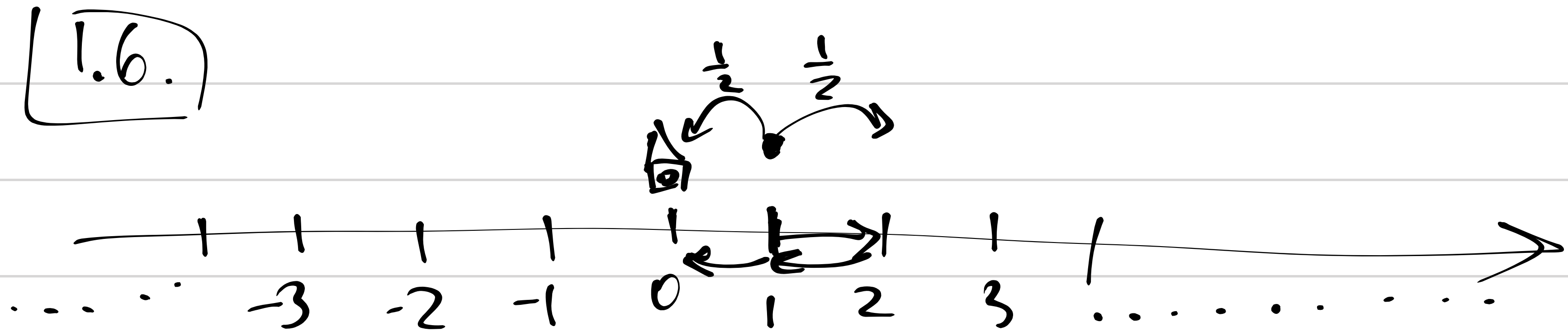
☺

1.6.



T — first moment of time when $X_T = 0$

Start: $X_0 = 1$

Calculate $P(T=k)$

$P(T=0) = 0$ $\qquad$ $P(T=2) \overset{?}{=} 0$

$P(T=1) = \frac{1}{2}$ $\qquad$ $P(T=3) \overset{?}{=} \boxed{\frac{1}{8}}$

RLL

$P(T=123) =$ hard $\qquad$ ......

LLR

(first visit to 0)

$P(T=0)$ $\quad$ $P(T=1)$ $\quad$ $P(t=2)$ .....

p_0 $\quad$ p_1 $\quad$ p_2 $\quad$ p_3 ...

$$\boxed{g(u) = p_0 \cdot 1 + p_1 \cdot u^1 + p_2 \cdot u^2 + p_3 \cdot u^3 + \dots}$$

$$g(u) = 0 + \frac{1}{2} \cdot u^1 + 0 \cdot u^2 + \frac{1}{8} \cdot u^3 + \dots$$

$$g(u) = E(u^T)$$

idea 1: pack $p_0, p_1, p_2 \dots$ into $g(u)$

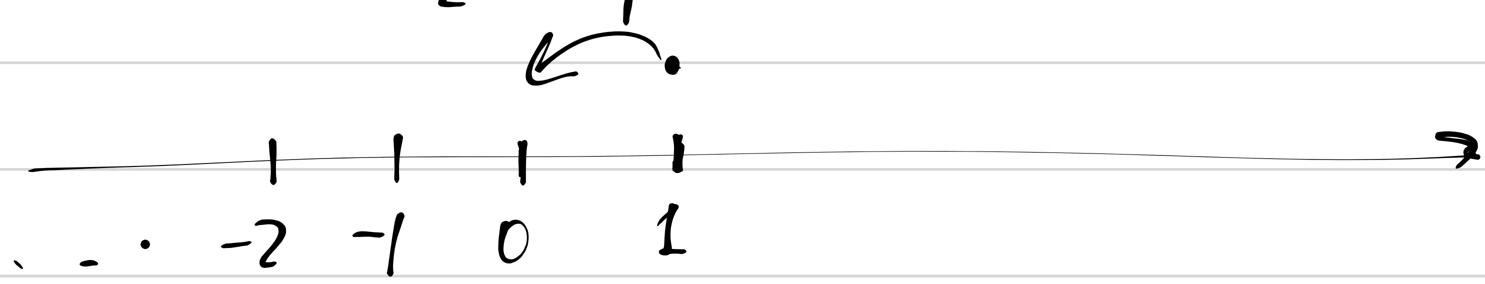
t	0	1	2	3	4	5	6	7	$\dots$
u^t	1	u^1	u^2	u^3	u^4	$\dots$			
$p(T=t)$	p_0	p_1	p_2	p_3	p_4	$\dots$			

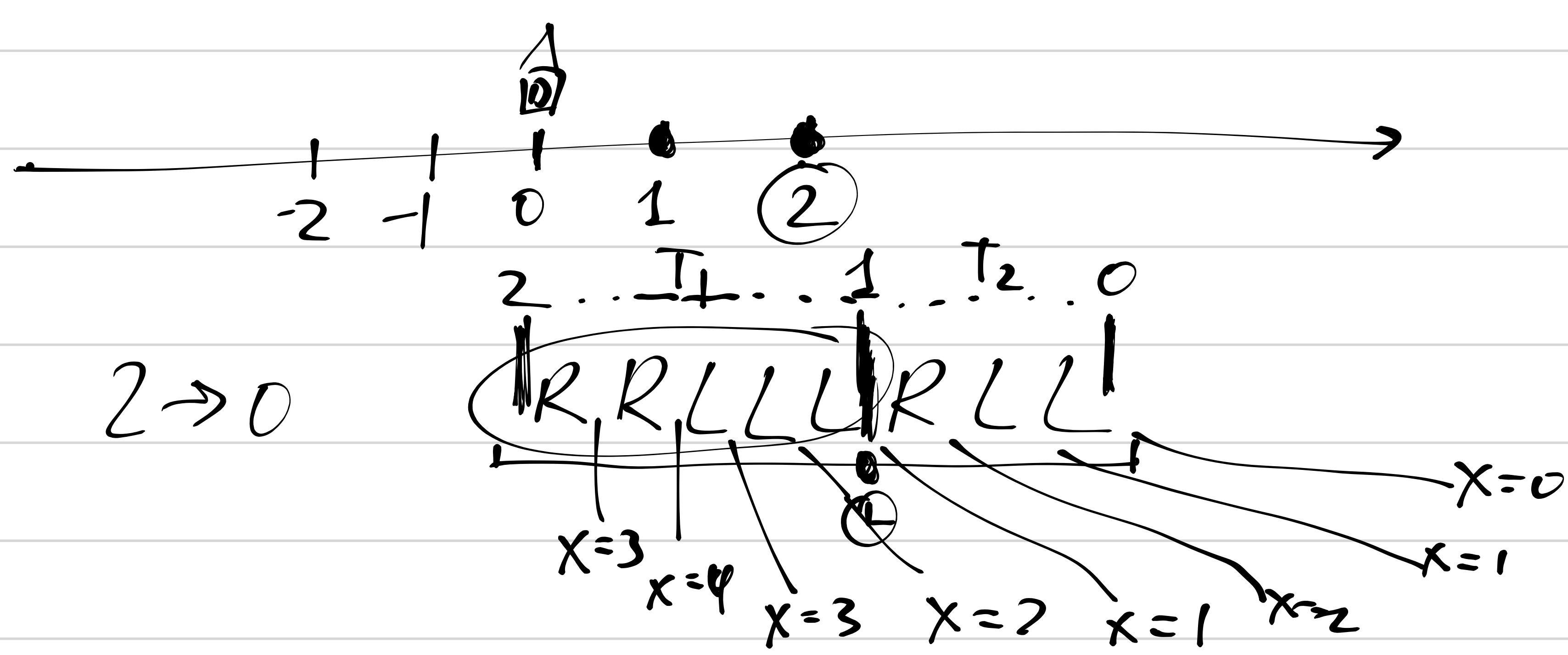
$$\boxed{E(u^T) = p_0 \cdot 1 + p_1 u^1 + p_2 u^2 + \dots}$$

→ Calculate only one function $E(u^T)$

→ extract all probabilities.

$$X_0 = 1 \quad \begin{array}{l} \xrightarrow{1/2} X_1 = 0 \quad \smile \quad T = 1 \\ \xrightarrow{1/2} X_1 = 2 \quad \frown \quad T\,? \end{array}$$



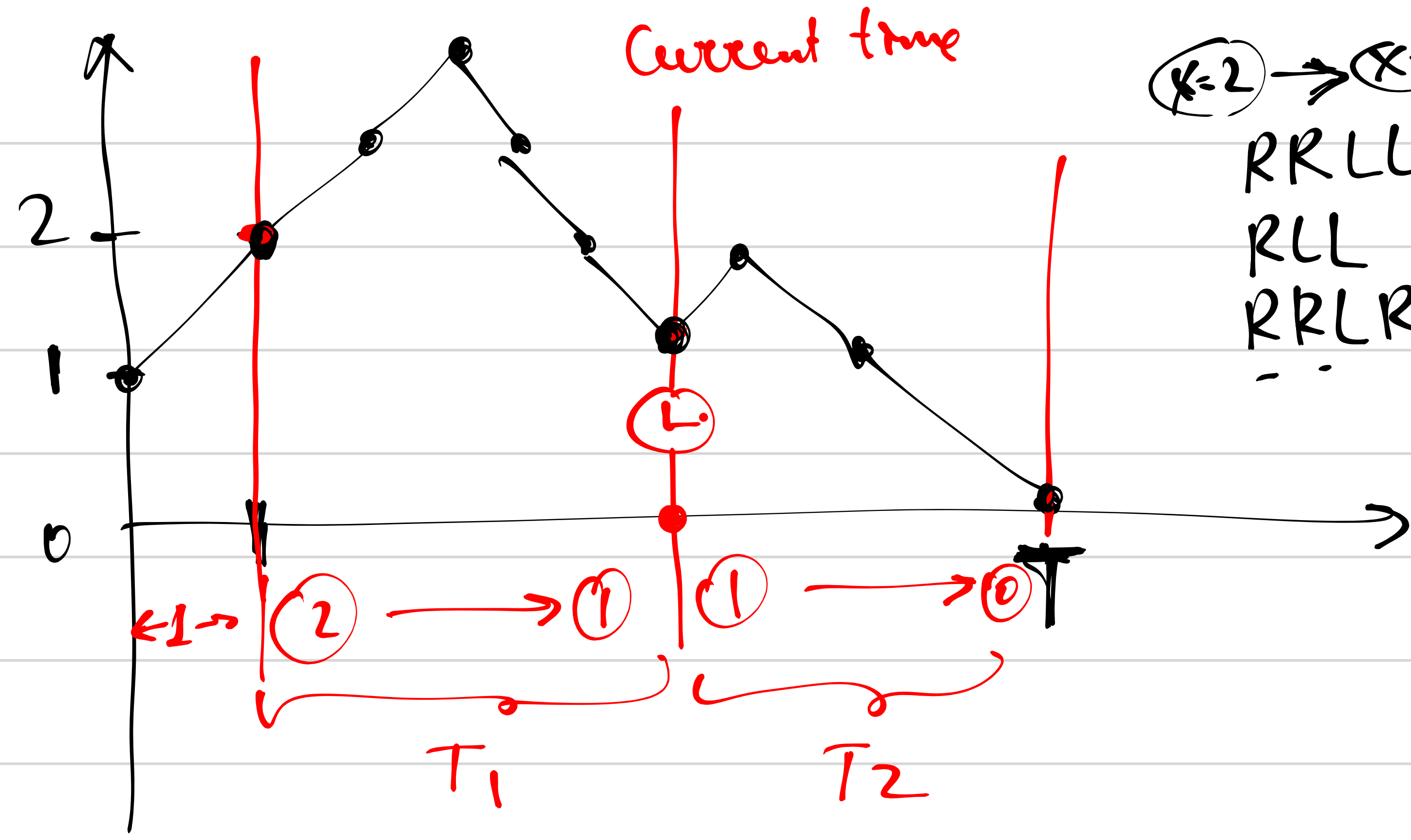


Idea 2
first step analysis

T_1, T_2 are independent

T_1, T_2, T identically distributed.

$$\boxed{X_0 = 1} \quad \begin{array}{l} \xrightarrow{1/2} X_1 = 0 \quad (T = 1) \\ \xrightarrow{1/2} X_1 = 2 \quad (T \to 1 + T_1 + T_2) \end{array}$$



$(X=2) \to (X=1)$

RRLLL

RLL

RRLRLLL

- - - - -

$$T \begin{cases} \xrightarrow{\frac{1}{2}} T = 1 \\ \xrightarrow{\frac{1}{2}} T \to 1 + T_1 + T_2 \end{cases} \qquad T_1, T_2 \sim T$$

if $\underline{X_2 = 2}$ then $T > T_2$

$T=1$

$$E(u^T) = \frac{1}{2} u^1 + \frac{1}{2} E\left(u^{1+T_1+T_2}\right)$$

!

$$\boxed{E(u^T)} = \frac{1}{2} \cdot u + \frac{1}{2} \cdot E(u) \cdot \boxed{E(u^{T_1})} \cdot \boxed{E(u^{T_2})} \quad !$$

T_1 and T_2 are indep.

$$\boxed{g(u)} = \frac{1}{2} \cdot u + \frac{1}{2} \cdot u \cdot \boxed{g(u)} \cdot \boxed{g(u)} \qquad (\cdot 2u)$$

$$2g \cdot u = u^2 + u^2 \cdot g^2$$

$$\underbrace{u^2 g^2 - 2gu + 1}_{(ug-1)^2} - 1 + u^2 = 0$$

$$(ug-1)^2 = 1 - u^2$$

$$ug - 1 = \pm\sqrt{1-u^2}$$

$$\boxed{g(u) = \frac{1 \pm \sqrt{1-u^2}}{u}}$$

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$(1+t)^{\alpha}$ = Taylor expansion?

$(1+t)^2 = 1 + 2t + t^2$

$(1+t)^3 = 1 + 3t + 3t^2 + t^3$

$\boxed{t = -u^2}$ $\boxed{n = \frac{1}{2}}$

$(1+t)^n = 1 + C_n^1 \cdot t + C_n^2 \cdot t^2 + C_n^3 \cdot t^3 + \ldots$

?

$$g(u) = \frac{1 \pm (1-u^2)^{\frac{1}{2}}}{u}$$

$$g(u) = \frac{1 + (1-u^2)^{\frac{1}{2}}}{u} = \frac{1 + 1 - C_{0.5}^1 \cdot u^2 + C_{0.5}^2 \cdot u^4 \ldots}{u}$$

$$g(u) = \frac{1 - (1-u^2)^{\frac{1}{2}}}{u} = \frac{1 - 1 + C_{0.5}^1 \cdot u^2 - C_{0.5}^2 \cdot u^4 + \ldots}{u}$$

we know initial terms.

$g(u) = 0 + \frac{1}{2} \cdot u + 0 \cdot u^2 + \frac{1}{8} \cdot u^3 + \ldots$

$p_1 \cdot u^1 + p_2 \cdot u^2 + p_3 \cdot u^3 + \ldots$ $\quad 7+1/2 = 4$

$g(u) = +C_{0.5}^1 \cdot u - C_{0.5}^2 \cdot u^3 + C_{0.5}^3 \cdot u^5 - C_{0.5}^4 \cdot u^7 \ldots$

$\ldots - C_{0.5}^6 u^{11} + \ldots$

$$C_5^3 = \frac{5!}{3!\,2!} = \frac{5 \cdot 4 \cdot 3}{3!}$$

$$\boxed{C_{\alpha}^{k} = \frac{\alpha \cdot (\alpha-1) \cdot (\alpha-2) \cdot \ldots \cdot (\alpha-k+1)}{k!}}$$

$$C_{0,5}^3 \stackrel{?}{=} \frac{0.5 \cdot (0.5-1) \cdot (0.5-2)}{3!}$$

$P(T=11) = -C_{0.5}^6$

$P(T=k) = |C_{0.5}^{(k+1)/2}|$

$$P(T=7) = -C_{0.5}^4 = -\frac{0.5 \cdot (0.5-1) \cdot (0.5-2) \cdot (0.5-3)}{4!}$$

$$P(T=123) = -C_{0,5}^{124/2} = -C_{0,5}^{62} =$$

$$= \frac{-0{,}5(0.5-1)\cdot(0.5-2)\cdot\ldots\ldots\cdot(0.5-61)}{62!} =$$

$$\frac{2^{62}}{2^{62}} \qquad = \boxed{\frac{1\cdot(2-1)\cdot(4-1)\cdot(6-1)\cdot\ldots\cdot(122-1)}{2^{62}\cdot 62!}}$$

$2n-1=123$

$n=62$

if k is even then $P(T=k)=0$

if $\boxed{k=2n-1}$

$$\boxed{P(T=2n-1) = \frac{(2n-2)\cdot(2n-4)\cdot(2n-6)\cdot\ldots 1}{2^n\cdot n!}}$$

2.4. $X \in \{0,1,2,3,4\ldots\}$

$$g(u) = E(u^X) = 0{,}1 + 0{,}2u + 0{,}15\cdot u^2 + \ldots$$

$P(X=0)\ ? = 0{,}1$ $\qquad E(u^X) = p_0\cdot u^0 + p_1\cdot u^1 + \ldots$

$P(X=1)\ ? = 0{,}2$

$P(X=2)\ ? = 0{,}15$

$P(X=3) \in\ ?$ unknown $\in[0;0.55]$

2.3

$$M(t) = MGF = E(\exp(tW)) = 1 + 2t + 7t^2 + 20t^3 + \dots$$

$$E(W)? \quad = M'(0) = \left[2 + 7\cdot 2\cdot t + 20\cdot 3\, t^2 + \dots\right]_{t=0} = 2$$

$$E(W^2)? \quad = M''(0) = \left[7\cdot 2 + 20\cdot 3\cdot 2\cdot t + \dots\right]_{t=0} = 14$$

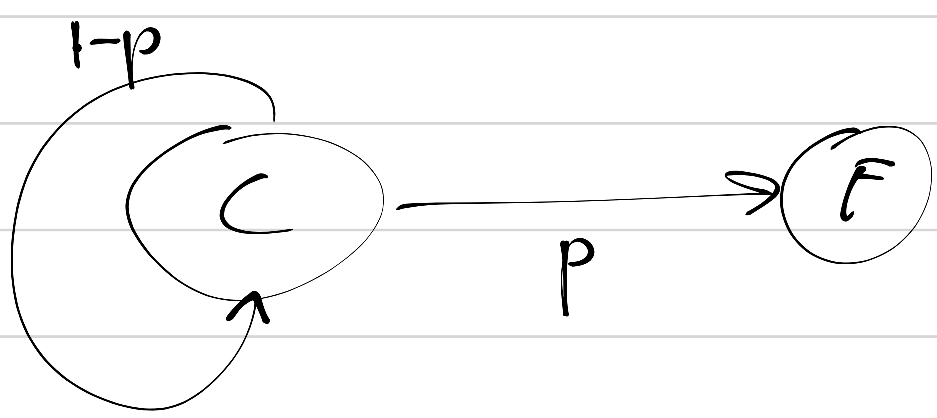
$$E(W^3)? \quad = M'''(0) = \left[20\cdot 3\cdot 2\cdot 1 + \dots\right]_{t=0} = 20\cdot 3!$$

$$Var(W) = 14 - 2^2 = 10$$

$$MGF(t) = 1 + k_1\cdot t + k_2\cdot t^2 + k_3\cdot t^3 + \dots$$

$$E(W^n) = k_n \cdot n!$$

POI



N

$$N \xrightarrow{p} N = 1 \qquad \exp(tN) = \exp(t)$$

$$N \xrightarrow{1-p} N \longrightarrow 1 + N_1 \qquad N_1 \sim N$$

$$E(\exp(tN)) = p\cdot \exp(t) + (1-p)\cdot E(\exp(1+N_1))$$

$$E(\exp(tN)) = \boxed{\begin{aligned} & p\cdot \exp(t) + (1-p)\cdot p\cdot \exp(2t) + \\ & \quad + (1-p)^2\cdot p\cdot \exp(3t) + \\ & \quad (1-p)^3 p\cdot \exp(4t) + \dots \end{aligned}}$$

$$E(\exp(tN)) = p\cdot \exp(t) + (1-p)\cdot \exp(t)\cdot \boxed{\left[p\cdot e^t + (1-p)\cdot p e^{2t} + (1-p)^2\cdot p\cdot e^{3t} + \dots\right]}$$

$$E(\exp(tN)) = p\cdot e^t + (1-p)\cdot \exp(t)\cdot E(\exp(tN))$$