Name, group no:

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1. Consider $MA(2)$ process given by

$$y_t = 5 + u_t + 2u_{t-1} + 4u_{t-2},$$

where (u_t) is a white noise with $\mathbb{V}\mathrm{ar}(u_t) = \sigma^2$.

(a) [1] Find the expected value $\mathbb{E}(y_t)$.

(b) [7] Find the autocorrelation function $\rho_k = \mathbb{C}\mathrm{orr}(y_t, y_{t-k})$.

(c) [2] Is the process (y_t) stationary?

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2. Consider $MA(2)$ process given by

$$y_t = 5 + u_t + 2u_{t-1} + 4u_{t-2},$$

where u_t are normal independent random variables with $\mathbb{V}\mathrm{ar}(u_t) = 4$.

You know that $u_{100} = 2$ and $u_{99} = -1$.

(a) [5] Find the 95% predictive interval for y_{101}.

(b) [5] Find the 95% predictive interval for $y_{1000001}$.

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3. The stationary process (y_t) has autocorrelation function $\rho_k = 0.2^k$ and expected value 100.

   (a) [7] Find the first two values of the partial autocorrelation function, ϕ_{11} and ϕ_{22}.

   (b) [3] Provide a possible linear recurrence equation for this process. Your equation may include y_t, its lags and a white noise process (u_t).

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4. Consider the equation $y_t = 5 + 2.5y_{t-1} - y_{t-2} + u_t$, where (u_t) is a white noise process.

   (a) [3] Find the roots of the corresponding characteristic equation.

   (b) [4] Rewrite the process as $A(L)(y_t - \mu) = u_t$. You should explicitely write the lag polynomial $A(L)$ and the value of μ.

   (c) [1] How many non-stationary solutions does the equation have?

   (d) [1] How many stationary solutions does the equation have?

   (e) [1] How many stationary solutions of the $MA(\infty)$ form with respect to (u_t) does the equation have?

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5. [10] The semi-annual (y_t) is modelled by $ETS(ANA)$ process:

$$\begin{cases} u_t \sim \mathcal{N}(0;4) \\ s_t = s_{t-2} + 0.1u_t \\ \ell_t = \ell_{t-1} + 0.3u_t \\ y_t = \ell_{t-1} + s_{t-2} + u_t \end{cases}$$

Given that $s_{100} = 3$, $s_{99} = -2$, $\ell_{100} = 100$ find 95% predictive interval for y_{102}.

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6. [10] The semi-annual (y_t) is modelled by $ETS(ANA)$ process:

$$\begin{cases} u_t \sim \mathcal{N}(0; 4) \\ s_t = s_{t-2} + 0.1u_t \\ \ell_t = \ell_{t-1} + 0.3u_t \\ y_t = \ell_{t-1} + s_{t-2} + u_t \\ \ell_0 = 100, s_0 = -3, s_{-1} = 3 \end{cases}$$

Check whether the process (y_t) is stationary.