# Stochastic processes and applications: Seminar #4

Maria Kirillova @makirill

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> **ℹ Info:** Do not be afraid to solve unfinished tasks at home!

## Conditional expectations

### Task 1

$X$ is a random variable following Poisson distribution with parameter $\lambda = 1$; random variable $Y$ is a random variable uniformly distributed on [1,2]. Assuming that X and Y are independent, find $\mathrm{E}(XY|X)$, $\mathrm{Cov}(XY, X), \mathrm{Var}(XY|X), \mathrm{Var}(XY)$.

### Task 2

Let $X$ and $Y$ be two jointly continuous random variables with joint probability density function

$$f(x, y) = \begin{cases} x + y & \text{if} \quad x, y \in [0, 1] \\ 0 & \text{otherwise.} \end{cases}$$

Find $f(x), f(y|x), \mathrm{E}(Y|X)$.

### Task 3

Random variables $X$ and $Y$ are independent and exponentially distributed with parameter $n$.

> **Question 1**
>
> (a) Find $\mathrm{P}(X|X + Y), \mathrm{E}(X|X + Y)$.
>
> (b) How the answer will change, if $X$ and $Y$ are following Poisson distribution with parameter $\lambda$?
>
> (c) Find $\mathrm{E}(X|X + Y)$ if $X \sim Exp(n_1), Y \sim Exp(n_2)$.

> **◆ Sources:**
>
> 1. Demeshev B., Problems on stochastic analysis https://github.com/bdemeshev
>
> 2. Brzeźniak Z. and Zastawniak T., Basic stochastic processes