

- The hedgehog Melissa starts at the vertex A of a triangle $\triangle ABC$. Each minute she randomly moves to an adjacent vertex with probabilities $\mathbb{P}(A \rightarrow B) = 0.7$, $\mathbb{P}(A \rightarrow C) = 0.3$, $\mathbb{P}(B \rightarrow C) = \mathbb{P}(B \rightarrow A) = 0.5$, $\mathbb{P}(C \rightarrow B) = 0.5$, $\mathbb{P}(C \rightarrow A) = 0$.
 - What is the probability that she will be in vertex C after 3 steps?
 - Write down the transition matrix of this Markov chain.
 - What is the expected time to get from the state A back to it?
- The number of players N who will win the lottery is a random variable with probability mass function $\mathbb{P}(N = k + 1) = \exp(-1)/k!$ for $k \geq 0$. Each player will get a random prize $X_i \sim U[0; 2]$. All random variables are independent. Let S be the sum of all the prizes.
 - Find $\text{Var}(S \mid N)$ and conditional moment generating function $M_{S|N}(u)$ for fixed value of N .
 - Find the unconditional moment generating function $M_S(u)$.

Note: you don't need to calculate the value in (c).

- Consider the stochastic process (X_n) , where X_0 is uniform on $[0; 1]$ and X_n is uniformly selected on $[0; X_{n-1}]$ given X_{n-1} .
 - Find $\mathbb{E}(X_n)$ and $\text{Var}(X_n)$.
 - Find the probability limit $\lim X_n$.
- Students arrive in the Grusha café according to the Poisson arrival process (X_t) with constant rate λ . Let's measure time in minutes.

The probability of no visitors during 5 minutes is 0.10.

 - Find the value of λ .
 - Plot the probability $\mathbb{P}(X_{2t} = X_t)$ as a function of t .
 - Plot the variance $\text{Var}(X_{2t} - X_t)$ as a function of t .
- The random variables X_1 and X_2 are independent and normally distributed, $X_1 \sim \mathcal{N}(1; 1)$, $X_2 \sim \mathcal{N}(2; 2)$. I choose X_1 with probability 0.4 and X_2 with probability 0.6 without knowing their values. Casino pays me the value Y that is equal to the chosen random variable. Let the indicator I be equal to 1 if I choose X_1 and 0 otherwise.
 - Express Y in terms of X_1 , X_2 and I .
 - Find $\mathbb{E}(I + Y \mid Y)$ and $\text{Var}(I + Y \mid Y)$.
 - Find $\text{Cov}(I, Y)$.

- The joint distribution of X and Y is given in the table

	$X = -1$	$X = 0$	$X = 1$
$Y = -1$	0.1	0.2	0.3
$Y = 1$	0.2	0.1	0.1

- Explicitly find the σ -algebra $\sigma(\max\{X + Y, 0\})$.
- How many elements are there in $\sigma(X - Y)$?