Привет !!! 2022-01-13

Упр Random walk

$$y_t = y_{t-1} + u_t + 1$$

$$u_t \sim N(0;4)$$

$$\mathcal{F}_t = \sigma(y_t, y_{t-1}, y_{t-2}, \ldots)$$

u_t indep of $\mathcal{F}_{t-1}$

y_1	y_2	y_3	y_4
5	7	9	10

$u_5 = y_5 - y_4 - 1$

u_6 не зав от $y_5, y_4 \ldots$

95%

a) $E(y_5 | \mathcal{F}_4)$, $Var(y_5 | \mathcal{F}_4)$, PI для y_5

predictive interval.

б) $E(y_6 | \mathcal{F}_4)$, $Var(y_6 | \mathcal{F}_4)$, PI для y_6

$$P(y_5 \in PI | \mathcal{F}_4) = 0.95$$

common PI $= [E(y_5 | \mathcal{F}_4) - \Delta; E(y_5 | \mathcal{F}_4) + \Delta]$

$$E(y_5 | \mathcal{F}_4) = E(y_4 + u_5 + 1 | \mathcal{F}_4) =$$

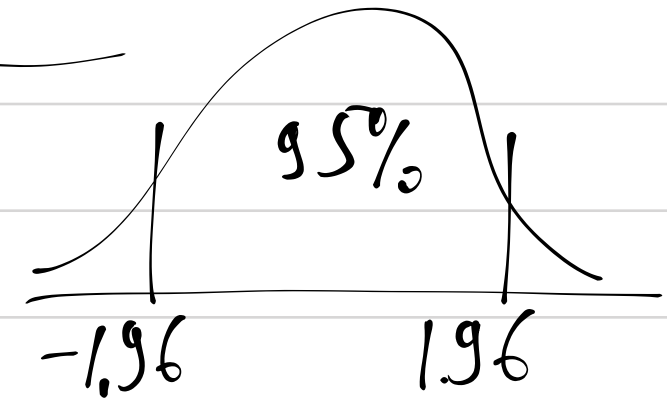
$$= E(\underbrace{y_4 + u_5 | \mathcal{F}_4}_{y_4 \text{ изв-н в } t=4}) + 1 = y_4 + E(\underbrace{u_5 | \mathcal{F}_4}_{\text{незав}}) + 1 =$$

$$= y_4 + \underbrace{E(u_5)}_{\text{по усл.}} + 1 = y_4 + 1$$

$$Var(y_5 | \mathcal{F}_4) = Var(y_4 + u_5 + 1 | \mathcal{F}_4) =$$

$$= Var(\underbrace{y_4 + u_5 | \mathcal{F}_4}_{y_4 \text{ изв-н в } t=4}) = Var(\underbrace{u_5 | \mathcal{F}_4}_{\text{незав}}) = Var(u_5) = 4$$

$$(y_5 \mid \mathcal{F}_4) \sim N(\underset{y_4+1}{11}; 4)$$



95% PI

$$[11 - q_L \cdot \sqrt{4}; 11 + q_R \cdot \sqrt{4}]$$

left quantile right quantile

$$[11 - 1.96 \cdot \sqrt{4}; 11 + 1.96 \cdot \sqrt{4}]$$

CI
conf. Int.

μ ← неизв. конст

$$0.95 = P(\mu \in CI) = P(\underset{\text{конст}}{\mu} \in [\hat{\mu}_L; \hat{\mu}_R]) \quad (y_1 \ldots; y_T)$$

PI
pred. int

$$0.95 = P(\underset{\text{сл. вел.}}{y_{t+1}} \in [\hat{y}_L; \hat{y}_R] \mid \underset{\text{информ.}}{\mathcal{F}_t})$$

Bayes int

б) $E(y_6 \mid \mathcal{F}_4)$, $Var(y_6 \mid \mathcal{F}_4)$, PI для y_6 с уч-м $\mathcal{F}_4$

$$E(y_5 + u_6 + 1 \mid \mathcal{F}_4) = E(y_4 + u_5 + 1 + u_6 + 1 \mid \mathcal{F}_4) =$$

$$= y_4 + 2 + E(u_5 + u_6 \mid \mathcal{F}_4) = y_4 + 2 + E(u_5) + E(u_6) = y_4 + 2 = 12$$

→ делаем подст. до получения изв-х (при $\mathcal{F}_4$) и нез-х от $\mathcal{F}_4$ величин

$$E(y_{4+h} \mid \mathcal{F}_4) = 10 + h$$

$$Var(y_6 \mid \mathcal{F}_4) = Var(\underbrace{y_4 + 2}_{} + u_5 + u_6 \mid \mathcal{F}_4) =$$

$$= Var(u_5 + u_6 \mid \mathcal{F}_4) = Var(u_5 + u_6) =$$

$$= 4 + 4 = 8$$

PI 95% $[12 - 1.96 \cdot \sqrt{8}\,;\, 12 + 1.96\sqrt{8}]$

Упр. RW (Random Walk) для $\ln y_t$

$$\underbrace{\ln(y_t)}_{z_t} = \underbrace{\ln(y_{t-1})}_{z_{t-1}} + u_t + 1$$

y_1	y_2	y_3	y_4
5	7	9	10
z_1	z_2	z_3	z_4
$\ln 5$	$\ln 7$		$\ln 10$

$z_t = \ln y_t$

$y_t = \exp(z_t)$

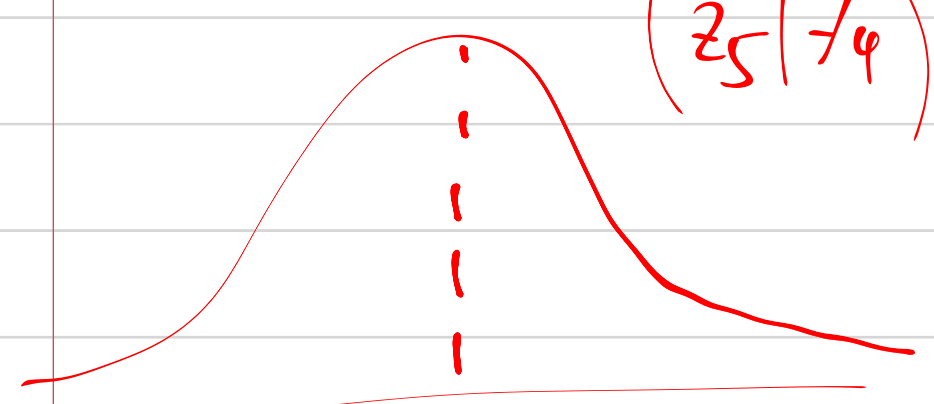
$u_t \sim N(0; 4)$

$\mathcal{F}_t = \sigma(y_t, y_{t-1}, \ldots)$

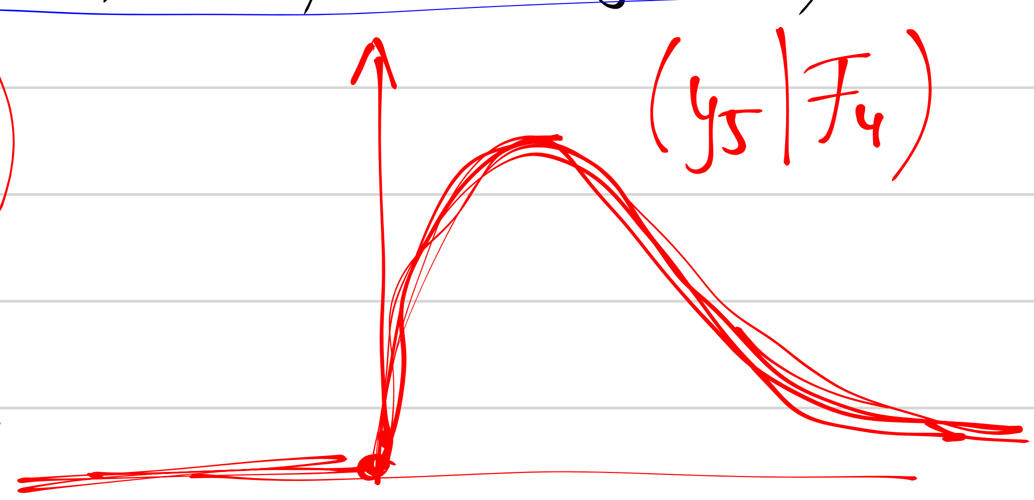
u_t не зав от $\mathcal{F}_{t-1}$

a) $E(y_5 \mid \mathcal{F}_4)$ б) $Med(y_5 \mid \mathcal{F}_4)$ c) $Mode(y_5 \mid \mathcal{F}_4)$

$(z_5 \mid \mathcal{F}_4)$



$(y_5 \mid \mathcal{F}_4)$



$E(z_5 \mid \mathcal{F}_4)$
$\shortparallel$
$Med(z_5 \mid \mathcal{F}_4)$
$\shortparallel$
$Mode(z_5 \mid \mathcal{F}_4)$

$$(z_5 \mid \mathcal{F}_4) \sim N(\underbrace{\ln 10 + 1}_{}; \;\;)$$

$N(z_4 + 1, 4)$

$y_5 = \exp(z_5)$

$$(y_5 \mid \mathcal{F}_4) \sim \exp(N(\ln 10 + 1; 4))$$

Med

$$P(y_5 > m \mid \mathcal{F}_4) = 0{,}5$$

$$P(\underbrace{\ln y_5}_{} > \ln m \mid \mathcal{F}_4) = 0{,}5$$

$$P(\underbrace{N(\ln 10 + 1; 4)}_{} > \underbrace{\ln 10 + 1}_{?} \mid \mathcal{F}_4) = 0{,}5$$

$$m = \exp(\ln 10 + 1) = 10 \cdot \exp(1) = 10e \approx 27$$

E

$$E(y_5 \mid \mathcal{F}_4) = \underline{E(\exp(z_5) \mid \mathcal{F}_4)} = \circledast$$

1

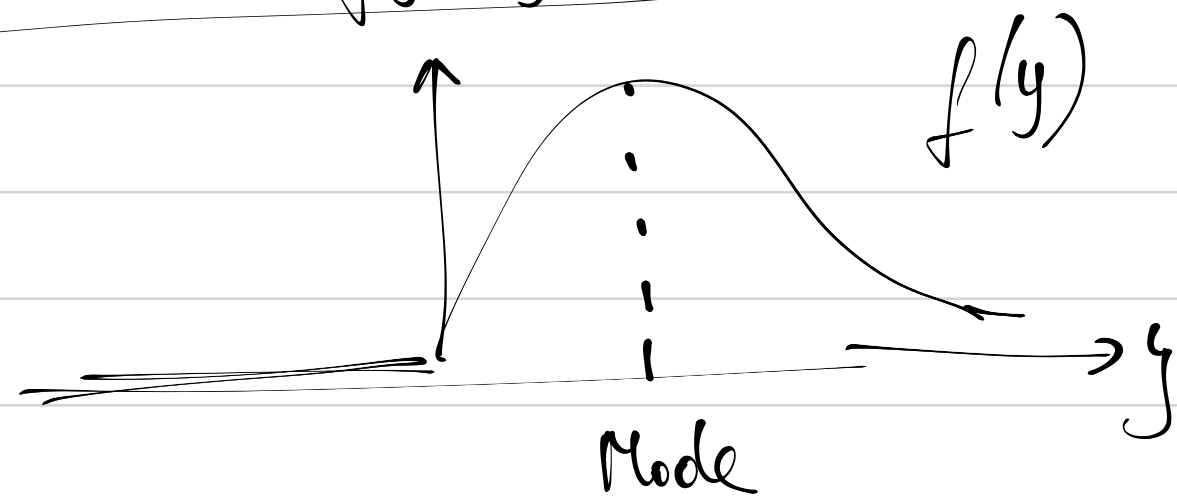
$$MGF_{z_5 \mid \mathcal{F}_4}(u) = E(\exp(u \cdot z_5) \mid \mathcal{F}_4)$$

→ wiki $\quad MGF_{N(\mu; \delta^2)}(u) = \exp\left(\mu \cdot u + \frac{\delta^2}{2} \cdot u^2\right)$

$$MGF(u) = \exp\left((\ln 10 + 1) \cdot u + \frac{4}{2} \cdot u^2\right)$$

$$\ast = MGF(1) = \exp(\ln 10 + 1 + 2) = 10 \cdot \exp(3) \approx 200$$

$\boxed{\text{Mode - макс } f_{y_5}(y)}$



Q. Как найти $f_Y(y)$?

A. Лайфхак! запишите $\boxed{f_Z(z)\,dz}$ и подставьте z.

важно!

$$f(z)\,dz = \frac{1}{\sqrt{2\pi\cdot 4}}\cdot\exp\left(-\frac{1}{2\cdot 4}\cdot\left(z-(\ln 10+1)\right)^2\right)\cdot dz =$$

$z_5 \mid \mathcal{F}_4 \sim \mathcal{N}(\cdot\,,\cdot)$ $\quad \sigma^2 \quad z=\ln y \quad \sigma^2 \quad \mu$

$$= \boxed{\frac{1}{\sqrt{8\pi}}\cdot\exp\left(-\frac{1}{8}\left(\ln y-(\ln 10+1)\right)^2\right)\cdot\frac{1}{y}\,dy} \;; \to \max_y$$

$$\boxed{\ln f(y) = \ln\frac{1}{\sqrt{8\pi}} - \frac{1}{8}\left(\ln y-(\ln 10+1)\right)^2 - \ln y \to \max}$$

$\ln y = \underset{\xi}{a}$ $\qquad -\frac{1}{8}\left(a-(\ln 10+1)\right)^2 - a \to \max_a$

a^*

y^*

мимо pandas sktime