Poisson process

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Consider the random process (X_t). The time t is continuous, $t \in [0; \infty)$. The random variable X_t counts the number of "arrivals" on $[0; t]$.
We assume that

1. $X_0 = 0$.
2. "Stationary increments". The number of arrival during any time interval $[t; t+h]$ depends only on the length h of the interval and not on starting time t.
3. "Independent increments".
4. For small time interval length h the probability of exactly one arrival is approximately proportional to h.

$$\P(X_{t+h} - X_t = 1) = \lambda h + o(h).$$

5. For small time interval length h the probability of two or more arrivals is negligible compared to h.

$$\P(X_{t+h} - X_t \geq 2) = o(h).$$

Let's recap that $o(h)$ is any function of h such that

$$\lim_{n \to \infty} \frac{o(h)}{h} = 0.$$

From the last two assumptions we deduce that $\P(X_{t+h} - X_t \geq 2) = 1 - \lambda h + o(h)$.