:

$$\begin{pmatrix} 0.2 & 0.1 & 0.7 & 0 & 0 & 0 & 0 & 0 \\ 0.4 & 0.6 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.3 & 0 & 0 & 0 & 0.7 \\ 0 & 0 & 0.3 & 0 & 0.7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.4 & 0 & 0.6 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

   (a) Draw the most beautiful graph for this chain. A fox or a cat is ok :)

   (b) Split the chain into classes and classify them as closed and not closed.

   (c) Classify the states as recurrent and transient.

2. The Lonely Queen is standing on the A1 field of the chessboard. She starts moving randomly according to chess rules.

   (a) How many moves on average will it take to go back to A1?

   (b) What proportion of her eternal life will the Queen spend on every field?

3. Joe Biden throws a die until six appears or until he says «Stop». The payoff is equal to the previous thrown number before the last throw. If six appears on the first throw Joe receives nothing. Joe maximizes the expected payoff.

   What is the best strategy and the corresponding expected payoff?

4. Ilya Muromets stands before the first stone. There are three roads behind the stone. And every road ends with a new stone. And there are three new roads behind every new stone. And so on. Every road is guarded with one-third probability by a three headed dragon Zmei Gorynich. Yes, there are infinitely many Zmeis Gorynichs.

   (a) What is the probability that Ilya will never meet Zmei Gorynich if Ilya chooses a road at random?

   (b) What is the probability that **there is** at least one Eternal Peaceful Journey without Zmei Gorynich?

5. Ilya and Zmei finally met and play with a coin. They throw a coin until the sequence HTH or TTH apears. Ilya wins if HTH appears and Zmei wins if TTH appears.

   (a) What is the probability that Ilya wins?

   (b) What is the expected number of throws?

   (c) What is the expected number of throws given that Ilya won?

Deadline: **2021-10-06, 21:00**.