Representing Uncertainty in Transformations of GLM Parameters

Bruce A. Desmarais

GLM Lab

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Interpretations by Transformation of θ_i

In the Generalized Linear Model

$$f(\mathbf{y}|\mathbf{X}, \boldsymbol{\theta}) = f(\mathbf{y}|\mathbf{X}, E[\mathbf{y}|\beta, \mathbf{X}], \gamma)$$

$$= f(\mathbf{y}|\mathbf{X}, h(\mathbf{X}\beta), \gamma) = f(\mathbf{y}|\mathbf{X}, h(\boldsymbol{\eta}), \gamma)$$

What's all this stuff?

The known quantities in the above model are $\mathbf{X}$ and $\mathbf{y}$. The quantities about which we are uncertain are the $\boldsymbol{\theta}$. Any interpretive quantity $(\Gamma(\hat{\boldsymbol{\theta}}, \mathbf{X}, \mathbf{y}))$, (such as the expected value, change in predicted probability etc.) is by necessity a function of $\hat{\boldsymbol{\theta}}$ and (possibly) $\mathbf{X}$ and $\mathbf{y}$. The same arguments that mandate communication of uncertainty regarding the regression coefficients can be invoked to advocate the communication of uncertainty regarding important $\Gamma(\cdot)$.

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The form of Uncertainty

The quantity used in standard likelihood analysis to summarize uncertainty regarding parameter estimates is the asymptotic sampling distribution of the MLE...Which is a multivariate normal distribution with mean vector located at the parameter estimates and the covariance matrix given by the expected inverse negative Hessian....This is where you get standard errors, confidence intervals p-vals etc.

Since $\Gamma(\hat{\boldsymbol{\theta}}, \mathbf{X}, \mathbf{y})$ is a function of $\hat{\boldsymbol{\theta}}$, which, because it is a function of $\mathbf{y}$ is a random variable, is itself a random variable...a function of

$$\hat{\boldsymbol{\theta}} \sim MVN(\hat{\boldsymbol{\theta}}, \hat{\Sigma})$$

The distribution of $\Gamma(\hat{\boldsymbol{\theta}}, \mathbf{X}, \mathbf{y})$ is fully determined by $MVN(\hat{\boldsymbol{\theta}}, \hat{\Sigma})$.

A Practical Approach

The distribution of $\Gamma(\hat{\boldsymbol{\theta}}, \mathbf{X}, \mathbf{y})$ cannot be written in general for for any reasonable collection of $\Gamma(\cdot)$, and for any given function it may be difficult or impossible to compute analytically. The following technique is a general approach that, by the central limit theorem, provides a good approximation with any $\Gamma(\cdot)$..caveats of course.

1. Identify a $\Gamma(\cdot)$ of interest.
2. Draw a large number of parameter vectors from $MVN(\hat{\boldsymbol{\theta}}, \hat{\Sigma})$
3. Compute $\Gamma(\hat{\boldsymbol{\theta}}, \mathbf{X}, \mathbf{y})$ for each draw, substituting each $\hat{\boldsymbol{\theta}}^{(i)}$ for $\hat{\boldsymbol{\theta}}$
4. The sample of $\Gamma(\hat{\boldsymbol{\theta}}, \mathbf{X}, \mathbf{y})$'s provides a great approximation to what you'd get if you worked through the math.