Stochastic processes and applications: Seminars #2-3

Maria Kirillova @makirill

HSE — September 19, 2023

Info: Do not be afraid to solve unfinished tasks at home!

1 Markov chain: (un)stationarity

Task 1

The most trusted buyer ships original iPhone from China to Russia with probability 0.75 and from China to UAE with probability 0.25. Sometimes with probability $\frac{2}{3}$ iPhone is defective and goes back to China to be fixed, otherwise it is sold in Russia. The buyer ships iPhone from UAE to Russia with probability 0.5 or sale it in UAE with probability 0.5. The buyer claims he uses Markov chain in his work.

Question 1

(a) Draw the graph and the matrix representations.

(b) Describe classes and states in this Markov chain.

(c) Find the probabilities with which an iPhone can be found in each country in the infinite trade.

Task 2

HSE student has a caring granny who cooked tasty pies with probability 0.7 when she see her grandson. HSE student gains 1 kilo each time he eats pies (of course, he can't refuse). At the beginning student's weight is 70 kilos.

Question 2

(a) Find the expected weight in an infinite granny game.

(b) Explain if there is a stationary distribution in this chain.

Task 3

The fair price of Sborbank in discrete stock market is somewhere between 100 and 101 rubles. If the current price is 100, the price grows by 1 with probability $\frac{9}{10}$, otherwise it goes down by 1. If the price is greater then 100, it grows by 1 w.p. $\frac{1}{3}$ and declines by 1 otherwise. If the price is lower than 100, it behaves controversially.

Question 3

(a) Draw the graph representation.

(b) Do you think this chain has some stationary distribution?

(c) Find the average time for the stock price to fall from 105 rubles to 98 rubles. (Hint: you need to decompose that long way into smaller ones and to use first step analysis).

2 Convergence

Task 4

Consider the sequence of random variables X_n from exponential distribution with parameter n.

Question 4

(a) Write down the distribution function.

(b) Find the probability limit for X_n

Task 5

Consider the random variable X and the sequence of random variables Y_n with $\mathrm{E}(Y_n) = \frac{1}{n}$ and $\mathrm{Var}(Y_n) = \frac{\sigma^2}{n}$.

Question 5

(a) Prove that $X_n \xrightarrow{p} X$

(b) Hint: you may need Triangle inequality and Chebyshev inequality.

Task 6

The random variables X_i are independent and uniformly distributed on [0; 1]. $Y_n = \min X_1, \dots X_n$. For Y_n:

Question 6

(a) find the almost sure limit;

(b) find the probability limit;

(c) find the distribution limit.

Sources:

1. Kelbert M., Sukhov Y., Probability and statistics in examples and problems
2. Cambridge course on Markov chains http://www.statslab.cam.ac.uk
3. Demeshev B., Problems on stochastic analysis https://github.com/bdemeshev