1. The hedgehog Melissa starts at the vertex A of a triangle ΔABC. Each minute she randomly moves to an adjacent vertex with probabilities $\mathbb{P}(A \to B) = 0.6$, $\mathbb{P}(A \to C) = 0.4$, $\mathbb{P}(B \to C) = \mathbb{P}(B \to A) = 0.5$, $\mathbb{P}(C \to B) = \mathbb{P}(C \to A) = 0.5$.

   (a) What is the probability that she will be in vertex C after 3 steps?
   (b) Write down the transition matrix of this Markov chain.
   (c) What is the expected time to get from the state A back to it?

2. The number of players N who will win the lottery is a random variable with probability mass function $\mathbb{P}(N = k + 1) = \exp(-1)/k!$ for $k \geq 0$. Each player will get a random prize $X_i \sim U[0; 1]$. All random variables are independent. Let S be the sum of all the prizes.

   (a) Find $\mathbb{V}\mathrm{ar}(S \mid N)$ and conditional moment generating function $M_{S|N}(u)$ for fixed value of N.
   (b) Find the unconditional moment generating function $M_S(u)$.

   Note: you don't need to calculate the value in (c).

3. Consider the stochastic process (X_n), where X_0 is uniform on $[0; 2]$ and X_n is uniformly selected on $[0; X_{n-1}]$ given X_{n-1}.

   (a) Find $\mathbb{E}(X_n)$ and $\mathbb{V}\mathrm{ar}(X_n)$.
   (b) Find the probability limit $\operatorname{plim} X_n$.

4. Students arrive in the Grusha café according to the Poisson arrival process (X_t) with constant rate λ. Let's measure time in minutes.

   The probability of no visitors during 5 minutes is 0.10.

   (a) Find the value of λ.
   (b) Plot the probability $\mathbb{P}(X_t = X_{10})$ as a function of t.
   (c) Plot the variance $\mathbb{V}\mathrm{ar}(X_t - X_{10})$ as a function of t.

5. The random variables X_1 and X_2 are independent and normally distributed, $X_1 \sim \mathcal{N}(1; 1)$, $X_2 \sim \mathcal{N}(2; 2)$. I choose X_1 with probability 0.3 and X_2 with probability 0.7 without knowing their values.

   Casino pays me the value Y that is equal to the chosen random variable.

   Let the indicator I be equal to 1 if I choose X_1 and 0 otherwise.

   (a) Express Y in terms of X_1, X_2 and I.
   (b) Find $\mathbb{E}(I \mid Y)$ and $\mathbb{V}\mathrm{ar}(I \mid Y)$.
   (c) Find $\mathbb{C}\mathrm{ov}(I, Y)$.

6. The joint distribution of X and Y is given in the table

	$X = -1$	$X = 0$	$X = 1$
$Y = -1$	0.1	0.2	0.3
$Y = 1$	0.2	0.1	0.1

   (a) Explicitely find the σ-algebra $\sigma(\min\{X + Y, 0\})$.
   (b) How many elements are there in $\sigma(X + Y)$?