Hi;   (2.2.a)

"unicorn" approach.   random term

ETS
ORBIT
UCM
...

$$y_t = \ell_{t-1} + b_{t-1} + u_t \quad + [s_{t-12}]$$

(obs.)

... past growth rate

past value of level (trend)

ARMA
VAR ...

don't think about the structure.

let the linear combination of white noises take care of the structure.

$$\boxed{y_t} = \boxed{\mu + u_t + \alpha_1 u_{t-1} + \alpha_2 u_{t-2} + \alpha_3 u_{t-3}}$$

Ex. ARMA = Auto Regression Moving Average.

ARMA(2,0) = AR(2)   2 lags of y_t 0 lags of u_t

$$1 \cdot y_t = 2 + 0.7\, y_{t-1} - 0{,}12\, y_{t-2} + 1 u_t$$

ARMA(2,1)

$$y_t = 5 + 0.6\, y_{t-1} - 0.08\, y_{t-2} + u_t + 0{,}7\, u_{t-1}$$

AR(2)   MA(1)

(!) Full assumptions.

ARMA-model.

notation.

$L \cdot y_t = y_{t-1}$

$L^{12} \cdot y_t = y_{t-12}$

$$P(L) = (1 - 0{,}5L + 0.06L^2)$$

$$P(L) \cdot y_t = 1 \cdot y_t - 0.5 y_{t-1} + 0{,}06 \cdot y_{t-2}$$

ARMA(p,q) - model.

① ARMA - equation

$$P(L) \cdot y_t = c + Q(L) \cdot u_t$$

② $P(L)$ - polyn. of degree p

$Q(L)$ - polyn of degree q.

③ $P(0) = 1$ $\quad Q(0) = 1$.

④ $P(L)$ and $Q(L)$ no common roots.

⑤ $u_t \sim$ white noise $\quad E(u_t) = 0$

$Var(u_t) = \sigma_u^2$

$Cov(u_t, u_s) = 0$ if $t \neq s$

by default

$u_t \sim N(0; \sigma_u^2)$ indep.

⑥ y_t is a $MA(\infty)$ wrt (u_t)

$$y_t = \mu + u_t + \alpha_1 u_{t-1} + \alpha_2 u_{t-2} + \alpha_3 u_{t-3} + \ldots$$

$$E(y_t) = \mu + 0 + 0 + 0 + 0 + \ldots + 0 \ldots = \mu$$

y_t is stat.

Ex. ARMA(2,0) $\boxed{\text{model.}}$ → ① ② ... ⑥ $\boxed{\text{All assumpt!}}$

with ⊛ $\boxed{y_t = \underline{2} + 0.7\, y_{t-1} - 0.12\, y_{t-2} + u_t}$

$u_t \sim N(0; 16)$

$P(L) \cdot y_t = c + Q(L)\, u_t$ ²

$Q(L) = 1$

$P(L) = 1 - 0.7L + 0.12L^2$

4 a) $Q(L)$? $P(L)$?

4 b) $E(y_t)$?

4 c) $MA(\infty)$ of y_t wrt (u_t) find first two terms

$$y_t = \mu + u_t + \alpha_1 u_{t-1} + \alpha_2 u_{t-2} + \dots$$

μ? α_1? α_2?

Idea 1: plug in into equation and solve the equations for $\alpha_1, \alpha_2, \dots$

Idea 2 use polyn.

Idea 3 reiterate original eq-n.

d) Is y_t stationary? (weak)

$E(y_t) = \mu$
$Var(y_t) = \sigma_y^2$
$Cov(y_t, y_s) = f(t-s)$

e) $Var(y_t)$, $Cov(y_t, y_{t-1})$, $Cov(y_t, y_{t-2})$

f) $y_1, \dots, y_{100}$

$y_{99} = 5$ 95% PI for y_{101}

$y_{100} = 6$ 95% PI for y_{102}

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b) $y_t = 2 + 0.7\, y_{t-1} - 0.12\, y_{t-2} + u_t$

$$E(y_t) = 2 + 0.7\, E(y_{t-1}) - 0.12\, E(y_{t-2}) + 0$$

Assump №6 →

$$\mu_y = 2 + 0.7\, \mu_y - 0.12\, \mu_y$$

$$\mu_y = \frac{2}{1 - 0.7 + 0.12} = \frac{2}{0.42}$$

$\boxed{\text{Reiterate!}}$

c) $y_t = 2 + 0.7\, y_{t-1} - 0.12\, y_{t-2} + u_t =$

$= 2 + 0.7\,(2 + 0.7\, y_{t-2} - 0.12\, y_{t-3} + u_{t-1}) - 0.12\, y_{t-2} + u_t$

$\mu + u_{t-2} + ?u_{t-3} \dots$

$= 3.4 + u_t + \boxed{0.7}\, u_{t-1} + 0.37\, \boxed{y_{t-2}} - 0.7 \cdot 0.12\, y_{t-3}$ ← $\mu + u_{t-3} + ?u_{t-4} + \dots$

α_1

$$= 3.4 + u_t + \boxed{0{,}7}\, u_{t-1} + 0{,}37\, y_{t-2} - 0{,}7 \cdot 0{,}12\, y_{t-3} =$$

$\mu + u_{t-2} + ?u_{t-3} \ldots$ $\quad \alpha_1 \quad \mu$

$$= 3.4 + u_t + 0{,}7 u_{t-1} + 0{,}37\left(2 + 0.7 y_{t-3} - 0.12 y_{t-4} + u_{t-2}\right) - 0{,}7 \cdot 0{,}12\, y_{t-3} = \ldots = \ldots = \ldots$$

$$= \underbrace{\mu}_{= \frac{2}{0{,}42}} + u_t + \underbrace{0{,}7}_{=\alpha_1} u_{t-1} + \underbrace{0{,}37}_{=\alpha_2} u_{t-2} + \ldots\ldots$$

$$y_t = \mu + u_t + \alpha_1 \cdot u_{t-1} + \alpha_2 u_{t-2} + \ldots$$

$$y_{t-1} = \mu + u_{t-1} + \alpha_1 \cdot u_{t-2} + \alpha_2 \cdot u_{t-3} \ldots$$

$$Cov(y_t, y_{t-1}) = \alpha_1 \cdot 1 + \alpha_2 \cdot \alpha_1 + \alpha_3 \cdot \alpha_2 + \alpha_4 \cdot \alpha_3 + \ldots$$

does not depend on t.

(y_t) is stationary!
(it follows from assump 6)

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e)

$$y_t = 2 + 0{,}7 y_{t-1} - 0{,}12\, y_{t-2} + u_t$$
$$u_t \sim N(0; 16)$$

$LHS = RHS$

$Var(y_t) = \gamma_0 = Cov(y_t, y_t)$

$Cov(y_t, y_{t-1}) = \gamma_1$

$Cov(y_t, y_{t-2}) = \gamma_2$

$\gamma_k = Cov(y_t, y_{t-k})$

$$\begin{cases} Cov(y_t, LHS) = Cov(y_t, RHS) \\ Cov(y_{t-1}, LHS) = Cov(y_{t-1}, RHS) \\ Cov(y_{t-2}, LHS) = Cov(y_{t-2}, RHS) \end{cases}$$

$\hookrightarrow \gamma_0, \gamma_1, \gamma_2$

Assump 6:

$y_t = \mu + u_t + \alpha_1 \cdot u_{t-1} + \ldots$

not corr with u_t

$\boxed{y_t} = 2 + 0.7\, y_{t-1} - 0.12\, y_{t-2} + u_t$

LHS RHS

$u_t \sim N(0; 16)$

LHS = RHS

$Cov(y_t, u_t) = Cov(\mu + u_t + \ldots, u_t) = \sigma_u^2 = 16$

$$\begin{cases} Cov(y_t, LHS) = Cov(y_t, RHS) \\ Cov(y_{t-1}, LHS) = Cov(y_{t-1}, RHS) \\ Cov(y_{t-2}, LHS) = Cov(y_{t-2}, RHS) \end{cases}$$

!

$\hookrightarrow \gamma_0, \gamma_1, \gamma_2$

$$\begin{cases} \gamma_0 = 0.7\gamma_1 - 0.12\gamma_2 + 16 \\ \gamma_1 = 0.7\gamma_0 - 0.12\gamma_1 + 0 \\ \gamma_2 = 0.7\cdot\gamma_1 - 0.12\gamma_0 + 0 \end{cases}$$