Univariate Differential Calculus

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Stats I

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What is Science?

Wikipedia defines contemporary science as "acquiring knowledge through the scientific method". Science is understood as a derivative. Let SM be the scientific method, $K(\cdot)$ be knowledge and Sc be science. According to Wikipedia.

$$Sc = \frac{d}{dSM}K$$

Within the context of political science, you will spend the next 50+ yrs of your life searching for the effect of some variable (X) (possibly vector valued) on politics (P). What you seek is the derivative

$$\frac{d}{dX}P$$

Therefore, to say you have no interest in derivatives is a tautology of the statement, "I have no interest in political science, or the scientific endeavor more generally".

What is a derivative?

If $f(\cdot)$ is a continuous function, the derivative of $f(\cdot)$ when $X = x$ is the rate of change of $f(\cdot)$ when $X = x$, and is given as the limit

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

If this limit does not exist, the function is not **differentiable**. Note something counter-intuitive about a derivative. *Generally*, if you have the rate of change of $f(\cdot)$ when $X = x$, this can only be used to compute the change in $f(\cdot)$ if X is increased (decreased) by zero. Why is this?

Lets Take a Few

Let

$$f(x) = x^2 - 5x$$

What is $f'(x)$ when $x = 3$? Show it using the limit.

Now derive $f'(x)$, leaving x undefined. Use the limit again.

Lets now show the general derivative of a polynomial.

$$f(x) = \sum_{i=1}^{n} a_i x^{r_i}, \ \ r \in \mathbb{R}$$

The Chain Rule

Memorize this rule

It is the most important definition in differential calculus. It is the bridge from knowledge regarding the derivative of simple functions, such as polynomials, *exp*() and *ln*() to knowing the derivative of any function.

$$\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$$

Mathematically, this is a thing of beauty. Since functions can be nested and rearranged conveniently, the fact that the chain rule involves two functions, generalizes it to an arbitrary number of functions. We will see some examples of this later.

Other Useful Definitions

The Product Rule

$$\frac{d}{dx}f(x)g(x) = f'(x)g(x) + g'(x)f(x)$$

What is the derivative of $(x+3)x^3$?

The Quotient Rule

$$\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

What is the derivative of $\frac{x^3}{x+3}$?

$log_b'(\cdot)$ and $exp'(\cdot)$

$$\frac{d}{dx}exp(f(x)) = f'(x)exp(f(x))$$

$$\frac{d}{dx}log_b(f(x)) = \frac{f'(x)}{f(x)ln(b)}$$

What is the derivative of $ln(f'(x))$?
What is

$$\frac{d}{d\beta_1}\ln\left(\prod_{i=1}^{n}F_i(\beta_1)^{y_i}(1-F_i(\beta_1))^{1-y_i}\right)$$

where $F_i(\beta_1) = \frac{1}{1+exp(-(\beta_0+\beta_1 x_i))}$.