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## DESIGN OF A WIRELESS SENSOR NETWORKING TEST-BED

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*Bjorn Deraeve • Roel Storms*

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Written on behalf of Mr. Luc Vandeurzen

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# *Abstract*

*Audio applications are on of the main subfields of digital signal processing. More specific, in audio and music one can use frequency modulation synthesis in order to make creative and totally different sounding tones. A theoretical background and practical approach with an Analog Devices DSP-board is reported here.*

*In the first two chapters it is explained how sound works and what aspects of frequency modulation are actually responsible for making a particular tone sound differently. Next a practical approach to add envelopes and effects to the tone is given.*

*Chapter ?? serves as introduction to ??. In this chapter some theoretical background and electronic designs of CPU architectures are explained. This chapter serves as a backbone for the practical implementations explored in chapter ??. Readers with a basic knowledge on this matter can skip this chapter and immediately start with chapter 6. The first part of this chapter handles the Analog Devices ADSP-21369 DSP board's initialisation routines and the second part focusses on the practical implementation of the signal creation and modulation algorithms.*

*Finally chapter ?? deals with the communication between the DSP-board and the Labview interface for the FM Synthesizer.*

*As a conclusion chapter ?? closes this report with some critical reflections on the implementation and team activities.*

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# Abbreviations

<b>ALU</b>	<b>A</b> rithmetic <b>L</b> ogic <b>U</b> nit
<b>ASIC</b>	<b>A</b> pplication <b>S</b> pecific <b>I</b> ntegrated <b>C</b> ircuit
<b>CPU</b>	<b>C</b> entral <b>P</b> rocessing <b>U</b> nit
<b>DAG</b>	<b>D</b> ata <b>A</b> ddress <b>G</b> enerator

# Chapter 1

## Sound nomenclature

*Written by Bjorn Deraeve*

### 1.1 Introduction

Sound is the human perception of little changes in air pressure. This oscillation of pressure, referred to as a sound wave, is a mechanical wave propagated through a solid, liquid or gas and is composed of frequencies within the range of hearing. For humans this is between about 20 Hz and 20 000 Hz, however the upper limit generally decreases with one's age. The changes in pressure must also be big enough in order to be audible.

The mechanical vibrations interpreted as sound waves are longitudinal waves. Any source makes the matter in the medium periodically displace and thus oscillating. The alternating pressure deviations from the equilibrium pressure are in the same direction as the energy propagation of the wave. The energy carried by the wave is converted between potential energy (extra compression) and kinetic energy (oscillations of the medium).

When a sound wave reaches the eardrum it starts to oscillate at a frequency corresponding to the sound waves frequency. These vibrations are detected by hair cells in the inner ear and are transduced into nerve pulses (action potentials) which transmit information about the sound to the brain.

Sound waves are often described as sinusoidal waves which are characterized by a wavelength and amplitude. The wavelength is directly related to the frequency as  $\lambda = \frac{v}{f}$ . The higher the frequency, the higher the perceived tone will be. In dry air the speed of sound is approximately 343 meters per second.

The sound wave causes a local pressure deviation from the ambient atmospheric pressure. To measure this deviation one uses the sound pressure level (SPL) scale. Since the human ear can detect sounds with a wide range of amplitudes (volumes) it is a logarithmic measure of the sound pressure at the deviation relative to a reference value. It is thus measured in an amount of decibels (dB) above the reference level. SPL is defined as:

$$L_P = 10 \cdot \log_{10}\left(\frac{p^2}{p_{ref}^2}\right) = 20 \cdot \log_{10}\left(\frac{p}{p_{ref}}\right) dB \quad (1.1)$$

where  $p$  is the rms value of the measured sound pressure and  $p_{ref}$  is the reference sound pressure,  $p_{ref} = 20$  microPa (rms), considered as the threshold of human hearing.

## 1.2 Sound characteristics

A sound is defined by several characteristics. The most important are pitch (dutch: 'toonhoogte'), timbre (dutch: 'klankkleur') and are briefly introduced in this section. Other characteristics are volume and the duration of the rising of the sound, of the persistence and of the damping.

### 1.2.1 Pitch and tone

For the human ear a pure tone is a sound with a constant frequency and timbre. This fundamental frequency, the number of perceived oscillations per second, is called the pitch of the sound. It is the frequency of the lowest tone present in the sound and is directly related to the frequency spectrum of the sound. Pitch allows the ordering of sounds on a frequency-based scale and is determined by the vibration frequency of the air. If air vibrations are not regularly there is no fixed frequency, such sound is called noise. On staves the pitch is indicated by the location of the note on the staff. The higher the tone the higher the note is positioned on the staff.



FIGURE 1.1: Position of notes on staves

A tone with a double pitch is said to be one octave higher and is the first upper tone. In addition to the ordinary meaning of tone and pitch, in Western music theory tone is also the abstract term of future tones to sound, represented by the letters A to G and derived symbols ( $A = 440\text{Hz}$ ). In some countries the fundamental tones are not indicated by letters but by the Greek diatonic scale (do-re-mi-fa-sol-la-si-do).

### 1.2.2 Timbre

Timbre is the quality of a sound that distinguishes different musical instruments and voices. Timbre or tone colour is determined by the fundamental frequency (pitch) and the present overtones, also known as upper partials or harmonics, so timbre is composed of many different frequency components. It is determined by the ratio of the strength of the overtone vibrations in the sound in proportion to the fundamental frequency of the sound.

#### 1.2.2.1 Spectral diagram

At this moment it is interesting to have a look at the spectral diagram of a sound. This diagram provides a link between the aural experience and visual representation of a sound. It does this by representing the energy (loudness, height of the line) and the frequency (position of the line) of the different frequency components within the timbre. It is unique to any sound and gives



audible information about that sound. Waveforms are very descriptive for the way air pressure changes with time but not for the actual timbre of a sound.

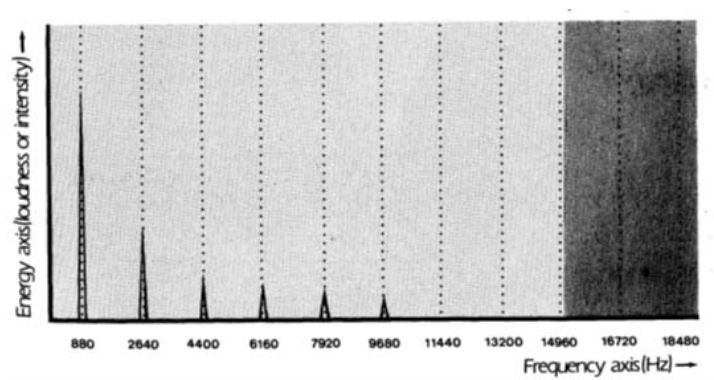


FIGURE 1.2: Spectral diagram of a timbre

How can we interpret figure 1.2? The line on the left is the fundamental frequency or first harmonic. The other frequencies visible in the spectrum, on the right of the first harmonic, indicate the other present frequencies in the sound. Most generally those are called 'partials'. If all partials are an integer multiple of the fundamental frequency they are called 'harmonics' or 'overtones' and the sound exists of the 'natural' harmonic frequencies only.

As we can see from the spectrum in figure 1.2, the partials are in the harmonic series. The note played in this figure was A = 880Hz, this is the first harmonic. The next harmonic is at 3 times 880Hz or 2640Hz. This is the third harmonic, and so on.

From the relative height of the lines (representing the energy or loudness) we can deduce that the harmonics become weaker.

### 1.2.2.2 Joseph Fourier

Today when musicians describe a timbre they talk about overtones, partials or harmonics, but it was the French mathematician and physicist Joseph Fourier who gave a mathematical basis to this idea with his work 'Théorie de la chaleur' which he presented in 1822. In this investigation of the use of Fourier series for problems of heat transfer and vibrations, he brought up the following simplified statement: each complex sound, such as that from a violin or trumpet, can be thought of as the sum of a collection of much simpler tones (sines), at frequencies which are an integer number multiples of the fundamental frequency (pitch). In other words: each complex sound can be resolved into mixtures of sine functions which may differ in amplitude and period but of which all periods are related as an integer multiple. This resolving into different components is called a Fourier transformation or analysis of the signal.

The spectrum as in figure 1.2 shows clearly the present frequencies (= partials, overtones or harmonics) and their energy level.



FIGURE 1.3: Sketch of Fourier

# Chapter 2

## Frequency modulation

*Written by Bjorn Deraeve*

### 2.1 Introduction

Thanks to new digital technology there was room for a new synthesis technique, Frequency Modulation Synthesis. The advantage of FM is that a very large number of different sounds can be produced with few elemental units (oscillators). In other words the timbral space is very large. The one requirement is the use of optimized computers, which is discussed in chapter ??.

#### 2.1.1 Basic concepts of modulation synthesis

It is important to know that in the synthesizer only one wave form, a sine wave, is stored. The trick of modulation synthesis is to remodel this wave until a desired sound is found.

An easy example is the square wave. Using additive synthesis a square wave can be built out of an infinite amount of sine functions. This example of additive synthesis also explains the Fourier transformation theory and is shown in figure 2.1.

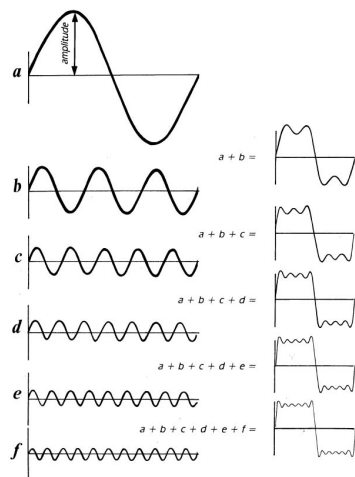


FIGURE 2.1: Additive synthesis of a square wave

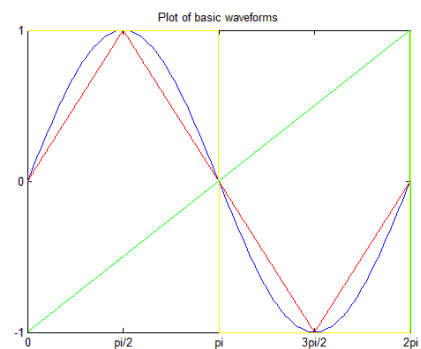


FIGURE 2.2: Basic waveforms

### 2.1.2 Oscillators and operators

The basic building blocks of FM synthesizers are operators. They are equivalent to oscillators in analogue synthesizers and produce the waveforms the user desires. Where an analogue oscillator produces a changing voltage the digital operator creates discrete samples according to a sine pattern. The operators invite the user to be creative in creating their own unique sound with the aid of different kinds of modulation techniques and waveforms. Knowing that most interesting sounds have many components, with the use of only four operators and modulation techniques one can generate sounds with much more than four partials.

### 2.1.3 Waveforms

To produce creative sounds it can be appropriate to combine sine waves with other waveforms. Synthesizers are typically able to produce sines, triangles, sawtooths and square waves. A square wave contains only odd-integer harmonic frequencies. To be an ideal wave, the signal should change from high to low instantaneously. However this is impossible to achieve in real-world systems because of the limited bandwidth, though good approximations can be made with additive synthesis. The Fourier series of the square wave is given in equation 2.1

$$x(t) = \frac{4}{\pi} \sum_{n=1,3,5,\dots}^{\infty} \frac{\sin(2\pi \cdot n \cdot f \cdot t)}{n} \quad (2.1)$$

$$x(t) = \frac{4}{\pi} \left( \sin(\omega t) + \frac{1}{3} \sin(3\omega t) + \frac{1}{5} \sin(5\omega t) + \dots \right) \quad (2.2)$$

Triangle waves contain just like square waves only odd harmonics of the pitch. The difference is that the higher harmonics lose their energy faster. Equation 2.3 gives the Fourier series to approximate a triangle wave using additive synthesis.

$$x(t) = \frac{8}{\pi^2} \sum_{n=1,3,5,\dots}^{\infty} (-1)^{(n-1)/2} \frac{\sin(2\pi \cdot n \cdot f \cdot t)}{n^2} \quad (2.3)$$

$$x(t) = \frac{8}{\pi^2} \left( \sin(\omega t) - \frac{1}{9} \sin(3\omega t) + \frac{1}{25} \sin(5\omega t) - \dots \right) \quad (2.4)$$

Finally sawtooth waves contain both even and odd harmonics of the fundamental frequency. The sound of it is even more raw. The equation for additive synthesis is given by equation 2.5.

$$x(t) = \frac{2}{\pi} \sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sin(2\pi \cdot n \cdot f \cdot t)}{n} \quad (2.5)$$

$$x(t) = \frac{2}{\pi} \left( \sin(\omega t) - \frac{1}{2} \sin(2\omega t) + \frac{1}{3} \sin(3\omega t) - \dots \right) \quad (2.6)$$

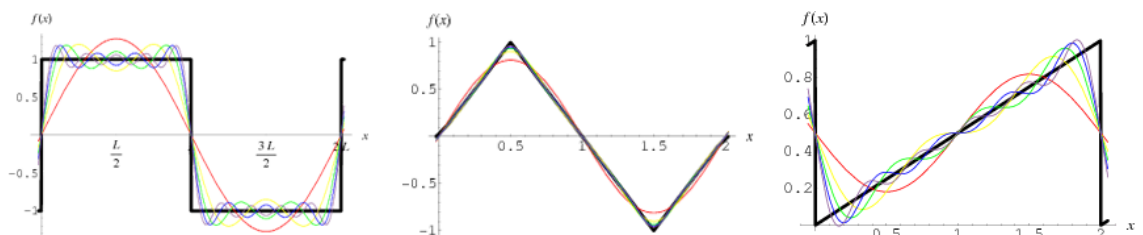


FIGURE 2.3: Square, triangular and sawtooth wave synthesis

Figures 2.4, 2.5 and 2.6 show the corresponding frequency responses of the square, triangle and sawtooth wave and confirm equations 2.1, 2.3 and 2.5.

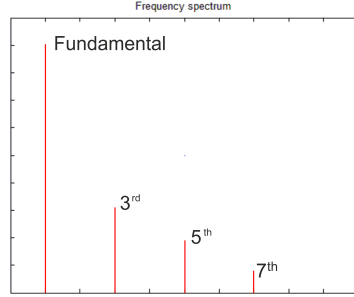


FIGURE 2.4: Frequency spectrum of square wave

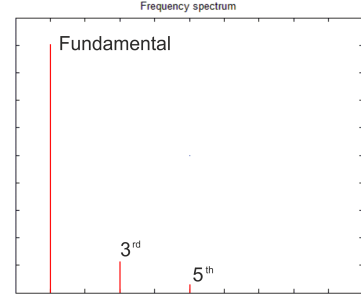


FIGURE 2.5: Frequency spectrum of triangular wave

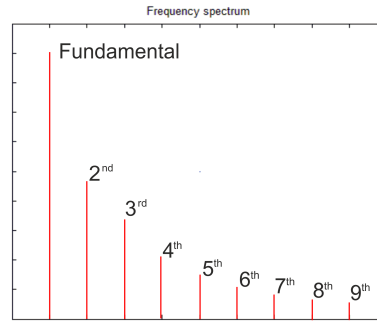


FIGURE 2.6: Frequency spectrum of sawtooth wave

## 2.2 Mathematical approach

The most basic form of frequency modulation is vibrato, in fact, FM is simply very fast vibrato. This possibility to create completely new sounds has been discovered by John Chowning at Stanford University in the mid-'60s. He found that when the frequency of the modulator increases beyond a certain point, the vibrato effect is replaced by a complex new tone.

We can create a vibrato sound by letting operator 2 cause a small change in frequency of operator 1. The rate and amount of frequency change is determined by operator 2. When the waveform of operator 2 increases the tone becomes brighter. This means that more harmonics are appearing in the frequency spectrum. Referring to the introduction section of this chapter, we remark again that FM is able to produce a wide variety of interesting spectra with only two basic oscillators.

Equations 2.7 and 2.8 are two basic sines. The instantaneous amplitude of the waves at any given point in time is called 'A' and are related to their gains 'a' (maximum amplitude of their cycle).

$$A_c(t) = a_c \sin(2\pi f_c t) \quad (2.7)$$

$$A_m(t) = a_m \sin(2\pi f_m t) \quad (2.8)$$

Equations 2.9 and 2.10 define the frequency modulation of the carrier wave  $A_c(t)$  by the modulation wave  $A_m(t)$ .

$$A_c(t) = a_c \sin((2\pi f_c + A_m) t) \quad (2.9)$$

$$A_c(t) = a_c \sin((2\pi f_c + a_m \sin(2\pi f_m t)) t) \quad (2.10)$$

Looking at equation 2.10 we see the modulated signal consists of a constant amplitude  $a_c$  and a time varying argument to the sine function, or phase function  $\phi(t)$ . This phase function is the addition of a constant ramp with slope  $2\pi f_c$  and a sinusoidal variation, see picture 2.7. The

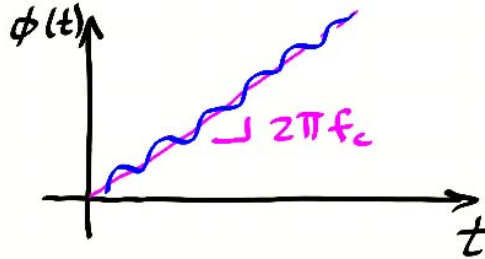


FIGURE 2.7: Phase function

phase function has two parameters that influence the nature of the modulated signal:  $a_m$  and  $f_m$ . The degree to which this variation is added to the constant ramp is given by the modulation index  $a_m$  or also referred to as  $I$ . So the modulation index describes the amount of frequency deviation.  $f_m$  is the modulation frequency and defines the rate at which the frequency deviation is to occur.

Now how does this influence the sound of a frequency modulated signal? First one must see that the side bands produced by FM lie around the carrier frequency, being plus or minus an integer multiple of the *modulator frequency* (the 1<sup>st</sup> parameter of  $\phi(t)$ ). This is expressed by equation 2.11 and visible in figure 2.8.

$$f_{sb} = 2\pi f_c \pm n \cdot 2\pi f_m \quad (2.11)$$

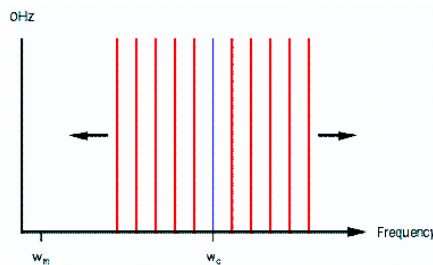


FIGURE 2.8: The position of the side bands

The other parameter of the phase function is the *amplitude of the modulator signal* and has a direct relation to the modulation index  $I$ . The modulation index is defined as the ratio of the change in carrier frequency to the modulator frequency:

$$I = \frac{\Delta\omega_c}{\omega_m} \quad (2.12)$$

Obviously the change in carrier frequency is determined by the amplitude of the modulator signal  $a_m$ . For any given modulator frequency, it is the modulation index (and thus the amplitude of the modulator) that defines the energy of each of the components in the spectrum. Also the height of the carrier frequency can be influenced by the value of  $I$ .

In fact the amplitude of the sidebands is controlled by a Bessel function  $J_k(a)$ . Equation 2.10 can be rewritten as an infinite summation given by equation 2.13, in which  $\theta$  corresponds to  $2\pi f_c$ ,  $a$  corresponds to  $a_m$  (strictly spoken I) and  $\beta$  corresponds to  $2\pi f_m$ :

$$\sin(\theta + a \sin \beta) = J_0(a) \sin \theta + \sum_{k=1}^{\infty} J_k(a) [\sin(\theta + k\beta) + (-1)^k \sin(\theta - k\beta)] \quad (2.13)$$

In this equation  $\sin(\theta)$  corresponds to the component based on the carrier wave and the series of  $\sin(\theta + k\beta)$  are for the side band components. Note that the amplitude of all the components is controlled by the Bessel functions.

First we will show exactly how the frequency of the modulator signal defines the position of the side band components: in equation 2.13 we see " $\sin(\theta)$ ", this leads to the frequency component of the carrier  $f_c$ , see picture 2.9. If  $k$  equals one, the sine functions between brackets lead to frequency components of the modulator,  $f_c + f_m$  and  $f_c - f_m$ . If  $k$  equals two components  $f_c + 2f_m$  and  $f_c + 2f_m$  are formed. And so on. The effect of the negative components will be ignored from now on<sup>1</sup>. Now we will have a look at how the Bessel functions control the amplitude of the

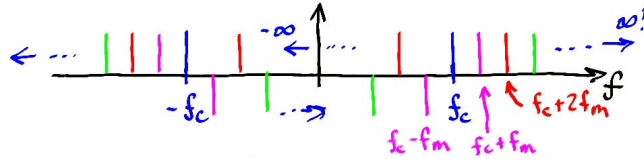


FIGURE 2.9: The position of the side bands

side bands. Equation 2.14 defines a Bessel function with order  $\alpha$ .

$$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + (x^2 - \alpha^2)y = 0 \quad (2.14)$$

As defined in equation 2.13, the order of the Bessel function corresponds to the sideband number. So figure 2.10 shows 6 Bessel functions as a function of modulation index for a specific sideband number.

Note that, if the modulation index is zero (no modulation), the amplitude of the carrier frequency is equal to 1 (black curve). As we look at sidebands that are farther away from the carrier frequency we see that it takes longer for the amplitude of that sideband to become non-zero (red, blue,... curves). So one Bessel curve shows that the amplitude of the  $n^{th}$  sideband depends on the modulation index. Or, for a given modulation index, the amplitude of the  $n^{th}$  sideband is given by a Bessel function of order  $n$ .

When the modulation index of the signals in figure 2.8 is increased the spectrum will look like figure 2.11:

Appendix A contains a simple labview program to display Bessel functions of different orders and more examples that summarize this section.

## 2.3 Bandwidth considerations

The bandwidth of the signal can be defined as the range of frequencies occupied by the signal. Although the sum series of sidebands is theoretically infinite, the modulation index will make sure that sidebands of higher frequency are very small and negligible.

Equation 2.15 gives an approximation of the bandwidth of a FM signal.

$$B = 2\pi f_m(1 + I) \quad (2.15)$$

<sup>1</sup>Since we deal with an infinite summation, the negative components will kind of mix into the positive components. This can be thought of as a kind of aliasing phenomenon.

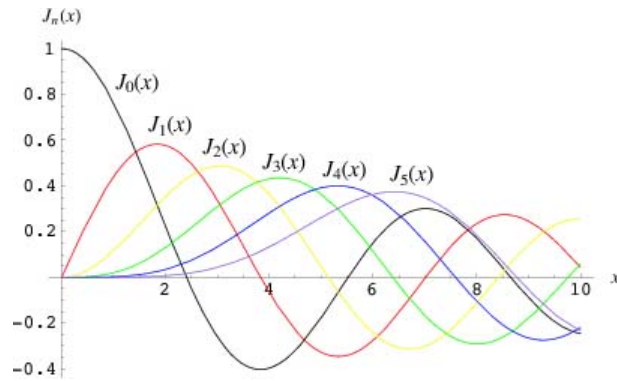


FIGURE 2.10: Bessel functions for orders  $n = 0, 1, 2, \dots$ . The order defines the curve, x-axis defines the modulation index and the y-axis returns the amplitude.

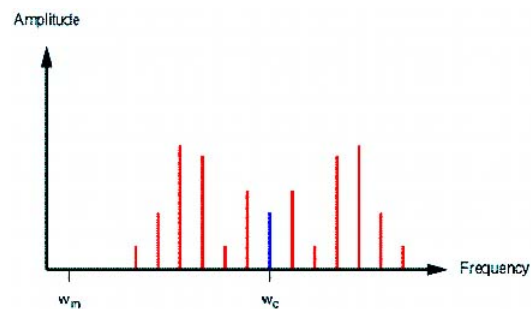


FIGURE 2.11: The amplitude of the sidebands.

For example: the bandwidth of a signal with a 300Hz modulator with  $I = 5$  will be  $2 \times 300\text{Hz} \times (1 + 5) = 3600\text{Hz}$ . This show again that with FM it is easy to create complex signals with a much higher bandwidth.

## 2.4 Practical implementation

Figure 2.12 shows the practical implementation of the FM theory in our synthesizer program. In this figure the first modulation mode is shown where operator 2 modulates operator 1 according to formula 2.10. The user can choose from this list of modulation types:

- 2  $\rightarrow$  1
- 3  $\rightarrow$  2  $\rightarrow$  1
- 4  $\rightarrow$  3  $\rightarrow$  2  $\rightarrow$  1
- 4  $\rightarrow$  3 + 2  $\rightarrow$  1

For more information on the implementation of the FM synthesizer program please read chapters ??, ?? and ??.

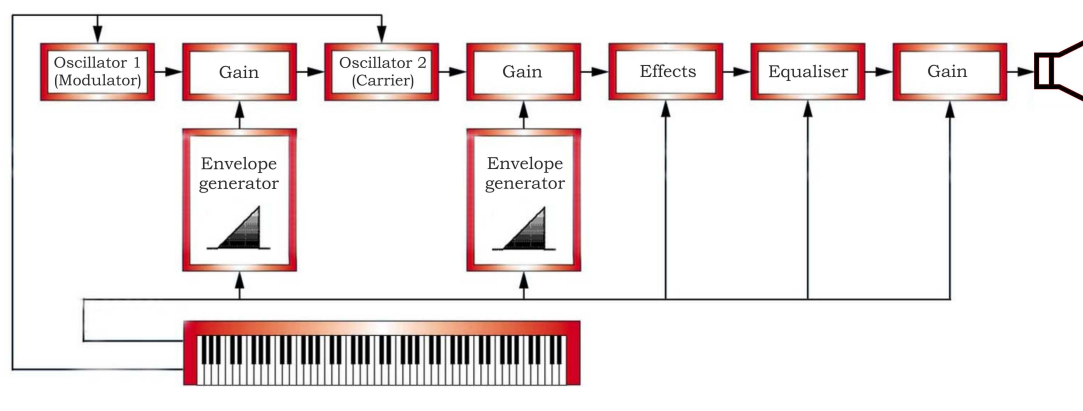


FIGURE 2.12: Functional design of the FM synthesizer program  
(First modulation type from the list above  
(Compare this figure with the program flow chart ?? discussed in chapter 6.)

## 2.5 Summary

We've showed that frequency modulation is a very powerful method for music synthesis. It is possible to generate sounds that are not obtainable by any other modulation technique.

The carrier frequency sets the center of the sideband cluster, the modulator frequency only sets the sideband positions.

It is the modulation index that determines the amplitude of the sidebands and the number of significant sidebands, doing so shaping the spectrum.



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## Appendix A

# Bessel functions and spectrum examples

### A.1 Modulation index $I = 0$

$$J_0(0) = 1 \text{ (carrier component)} \quad J_n(0) = 0 \text{ for } n \geq 1 \text{ (sidebands)}$$

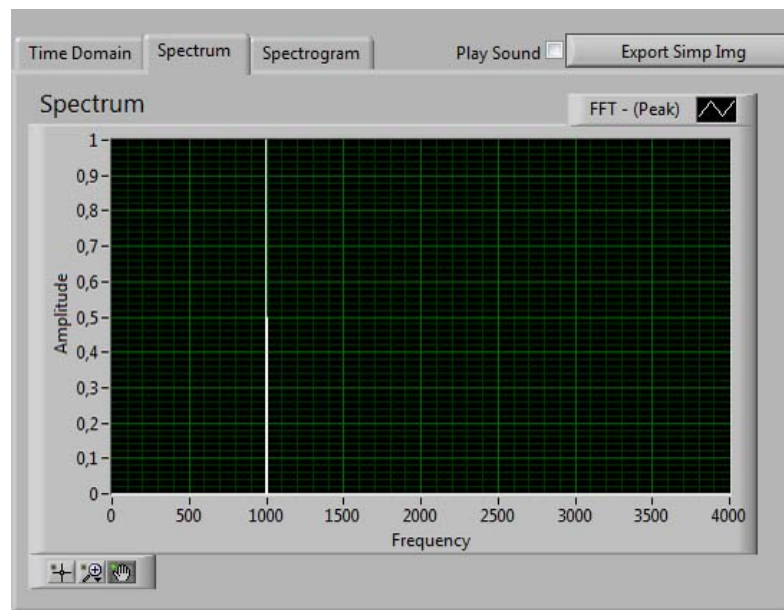


FIGURE A.1: Spectral diagram of a pure 1000Hz signal. Naturally there are no sidebands.

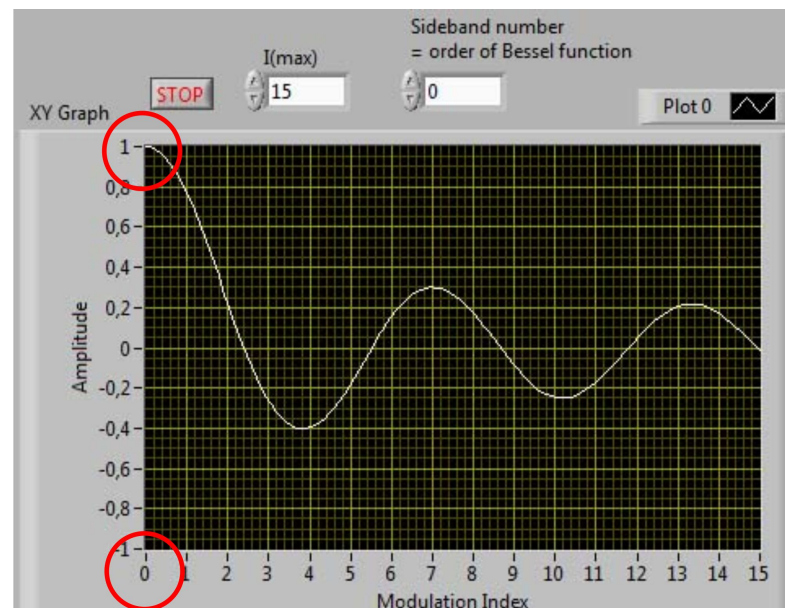


FIGURE A.2: Bessel function of the carrier component (order 0) Remark the amplitude is equal to 1 for  $I = 0$ :  $J_0(0) = 1$

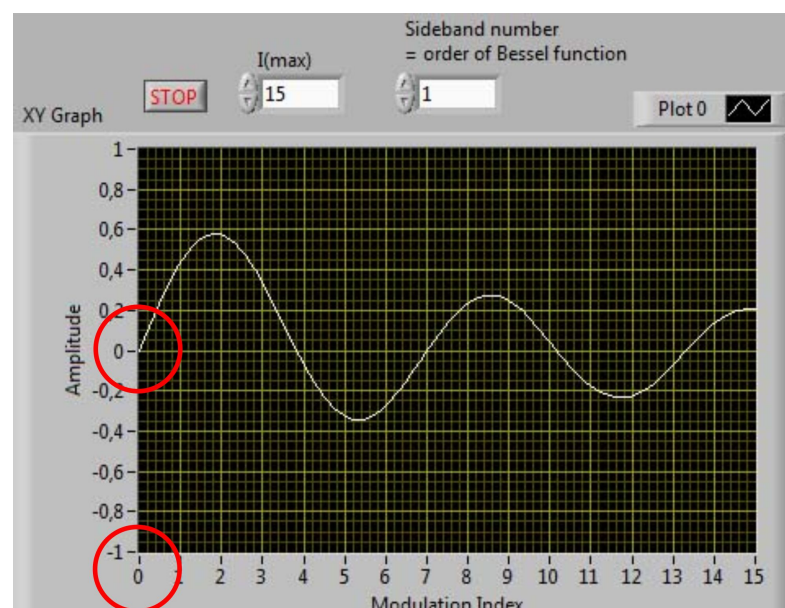


FIGURE A.3: Bessel function of the first sidebands component (order 1) Remark the amplitude is equal to 0 for  $I = 0$ :  $J_1(0) = 0$

## A.2 Modulation index $I = 1$

$J_0(1) < 1$  (carrier component begins to decline)

$J_n(1) \geq 0$  (sidebands begin to increase)

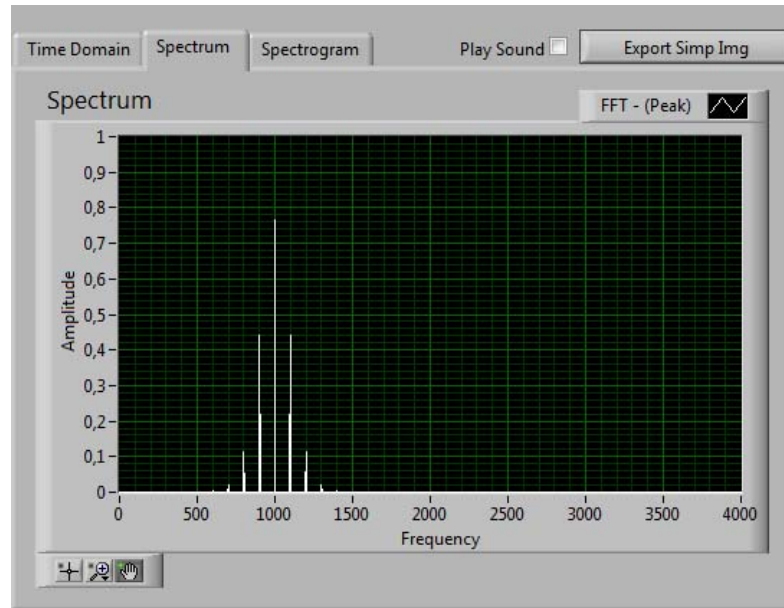


FIGURE A.4: Spectral diagram of a 1000Hz signal modulated with a 100Hz signal and  $I = 1$ .

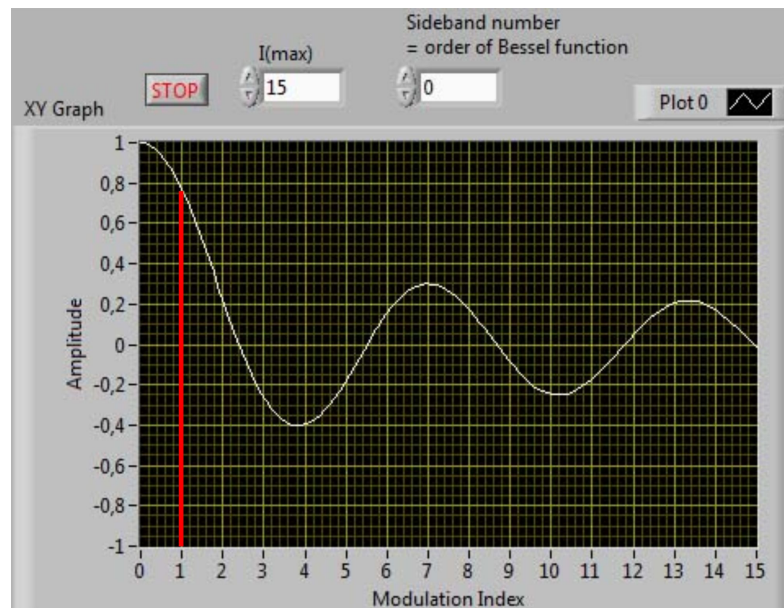


FIGURE A.5: Bessel function for the carrier (order 0) and  $I = 1$  shows that the amplitude of the carrier component decreases.

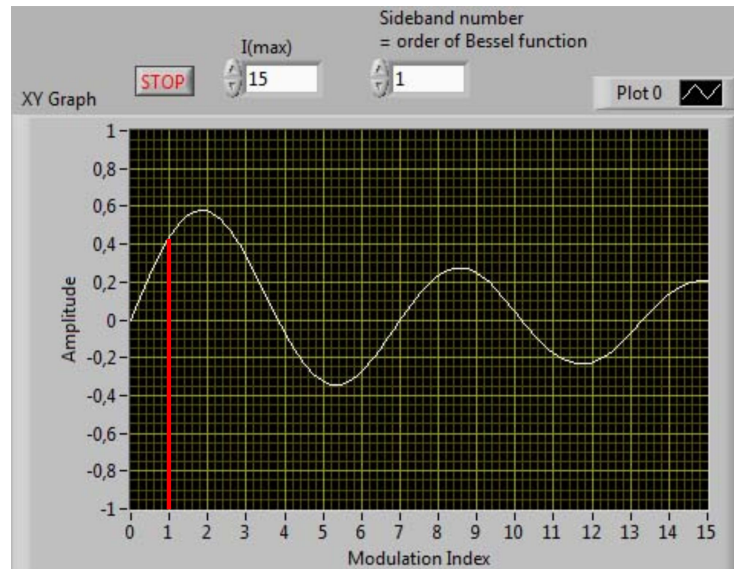


FIGURE A.6: Bessel function of the first sideband component (order 1) returns an amplitude of 0.4 if  $I = 1$ . Compare this value with figure A.4

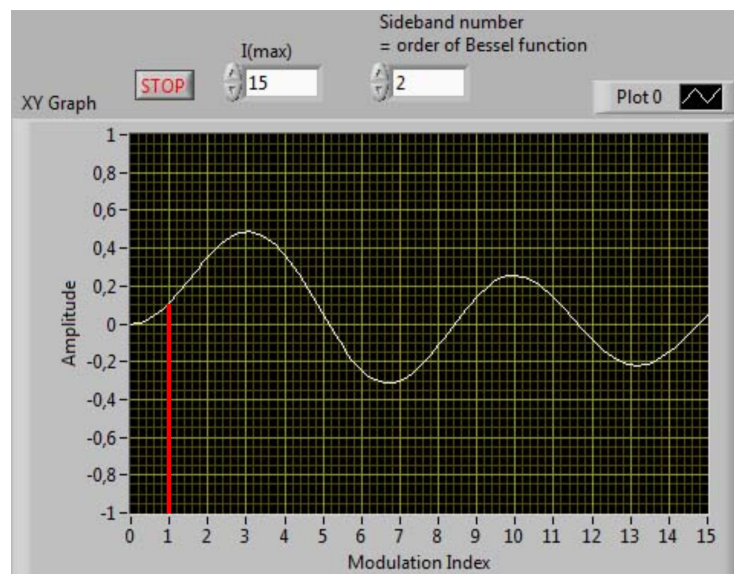


FIGURE A.7: Bessel function of the second sideband component (order 2) returns an amplitude of 0.1 if  $I = 1$ . Compare this value with figure A.4

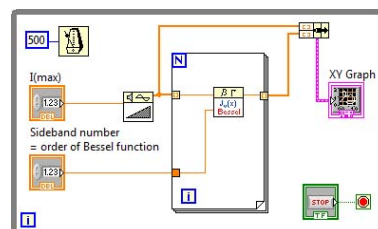


FIGURE A.8: Simple labview program to display Bessel functions of the first kind