Name, group no:

. . . .

1. Consider the process
$$y_t = t^2 + u_t + 4u_{t-1} + 2u_{t-2},$$
where (u_t) is a white noise with $\mathbb{V}\text{ar}(u_t) = \sigma^2$.

   (a) [1] Find the expected value $\mathbb{E}(y_t)$.

   (b) [7] Find the autocorrelation function $\rho_k = \mathbb{C}\text{orr}(y_t, y_{t-k})$.

   (c) [2] Is the process (y_t) stationary?

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2. Consider the process

$$y_t = t^2 + u_t + 4u_{t-1} + 2u_{t-2},$$

where u_t are normal independent random variables with $\mathbb{V}\mathrm{ar}(u_t) = 4$.

You know that $u_{10} = 110$ and $u_9 = 90$.

(a) [5] Find the 95% predictive interval for y_{11}.

(b) [5] Find the 95% predictive interval for $y_{1000000}$.

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3. The stationary process (y_t) with expected value 100 has autocorrelation function with $\rho_1 = 0.5$, $\rho_2 = 0.1$ and $\rho_k = 0$ for $k \geq 3$.
   (a) [6] Find the first two values of the partial autocorrelation function, ϕ_{11} and ϕ_{22}.
   (b) [4] Provide a possible equation for this process. Your equation may include y_t, its lags and a white noise process (u_t).

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4. Consider the difference equation $y_t - 0.7y_{t-1} + 0.1y_{t-2} = u_t - 0.5u_{t-1}$, where u_t are independent and normal $\mathcal{N}(0; 1)$.

   (a) [3] How many stationary and non-stationary solutions does the equation have?

   (b) [7] Write a *more simple* difference equation with the same set of stationary solutions.

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5. [10] The semi-annual (y_t) is modelled by $ETS(AAA)$ process:

$$\begin{cases} u_t \sim \mathcal{N}(0; 9) \\ b_t = b_{t-1} + 0.2u_t \\ s_t = s_{t-2} + 0.1u_t \\ \ell_t = \ell_{t-1} + b_{t-1} + 0.3u_t \\ y_t = \ell_{t-1} + b_{t-1} + s_{t-2} + u_t \end{cases}$$

You know that $s_{100} = 3$, $s_{99} = -2$, $\ell_{100} = 100$, $b_{100} = 1$.

Find 95% predictive interval for y_{102}.

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6. [10] The semi-annual (y_t) is modelled by $ETS(AAA)$ process:

$$
\begin{cases}
u_t \sim \mathcal{N}(0; 9) \\
b_t = b_{t-1} + 0.2u_t \\
s_t = s_{t-2} + 0.1u_t \\
\ell_t = \ell_{t-1} + b_{t-1} + 0.3u_t \\
y_t = \ell_{t-1} + b_{t-1} + s_{t-2} + u_t
\end{cases}
$$

You know that $s_{100} = 3$, $s_{99} = -2$, $\ell_{100} = 100$, $b_{100} = 1$.

Decompose $y_{101} = 105$ into trend component, seasonal component and residual.