4.1 : 2×2 Contingency Table

	class 1	class 2	
Pop. 1	O_{11}	O_{12}	n_1
Pop. 2	O_{21}	O_{22}	n_2
	C_1	C_2	N

random

Test: T_1

random

where $$T_1 = \frac{\sqrt{N}\,(O_{11}O_{22} - O_{12}O_{21})}{\sqrt{n_1 \cdot n_2 \cdot C_1 \cdot C_2}}$$

$H_0: P_1 \geq P_2$
$H_a: P_1 < P_2$

$C = \{T_1 : T_1 < z_\alpha\}$

or

$P.V = P(Z < T_1)$

$H_0: P_1 \leq P_2$
$H_a: P_1 > P_2$

$C = \{T_1 : T_1 > z_{1-\alpha}\}$

or

$P.V = P(Z > T_1)$

$H_0: P_1 = P_2$
$H_a: P_1 \neq P_2$

$C = \{T_1 : T_1 < z_{\alpha/2} \text{ or } T_1 > z_{1-\alpha/2}\}$

or

$P.V = 2P(Z < T_1)$ or
$P.V = 2P(Z > T_1)$
+ the smaller

Fisher's Exact Test | cont. 4.1 | (2)

(a) Small Sample — If both fixed

$n \leq 20$

	c_1	c_2	
r_1	x	$r-x$	r
r_2	$c-x$	$N-r-c+x$	$N-r$
	c	$N-c$	N

(column totals fixed; row totals fixed)

$$T_2 = x \quad \begin{cases} P(T_2 = x) = \dfrac{\binom{r}{x}\binom{N-r}{c-x}}{\binom{N}{c}} & x = 0, 1, \ldots, \min(r, c) \\ = 0 & \text{for all values of } x \end{cases}$$

(b) Large Sample

$n > 20$

$$T_3 = \frac{x - \frac{rc}{N}}{\sqrt{\dfrac{rc(N-r)(N-c)}{N^2(N-1)}}}$$

cont. 4.1

(a) Small Sample ($n \leq 20$)

$H_0: p_1 \geq p_2$
$H_a: p_1 < p_2$

$P.V = P(T_2 \leq x)$

(1)

$H_0: p_1 \leq p_2$
$H_a: p_1 > p_2$

$P.V = P(T_2 \geq x)$

(2)

$H_0: p_1 = p_2$
$H_a: p_1 \neq p_2$

- use (1) & (2)
- double them
- the smaller

(b) Large Sample ($n > 20$)

$C = \{T_3 : T_3 \leq z_\alpha\}$

or

$P.V = P(Z \leq T_3)$

add 0.5 to T_3 on Page 2

(1)

$C = \{T_3 : T_3 \geq z_{1-\alpha}\}$

or

$P.V = P(Z \geq T_3)$

subtract 0.5 from T_3 on Page 2

(2)

$C = \{T_3 : T_3 \leq z_{\alpha/2}$ or $T_3 > z_{1-\alpha/2}\}$

or

- use (1) & (2)
- Double it
- the smaller

Cont. 4.1

The Mantel-Haenszel Test

	col 1	col 2	
row 1 (Fixed)	X_i	$r_i - X_i$	r_i
row 2	$c_i - X_i$	$N_i - r_i - c_i + X_i$	$N_i - r_i$ (Fixed)
	c_i		N_i

Test stat:

$$T_4 = \frac{\sum X_i - \sum \frac{r_i c_i}{N_i}}{\sqrt{\sum \frac{r_i c_i (N_i - r_i)(N_i - c_i)}{N_i^2 (N_i - 1)}}}$$

If both random use:

$$T_5 = \frac{\sum X_i - \sum \frac{r_i c_i}{N_i}}{\sqrt{\sum \frac{r_i c_i (N_i - r_i)(N_i - c_i)}{N_i^3}}}$$

$H_0: P_{1i} \geq P_{2i}$

$H_a: P_{1i} \leq P_{2i}$ for all i

and $P_{1i} < P_{2i}$ for some i

$C = \{T_4 : T_4 < z_\alpha\}$

$C = \{T_5 : T_5 < z_\alpha\}$

$H_0: P_{1i} \leq P_{2i}$

$H_a: P_{1i} \geq P_{2i}$ for all i

and $P_{1i} > P_{2i}$ for some i

$C = \{T_4 : T_4 > z_{1-\alpha}\}$

$C = \{T_5 : T_5 > z_{1-\alpha}\}$

$H_0: P_{1i} = P_{2i}$

$H_a: P_{1i} > P_{2i}$ for some i

or

$P_{1i} < P_{2i}$ for some i,

but not both

or

p.value

$p.v = P(z < T_4)$

+ add 0.5 to T_4

$p.v = P(z < T_5)$

+ NO 0.5

(1)

$p.v = P(z > T_4)$

- subtract 0.5 from T_4

$p.v = P(z > T_5)$

+ NO 0.5

(2)

- use (1) & (2)
- Double it
- the Smaller