Higher Spin Gravity and Symmetries

Başar Deniz Sevinç*
Middle East Technical University, Ankara, Turkey
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In this review, the study of Higher Spin Gravity and Symmetries is motivated. First, we construct the higher spin currents from the free scalar CFT. Then, using the celebrated AdS/CFT conjecture, the relation between higher spin gauge fields and the vector field in the $O(n)$ model is inspected.

Keywords: Higher Spin Field, AdS/CFT, Higher Spin Currents

I. INTRODUCTION

The study of Higher Spin Fields are a prominent subject due to the relations with quantum gravity. The founding idea of Higher Spin Gravity is to find the most general gauge theories that can incorporate any spin. The infinite symmetries arising from higher-spin is believed to cure quantum behavior of gravity theories. Thus, making itself a candidate for a potential quantum gravity theory. The search for a quantum gravity has given birth to fields such as supergravity, string theory. Supergravity includes massless particles with spin s=2, graviton, s=3/2, gravitino and s=1 gauge bosons. The particle spectrum of string theory has tower of massive higher spin states for self-consistency. On the other hand, the Higher-Spin Gravity (HiSGRA) theories involve any spin and inherently have graviton in them. Figure 1 displays the spins belonging to different theories.

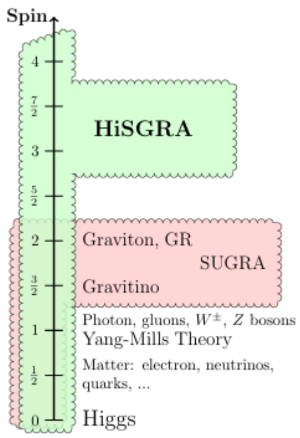

Figure 1. The place of HiSGRA and other theories with respect to spin[1]

On the other hand, since the birth of the AdS/CFT conjecture [2] [3], the role of the higher spin gauge fields in this scheme are being investigated. The conjecture gives clues that particles with spin $s > 2$ should be involved to have an ultimate theory.

In addition, the HiSGRA is believed to be reside in between string theory and Supergravity. We know that the mass squared of string excitations, is proportional to: $[m]^2 \sim \frac{N}{\alpha'}$. Where N is the excitation level and α' is the string tension. If we take the limit where $\alpha' \to 0$ we recover all the massless modes of Supergravity. And the limit $\alpha' \to \infty$, is believed to be the massless excitations of HiSGRA.

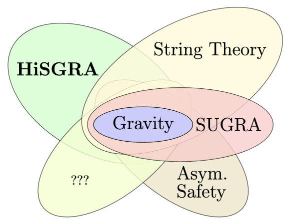

Figure 2. HiSGRA with other theories and gravity [1]

The motivation of the today Higher Spin Gravity is to find the all possible interaction that we can have in a consistent manner. It is still an open question if there exist non-trivial all kinds of couplings among higher spin fields such that when the deformed gauge algebra is non-Abelian. Apart from the symmetries, for example, there cannot be an interaction term of gravitino and photon because it enables faster than light communication. This type of inconsistencies should be eliminated. Thus, the goal is to write down a fully consistent theory of s>2 particles while involving gravity. In order to fully understand the symmetry content, the higher-spin currents should be investigated, which we will give an introduction in this article.

Another good feature about these theories is that it has much larger symmetry group than CFT's, which is believed to potentially cure UV divergences in gravity. Thus, it is a potential theory to describe gravity in the quantum sense. The figure 2 displays exactly this. In this review, we will first give a brief historical introduction. Then, we will construct higher spin-currents from

* Correspondence email address: basar.sevinc@metu.edu.tr

free scalar CFT. After having familiarity with the construction, we will move onto show the correspondence between higher spin gauge field model and the vector model $O(n)$.

II. HISTORY

The formulation of higher spin fields is a highly nontrivial one and has a long history. But it is believed to have been started by Fierz and Pauli in 1939 [4]. They constructed the massive consistent equations of motion for higher spin fields. 35 years later to that, Sing-Hagen Lagrangian was constructed, which is the action for Fierz and Pauli equations of motion. [5] Then in 1978, Fronsdal [6] formulated the Lagrangian for the massless case. Only the spin s-2 and s fields could survive, while all the other fields are decoupled. Furthermore, for s = 2, the Fronsdal Lagrangian becomes the linearized Einstein-Hilbert Lagrangian. In 1990, Vasiliev formulated the exact non-linear equations of motion for the theory, which admits an AdS vacuum solution. [7] [8] [9] [10]However, we will not derive them here because, as also [11] says: "The shortest route to the Vasiliev equations is 40 pages."

III. HIGHER SPINS FROM FREE SCALAR CFT

Here we will calculate higher spin currents and inspect the charges using the Free Scalar CFT. Euclidean signature will be assumed throughout this review. The action for the model is:

$$S = \int d^d x \; (\partial_\mu \phi)^2 \tag{1}$$

We know that this Lagrangian obeys the conformal algebra, given by 4 generators: $M_{\mu\nu}, K_\mu, D, P_u$. They are the angular momentum, special conformal, dilation and translation generators respectively. However this model has a larger symmetry algebra, which is the Higher Spin Algebra. To understand this, one can compare them by counting them term by term respectively. For example, we can construct a Higher-Spin(HS) current by:

$$J_{\mu_1\mu_2\ldots\mu_s} = \sum_{k=0}^{s} c_{sk} \partial_{\{\mu_1\ldots.\mu_k} \phi \partial_{\mu_{k+1}\ldots.\mu_s\}} \phi \tag{2}$$

where the curly brackets are for traceless symmetrization, s are even integers and c_{sk} are coefficients which can be calculated. As an answer to the previous issue on counting, one can think of 1 HS operator corresponding to each of the even spin current operator. In addition, these currents correspond to the irreducible spin s representations of the symmetry group $SO(d)$. The spin should be even because the field ϕ was declared to be a singlet beforehand in this theory.

One can fix the coefficients by the conservation equation:

$$\partial^{\mu_1} J_{\mu_1\mu_2\ldots\mu_s} = 0 \tag{3}$$

As one can infer, there are a lot of indexes in higher-spin formalism and these can be untraceable at some point. So, to eliminate this it is useful to pass to index free notation by introducing an auxiliary polarization vector ϵ^μ. We know represent the current as:

$$J_s(x,\epsilon) = J_{\mu_1\mu_2\ldots\mu_s} \epsilon^{\mu_1} \ldots. \epsilon^{\mu_s}$$

Where s is the spin. It is convenient to choose ϵ to be null. This is to project out the trace terms as we have constructed the current in equation 2. In anywhere of the calculation, one can pass to the index notation by introducing covariant derivatives:

$$J_{\mu_1\mu_2\ldots\mu_s} \propto D_{\mu_1} \ldots D_{\mu_s} J_s(x,\epsilon) \tag{4}$$

where the covariant derivatives are in the ϵ space:

$$D_\mu = \left[(\frac{d}{2} - 1 + \epsilon^\nu \frac{\partial}{\partial \epsilon^\nu}) \frac{\partial}{\partial \epsilon^\mu} - \frac{1}{2} \epsilon_\mu \frac{\partial}{\partial \epsilon^\nu} \frac{\partial}{\partial \epsilon_\nu} \right] \tag{5}$$

Than the conservation equation can be rewritten in the form:

$$\partial^\mu D_\mu J_s(x,\epsilon) = 0 \tag{6}$$

In this language of spin polarization vector, spin current operators can be expressed as:

$$J_s(x,\epsilon) = \sum_{k=0}^{s} c_{sk} (\epsilon\partial)^k \phi (\epsilon\partial)^{s-k} \phi = \phi f_s(\epsilon \overleftarrow{\partial}, \epsilon \overrightarrow{\partial}) \phi \tag{7}$$

The directions on the partial derivatives denote the direction the derivative is acting on. If one inserts this into the conservation equation, one gets the Gegenbauer differential equation for f_s. Thus f_s obeying the Gegenbauer polynomials.

We can now construct the charges of the currents. It is known that the usage of conformal killing vector coupled to the energy momentum tensor which has the conformal dimension $\Delta = d$ a way to derive conserved current and so the charge. In analogous to this, we now need conformal killing tensors of spin $(s-1)$ $\xi^{\mu_1\mu_2\ldots\mu_{s-1}}$ coupling to current. A conformal killing tensor obeys the condition:

$$\partial_{(\mu_1} \xi_{\mu_2\ldots\mu_s)} = \frac{s-1}{d+2s-4} g_{(\mu_1\mu_2} \xi_{\mu_3\ldots\mu_s)} \tag{8}$$

Now we can write the currents associated to these and derive the charges the usual way by integrating the 0^{th} component on the $(d-1)$ dimensional space.

$$J_\mu^{\xi_{s-1}} = J_{\mu_1\mu_2\ldots\mu_s} \xi^{\mu_2\ldots\mu_s} \tag{9}$$

and thus:

$$\partial^\mu J_\mu^{\xi_{s-1}} = 0 \tag{10}$$

One should note that for $s = 2$ these currents constitute to the generators of the conformal algebra which we have mentioned in the beginning.

The action of the charges on the fields are as usual:

$$[Q_s, \phi] \sim \xi^{\mu_1 \dots \mu_{s-1}} \partial_{\mu_1} \dots \partial_{\mu_s} \phi \tag{11}$$

Thus, one gets higher derivative symmetries for higher spins. In addition, by inspection there is one charge for each conformal Killing tensor. As a result, the number of HS spin generators at each spin s is the dimension of the representation at spin s. We will show more on this in the $O(n)$ model. Perhaps the most interesting fact on higher spin is that generally, if one takes the commutator of charges [27] of same spin:

$$[Q_4, Q_4] \sim Q_2 + Q_4 + Q_6 \tag{12}$$

it generates charges of higher spin. In this case, once a spin-4 charge is present in the theory, you inherently have charge of spin 6. Consequently, one needs an infinite tower of charges to get a closed algebra and the algebra is infinite dimensional intrinsically.

The case for spin-4 charge is explicit from the construction, one can ask if there are any other CFT with exact higher spin symmetry. In [12] by Maldacena and Zhiboedov, it has been shown that in a CFT with dimension bigger than 3 at least one HS current J_s with $s > 2$, it is a free CFT in disguise. That is, all correlation functions with the stress tensor and the conserved currents collide with those of a free CFT. It is also shown in this article that single higher spin conserved current implies the existence of an infinite number of higher spin conserved currents as aforementioned in 12.

IV. FREE O(N) VECTOR MODEL AND ADS

Let us consider the most prominent theory for discussing magnets, which is the free $O(n)$ Vector Model. Its action is given by:

$$S = \int d^d x \frac{1}{2} (\partial_\mu \phi^i)^2 \tag{13}$$

Where our field transforms in the fundamental representation. The HS current in this case is similar to the case in free scalar CFT:

$$J_s^{ij}(x, \epsilon) = \sum_{k=0}^{s} c_{sk} (\epsilon\partial)^k \phi^i (\epsilon\partial)^{s-k} \phi^j \tag{14}$$

Let us break down the HS currents into different representations.

$$J_s + J_s^{\{ij\}} + J_s^{[ij]} \tag{15}$$

We can have even spin $O(n)$ singlets, even spin in the symmetric traceless representation and antisymmetric representation in the adjoint representation of $O(n)$ respectively. As an example, conformal energy momentum tensor of this theory is the $O(n)$ singlet, and a single trace operator.

Now let us move on to start using AdS/CFT correspondence. We declare that we are only interested in correlation functions that are $O(n)$ invariant functions. Define the single trace operators. In this theory, a single trace operator has sum over a single index, which is i. Thus, there only exists one 0-spin single trace operator in this theory: $J_0 = \phi^i \phi^i$. Recalling the conformal dimension of this field is: $\Delta_\phi = \frac{d-2}{2}$, than the J_0 has $\Delta_{J_0} = d - 2$. And the multi-trace operators should have more than two ϕ^i sum. An example would be: $\phi^i \phi^i (\phi^j \partial^\mu \phi^j)$, which is a double trace operator.

We can write down all the single trace operators with the dimensions:

$$J_0 + \sum_{s=2,4,6\dots} J_s \tag{16}$$

$$(\Delta, S): \ (d-2, 0) + \sum_{even\ s} (d - 2 + s, s) \tag{17}$$

where Δ is the conformal dimension and S is the spin. This result coincides with the results of the Flato and Fronsdal in 1980 [13].

Now assume that AdS/CFT correspondence holds to its full extent so that we can move on from the boundary, what we established here, to AdS space. The singlet sector in CFT is dual to some gravitational theory in AdS. Thus, single trace operators in CFT, correspond to the single particle states in the Bulk. In other words, conserved current spectrum is dual to the gauge field spectrum in the bulk. For example, an application of AdS/CFT is that conserved spin-2 current J_2 is dual to graviton in the bulk.

Generalizing this to spin-s:

$$J_s : \partial J_s = 0 \leftrightarrow \quad \delta\phi_s \sim \nabla \epsilon_{s-1} \tag{18}$$

where ϕ_s is the massless gauge field. The left hand side is CFT and right hand side is AdS. As an example to the work here done:

$$J_0 = \phi^i \phi^i \leftrightarrow \phi \tag{19}$$

where ϕ is the scalar field, with fixed $[m_\phi]^2 = \Delta_{J_0}(\Delta_{J_0} - d)/l^2$ where l is the AdS radius. Again, the left hand side is CFT and right hand side is the bulk.

The spectrum in equations 18 and 19 matches with the spectrum in Vasiliev's minimal bosonic theory in

AdS_{d+1}. The CFT and AdS spectrum matches. But how about interactions? The interaction terms with the multi-trace operators are a pioneering study of field. For recent studies, one can see Witten's article: [14] He argues here that multi-trace interactions in quantum field theory on the boundary of AdS space can be incorporated into the AdS/CFT correspondence by using a more general boundary condition for the bulk fields than has been considered ever before. The trace he argues in the beginning is of a matrix valued field: $O = \frac{1}{N}Tr[F_\alpha(\Phi)]$, where F_α are considered to be polynomial functions in many applications.

As interactions in our case, in CFT, correlation functions are the primary concern. And in free CFT case, this is done by the J_s currents. In AdS dual of this, one should resort to Witten Diagrams, in which the propogator can go from bulk to boundary, bulk to bulk, or visa versa. One can easily see that since the correlation functions in CFT are non-trivial, interactions in AdS space HS theory should be non-trivial. In other words, in light of the equation 18, there should be consistent massless interactions in the bulk. In fact, Vasiliev's theory provides these consistent massless higher derivative interactions. In 2002, it was conjectured that the singlet sector of critical 3 dimensional $O(n)$ model with the $(\phi^i\phi^i)^2$ interaction is dual, in the large N limit, to the minimal bosonic theory in AdS_4 containing massless higher spin gauge fields with the constraint the spin should be even [15].

V. FUTURE OUTLOOK

Here we want to mention recent progress and future directions with respect to HiSGRA. First, HiSGRA in low dimensions. The HiSGRA theories in 2 dimensions is not explored. For example, we know that Jackiw-Teitelboim gravity is a BF type theory. As in recent work, [16] one can construct a HS extension of Jackiw-Teitelboim gravity as a higher spin gravity in 2-dimensions. In addition, it is believed to have the potential to build a bulk dual of the Sachdev–Ye–Kitaev model.[17]

Secondly, slightly broken higher spin symmetry. This idea is powerful enough to fix all 3-4 point correlation functions. [18]. The slightly broken higher-spin symmetry is a new type of infinite-dimensional symmetry that extends the conformal symmetry at least in three dimensions. In vector models, it can be analogous to Virosoro symmetry.

The higher spin symmetry of the $\mathcal{N} = 4$ SYM can be explored. An example of this is [19]. It does not follow the slightly broken higher spin symmetry since the conservation of $\mathcal{N} = 4$ SYM higher spin currents is not broken by the double trace operators built of the higher spin currents. The spectrum of operators of this theory contains an infinite tower of operators on top of the currents. The currents here can be organized into a multiplet of higher-spin algebra. Moreover, understanding the higher-spin breaking can shed light on the tensionless limit of string theory.[20]

The HiSGRA also touches pure mathematics. Conformal geometry is one of the oldest branches of mathematics. The most profound question in this area is the calculation of conformal invariants and conformal invariant operators. Conformal HiSGRA has its own way to deal with these problems. For example, in [21] they fix the conformal invariant wave operator in d = 4 for s = 3 up to linear order in the Riemann tensor on generic Bach-flat backgrounds.

On the condensed matter side, there is the possibility to apply the (slightly broken or broken) higher spin techniques on fractons, quantum Hall effect, and massive Chern-Simons theories. For fractons, one can see [22]. In addition, it is searched in [23] if the anyons can be described by a quantum field theory that has tensorial and tensor-spinor particles and fields including gravity. The Vasiliev's master equation formalism is the starting point in this article.

Finally, higher spin gravity should have potential applications on the cosmology side. This has become more prominent after the black hole mergers detected by LIGO [24]. In the black hole scattering problem, the major challange is to identify the correct QFT of massive spinning field to calculate the correct scattering amplitude. There has been publications on scattering amplitudes of Kerr black holes emitting a graviton at three points [25]. To calculate more accurate scattering amplitudes higher order scatterings between massive fields of arbitrary spin and two or more gravitons are needed. However, these amplitudes turn out to be having a contact term ambiguity. The higher-spin gravity enters the picture here. The correct action, so the potential should be formulated with involving higher-spin fields. For example, a gauge invariant description of massive spin 3 particle is established in [26] in a Ricci flat gravitational background.

VI. CONCLUSIONS

In this review we have given motivation on the study of Higher-Spin Gravity. The massless version is believed to be residing in a limit of string theory. HiSGRA, incorporating any spin, has graviton inherently and the infinite symmetry due to any spin can give rise to toy Quantum Gravity theories. Next, we have mentioned a number of the key points in the history of HiSGRA. An important remark to be made here is that in the end of the Vasiliev's article[7], he is in await for a full action regarding his totally consistent non-linear, higher-spin equations of motion.

We have derived the higher spin currents from free scalar CFT, which involved conformal killing tensors. The nature of infinite tower of charges was revealed. After that, we moved on with the magnet model which is the $O(n)$ vector model. The higher-spin currents were also established here and then by using AdS/CFT duality it has been shown that the single and multi trace operators on CFT should match the spectrum of the single and multi particle states on the AdS side.

The study of higher spin gravity and symmetries have a bright future considering its potential applications. Some of the mentioned potential applications were on cosmology, condensed matter and pure mathematics. On cosmology it is believed to have applications on black hole scattering, after the detection of gravitational waves. On condensed matter, it can help the research of fractons, which is the prominent are of research in the field. On pure mathematics higher spin gravity theories can calculate conformal invariants and operators. In conclusion higher-spin gravity and symmetries can give us new ways to deal with modern problems in cutting-edge research.

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