1. [10 points] Let (X_t) be independent identically distributed random variables with $\mathbb{P}(X_i = -1) = 0.7$ and $\mathbb{P}(X_i = +1) = 0.3$. Consider the sum $S_t = X_1 + X_2 + \ldots + X_t$.

   (a) [3] Is S_t a martingale?

   (b) [7] Find all constants c such that $M_t = \exp(cS_t)$ is a martingale.

2. [10 points] Let
$$Y_t = \exp\left(-6t^3 + \int_0^t f(s)\,dW_s\right),$$
where f is some deterministic function.

   (a) [5] Using Ito's lemma find dY_t.

   (b) [5] Find at least one function f such that Y_t is a martingale.

3. [10 points] Consider the sequence S_n,
$$S_n = \sum_{i=1}^{n} (W_{ti/n} - W_{t(i-1)/n})^3.$$

   (a) [3] Find $\mathbb{E}(S_n)$.

   (b) [5] Find the limit of $\mathrm{Var}(S_n)$ when $n \to \infty$.

   (c) [2] Find the mean square limit of S_n.

4. [10 points] Consider the process
$$I_t = t^2 + t\int_0^t W_u^3 dW_u.$$

   (a) [3] Find $\mathbb{E}(I_t)$.

   (b) [3] Find $\mathrm{Var}(I_t)$.

   (c) [4] Find $\mathrm{Cov}(I_t, W_t)$.

5. [10 points] Consider two-period binomial model with initial share price $S_0 = 600$. Up and down share price multipliers are $u = 1.2$, $d = 0.9$, risk-free interest rate is $r = 0.03$ per period.

   The option pays you the minimal share price $X_2 = \min\{S_0, S_1, S_2\}$ at $t = 2$.

   (a) [4] Find the risk neutral probabilities.

   (b) [6] Find the current price X_0 of this option.

6. [10 points] Consider the framework of Black and Scholes model with riskless rate r, volatility σ and initial share price S_0.

   Find the current price X_0 of an option that pays you 1 dollar at fixed time T if $S_T/S_{T/2} \geq S_{T/2}/S_0$.