Привет Hi

9 ha = 3 months × 3 has 4/9

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Exer (*) $y_t = 0{,}5 y_{t-1} - 0.06 y_{t-2} + u_t$ $u_t \sim$ white noise

a) lag polynomial, characteristic polynomial.

One stat. sol-n
b) Are there any stationary sol-s of (*)

MA(∞) wrt ($u_t$)
c) can stationary solution (if it exists) be written as MA(∞) wrt ($u_t$) ?

$(1 - 0{,}5L + 0{,}06L^2)\, y_t = u_t$ roots $\ell_1 = \frac{1}{0{,}2}$ $\ell_2 = \frac{1}{0{,}3}$

$P(\ell) = 1 - 0{,}5\ell + 0{,}06\,\ell^2$ lag polynomial of AR part

charact. polynom. $y_t - 0{,}5 y_{t-1} + 0{,}06 y_{t-2} = 0$ (no $u_t$) (const)

let's find simple solutions $y_t = \lambda^t$

$\lambda^t - 0{,}5\lambda^{t-1} + 0.06 \cdot \lambda^{t-2} = 0$ roots

$\mathrm{char}(\lambda) = \lambda^2 - 0{,}5\lambda + 0{,}06$ $\lambda_1 = 0{,}2$ $\lambda_2 = 0{,}3$

characteristic polynomial of AR part

Th equation $P(L) \cdot y_t = c + Q(L) \cdot u_t$, $u_t$ is a white noise.

If lag polynomials of AR and MA part have no common roots then there 3 cases:

① No stationary solutions ( there is a root of lag poly $|\ell| = 1$; there is a root of char poly $|\lambda| = 1$ )

② Unique stationary solution of the form
$y_t = \mu + u_t + c_1 u_{t-1} + c_2 u_{t-2} + \ldots$ — MA(∞) wrt ($u_t$)

All roots $|\ell| > 1$
All roots $|\lambda| < 1$

③ Unique stationary solution of —//—
$y_t = \mu + v_t + c_1 v_{t+1} + c_2 v_{t+2} + \ldots$ — MA(∞) wrt ($v_t$) where $v_t$ is a white noise different from $u_t$

No $|\ell| = 1$, $\exists |\ell| < 1$

No $|\lambda| = 1$, $\exists |\lambda| > 1$

ex $\quad y_t = 6y_{t-1} - 10y_{t-2} + u_t + u_{t-1}$ $\quad u_t \sim$ white noise

One $\quad$ a) how many stationary solutions are there?

No $\quad$ b) can the stationary solut-n be written as MA($\infty$) wrt ($u_t$)?

char-c poly of AR part: $\lambda^2 - 6\lambda + 10 = 0$

MA part: $\lambda + 1 = 0$

roots AR part: $\lambda^2 - 6\lambda + 9 + 1 = 0 \quad (\lambda-3)^2 = -1$

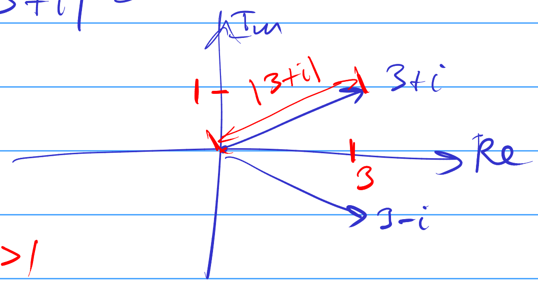
MA part: $\lambda_1 = -1 \quad\quad \lambda - 3 = \pm i$

$\lambda_1 = 3 - i$

$\lambda_2 = 3 + i$

no common roots of AR and MA part.

$|3-i| = |3+i| =$




$Re(3+i) = 3$

$|3+i| = \sqrt{3^2+1^2} = \sqrt{10} > 1$

$|3-i| = \ldots\ldots = \sqrt{10} > 1$

ex

non-causal $y_t$

$y_t = u_t + \frac{1}{2}u_{t+1} + \left(\frac{1}{2}\right)^2 u_{t+2} + \left(\frac{1}{2}\right)^3 u_{t+3} \ldots \ldots \quad u_t \sim$ WNoise

Yes $\quad$ a) is this a stationary process?

No $\quad$ b) is it MA($\infty$) wrt ($u_t$)?

c) $Cov(y_t, y_{t-1})$?

causal $y_t$

def $\quad$ MA($\infty$) wrt to ($u_t$) : $y_t = \mu + u_t + c_1 \cdot u_{t-1} + c_2 \cdot u_{t-2} + c_3 u_{t-3} + \ldots$

a) $E(y_t) = 0 + \frac{1}{2}0 + \ldots\ldots = 0$

$Var(y_t) = \sigma_u^2 + \left(\frac{1}{2}\right)^2 \sigma_u^2 + \left(\frac{1}{2}\right)^4 \cdot \sigma_u^2 + \left(\frac{1}{2}\right)^6 \sigma_u^2 + \ldots\ldots = const = \frac{\sigma_u^2}{1-(1/2)^2} = \frac{b_1}{1-q}$

b) NO

c) $\text{Cov}\left(u_t + \frac{1}{2}u_{t+1} + \left(\frac{1}{2}\right)^2 u_{t+2} + \left(\frac{1}{2}\right)^3 u_{t+3} + \ldots,\right.$

$\left. u_{t-1} + \frac{1}{2}u_t + \left(\frac{1}{2}\right)^2 u_{t+1} + \left(\frac{1}{2}\right)^3 u_{t+2} + \left(\frac{1}{2}\right)^4 u_{t+3} + \ldots\right) =$

$$= \frac{1}{2}\sigma_u^2 + \frac{1}{2}\cdot\left(\frac{1}{2}\right)^2\cdot\sigma_u^2 + \left(\frac{1}{2}\right)^2\cdot\left(\frac{1}{2}\right)^3\cdot\sigma_u^2 + \left(\frac{1}{2}\right)^3\cdot\left(\frac{1}{2}\right)^4\sigma_u^2 + \ldots =$$

$$= \frac{\frac{1}{2}\sigma_u^2}{1-\left(\frac{1}{2}\right)^2} = \frac{2\sigma_u^2}{3}$$

ARMA(2,1) model / process → $y_t = ? + u_t + ?u_{t-1} + ?u_{t-2} + \ldots$

$1\cdot y_t = ?y_{t-1} + ?y_{t-2} + c + 1\cdot u_t + ?u_{t-1}$

+ $(y_t)$'s stationary, MA(∞) wrt $(u_t)$

+ $P(L)y_t = Q(L)\cdot u_t + c$ — $P(L)$ and $Q(L)$ have no common roots.

$y_t$ is MA(∞) wrt $(u_t)$ ⟹ $y_t$ is stationary.

Ex.

ARMA(2,0) model = AR(2) model

with equation $y_t = 5 + 0.3y_{t-1} + u_t$, $u_t \sim$ w. noise.

$u_t \sim N(0;16)$

a) autocorr function $\rho_k$? partial autocorr-n $\varphi_{kk}$?

b) $y_{100} = 10$ PI 95% for $y_{101}$?

c) $y_{100} = 10$ PI 95% for $y_{100000}$?

d) represent $y_t$ as MA(∞) wrt $(u_t)$.

a) $$\text{Corr}(y_t, y_{t-k}) = \frac{\text{Cov}(y_t, y_{t-k})}{\sqrt{\text{Var}(y_t)\text{Var}(y_{t-k})}} = \frac{\text{Cov}(y_t, y_{t-k})}{\text{Var}(y_t)} = \frac{\gamma_k}{\gamma_0}$$

$$y_t = 5 + 0{,}3\, y_{t-1} + u_t$$

$\mu + u_{t-1} + ?u_{t-2} + ?u_{t-3} + \dots$

$\gamma_0$:

$$\underline{Var(y_t)} = 0{,}3^2\, \underline{Var(y_{t-1})} + \underbrace{Var(u_t)}_{16} + 2 \cdot 0{,}3\, \underbrace{Cov(y_{t-1}, u_t)}_{0}$$

$$\gamma_0 = 0{,}09 \cdot \gamma_0 + 16 \qquad \gamma_0 = \frac{16}{0.91}$$

$$Cov(y_{t-1}, y_t) = Cov(y_{t-1}, 5 + 0{,}3\, y_{t-1} + u_t)$$

$$\gamma_1 = 0{,}3\, \gamma_0 + 0$$

$$Cov(y_{t-2}, y_t) = Cov(y_{t-2}, 5 + 0{,}3\, y_{t-1} + u_t)$$

$$\gamma_2 = 0{,}3 \cdot \gamma_1 + 0$$

$$\gamma_3 = 0{,}3 \cdot \gamma_2$$

$$\vdots$$

$$\rho_1 = Corr(y_t, y_{t-1}) = \frac{\gamma_1}{\gamma_0} = 0{,}3$$

$$\rho_2 = Corr(y_t, y_{t-2}) = \frac{\gamma_2}{\gamma_0} = 0{,}3^2$$

$$\vdots$$

$$\rho_k = 0{,}3^k$$

$$y_t \quad y_{t+1} \quad y_{t+2} \quad \dots \quad y_{t+k} \qquad Corr = 0{,}3^k$$

$$\varphi_{kk} = p\,Corr(y_t, y_{t-k};\ y_{t-1}, y_{t-2}, \dots y_{t-k+1})$$

$$\varphi_{11} = p\,Corr(y_t, y_{t-1};\ \emptyset) = Corr(y_t, y_{t-1}) = 0{,}3 = \rho_1$$

$$\varphi_{22} = p\,Corr(y_t, y_{t-2};\ \underset{\text{fixed}}{y_{t-1}})$$

$$y_t = 5 + 0{,}3\, y_{t-1} + u_t$$

decomposition

$$y_t = \alpha + \varphi_{21} \cdot y_{t-1} + \varphi_{22} \cdot y_{t-2} + w_t$$

$$\boxed{y_t = 5 + 0{,}3\, y_{t-1} + 0 \cdot y_{t-2} + u_t}$$

$$\begin{cases} Cov(y_{t-1}, u_t) = 0 \\ Cov(y_{t-2}, u_t) = 0 \end{cases}$$

$$\begin{cases} Cov(y_{t-1}, w_t) = 0 \\ Cov(y_{t-2}, w_t) = 0 \end{cases} \Rightarrow \varphi_{22}\,?$$

$\alpha = 5$

$\varphi_{21} = 0{,}3$

$\varphi_{22} = 0$