

Micro-Level Interpretation of Exponential Random Graph Models with Application to Estuary Networks

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The exponential random graph model (ERGM) is an increasingly popular method for the statistical analysis of networks that can be used to flexibly analyze the processes by which policy actors organize into a network. Often times, interpretation of ERGM results is conducted at the network level, such that effects are related to overall frequencies of network structures (e.g., the number of closed triangles in a network). This limits the utility of the ERGM because there is often interest, particularly in political and policy sciences, in network dynamics at the actor or relationship levels. Micro-level interpretation of the ERGM has been employed in varied applications in sociology and statistics. We present a comprehensive framework for interpretation of the ERGM at all levels of analysis, which casts network formation as block-wise updating of a network. These blocks can represent, for example, each potential link, each dyad, the out- or in-going ties of each actor, or the entire network. We contrast this interpretive framework with the stochastic actor-based model (SABM) of network dynamics. We present the theoretical differences between the ERGM and the SABM and introduce an approach to comparing the models when theory is not sufficiently strong to make the selection a priori. The alternative models we discuss and the interpretation methods we propose are illustrated on previously published data on estuary policy and governance networks.

Introduction

When the interests and ambitions of autonomous organizations overlap in problems of public policy, these groups self-organize into a decentralized and dynamic policy network. This is a complex, interdependent process, whereby coordination and partner selection depends upon the attributes of the relevant actors as well as the evolving structure of the network itself. The study of exchange, coordination, and support among policy actors in such networks is essential to building a comprehensive understanding of modern policy problems because organizations rarely undertake policymaking in a vacuum. Moreover, it is uncommon that all of the relevant policymaking entities are organized within an explicit hierarchy. In the common situation that matters of public policy affect and require the participation of varied and decentralized actors, policy is made by these self-organizing policy networks. Therefore, understanding the policymaking process is, in part, a problem of network analysis. Though still a young and burgeoning field, a number of well-developed

methods for the study of networks have emerged in recent years. The exponential random graph model (ERGM) (Cranmer & Desmarais, 2011; Holland & Leinhardt, 1981; Wasserman & Pattison, 1996) is one such method, which is extremely flexible in specifying models of network generation. Throughout the past two decades, a number of approaches to the interpretation of ERGM results have been proposed and applied. We provide a unified framework that encompasses interpretation of ERGMs at many different levels of concentration (e.g., individual ties, individual actors, groups of actors, and the entire network). We contrast our proposed approach to ERGM interpretation with the interpretation of stochastic actor-based models (SABM) network models (Snijders, van de Bunt, & Steglich, 2010). Our proposed methodology is illustrated through the reanalysis of Berardo and Scholz's (2010) study of estuary networks.

The central characteristic of network data that is both scientifically interesting and statistically challenging is that the probability of a tie forming between any two actors in the network (e.g., a contract between two city governments) often depends upon the structure of the rest of the network (Cranmer & Desmarais, 2011). Additionally, the likelihood of two actors tying can also depend upon actor attributes (i.e., covariates). To provide a concrete policy example of the dual presence of interdependence and covariate theory, consider the dynamics of partner selection within an interorganizational network. Scholars have theorized that disproportionately highly connected actors in interorganizational networks serve as key coordinators within the network (Berardo & Scholz, 2010; Hagen, Killinger, & Streeter, 1997). From this, Berardo and Scholz (2010) deduce that highly connected policy organizations are likely to be sought after because of their coordinating potential. In a parallel line of reasoning, Berardo and Scholz (2010) also hypothesize that actors are likely to seek out trustworthy partners. These hypotheses represent (i) theory about interdependence in the network (the connectedness of an actor begets connections), and (ii) a theory about the dependence of the network on actor attributes (trustworthiness attracts partnerships).

One method that is flexible enough to accommodate a wide range of theories of network generation is the ERGM. The ERGM models a network as a single multivariate observation in which the components of the network depend on covariates as well as endogenous dependencies among the ties. An extension proposed by Robins and Pattison (2001) and developed further by Hanneke, Fu, and Xing (2010), the temporal ERGM (TERGM), extends the ERGM such that it may analyze a discrete time series of network observations. Taken together, the ERGM/TERGM provides a powerful analytical framework that can be applied almost ubiquitously to political and policy networks.¹ To elucidate the great flexibility of the ERGM, we briefly review its mathematical form. Interested readers who would like a more extensive background treatment of the ERGM are referred to the Robins et al. article in this issue. The ERGM takes the form of a probability distribution that gives the probability of observing the entire network of n actors, which we represent as Y , an $n \times n$ matrix with $Y_{ij} = 1$ if there is a tie from i to j and 0 otherwise. The probability of observing any of the possible realizations of the network as given by an ERGM is

$$P(Y) = \frac{\exp\left(\sum_{j=1}^k \theta_j \Gamma_j(Y)\right)}{\sum_{Y^* \in \mathcal{Y}} \exp\left(\sum_{j=1}^k \theta_j \Gamma_j(Y^*)\right)} \quad (1)$$

The Γ_j are network statistics that are specified to measure features of the network that are hypothesized to influence the likelihood of observing a particular realization of the network (e.g., the number of reciprocal pairs of organizations in a policy network).² The θ are parameters, similar to regression coefficients, that give the effects of the respective network statistics on the likelihood of observing a particular realization of the network. The larger θ_j , the higher the likelihood of observing a network with a high value of Γ_j . These statistics can be adapted to capture the interdependence within the network as well as the dependence of the network on other features of the actors or their relationships. Once the parameters are estimated, it is critical to accurately interpret the results. This can present a problem because there is little consensus on how ERGMs should be interpreted. We begin by reviewing the variety of interpretation approaches that have been developed for and applied to ERGMs. We then offer a unified and adaptable approach to ERGM interpretation that encompasses many of the previous as well as unexplored special cases.

Alternative Approaches to Interpreting ERGM

One factor that has inhibited the use of ERGMs by political and policy scientists is the misconception that ERGMs can only provide inferences at the network level. Researchers whose substantive interest lies in actor- and relationship-level effects have shied away from the ERGM in favor of techniques such as the SABM (Andrew, 2010; Berardo & Scholz, 2010), widely perceived as more appropriate for drawing actor-level inferences. The approach to inference and hypothesis testing with the ERGM, as recently applied within the political and policy sciences, can be summarized by a three-step process. First, develop network statistics that represent theorized network effects (e.g., reciprocity, transitivity, and homophily with respect to attributes of actors in the network). Second, estimate an ERGM parameterized with the respective statistics. Third, use hypothesis tests on the corresponding parameter estimates to assess whether the given relational tendencies are significant features of the process that generated the network (see, e.g., Lazer, Rubineau, Chetkovich, Katz, & Neblo, 2010; Cranmer, Desmarais, & Menninga, 2012; Goodreau, Kitts, & Morris, 2009). Though parsimonious and useful for network-level inference, this approach to interpretation only taps the surface of the information available from ERGM estimates.

In contrast to these more recent examples, which focus on network-level inferences, an earlier literature on the ERGM presented the model, often referred to as the p^* model, as a method to model the formation of individual ties in the network, conditioned on the other ties in the network (Wasserman & Pattison, 1996). Specifically,

$$P(Y_{ij} = 1 | Y_{-ij}, \theta) = \text{logit}^{-1} \left(\sum_{h=1}^k \theta_h \delta_h^{(ij)}(Y) \right),$$

where Y_{-ij} indicates the network excluding Y_{ij} , $\delta_h^{(ij)}(Y)$ is equal to the change in Γ_h when Y_{ij} is changed from zero to one, and logit^{-1} is the inverse logistic function such that $\text{logit}^{-1}(x) = 1/(1 + \exp(-x))$. Within this framework, interpretation of the ERGM corresponds to the interpretation of a logistic regression model of tie formation, where some of the covariates represent measurements on structures incorporating other ties in the network. Continuing the example from above, if Y_{ij} is dyad within a directed policy network whereby organization i is considering using organization j as a resource in solving a problem by forming a tie with organization j , and Γ_h is designed to measure the popularity of j as a resource using a statistic called in-two-stars, then $\delta_h^{(ij)}(Y)$ would be the number of other organizations sending ties to j . Thus, in the tie-formation interpretation presented by Wasserman and Pattison (1996), the probability of i forming a tie with j would depend, in part, on the number of other in-going ties accumulated by j .

Pattison and Robins (2002) develop a neighborhood-level approach to the interpretation of ERGMs. As the basis of their method, Pattison and Robins (2002) note that most “global” network statistics designed for the specification of ERGMs are actually counts of link patterns within small neighborhoods of the network. For example, to measure the tendency toward reciprocity in a network, researchers commonly employ the count of mutual dyads in the network: the number of dyads ij in which there is a link from i to j and from j to i . This can be seen as an aggregation over social processes within neighborhoods defined as dyads, rather than as a global network statistic. Pattison and Robins (2002) focus primarily on neighborhood-level processes, and then examine how the interaction among and aggregation of neighborhoods gives rise to global network structure.

Again in keeping with our example of disproportionately central organizations, we first theorize about the motives of the senders to highly central organizations—namely, that the highly connected organizations serve a critical coordination role. Our immediate focus is on the local neighborhood of the highly central organization and those to which it is directly tied. Once we focus on the aggregation of this process over a complete network of potential senders and central organizations, we arrive at the expectation of a network characterized by the presence of a few disproportionately highly connected organizations. An excellent example of neighborhood-oriented interpretation can be found in the recent policy networks literature: Shrestha (2010) considers the process of contracting between local governments in a Florida county. One hypothesis is that, reciprocal service contracts, where each city in a dyad contracts the same service from the other city, can serve the purpose of creating geographic efficiencies in service provision. From this neighborhood (i.e., dyad) expectation, Shrestha (2010) develops a network-level hypothesis of a positive reciprocity effect in an ERGM applied to the intercity contractual network under study.

We aim to unify the literature on ERGM interpretation by developing and explicating a framework for interpreting the ERGM that applies at many different levels, including network, actor, dyad, and individual tie levels, a framework that, we hope, will increase the applied utility of the ERGM to political and policy scientists. We begin by manipulating equation (1) to derive the conditional distribution of any block in the network given the state of the rest of the network. For instance, we can derive the probability of a set of ties sent to an actor given the rest of the network. We show how, when combined with insight from the theory of Gibbs sampling, the form of the ERGM implies a block-wise dynamic updating process, which can be interpreted, for example, at the actor or dyad (link) level. We contrast this dynamic interpretation of the ERGM with the assumptions underlying the SABM. Through a replication analysis of Berardo and Scholz's (2010) study of estuary networks, we illustrate these interpretation methods as well as a comparison of the ERGM and TERGM to the SABM for the purposes of fitting dynamic network models.

Dynamic Network Formation in the ERGM

In the social sciences, longitudinal network data are typically gathered at several discrete points in time. In most such datasets, the time intervals at which the data are gathered, even if regular, are "coarse," meaning that a substantively significant amount of time has passed between waves of data collection. In terms of measurement, the data appear as several cross-sectional recordings of a given network, tacitly implying that all ties within a given period are measured or change simultaneously. However, relational data are typically generated dynamically; relationships change sequentially in continuous time, and changes are not typically clustered around the times of data collection. Moreover, it rarely makes theoretical sense to conceptualize ties as being dependent upon each other *and* generated simultaneously because dependence arises when individual components of the network are generated conditional upon the rest of the network (i.e., in sequence). In this section, we describe sequential micro-level network generating processes that are consistent with a given ERGM.

We draw upon the theory of Gibbs sampling (Geman & Geman, 1984) to describe a useful class of sequential network generating processes that are consistent with any given ERGM.³ Let the adjacency matrix representing the network be partitioned into m disjoint groups or blocks. These blocks could be (i) each potential tie in the network, (ii) each dyad in the network, (iii) blocks of outgoing ties from each node in the network, (iv) blocks of incoming ties to each node in the network, (v) sub-networks of groups of actors (i.e., clusters), or (vi) any other conceivable partition of the network. Within the context of policy networks, it could be assumed that organizations in a coordination network sequentially update their outgoing ties to reflect the state of coordination hubs within the network.

Because we believe the actual data generating process to be sequential, we assume there is some probabilistic process that determines an updating sequence for the m blocks of the partition ($S(m)$). For instance, if there are three blocks in the

partition, $S(m)$ probabilistically generates sequences from $\{(1, 2, 3), (1, 3, 2), (2, 1, 3), (2, 3, 1), (3, 1, 2), (3, 2, 1)\}$.⁴ Considering the example of city-to-city service provision (e.g., Shrestha, 2010), it would be plausible to assume that cities evaluate their needs for services in accordance with the scheduling of their budget/fiscal processes, and update their incoming ties (i.e., purchases from other cities) in a sequence that corresponds to the alignment of cities' budgeting schedules. Block Gibbs sampling is a probabilistic process that constructs a full joint distribution (i.e., a full network) by the sequential sampling of subsets (i.e., blocks), conditioned on the rest of the variables (Gill, 2007). Because the conditional distributions of the blocks are properly deduced from the ERGM and the description of $S(m)$ is consistent with a stationary sequence, the interpretation approach we describe is fully consistent with the process of block Gibbs sampling. It follows then, given that the network has gone through a large number of updating cycles according to sequences drawn from $S(m)$ and ERGM-based conditional distributions, that Y will have an ERGM distribution. In other words, an ERGM appropriately describes a network that has been sequentially updated by probabilistically generating one block conditional upon the other blocks, whereby the "conditioning" is based on conditional distributions derived from the ERGM.

In order to implement interpretation based on sequential generation, we need to compute the conditional probability of one block of the network given the rest of the network. This computational process can be stated generally. Let Y_b be the b^{th} block of the network, \mathcal{Y}_b be the set of possible realizations of Y_b (i.e., the set of every possible link configuration of a network with the same number of nodes), and Y_{-b} be the rest of the network not represented by the b^{th} block. The probability of any particular block (Y_b) given the rest of the network (Y_{-b}) is

$$P(Y_b | Y_{-b}) = \frac{\exp\left(\sum_{j=1}^k \theta_j \Gamma_j(Y_b \cup Y_{-b})\right)}{\sum_{Y_b^* \in \mathcal{Y}_b} \exp\left(\sum_{j=1}^k \theta_j \Gamma_j(Y_b^* \cup Y_{-b})\right)}, \quad (2)$$

where the notation $Y_b^* \cup Y_{-b}$ stands for the complete network created by holding Y_{-b} constant and inserting Y_b^* into the b^{th} block of Y . The conditional probability of a block can be used to probe exactly how ties in the network depend upon each other, according to the results from a given ERGM. For example, as we illustrate below, it can be used to compute the probability that a dyad forms an asymmetric relationship. Also, to give another example, to untangle actor-level tendencies, this approach can be used to compute the likelihood that an actor sends or receives additional ties given that it is already sending or receiving a specified number of ties. The application of our proposed interpretation method is made easy by our companion software described in the Software Appendix. Note that the same interpretation procedures could be trivially applied to TERGM results. The TERGM (i.e., the general form described by Robins & Pattison [2001] and Desmarais and Cranmer [2012], not the conditional independence form proposed by Hanneke et al. [2010]), implies an ERGM for each time point, so the methods of interpretation we develop are applicable to a selected time point in a time series of networks modeled with a TERGM.

Dynamically Interpreted ERGMs and SABMs

The SABM (Snijders et al., 2010) is a well-developed method capable of modeling dynamic network formation that is often viewed as an alternative to the ERGM and has seen widespread use because of its actor-oriented approach. The SABM offers a dynamic approach to the study of networks with a focus and natural interpretation at the node level.⁵ In this section, we compare the dynamically interpreted ERGM with the SABM. Note that we do not see these two models as in opposition, and it is certainly not possible to identify one of the two models as being most appropriate for all dynamic network analysis problems. The SABM and ERGM are similar models. Indeed, if the dynamic process assumed in the SABM is iterated for a long period of time, the network distribution in the SABM will converge to an ERGM (Snijders, 2001). This emphasizes the fact that one interpretation of the ERGM is, at least in the long run, consistent with the network dynamics explicitly assumed in the SABM. As we illustrate below, in a given application, it is possible to compare them empirically rather than assume one *a priori*.

The SABM assumes a specific process by which the network is dynamically updated.⁶ The SABM begins by assuming that ties in the network change one at a time. An actor is selected according to an estimated rate of change function and the selected actor can change, at most, one outgoing tie. Formally, let $Y^{(ij)}$ represent the network in which the ij^{th} element of the adjacency matrix is toggled (i.e., a tie is formed where none existed, or an existing tie is dissolved). In the SABM, the probability that actor i , when the opportunity arises, changes its relationship with j is proportional to $\exp(\sum_{h=1}^k \theta_h \Gamma_{ih}(Y^{(ij)}))$, where $\sum_{h=1}^k \theta_h \Gamma_{ih}(Y^{(ij)})$ is referred to as the “objective function” (Snijders et al., 2010) and includes parameters θ and network statistics $\Gamma()$, much like the ERGM. In other words, the objective function in a SABM is a linear combination of network statistics that change with actor i ’s outgoing ties, weighted by real-valued parameters. There is an additional equation in the SABM that determines the rate at which actors consider changing one of their outgoing ties between observations of the network. These rate functions can be actor specific or apply to the network as a whole.⁷

The dynamic process by which ties are formed and dissolved in the SABM is more specific and explicit than the dynamic interpretation of ERGMs described above. There are two important potential benefits to the network formation processes in the SABM. First, if the process assumed in the SABM is very close to what the researcher theorizes is taking place, then the model more closely represents theory. Second, SABMs can be more robust in terms of parameterizing stable network distributions and estimating the parameters of the model. In the between-time-point simulation process, the SABM simulates tie changes that depend upon existing, previously simulated, ties. At no point does the SABM simultaneously generate interdependent ties. The ERGM, on the other hand, treats tie observations simultaneously, which can result in the problem of degeneracy, whereby the distribution of networks is concentrated in a few, empirically unrealistic networks (Snijders, 2011). One benefit of assuming a specific, sequential network generation

process in the SABM is that it avoids the degeneracy problem encountered with the ERGM (Snijders, 2011). However, these two strengths do not render the SABM more appropriate for all applications. First, theory may not be strong enough to establish a clear preference for the process assumed in the SABM. Second, it is always possible to check whether an ERGM specification is degenerate; it is often possible, as we illustrate below, to specify ERGMs that are not degenerate. There has even been work to develop approaches to specifying ERGMs that avoid degeneracy (Snijders, Pattison, Robins, & Handcock, 2006).

The SABM is designed to model a panel of network observations, such that the explicit stochastic process modeled is the change in the network from one period in time to the next. The ERGM, by specifying Γ to incorporate measurements on past networks (e.g., a tie-wise covariate that indicates whether Y_{ij} existed in the previous period), can also condition the distribution of networks at any given time on past realizations of the network (Desmarais & Cranmer, 2012; Hanneke et al., 2010; Robins & Pattison, 2001). There is a subtle difference in the distributions under study in the SABM and the ERGM conditioned on previous iterations of the network. In the temporally conditioned ERGM, what Hanneke et al. (2010) term the TERGM, the ERG distribution of the networks at times $k + 1, k + 2, \dots$ are assumed to be in their stationary distribution conditional on the first k networks in the time series. Because these first k networks are assumed fixed and are not modeled as stochastic outcomes, there is no implicit or explicit assumption about whether they are or are not drawn from the stationary distribution of the series of networks. The SABM focuses on change in the network from one period to the next. Conditioning on the initial period in the panel, the stochastic process that is modeled in the SABM is the change in the network. Thus, the SABM does not explicitly assume that the networks on hand were sampled from the stationary distribution of networks.

To compare the difference to a methodology that is perhaps more familiar to social scientists, we draw an analogy regarding the difference between the ERGM and SABM to autoregressive time series analysis with and without a unit root (Williams, 1992). Consider a classic autoregressive form: $y_t = \rho y_{t-1} + \varepsilon_t$. When $0 < \rho < 1$, y is a stationary time series with positive autocorrelation. However, if $\rho = 1$, y is said to have a unit root, and the difference $y_t - y_{t-1}$ is stationary, but y_t is not. It is likely that neither the ERGM nor the SABM posit a more or less plausible assumption about stationarity *a priori*, though it is important to scrutinize this difference. Berardo & Scholz (2010) use the SABM to model change in interorganizational networks involved in estuary policy, which they describe as a “newly emerging arena” (p. 639). They cite this characteristic as a reason to prefer the SABM. Though we would not go as far as to say that the structure of stationarity can be determined strictly through theory, it is important to be mindful of features of the data that might render one process more likely than another, and thus serve as a reason to rigorously empirically scrutinize whether the SABM or (T)ERGM more accurately recovers the network conditioned on past realizations.

One other major difference between the SABM and the ERGM is that the SABM includes a feature that models actor attributes termed “behavior.” The behavior equation is specified as a regression model that includes both other covariates (i.e.,

node attributes) and functions of the network. As a starting point for the behavior model, consider a standard regression model for behavior at time t , which includes other node attributes as well as measurements on the network at time $t - 1$ (e.g., actor centrality, the number of hops from other actors with similar behavior). Instead of simply including measurements on the network at $t - 1$, while new intermediate networks are simulated in the SABM estimation procedure, measures of the simulated networks are used in the behavior model. Similarly, because a stochastic model of behavior is specified, if the behavior being modeled is incorporated in the network generation model, intermediate node behavior is simulated and incorporated in the intermediate network generation steps (Steglich, Snijders, & Pearson, 2010). Thus, the SABM estimates the network and behavior equation via an innovative simulation of the coevolution of the network and actor attributes (Snijders et al., 2010). This is certainly a powerful feature of the SABM. However, a viable alternative is to use a conventional approach to modeling behavior based on realizations of past networks and networks based on realizations of past behavior. There is no reason to believe *a priori* that a model of behavior based on simulated past networks and a network model that includes simulated previous behavior will necessarily outperform the computationally simpler approach. This ultimately depends upon the accuracy of the intermediate simulations and should be empirically scrutinized.

For the sake of avoiding model-dependent conclusions (Ho, Imai, & King, 2007), it is always good practice to consider multiple alternative models for the same process. Nonetheless, we identify the following four points as the most salient theoretical considerations in comparing the SABM and the (T)ERGM:

- *Does the network change by one actor changing one tie at a time?* The SABM makes a very specific assumption about the process of network evolution. In contrast, there are many component-by-component updating processes that are consistent with the ERGM. If the conception of the network evolution process in the SABM is similar to that theorized for the network under study, the SABM may be preferred.
- *Do the network and behavior co-evolve nearly simultaneously?* Because we do not observe the intermediate changes in the network and node attributes between panel network observations, it is not possible to condition the network at t on very recent behavior or condition behavior at t on very recent iterations of the network. The SABM offers the option to co-condition on simulated networks and behavior. If the *particular* process of coevolution represented by the SABM behavior component is theoretically appealing, the SABM may be preferred.
- *Is there a strong reason to suspect non-stationarity in the network series?* If there is strong trending in relevant network statistics or a substantial lack of intertemporal consistency in the network structure, it would be reasonable to suspect nonstationarity, in which case the SABM may be preferred.⁸
- *Are there prohibitive degeneracy problems with ERGM?* With the innovations developed by Snijders et al. (2006), and careful consideration of multiple approaches to incorporating structural effects of interest, the degeneracy problem in ERGMs

can usually be overcome. However, if it is not possible to identify a theoretically satisfying ERGM that does not exhibit degeneracy, the SABM may be preferred.

These points are primarily intended to provide a succinct focus on the major differences between the SABM and the ERGM. If, for some reason, it is infeasible to empirically compare the performance of the two models, or to report both of them in the interest of robustness, these points can be used to select between the two *a priori*. However, because theory will rarely be strong enough to definitively inform model choice, we strongly suggest empirical comparison as an approach to evaluating the merits of the SABM and ERGM in a given application. We illustrate the comparison process in the application below.

Empirical Application: Reanalysis of Berardo and Scholz (2010)

To illustrate the interpretation approach that we advocate and highlight the differences between the dynamically interpreted ERGM/TERGM and SABM, we reanalyze data gathered by Schneider, Scholz, Lubell, Mindruta, and Edwardsen (2003) and analyzed using the SABM by Berardo and Scholz (2010). The data measure relationships between relevant policy actors, including a variety of governmental and nongovernmental organizations, involved in water usage and environmental policies relevant to 10 estuaries. There are no connections across estuaries, and there are a total of 194 organizations involved in all of the networks. The data were gathered by surveying experts in each agency involved and asking them,

Please think about three people on whom you have relied most heavily in dealing with estuary issues during the past year. Consider the full range of stakeholders, including government agencies, interest groups, and local officials. Please write the name of the organization your contact works with in the space provided.

Data were gathered in two waves: 1999 and 2001. The network is constructed by drawing a directed tie from the agency of the respondent to the agency of the reported contact. The networks for the 10 estuaries were represented in a single adjacency matrix using structural zeros to capture the fact that there are no cross-estuary ties. The result is 10 adjacency matrices to subject to statistical network analysis and, importantly, assumed to be generated according to the same model. For further details on data gathering and network coding, see Schneider et al. (2003) and Berardo and Scholz (2010).

Before proceeding with our empirical analysis, we briefly consider three reasons that Berardo and Scholz (2010) cite for using the SABM rather than the ERGM. We agree that all of these reasons justify a suspicion that the SABM will be more appropriate. However, none of them are conclusive. First, Berardo and Scholz (2010, p. 638) claimed that the ERGM is unable to account for the history of the network, whereas the SABM can control for previous realizations of the network. The temporal extensions developed by Robins and Pattison (2001), Hanneke et al. (2010), and Desmarais and Cranmer (2012) can easily account for previous realizations of the

network. In a TERGM, the network statistics (Γ) may include functions of earlier networks (see Cranmer & Desmarais, 2011; Cranmer et al., 2012; and Desmarais and Cranmer, 2011a for applied examples). Second, Berardo and Scholz (2010, p. 639) argue that the ERGM assumes that each relationship maximizes the utility of the relevant actor given the rest of the network. The implication is that the ERGM may carry a, sometimes unwanted, assumption that the ties in the network represent a static equilibrium given the actors' utility functions. However, both the ERGM and SABM are probabilistic models that assume networks are generated in proportion to the frequency of a set of network statistics selected by the researcher. As noted above, the ERGM assumes the network under study is drawn from the stationary distribution. In contrast, the SABM assumes the network *changes* according to a stationary process. Neither assumption is more or less warranted *a priori*. Finally, Berardo and Scholz (2010, p. 640) point out that the ERGM may seem to be limited compared with SABM, because the SABM model includes an optional "behavior" component that can model the coevolution of a network and an actor attribute. As we note above, this is definitely an innovative feature of SABM, but there is no clear reason to assume that the near-simultaneous feedback represented in the SABM is closer to reality than the more delayed conditioning that can be represented by modeling based on past observations.

In light of the fact that the *a priori* selection of the SABM over the ERGM can be an ambiguous choice, we reconsider the analysis of Berardo and Scholz (2010) using both methods and contrast their advantages. We do not stop at comparing the SABM and the ERGM. We also include a specification of the network dynamics that renders estimation and interpretation simpler than in SABM or ERGM, yet is still consistent with a network that evolves such that ties form dependent on ties previously formed in the network. Hanneke et al. (2010) proposed a special case of the TERGM in which the probability of a tie forming at time t depends only upon functions of the network observed at previous time points. Another way to state the foundational assumption underlying this special case of the TERGM is that ties formed at time t are assumed to form independently conditional upon features of the network measured at previous points in time. This differs from an ERGM that also includes statistics from past networks, including the dynamical ERGM proposed by Robins and Pattison (2001), and the more general TERGM initially presented by Hanneke et al. (2010). Consider again Shrestha's (2010) hypothesis that there is reciprocity in city-to-city contracting networks. In this special case of the TERGM proposed by Hanneke et al. (2010), this would be operationalized by testing whether, in the current time point, cities are more likely to provide services to those cities from which they purchased services in the previous time point than those cities from which they did not purchase services in the previous time point. Whereas the ERGM simultaneously models the buildup of dependent ties over an unobserved dynamic process, and the SABM models dependent ties over a simulated, and also unobserved, dynamic process, in this special case of the TERGM, it is assumed that all of the ties on which the current ties were conditioned were observed prior to observing the current iteration of the network.

The independence assumption in the TERGM of Hanneke et al. (2010) is, of course, dubious. The benefits of using the TERGM in this form are (i) that it avoids

degeneracy entirely and (ii) the model is optimally estimated using logistic regression. We discussed above how the sequential conditioning of *simulated* new ties on *simulated* past ties in the SABM makes estimation more stable than modeling simultaneous dependence in the ERGM. The TERGM represents the extreme of this phenomena—further simplifying the dependence structures by eliminating the intermediate simulations present in the SABM. Though the contemporaneous independence assumption is certainly questionable, there is always a trade-off between adding complexity to a model and maintaining a specification for which computation and interpretation are tractable. Also, note that we are not comparing the TERGM to all possible models that relax the assumption that contemporaneous ties are formed independently. Rather, we are specifically comparing the TERGM to specific forms of the ERGM and the SABM in the current application. Though the general assumption that ties are formed contemporaneously may not be accurate, it is certainly plausible that the simple intertemporal dependence assumed in the TERGM is closer to the true data generating process than the selected contemporaneous dependencies embedded in the ERGM and/or the specific dynamical process assumed and simulated in the SABM.

The Homogeneity of Estuary Network Dynamics

The substantive research objective of the empirical application is to analyze and identify features of coordination in local estuary networks that are common and generalizable across localities, and therefore generalizable across networks. This serves as the primary justification for pooling the 10 networks into a sample and estimating a single set of parameters that covers all of the networks. Additionally, to the degree that the policy networks were generated through a common process, pooling the networks provides greater statistical leverage in precisely estimating the network dynamics. However, we cannot be certain that a homogenous process underlies the generation of all of the policy networks in our sample. Furthermore, we cannot be certain that we have identified structural components sufficient to explain empirical anomalies in coordination. To be confident that we do not include inexplicably aberrant estuary networks in our sample, we conduct an extensive outlier detection exercise.

We use two approaches to identify outlying networks. First, using the force-directed placement algorithm proposed by Fruchterman and Reingold (1991), which places connected nodes closer to each other and attempts to avoid overlapping ties, we visualize each of the 10 policy networks. We depict each of the networks as measured in 2001, because each of the models we use conditions on the network in 1999, treating it as fixed rather than stochastic. The networks are given in Figure 1. The four networks in the upper left of the figure, also indicated by bold-font titles, appear to be structurally distinct from the other six. They all exhibit fewer connections and a higher proportion of nodes that are far from the central interconnected component of the network. Though this is a useful first step, it is incomplete, because it is still possible that the visual outliers are generated by the same stochastic process

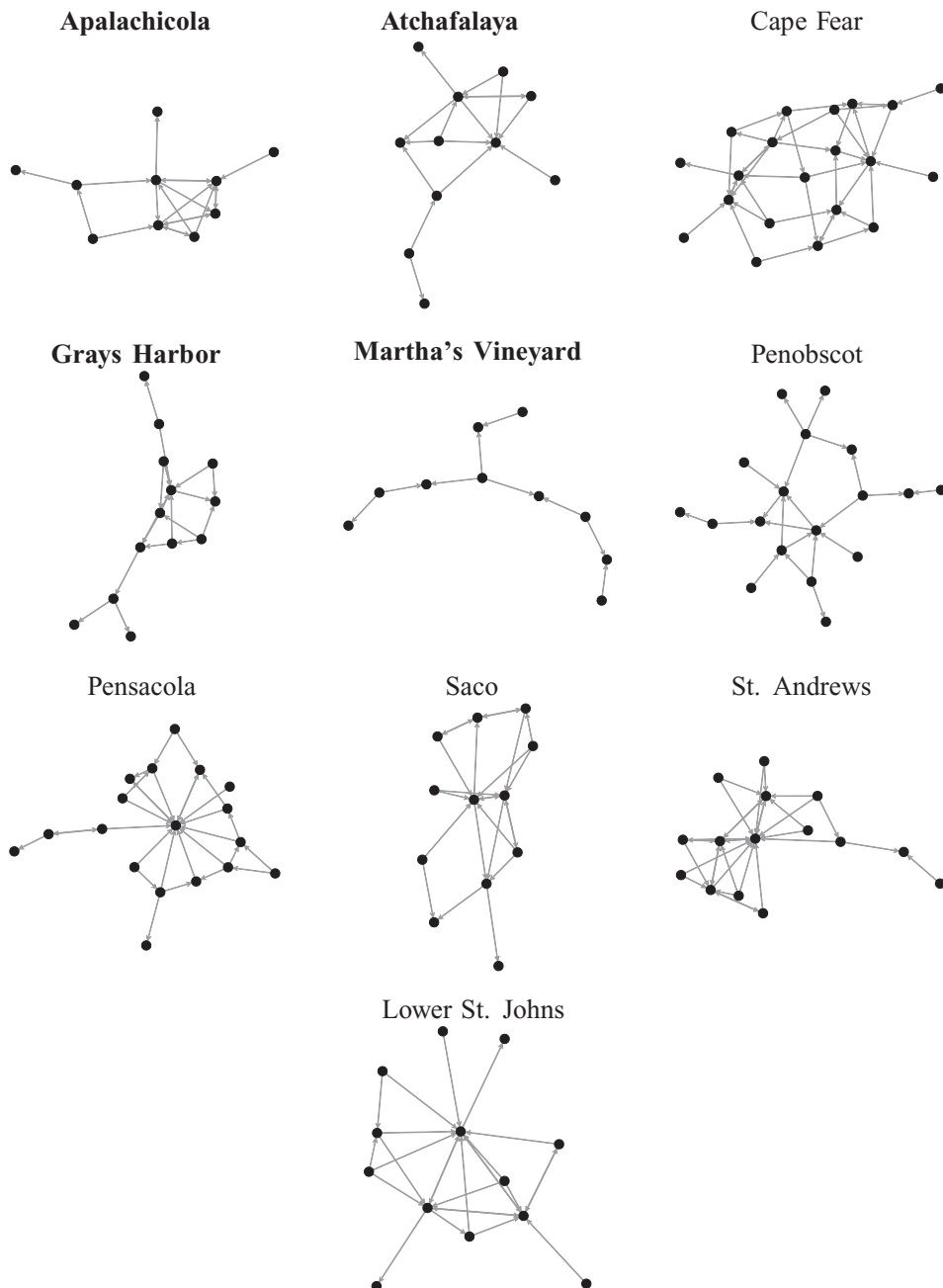


Figure 1. Plots of All Ten Networks Included in the Original Sample Analyzed by Berardo and Scholz (2010). The Networks Titled in Bold Font Appear Structurally Distinct from the Others.

Table 1. Coefficient Estimates, with Standard Errors in Parentheses, from the SABM, ERGM, and TERGM Models of Estuary Networks, Respectively

Network Dynamics	SABM		ERGM		TERGM	
Density	-3.225*	(0.197)	-1.896*	(0.397)	-5.94	(4.448)
Reciprocity	0.871*	(0.23)	0.473*	(0.144)	0.903*	(0.241)
Transitivity	0.115	(0.098)	0.466*	(0.167)	-0.014*	(0.002)
Popularity	0.987*	(0.093)	0.588*	(0.117)	0.29*	(0.034)
In-3-Star	—	—	-0.097*	(0.034)	—	—
Out-2-Star	—	—	-0.816*	(0.197)	—	—
Memory	4.927*	(0.546)	0.948*	(0.083)	10.814*	(1.199)
Trust-Out	0.032	(0.063)	-0.071*	(0.008)	-0.064	(0.048)
Trust-In	0.015	(0.046)	0.026*	(0.007)	-0.089	(0.048)
Trust-Sim	0.039	(0.533)	0.051*	(0.012)	0.091	(0.057)
Prodev-Out	-0.02	(0.153)	0.148*	(0.008)	0.065	(0.083)
Prodev-In	0.277*	(0.115)	0.057*	(0.007)	-0.085	(0.096)
Prodev-Sim	0.194	(0.127)	-0.036	(0.025)	-0.092	(0.093)
Gov-Out	-0.048	(0.063)	0.063	(0.07)	-0.048	(0.199)
Gov-In	0.019	(0.041)	0.219*	(0.046)	0.514*	(0.198)
Gov-Sim	0.477	(0.776)	-0.224*	(0.06)	-0.291	(0.196)
Behavior Dynamics	SABM/Behavior		OLS			
Intercept (Rate)	4.987*	(1.406)	2.937*	(1.117)	—	—
Trust Tendency	0.03	(0.061)	—	—	—	—
Alters' Similarity	2.702	(2.539)	0.019	(0.086)	—	—
Trust 1999	-0.098	(0.079)	0.521*	(0.097)	—	—
Prodev	-0.005	(0.05)	-0.036	(0.147)	—	—
Gov.	0.104	(0.12)	0.182	(0.354)	—	—
Predictive R^2	0.047		0.152			

*Indicates the estimate is statistically significant at the common 0.05 level (two-tailed). Bold font indicates that in each model, the estimate is statistically significant and has the same sign. The MCMC-MLE for the ERGM model was run for five iterations with 50,000 draws to approximate the normalizing constant in each iteration. The SABM model was estimated by the method of simulated moments, with 5,000 iterations in phase three and four subphases in phase two. All *t*-statistic diagnostics indicate convergence of the SABM estimates.

ERGM, exponential random graph model; SABM, stochastic actor-based models; TERGM, temporal exponential random graph model; MCMC-MLE, Markov chain Monte Carlo maximum likelihood.

as the other networks but are simply conditioned on structurally distinct fixed data (e.g., the networks in 1999, actor-level covariates).

The second diagnostic test for outlying networks that we employ directly examines the degree to which each estuary exhibits a different stochastic process generating the network in 2001. We first estimate parameters for the SABM model specified by Berardo and Scholz (2010) and replicated in Table 1. We then project the distances between the parameter estimates for the 10 estuary networks into two-dimensional space using classical multidimensional scaling (Gower, 1966), which finds positions that minimize the sum of squared differences between the distance matrix defined on the inferred positions and the distance matrix defined on the original multivariate points (i.e., the parameter estimates). This approach is useful for projecting points defined in a very high dimensional space, such as the 20 parameters estimated in the SABMs, into a lower dimensional space that can be intuitively visualized while maintaining the relative positions of the points. The results of this

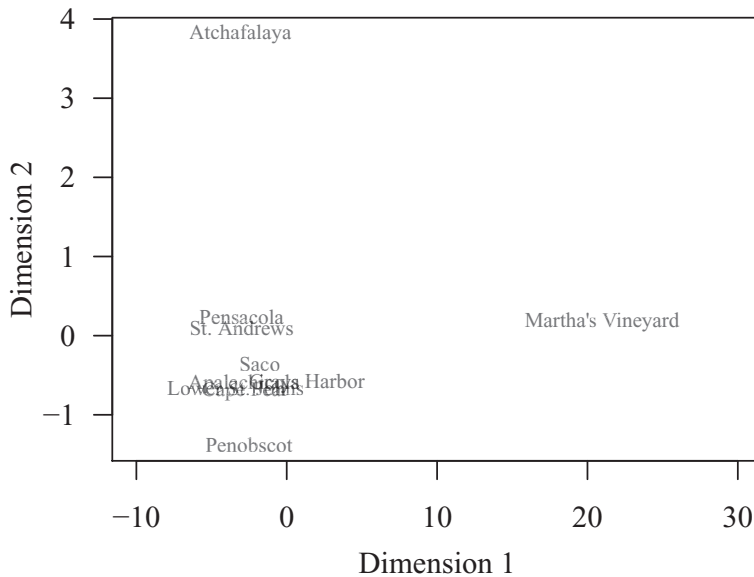


Figure 2. Depiction of Estuary-By-Estuary Differences in SABM Parameter Estimates by Classical Multidimensional Scaling. Estuaries Appearing Closer Together in the Two-Dimensional Space Produce More Similar Results with Respect to the Single-Network SABM Estimates. SABM, stochastic actor-based models.

exercise are depicted in Figure 2. The two networks that stand out as outliers based on the multidimensional scaling diagnostic include Atchafalaya and Martha's Vineyard. The other eight networks cluster in the lower left section of the two-dimensional space, indicating that they produce relatively similar estimates with respect to the SABM specification developed by Berardo and Scholz (2010). Because both of these networks also appeared visually distinct from the others, we exclude them from the analyses going forward and restrict our sample to the remaining eight estuary networks.

Model Specification and Results

We estimate three models of network formation and two models of behavior based on the specification in Berardo and Scholz (2010): an exact replication of their SABM model using the R package RSiena, a cross-sectional ERGM, a TERGM, and a least squares model of behavior. Each of the specifications includes effects designed to test the same hypotheses articulated by Berardo and Scholz (2010).

There are three actor-level independent variables of interest: *trust*, which measures the respondent's trust in other stakeholders on an 11-point scale; *government actor*, an indicator for whether the actor works for a government agency; and *prodevelopment*, which is a seven-category measure of how prodevelopment (7) or pro-environmental (1) an actor self-reports themselves to be. For each of these three variables, sender, receiver, and similarity effects are included in the model

specification. Sender (out) effects are the sum of each actor's out-degree (i.e., the number of outgoing connections sent by the actor) multiplied by the sender's covariate value. Similarly, receiver effects are the sum of each actor's in-degree multiplied by the sender's covariate value. Lastly, similarity is measured as the sum of Y_{ij} multiplied by negated absolute difference between i and j 's covariate values, over all ij .⁹

The other set of effects we include in each model are network effects. First, we include effects that account for density of the networks under study. Density is accounted for with the out-degree of the sender in the SABM. This controls for the average out-degree, which, when aggregated over all the actors, accounts for the average number of ties sent throughout the network. Analogously, the count of the number of ties in the network is included as a statistic in the ERGM and TERGM.

One of the central hypotheses derived by Berardo and Scholz (2010) is that actors will prefer to tie to actors that have many ties already in order to take advantage of highly connected actors' roles as coordination hubs. This is termed a popularity effect. In the SABM, the square root of the in-degree of the alter (i.e., the actor receiving the tie from the actor deciding to form a tie [ego]) in the SABM model and in-two-stars in the ERGM and TERGM. In the ERGM, the statistic is computed $\Gamma_{2S}(Y) = \sum_i \sum_{i \neq j} Y_{ji} Y_{ki}$. In the TERGM, the cross-time in-two-stars statistic is given by a receiver covariate equal to the in-degree of the potential tie recipient in 1999. The estimated coefficients corresponding to these effects are expected to be positive.

Berardo and Scholz (2010) explain how organizations in estuary networks can benefit from mutual coordination of policy in order to take full advantage of information, equipment, and personnel exchange. This constitutes a strong argument for the expectation that the estuary networks will be characterized by reciprocity. We control for reciprocity in each model. In the SABM, for the decision of i to send a tie to j , this is an indicator of whether there is currently a tie from j to i . We account for this in the ERGM with the negated count of asymmetric dyads, recognizing that a reciprocal network will exhibit fewer asymmetric dyads.¹⁰ In the TERGM, reciprocity is measured as a covariate that measures whether a reciprocal tie existed in 1999. We expect that the coefficients corresponding to the reciprocity statistics will be positive.

Berardo and Scholz (2010) also posit that the estuary networks exhibit transitivity whereby dense interconnected clusters tend to form. We include statistics to account for triad closure. In the SABM, the count of the number of transitive triads created by a potential tie is included. In the ERGM, we include the count of the number of ties in the network that are included in a closed triangle.¹¹ In the TERGM, transitivity is measured by a covariate that gives the number of shared partners between i and j in 1999. We expect that the coefficients associated with these statistics will be positive.

Lastly, each model is estimated using the 2001 estuary networks conditioned on the realization of those networks in 1999. This conditioning accounts for the history of the network to the extent possible with only two time points. In the SABM, temporal conditioning is accomplished by explicitly treating the process as change from one period to the next and estimating the rate of change. In the ERGM and

TERGM models, intertemporal conditioning is accomplished with a dichotomous tie-wise covariate that equals 1 if there was a tie in 1999 and -1 otherwise.

The behavior specifications, which model the response on the *trust* scale in 2001, include four substantive effects. First, the Alters' Trust effect is the average similarity of the 1999 value of Trust for each node's neighbors in the network. This first effect takes advantage of the intermediate simulations in the SABM estimation. Every time a new tie is created or extant tie eliminated in the SABM, it is possible that the Alters' Trust effect also changes. In the ordinary least squares (OLS) model of trust in 2001, we fix the Alters' Trust effect to be the average similarity of Trust for each node's neighbors in the 1999 network. The Trust 1999 effect is a node-level covariate, which accounts for autocorrelation in Trust. Also, the effects of prodevelopment beliefs and the government actor indicator are included in the behavior model. Note, the OLS model of trust is estimated independent of any of the network models. Thus, unlike in the SABM, Trust in 2001 is assumed to depend on the network in 1999 but at no more recent point in time.

Last, there are two structural effects included in the ERGM that are not incorporated in the other two specifications. First, we include the count of in-three-stars. Again, Berardo and Scholz (2010) hypothesize that the estuary networks should be characterized by a few large coordinating hubs. In network terminology, this means there should be a few large in-stars. In the context of the ERGM, Robins, Pattison, and Woolcock (2004) show that this pattern can be captured by including in-three-star statistics as well as in-two-stars. Second, out-two-stars models the variation in out-degree in the network. Because the survey directed senders to establish three ties, we expect that there will be lower variance in out-degree, as evidenced by a negative out-two-star statistic, than if no such direction were provided. We include the out-two-stars statistic and expect it to have a negative coefficient. We attempted to estimate the ERGM without these last two statistics. However, the estimated network distribution is degenerate without them. In Figure 3, we present diagnostics for the degeneracy of the ERGM estimates reported in Table 1. We simulated 500,000 networks from the model to assure that the results were not a degenerate distribution. For most of the network statistics, the simulated values vary evenly around the observed values, and there is no trend in any of the simulations. The simulations indicate the model is not degenerate.

As can be seen in Table 1, results vary across the three models. To avoid what Ho et al. (2007) refer to as model dependence, whereby the results of hypothesis tests depend upon a hard selection among plausible models, we only consider results that are consistent across models to be reliable. There are four effects that are similar across the three models. First, in each model, there is strong evidence that organizations reciprocate ties in the estuary network. Also, the measure of popularity has a positive and statistically significant coefficient in each model, which provides support for Berardo and Scholz's (2010) hypothesis that organizations prefer to form ties with those that are already central in the network. Third, though the rate of change effect used to measure memory in the SABM is not technically comparable with the lagged ties used in the ERGM/TERGM, in both the ERGM and TERGM, we see evidence for positive autocorrelation in tie values. Fourth, the receiver effect of

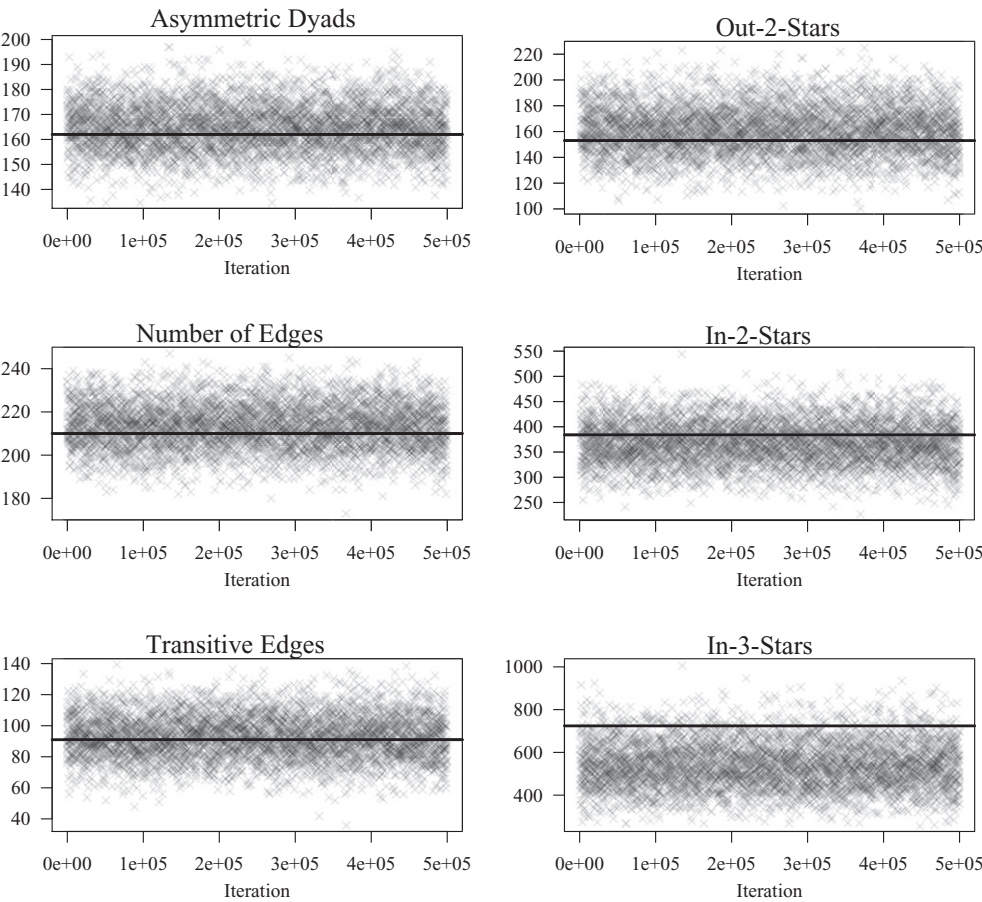


Figure 3. Simulations from the Full ERGM Model Reported in Table 1. We Simulate a Markov Chain of 500,000 Networks from the Model with the Parameter Estimates. We Depict the Measures of All of the Network Statistics Included in the ERGM. Specifically, We Use the `simulate.ergm()` Function in the R Package `ergm` with a Burn-In of 1,000 Iterations and 1,000 Tie Perturbations Between Sampling Iterations. The Solid Black Lines Are Placed at the Observed Values of the Network Statistics. ERGM, exponential random graph model.

the government actor variable is positive in each model and statistically significant in both the ERGM and TERGM, which means that government organizations tend to receive more ties than others.

The pitfalls of model dependence are evident in this analysis. A number of substantive inferences differ based on which model is used. For instance, Trust-In is an independent variable of primary interest to Berardo and Scholz (2010), because it captures the extent to which organizations are more likely to coordinate with those that are considered more trustworthy. Trust-In has a positive but nonsignificant effect in the SABM, while the ERGM finds a positive and significant effect, and the coefficient is negative and not significant in the TERGM. The results of a number of other effects are statistically significant in the ERGM, whereas they are not in the

SABM and TERGM. Again, we have no reason to prefer any of the three models *a priori*, so we accept the null hypothesis of no effect when there is substantial across-model heterogeneity in results.

Our models of behavior do not shed any consistent light on the dynamics underlying interorganizational trust in the estuary networks. The one substantive effect that is statistically significant is that of Trust in 1999 in the least squares model of behavior, which simply indicates that actors who were more trustworthy in 1999 also tended to be so in 2001. An effect that is in the same direction as hypothesized and found by Berardo and Scholz (2010) in both behavior models we estimate. However, Alters' Similarity is not statistically significant. This statistic measures the degree to which actors exhibit similar trustworthiness to those with which they are tied. This effect was positive and significant with the full sample of 10 networks. However, once the outlying networks are excluded, the effect is no longer statistically significant. This suggests the null hypothesis of no effect may be rejected with a larger sample of structurally similar estuary networks.

Though we have identified which effects are statistically significant in the hypothesized direction across the three models, our interpretation of those effects is far from complete. Simply noting that the networks on hand were unlikely to have been generated by a model with a coefficient of zero does not tell us anything about the substantive significance of the results. For each of the three effects that are consistent across the three models (reciprocity, popularity, and memory), there are three candidate models we can use to assess the related dynamics of the estuary network. We use the best fitting of the three models to conduct our substantive interpretations, because the best fitting model produces predictions that most closely accord with the observed data. Thus, before turning to the substantive interpretations, we must determine which of the three models provides the better fit.

Simulation-Based Model Comparison

As we describe above, the ERGM, TERGM, and SABM all constitute alternative generative models of the estuary networks, which embed subtly differing behavioral assumptions. There are three potential complications to the straightforward comparison of the results. First, each model in our analysis is estimated using a different method; SABM models are estimated by simulated method of moments, the ERGM is estimated by Markov chain Monte Carlo maximum likelihood, and the TERGM is estimated by maximum likelihood. Second, there are slightly different operationalizations of the common effects across the models. Third, each model has a different number of parameters feeding into the network-generation model; the TERGM has the fewest, the ERGM has the additional in-three and out-two-star parameters, and the behavior equation feeds into the generative network model in the SABM. Thus, in order to compare the three models, we need to employ a method that is robust to these three complications. The overall objective with each model is to provide a parsimonious and accurate explanation of the data on hand, which can be extrapolated to other similar networks.

We overcome the challenges to model comparison by using out-of-sample methods in which we randomly split the sample of networks, re-estimate each model on one half of the networks (i.e., the training set), and predict the held-out networks using the trained models. The first two challenges are overcome, because the result of the estimation of each model, even considering different operationalizations and estimation methods, is a probability distribution that can be used to simulate the held-out networks. Thus, though each model is different in form, a critical commonality is that they all imply a probability distribution for the networks in 2001. The third challenge is overcome because out-of-sample methods can be used to compare models with different numbers and types of parameters without *post hoc* corrections for the different parameterizations (Desmarais, 2012; Smyth, 2009). Out-of-sample model fit can decrease if erroneous parameters are added to a model (Ward, Greenhill, & Bakke, 2010). An additional benefit of using out-of-sample performance measures is that they directly assess the generalizability of the results to networks outside of the training sample by predicting the structure of networks that are assumed to be comparable with those in the training sample but were excluded from model estimation (Ward & Hoff, 2007). This last point emphasizes the importance of the outlier detection phase of the analysis—out-of-sample assessments of model fit would be inapplicable if there were substantial heterogeneity in the models that generated the networks in the sample.

In order to compare the out-of-sample predictive performance of our three models, we perform five iterations of the following data-splitting exercise. In each iteration, the sample of eight networks is split into a training sample of four networks and a validation sample of four networks. We re-estimate the three models on the training sample and then, based on the initial networks and covariate values for the validation sample, we simulate 500 predicted networks for each of the validation networks using the parameter estimates derived from the training sample. As a baseline, we also include an Erdős-Rényi random graph, in which the existence of every tie in the network is given by a simple Bernoulli distribution (Erdős & Rényi, 1960). There is a single parameter estimated in the Erdős-Rényi model—the probability of a tie. This serves as a check as to whether the complexity of the network analytic models is warranted. More complex models may impose erroneous structure on the data generating process, which will be evidenced in predictive experiments by reduced predictive performance (Shmueli, 2010). We do not argue that the process of estuary network generation is not a complex one; rather, we may not have imposed the correct forms of complexity on the models through our specification choices. For each iteration of the predictive experiments, the tie probability in the Erdős-Rényi model is estimated as the proportion of existing ties in the training networks.¹²

In terms of the network dynamics, we evaluate model fit based on the mean absolute difference between network statistics generated in the simulation and the network statistics computed on the validation networks. Lower values indicate better fit. Predictive R^2 is used to evaluate the fit of the behavior components; larger values indicate better fit. We consider several metrics, all related to the network statistics included in the models, to evaluate fit: mutuality (reciprocity) and transitivity at the

network level (Butts, 2010), the in- and out-degree of each node in the network, and the individual tie values. The results are depicted in Figure 4. The ERGM exhibits the lowest prediction error in three of the five predictive metrics, with TERGM having the lowest prediction error in predicting in-degree and the Erdős–Rényi graph exhibiting the lowest prediction error in predicting transitivity. The SABM does not perform well relative to the other models in any of the prediction tasks. In terms of the behavior component, the ordinary least squares model outperforms the SABM at predicting the values of Trust in 2001 for the held-out nodes. However, neither of the behavior models are particularly illuminating, so the choice between these two models is less consequential.

The results of the predictive exercise allow us to evaluate the broad relative theoretical merits of the SABM and ERGM in the context of the application to estuary networks. First, in our theoretical review of estuary networks, drawing on Berardo and Scholz (2010), we have discussed the importance of the sending and receiving dynamics within the network, through the guise of popularity and the coordination role of central actors. The prediction of the in- and out-degrees assesses the models' accuracy in recovering the sending and receiving activity in the network. Second, we also discussed the theoretical relevance of reciprocity and transitivity in reinforcing direct ties between organizations. We therefore have a substantive justification to prefer a model that accurately recovers these direct bonding dynamics. Third, the link is the basic building block of the network. The link prediction metric directly evaluates the models' abilities to accurately reproduce the relationships formed between organizations. An added benefit of the out-of-sample nature of our diagnostics is that they directly assess the generalizability of model inferences. The predictions are performed on networks comparable with those used to estimate the models but were iteratively excluded from the training sample. Thus, the substantive motivation to prefer the model with the best out-of-sample predictive performance along the metrics we have identified is that it exhibits the most accurate generalizable data generating process when evaluated with respect to the generative features of the estuary network data that are of most pertinent theoretical interest.

Bridging, Bonding, and Memory in Policy Partnerships

Because the out-of-sample fit diagnostics generally indicate that the ERGM fits the data better than both the TERGM and SABM, we interpret the consistent effects—reciprocity, popularity, and memory from the ERGM. These effects map nicely onto the categories of dyadic, actor, and tie-level interpretations, respectively. To interpret each effect, we randomly select 100 units at the appropriate level (e.g., ties, actors, and dyads), then compute intuitive probabilities to interpret based on those effects, conditioning the computed probabilities on the rest of the estuary network in 2001. Simple, fully contained examples of how to compute these interpretations are given in the Software Appendix. Also, the full replication code and data are available at <http://people.umass.edu/bruced/> for readers who would like to replicate the interpretation graphics in Figure 5 exactly.

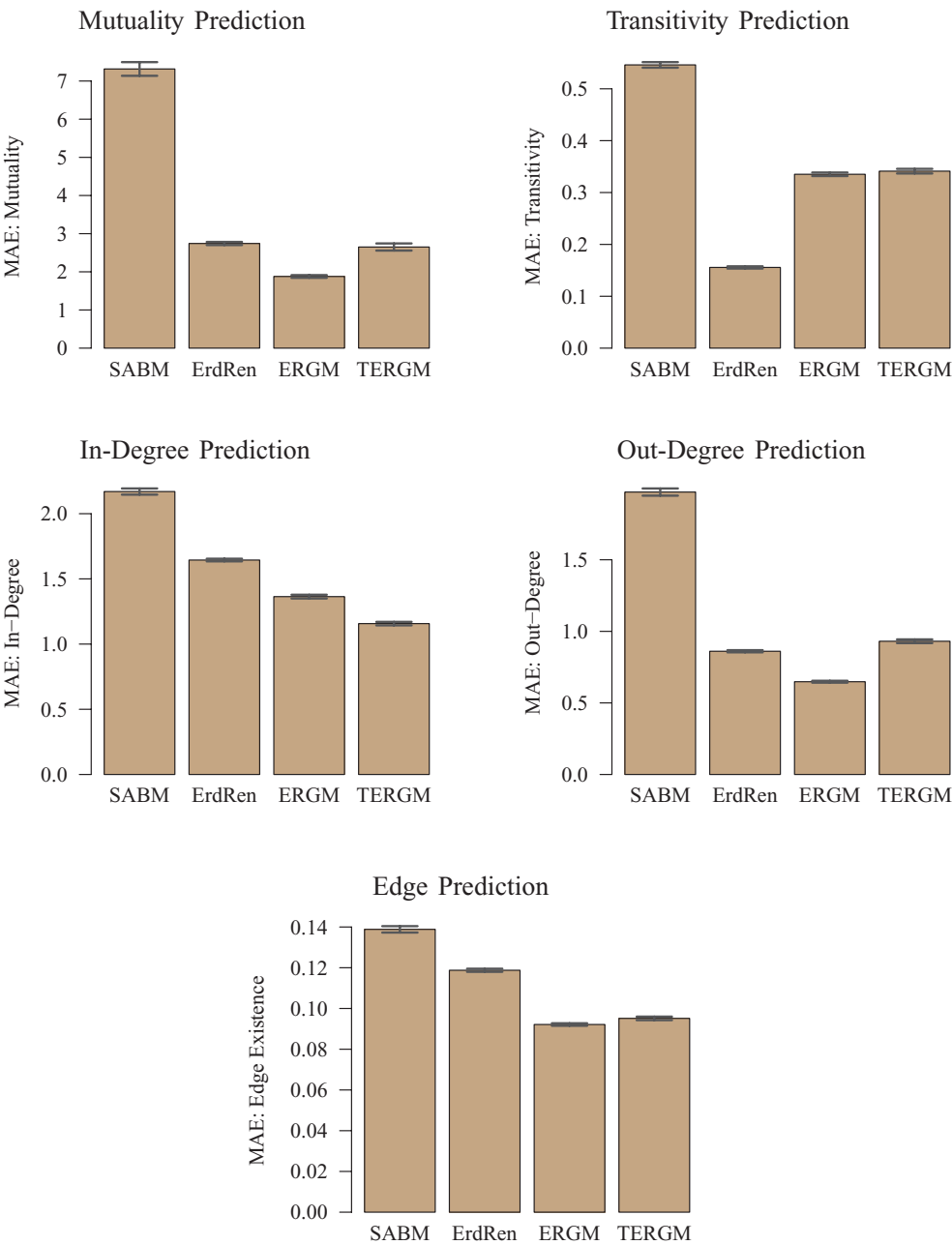


Figure 4. Out-of-Sample Predictive Performance. Barplots Depict Mean Absolute Error in Predicting the Respective Quantities in the Observed Validation Networks Using Networks Simulated from the Respective Models Estimated Using the Training Networks. Error Bars Depict 95% Confidence Intervals Around the Average Prediction Error Over All of the Simulations. ErdRen, Erdős Rényi; ERGM, exponential random graph model; SABM, stochastic actor-based models; TERGM, temporal exponential random graph model.

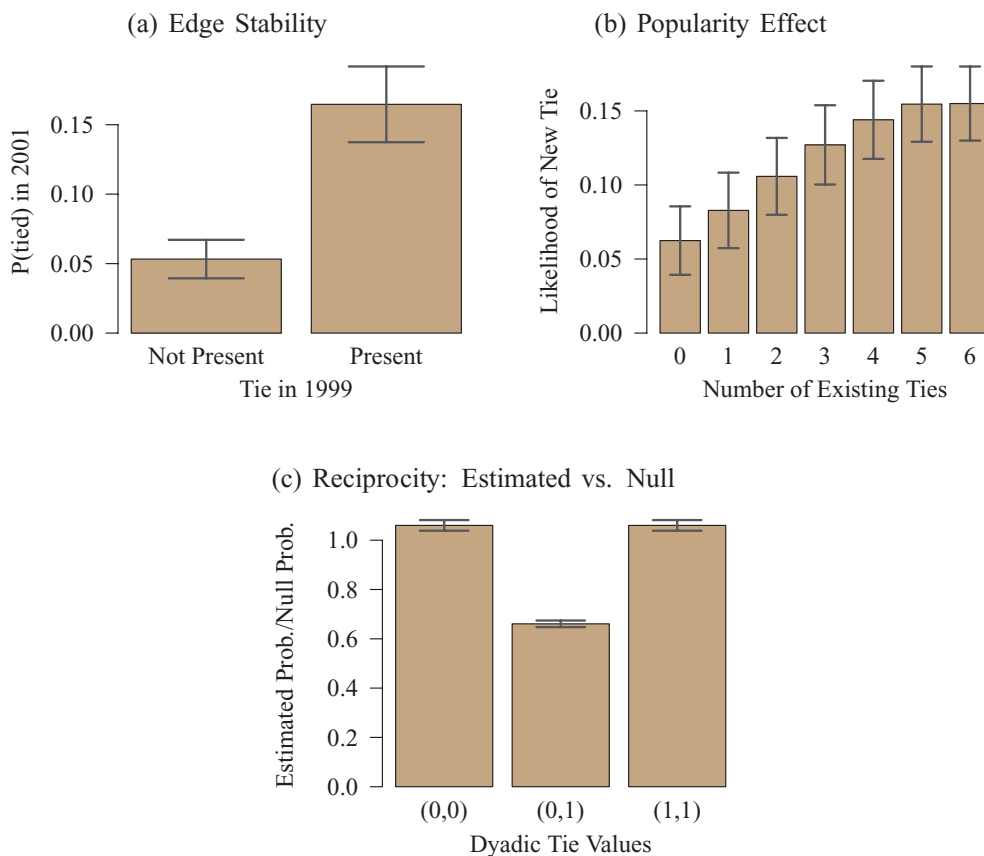


Figure 5. Interpretation at Different Levels. Barplots Depict the Mean of Respective Quantities Computed from the Full ERGM Estimates in Table 1. Error Bars Depict 95% Confidence Intervals over 100 Randomly Selected Subgraphs. Note, in the Reciprocity Plots (0,1) Represents the Average Over the Two Asymmetric Configurations in the Dyad. ERGM, exponential random graph model.

The two main theoretical categories of network effects posited by Berardo and Scholz (2010) are those that serve to *bridge* organizations in the estuary network, creating indirect ties between actors in the network, and those that serve to *bond* organizations, strengthening direct ties. When organizations seek to connect with those that are already connected to many other organizations, they are either implicitly or explicitly seeking a coordination bridge to the popular organization's neighbors. The popularity effect provides an assessment of the presence of this process within the estuary network. Panel (b) in Figure 5 gives the probability of an additional tie being formed with a recipient organization given the number of ingoing ties that have already been sent to that organization, averaged over 100 randomly selected recipient organizations. When an organization is first accumulating ties, the probability of a link being sent to a recipient organization increases approximately 5 percent when three more existing ingoing ties are added, but this effect tapers off as the number of existing ties approaches five. There is approximately a 6 percent

chance a sender will choose to send a tie to a candidate organization if that potential recipient has no existing ingoing ties. The chance of a tie increases to 16 percent if the candidate recipient organization already has six existing ties. From this, we see that organizations in the estuary networks have a strong tendency to seek out connections with popular organizations, thus establishing indirect bridges to many others in the network.

The other broad network process theorized by Berardo and Scholz (2010) is bonding, which is constituted by the reinforcement of direct ties. Reciprocity, also referred to as mutuality, is one mechanism by which direct ties are strengthened. An asymmetric dyad in the estuary networks under study indicates that one organization relies heavily on the other but not the reverse. This bond is weaker than one in which the two organizations mutually recognize each other as important resources. In panel (c) of Figure 5, we illustrate what the sign and magnitude of the reciprocity parameter estimate in the ERGM model mean for the likelihood of asymmetric ties. The first step in computing this quantity is to set the reciprocity parameter to zero (i.e., the null value) and, for 100 randomly selected dyads, compute the probability of a dyad with no ties (0,0), an asymmetric dyad (0,1), and a mutual dyad (1,1). For those same dyads, we then computed the respective probabilities, using this time the estimated reciprocity parameter. The ratio of the estimated probabilities (i.e., estimated/null) is presented in panel (c) of Figure 5. The probability of an asymmetric dyad is approximately 40 percent lower using the estimated parameter value, as compared with setting it to zero. This can be interpreted by saying that the probability of an asymmetric dyad is roughly 40 percent lower than it would be if there were no tendency for organizations to avoid weak bonds in forming estuary policy partnerships.

One salient feature of the estuary networks, as discussed by Berardo and Scholz (2010), is the period-to-period maintenance of ties between organizations. Using the estimates from the ERGM model, we can directly assess whether a tie between organizations is more likely in the current period if those two organizations were tied in the previous period. To do this, we randomly select 100 directed dyads ij , set the value of the link in 1999 to 0, and evaluate the probability of a link in 2001, then set the value of the link in 1999 to 1, and evaluate the probability of a link in 2001. Panel (a) in Figure 5 gives the results from this exercise. We find that, on average, the probability of a link in 2001 is slightly above 5 percent if there was no link in 1999, and slightly above 15 percent if there was a link in 1999. According to the ERGM estimates, a link in 1999 increases the probability of a link in 2001 approximately threefold. At a retention probability of 15 percent over 2 years, ties between organizations are certainly not permanent. However, it is also not the case that organizations select completely new partners in estuary policy.

Conclusion

We have provided a unified framework within which ERGMs can be interpreted at multiple levels, including the tie, dyad, and node levels. Also, we derive this interpretation approach as a dynamic process of constructing a network through the

conditional generation of blocks that correspond the units (i.e., level) of interest. As such, the ERGM can model the process by which any given block in the network changes conditional on the rest of the network; a powerful and general paradigm for modeling networks that change over time and interpreting effects of interest at the micro level.

The dynamic interpretations of the ERGM and TERGM are quite similar to the dynamic model implicit to the SABM; the SABM being the dominant model used by political and policy scientists to analyze dynamic networks with a substantive interest in actor- and relation-level effects. We have shown that, though similar, there are some subtle differences between the SABM and (T)ERGM. Generally speaking, the SABM makes more specific assumptions about the dynamic generation of networks than those that are implied by the ERGM. Depending on the application at hand, a more general (ERGM/TERGM) or more specific (SABM) model may be theoretically preferable, but often the choice between models will not be obvious *a priori*.

In our reanalysis of Berardo and Scholz's (2010) work on estuary networks, we demonstrated the use of out-of-sample predictive metrics to evaluate the relative merits of the SABM, ERGM, and TERGM. Out-of-sample methods are very general because they can be used to compare any methods that can be manipulated to predict the held-out data. Also, out-of-sample methods are of direct inferential import because they constitute a way to assess the generalizability of results to data that was not used to estimate the model. For this specific application, we found that the ERGM provided a better fit to the estuary networks than the TERGM (second-best fit), random graph (third best), or SABM (fourth best). This offered us a convenient justification for continuing to illustrate our ERGM interpretation method with the ERGM results, but we must emphasize that our predictive experiments *do not* speak to the relative merits of the SABM, TERGM, and ERGM in applications beyond our current analysis.

By considering three alternative models of the estuary network data, we highlighted that a number of the hypothesis tests were model dependent. However, there were three effects that were consistent across all tests. First, policy organizations tend to seek out popular partners in what Berardo and Scholz (2010) refer to as a bridging process. Second, pairs of organizations tend to avoid asymmetric (i.e., weak) relationships, in an effort to strengthen their bonds. Third, there is notable persistence in ties across time, with a tie in 2001 being approximately three times as likely if the same tie existed in 1999. The first two findings are in accordance with the results of Berardo and Scholz (2010) and replicated with a different model—the ERGM. The last finding is novel with respect to past analyses of the estuary network data. With all three effects, we illustrate how our proposed interpretation methods can be applied to clearly explicate the substantive significance of ERGM results.

Looking forward, we hope that our explication of the dynamic process consistent with the ERGM and TERGM, as well as our approach to diagnostic testing of model fit, will make the ERGM and TERGM more useful to applied researchers interested in effects at the actor, dyadic, and tie levels. To aid this process, we have developed companion software, described in the Software Appendix, that makes micro-level interpretation of the ERGM or TERGM simple to implement.

Software Appendix

We have implemented methods that interface with the ERGM package in R to compute the conditional probabilities of individual ties, dyads, outgoing ties coming from a node, and ingoing ties going to a node. They are available in the file `InterpretationFunctions.R` in the replication archive at <http://people.umass.edu/bruced/>. The functions `pNode`, `pDyad`, and `pEdge` compute probabilities at the node, dyad, and tie levels, respectively. The functions we provide take estimates from an ERGM model as arguments for the conditioned and conditioning network blocks. Below we illustrate how this interpretation method, implemented using these functions in R, can be used to produce lower level conditional interpretations of ERGM results.

We begin by considering the tie-level function, as it is a single predicted probability.

```
# Read in the ERGM Library
library(ergm)
# Read in the SNA library
library(sna)
# Set the appropriate working directory
# setwd("/Users/username/Desktop/Research/PSJ/Code/")
# Read in the Interpretation Functions
source("InterpretationFunctions.R")
# Set the random number seed
set.seed(5)
# Create an artificial network of 20 nodes
# See rgraph for more details
# Create an omitted sender feature to create omitted
sender effects
send <- cbind(rnorm(20))
# Create an omitted receiver feature to create omitted
receiver effects
rec <- t(cbind(rnorm(20)))
# Create omitted homophily to induce reciprocity
assort <- as.matrix(dist(rnorm(20),diag=T,upper=T))
# Create edge probabilities
prob <- pnorm(-1.5+send+rec-.1*assort)
art_net <- network(rgraph(20,tprob=prob))
# Estimate an ERGM with edges, out and in two stars, and
reciprocity gest <- ergm(art_net ~ edges +
istar(2)+ostar(2) + mutual) summary(gest)
# Compute the probability of an edge from node one to
node two prob <- pEdge(ergm_formula = "net ~ edges +
istar(2)+ostar(2) + mutual",theta = coef
(gest),i=1,j=2,net=art_net)
# Print prob in the R console
```



```
> prob
theta
[1,] 0.2884154
```

The `pEdge` function takes several arguments. The first argument, “`ergm_formula`” is the formula used to estimate the ERGM model given in quotation marks (i.e., as a character object), with “`net`” substituted for the name of the network. The second argument, “`theta`,” is the vector of ERGM parameter estimates. These can be obtained by wrapping `coef()` around the ERGM object (i.e., “`gest`”). The arguments “`i`” and “`j`” indicate the sender and receiver nodes for which the probability of a tie is to be computed, respectively. Lastly, “`net`” is the network for which “`net`” was substituted in the first argument. The function returns the probability that there is a tie from i to j , given the ERGM specification, parameter estimates, and the rest of the network.

Now we turn to the function, `pDyad`, which computes the probability distribution of a dyad (i.e., the joint distribution of the two ties in a dyad), given the rest of the network.

```
# Compute the dyad distribution
prob_dyad <- pDyad(ergm_formula = "net ~ edges + istar(2)+
  ostar(2) + mutual",
theta = coef(gest), i=1, j=2, net=art_net)
# Print it in the R console
> prob_dyad
j->i = 0 j->i = 1
i->j = 0 0.18257685 0.5446489
i->j = 1 0.05202021 0.2207540
```

The arguments to the `pDyad` function are the same as those to `pEdge`, except the “`i`” and “`j`” represent both ties in the dyad rather than a single tie. The matrix returned gives the probability of all four possible combinations of the tie from “`i`” to “`j`” and the tie from “`j`” to “`i`,” given the model and the rest of the network. The row and column labels indicate the state of each tie in the respective dyad outcome. For instance, looking at the bottom right cell, we see that there is a 0.2207540 probability that there is a tie from i to j and one from j to i .

Lastly, we illustrate the `pNode` function, which computes the joint probability of a group of outgoing or incoming ties from/to a node.

```
# Compute the joint distribution of three out-going ties
prob_out <- pNode(ergm_formula = "net ~ edges + istar(2)+
  ostar(2) + mutual",
theta = coef(gest), node=4, others=c(3,1,8), nodeSend=T,
net=art_net)
> prob_out
prob Receiver3 Receiver1 Receiver8
0.18545841 0 0 0
```

```

0.08331782 1 1 1
veci 0.30604097 1 0 0
veci 0.03406209 0 1 0
veci 0.08197371 0 0 1
veci 0.08413425 1 1 0
veci 0.20247721 1 0 1
veci 0.02253554 0 1 1

```

The arguments to `pNode` differ from those to the two previous functions. There are three actor-related arguments. The argument “node” indicates the single sender or receiver in the block. The “others” argument is a vector of recipient nodes in the case that “node” is a sender and sender nodes in the case that “node” is a receiver. The argument “nodeSend” is a logical indicator of whether “node” is a sender (T) or receiver (F). The matrix returned has a number of columns equal to the length of “others” plus one. The first is the probability of the blockwise outcome given the model and the rest of the network, and the other columns indicate the state of the ties between “node” and “others.” For example, from the first row, we see that there is a 0.18545841 probability that there are no ties from node 4 to nodes 3, 1, and 8.

The quantities derived with these functions can be used to compute the interpretation metrics given in Figure 2 as well as many other measures. For instance, to compute the “Edge Stability” metric, `pEdge` is used to compute the probability of a tie given a tie in the previous period versus no tie in the previous period. Confidence intervals are derived by computing the probabilities over 100 randomly selected directed dyads. Also, the “Popularity Effect” is computed by (i) randomly selecting a target node, (ii) randomly fixing a number of ingoing ties corresponding to the x -axis in the “Popularity Effect” plot, and (iii) using `pNode` to compute the average probability that the nonfixed ingoing ties to that node exist.

The following Rcode produces a plot of the form of panel (c) in Figure 5.

```

### Illustrate the dyadic (i.e., reciprocity)
interpretation plot ###
# Create Matrices to store the dyad probabilities
dyadPrEst <- matrix(0,100,3)
dyadPrNull <- matrix(0,100,3)

# define parameters to use in the estimated probabilities
thetaEst <- coef(gest)
# define parameters to use in the null probabilities
thetaNull <- coef(gest)
thetaNull[4] <- 0

# Iterate through 100 randomly selected dyads
set.seed(5)
for(i in 1:100){

```

```

# select dyads for the ith iteration
dyadi <- sample(1:20,2,rep=F)

# estimate dyad probabilities according to the estimated parameters
prEst <- pDyad(ergm_formula = "net ~ edges + istar(2)+
ostar(2) + mutual",
  theta = thetaEst,i=dyadi[1],j=dyadi[2],net=art_net)

# estimate dyad probabilities setting the mutuality
parameter to zero
prNull <- pDyad(ergm_formula = "net ~ edges + istar(2)+
ostar(2) + mutual",
  theta = thetaNull,i=dyadi[1],j=dyadi[2],net=art_net)

# store the respective probabilities in the storage
matrices
dyadPrEst[i,1] <- prEst[1,1]
dyadPrEst[i,2] <- (prEst[1,2]+prEst[2,1])/2
dyadPrEst[i,3] <- prEst[2,2]
dyadPrNull[i,1] <- prNull[1,1]
dyadPrNull[i,2] <- (prNull[1,2]+prNull[2,1])/2
dyadPrNull[i,3] <- prNull[2,2]
}

# Function to easily compute 95% confidence intervals
organized for barplots
mu_ci <- function(x){
  cix <- t.test(x)$conf.int
  c(cix[1],mean(x),cix[2])
}

# Compute the ratios
dyadSumr <- apply(dyadPrEst/dyadPrNull,2,mu_ci)

# Read in necessary library
library(gregmisc)

# Identify file, will write to working directory
filei <- "dyadRatio.pdf"

# Initialize pdf file
pdf(filei,height=3.5,width=5.25,pointsize=13,family="Times")

# Set aesthetic graphic parameters
par(las=1,mar=c(4,5,1,1),cex.lab=1.25,cex.axis=1.25)

# Labels for x-axis
colnames(dyadSumr) <- c("(0,0)","(0,1)","(1,1)")

```

```
# Draw the bars and confidence intervals
barplot2(dyadSumr[2,],col="tan",plot.ci=T,ci.l=dyadSumr
[1,],ci.u=dyadSumr[3,],
ci.lwd=2,ci.col="grey40", ylab="")

# Title the axes
title(ylab="Estimated Prob./Null Prob.",line=3.5)
title(xlab="Dyadic Edge Values", line=2.7)

# Stop writing to filei
dev.off()
```

A plot depicting the reciprocity effect should now be in the file “dyadRatio.pdf,” which will be in the working directory.

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Notes

1. The ERGM/TERGM framework we discuss here is limited by an inability to accommodate networks with valued ties (i.e., ties must be binary, either present or absent), but recent extensions by Wyatt, Choudhury, and Bilmes (2010), Desmarais and Cranmer (2011b), and Krivitsky (2011) may provide the key to overcoming this limitation.
2. Note, one assumption implicit to the ERGM is that the researcher has a strong theoretical grasp of the generative processes in the network under study and can define the Γ to accommodate these processes. This is one potential drawback when compared with less supervised methods of accounting for network dependencies, such as the latent space approach (Hoff, Raftery, & Handcock, 2002) or quadratic assignment procedure (see Robins et al., this issue). We recognize that star and transitivity effects can account for variance introduced through omitted sender/receiver and homophily effects, respectively, but the quality of inferences in the ERGM are dependent upon the quality of the specification. If the researcher has little theory about network processes, and is only interested in accounting for unmeasured dependencies, loosely conceived, a less supervised method than ERGM would probably be appropriate. However, when theory is strong, and there is direct interest in testing for specific network dependencies, these less supervised methods will not be useful and a method such as ERGM will be preferable.
3. We are careful in our use of the phrase “consistent with.” As with any statistical model, there are numerous processes that are consistent with the model. Take for instance, a simple linear regression model. The model implies a bivariate normal distribution, which holds if x causes y , y causes x or some

mix of both. Except under strictly controlled experimental conditions, x is *assumed* to be exogenous. It is not possible for a model to imply or identify one and only one generating process, even if it is derived with one process in mind. Thus, it is the role of the theory and the researcher to determine which sequential process is most likely or interesting in a given application.

4. When we say probabilistically, note that we do not imply a *uniform random* process. It is possible that some sequences are more prevalent than others.
5. There are actually several options implemented for the process assumed by SABM, but we limit our attention to the most common actor-oriented process and the one used by Berardo and Scholz (2010), whose replicated work we consider below.
6. In the interest of clarity, we compare ERGM with the form of SABM that is used almost exclusively, but we note that there are extensions to the basic SABM. First, there is a tie rather than actor-based version of the model. Second, a number of statistics for the SABM have been designed to explicitly model tie dissolution. These two extensions have seen little application to date.
7. It is important to note that this objective function differs substantially from a conventional rational choice conception of a utility function. In the SABM objective function, every node in the network shares the objective of increasing network statistics that correspond to positive parameter values and decreasing network statistics that correspond to negative parameter values. If interpreted as a utility function, this implies that every node in the network has the *same* utility function.
8. Note, to our knowledge, formally diagnosing nonstationarity in a time series of networks is an open problem. This might be particularly challenging to diagnose in a short series of networks.
9. In the SABM, the absolute difference is normalized by the maximum absolute difference across all dyads in the network.
10. It is perhaps more conventional to use the count of mutual dyads to measure reciprocity in the ERGM. However, we found that this statistic was more prone to degeneracy.
11. This measure is less prone to degeneracy than a triangle count and is implemented by fixing the exponential weight in the geometrically weighted edgewise shared partner statistic (i.e., GWESP) to zero (Goodreau, Handcock, Hunter, Butts, & Morris, 2008).
12. Our use of the term Erdős–Rényi deviates only trivially from the original definition stated by Erdős and Rényi (1960). They presented their random graph as having a binomial distribution with $\binom{n}{2}$ possible links, where n is the number of nodes. Of course, there are $2^{\binom{n_i}{2}}$ possible links in the validation network, where n_i is the number of nodes in the i^{th} validation sample, because the networks with which we are working are directed.

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