

Stochastic processes and applications: Seminar #1

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i | **Info:** Do not be afraid to solve unfinished tasks at home!

1 Markov chain: the first meeting

Task 1

A HSE student lives in two states: sleeping and studying. If he or she sleeps, a student will continue to sleep with a probability equal to 0.25, otherwise a student will start to study. If a student studies, the probabilities to continue studying and to start sleeping are equal.

Question 1

- (a) Write the transition matrix of this Markov chain.
- (b) Draw the graph representation.
- (c) What is the probability that a Sleeping Student will be a Studying Student after 1 turn? After 2 turns?
- (d) We know that now student is sleeping with probability $p = \frac{2}{3}$. Find the probabilities of sleep and study after 1 and 2 turns. Try to do it with **matrix** operations.

Task 2

The most trusted buyer ships original iPhones from China to Russia with probability 0.75 and from China to UAE with probability 0.25. He never sends it to China from anywhere. The buyer ships iPhones from Russia to UAE with probability 0.25 and from UAE to Russia with probability 0.5. The buyer claims he uses Markov chain in his work.

Question 2

- (a) Draw the graph representation of this Markov chain.
- (b) Write the transition matrix of this Markov chain.
- (c) What is the probability that a brand new iPhone from the factory will be in Russia after 1 shipment? After 3 shipments?

2 First step analysis

Task 3: The model of a completely unteachable student

Students in NOTHSE University try to pass the Exam. Somehow with probability 0.7 a student may pass it and finish his or her education. Otherwise a student goes to retake. Students are unteachable and the University allows an infinite number of retakes. So this process can be described with a Markov chain.

Question 3

- (a) Draw the graph representation of this Markov chain.
- (b) What is the probability to finish the education?
- (c) Use **first step analysis** to find the average number of retakes per student in this University.

Task 4

HSE student starts at home. He or she goes for a walk for 1 hour with probability 0.25 or with probability 0.75 spend 1 hour to go to HSE. In HSE the student studies for 1 hour with probability 0.25 or returns home otherwise.

Question 4

- (a) Write the transition matrix of this Markov chain.
- (b) Draw the graph representation.
- (c) Use **first step analysis** to find how many hours the student spends before he or she returns home on average.

Task 5

Alice and Bob toss a coin, writing down the results. If the last 3 tosses are Head, Head and Tail, Alice wins. If the last 3 tosses are Tail, Head and Head, Bob wins.

Question 5

- (a) Is it useful to work with matrix representation in this case?
- (b) Draw the graph representation. Who is likely to win the game?
- (c) Use **first step analysis** to find the probability of Alice's win.
- (d) Find the probability to finish the game after 4 tosses.



Sources:

1. Kelbert M., Sukhov Y., Probability and statistics in examples and problems
2. Cambridge course on Markov chains <http://www.statslab.cam.ac.uk>
3. Demeshev B., Problems on stochastic analysis <https://github.com/bdemeshev>