1.2.
p - probability of "head"
N - total number of throws
$E(N)$, $E(N^2)$, $E(N^3)$
$Var(N) = E(N^2) - (E(N))^2$

To find all these $E(N)$, $E(N^2)$, $E(N^3)$ ... it is sufficient to find one fancy function $E(\exp(tN))$

Two states:
(C) = the game continues
(F) = the final state

P(C -> F) = p
P(C -> C) = 1 - p

P(F -> F) = 1
P(F -> C) = 0

We will use the function $E(\exp(tN)) = M(t)$ to calculate $E(N)$, $E(N^2)$, $E(N^3)$....

By definition $M(t) = E(exp(tN))$

We will find two functions: $M_F(t)$ and $M_C(t)$, where the letter F or C denotes the initial state.

If we start at F then we need $N = 0$ moves to reach F.
$M_F(t) = E\exp(tN) = E\exp(0) = E(1) = 1$
We really don't use this function, just to get the idea.

Now let's move to the hard case:

(1) $M_C(t) = p \cdot \text{make one move and arrive at F} + (1-p) \cdot \text{make one move and arrive at C}$

(2) $M_C(t) = p \cdot \exp(t \cdot 1) + (1-p) \cdot \text{make one move and arrive at C}$

(3) $M_C(t) = p \cdot \exp(t \cdot 1) + (1-p) \cdot E(\exp(t(1+N)))$

(4) $M_C(t) = p \cdot \exp(t \cdot 1) + (1-p) \cdot E(\exp(t)\exp(tN)))$

(5) $M_C(t) = p \cdot \exp(t \cdot 1) + (1-p) \cdot \exp(t) E(\exp(tN))$

(6) $M_C(t) = p \cdot \exp(t \cdot 1) + (1-p) \cdot \exp(t) M_C(t)$

We solve the equation (6) for $M_C(t)$:

$$M_C(t) = \frac{pe^t}{1-(1-p)e^t}$$

Hence,

$$M_C(t) = E(\exp(tN)) = \frac{pe^t}{1-(1-p)e^t}$$

This function is not our final goal, we will use it to calculate all the expected values...

Let's look at the derivatives of this $M_C(t)$ function:

$M_C'(t) = E(N\exp(tN))$

$M_C''(t) = E(N^2\exp(tN))$

$M_C'''(t) = E(N^3\exp(tN))$

If we plug in $t=0$ we see that there is a fancy way to calculate expected values:

$M_C'(0) = E(N)$

$M_C''(0) = E(N^2)$

$M_C'''(0) = E(N^3)$

In our case

$$E(N) = M_C'(0) = \frac{pe^t(1-(1-p)e^t) - pe^t(-(1-p)e^t)}{(1-(1-p)e^t)^2} = \frac{p^2 + p(1-p)}{p^2} = 1/p$$

I hope you can take the two remaining derivatives :)

1.4.

Initially the pot is empty.
1, 2, 3 -> the corresponding sum is added into the pot.
4, 5 -> you take the pot with money and the game ends
6 -> you get nothing and the game ends

a) What is probability that the game eventually ends by 6?
by symmetry: 1/3
b) Expected duration of the game?
N - random number of throws

First step equation.

$$E(N) = 3/6 \cdot 1 + 3/6 \cdot E(N+1)$$

$E(N) = a$

$$6a = 3 + 3(a+1)$$

$a = 6/3 = 2$

c) the main question!
X - the random payoff we get

$E(X)$?

First step equation:

$$E(X) = 3/6 \cdot 0 + 1/6 \cdot (E(X) + 1 \cdot 2/3) + 1/6 \cdot (E(X) + 2 \cdot 2/3) + 1/6 \cdot (E(X) + 3 \cdot 2/3)$$

You can solve it!

Another good idea!

$$E(X) = (E(N) - 1) \cdot (1 + 2 + 3)/3 \cdot 2/3$$