Univariate Integral Calculus

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Stats I

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What is an Integral?

- *p*-values and confidence intervals are integrals
- Moments, including the mean, variance, skew and kurtosis are derived via integration
- Expected utility in game-theoretic models is derived using integration if uncertainty is represented by a continuous distribution
- More generally, in science, integrals are necessary when rigorously discussing any aggregate feature of the world.

So..What is an integral?

1. The *indefinite* integral, often referred to as the anti-derivative, of $f(x)$ is used to find the function $F(x)$, the first derivative of which is $f(x)$.
2. Also, the *definite* integral of $f(x)$ from a to b is used to find the area under $f(x)$ between a and b.

The fundamental theorem of calculus *lite*. If

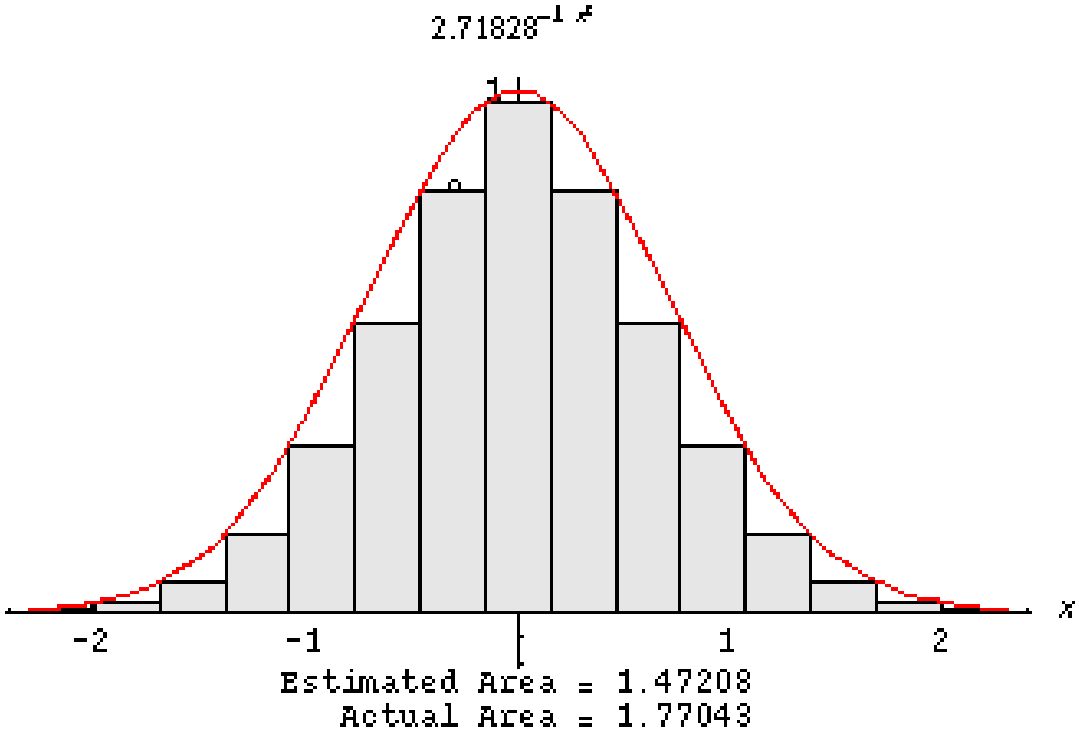
$$f(x) = g'(x)$$

then

$$\int_a^b f(x) = g(b) - g(a) = g(x)|_{x=a}^{x=b}$$

As long as $f(\cdot)$ is continuous and defined on the closed interval $[a, b]$.

The Riemann Sum



What can we do with this to get an integral?

Let's find some

What is

$$\int x^2 dx$$

$$\int_a^b (x^2 - x)^2 dx$$

$$\int \sum_{i=1}^{n} a_i x^{r_i} dx$$

$$\int Ce^{(-x^2)} dx$$

Trick I: Integration by Substitution

$$\int_a^b f(g(t))g'(t)dt = \int_{g(a)}^{g(b)} f(x)dx$$

Example

$$\int_a^b (x^2 - x)^2(10x - 5)dx$$

Trick II: Integration by Parts

$$\int_a^b f(x)g'(x)dx = g(x)f(x)|_{x=a}^{x=b} - \int_a^b f'(x)g(x)dx$$

Example

$$\int_a^b xe^x dx$$

Integrability and its Challenges

- Integration is a mathematical art form
- Becoming a good integrator requires practice...lots of it
- Non-Integrability prevents the derivation of useful general rules
- Many statisticians, mathematicians and political methodologists have earned their salary and tenure through the discovery of efficient approximations of non-integrable functions.
- As Riemann is resurrected, analytic integration is becoming much less important in scientific applications. *This is motivation to study it harder*

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