1. The hedgehog Melissa starts at the vertex A of a triangle ΔABC. Each minute she randomly moves to an adjacent vertex with probabilities $\mathbb{P}(A \to B) = 0.7$, $\mathbb{P}(A \to C) = 0.3$, $\mathbb{P}(B \to C) = \mathbb{P}(B \to A) = 0.5$, $\mathbb{P}(C \to B) = \mathbb{P}(C \to A) = 0.5$.
   (a) What is the probability that she will be in vertex B after 3 steps?
   (b) Write down the transition matrix of this Markov chain.
   (c) What proportion of time Melissa will spend in each state in the long run?

2. The number of players N who will win the lottery is a random variable with probability mass function $\mathbb{P}(N = k) = 7 \cdot 0.3^k/3$ for $k \geq 1$. Each player will get a random prize $X_i \sim U[0;1]$. All random variables are independent. Let S be the sum of all the prizes.
   (a) Find $\mathbb{E}(S \mid N)$ and conditional moment generating function $M_{S|N}(u)$.
   (b) Find the unconditional moment generating function $M_S(u)$.
   (c) What is the probabilistic meaning of $M_S''(0) - (M_S'(0))^2$?

   Note: you don't need to calculate the value in (c).

3. Consider the stochastic process (X_n), where X_0 is uniform on $[0;2]$ and $X_n = (1 + X_{n-1})/2$.
   (a) Find $\mathbb{E}(X_n)$ and $\mathbb{V}\text{ar}(X_n)$.
   (b) Find the probability limit $\operatorname{plim} X_n$.

4. Students arrive in the Grusha café according to the Poisson arrival process (X_t) with constant rate λ.

   The probability of no visitors during 5 minutes is 0.05.
   (a) Find the value of λ.
   (b) Find the variance and expected number of arrivals between 5 pm and 8 pm.
   (c) What is the probability of exactly 5 arrivals between 5 pm and 8 pm?

5. The random variables X_1 and X_2 are independent and normally distributed, $X_1 \sim \mathcal{N}(1;1)$, $X_2 \sim \mathcal{N}(2;2)$. I choose X_1 with probability 0.3 and X_2 with probability 0.7 without knowing their values.

   Casino pays me the value Y that is equal to the chosen random variable.

   Let the indicator I be equal to 1 if I choose X_1 and 0 otherwise.
   (a) Express Y in terms of X_1, X_2 and I.
   (b) Find $\mathbb{E}(Y \mid I)$, $\mathbb{V}\text{ar}(Y \mid I)$.
   (c) Find $\mathbb{E}(Y)$ and $\mathbb{V}\text{ar}(Y)$.

6. The joint distribution of X and Y is given in the table

	$X = -2$	$X = 0$	$X = 2$
$Y = -1$	0.1	0.2	0.3
$Y = 1$	0.2	0.1	0.1

   (a) Explicitely find the σ-algebra $\sigma(X)$.
   (b) How many elements are there in $\sigma(X \cdot Y)$?