h.4

(9.5)

Apr-T-86

M

2 Ch. 2

1.5 Ch. 3 (3.1, 3.2, 3.3, 3.4)

1.5 Ch. 4 (4.1, 4.2, 4.3, 4.5)

Test #2 (Next Monday)

open book — open note

Data:

class	1	2	----	C	
frequency	o_1	o_2	----	o_C	N Sample size

Test

$H_0: F(x) = F^*(x)$ where $F^*(x)$ is completely specified

$H_a: F(x) \neq F^*(x)$ for some x.

} except possibly some unknown parameters.

Under H_0, compute the expected frequency in each cell

$E_i = N \cdot P_i = N \cdot P(\text{the outcome in class } i \text{ under } H_0)$

Test statistic

$$T = \sum_{1}^{C} \frac{(o_i - E_i)^2}{E_i}$$

$$C = \{T : T \geq c\}$$

For large sample $T \sim$ chi-square with d.f $= C-1$ if $F^*(x)$ is completely specified.

$C-1-k$ if k is the # of parameters to be estimated for H_0

$F^*(x) \sim N(\mu, \sigma^2)$

Reject H_0 if $T \geq \chi^2_{1-\alpha}$ where

$\chi^2_{1-\alpha}$ is the $(1-\alpha)^{th}$ quantile of the chi-square.

Toss a coin 100 times and record # of heads

Repeat this experiment 5 times.

Define

X = # of heads among 100 tosses

	1	2	3	4	5	← # of heads each time (among 100 toss)
Data: Frequency	40	35	55	70	80	
E_i	50	50	50	50	50	

Under H_0 $p_i = 1/2$ for $i = 1,2,3,4,5$

$E_i = 100 \cdot p_i = 100 \cdot \frac{1}{2} = 50$ $\quad i = 1,2,3,4,5$

Test statistic

$$T = \frac{(40-50)^2}{50} + \cdots + \frac{(30-50)^2}{50} = 23$$

$T \sim$ chi-square $(d.f = 5-1 = 4)$

since

$23 > \chi^2_{.95}(d.f=4) = 9.488$, we reject H_0

P-value $= P(\chi \geq 23)$, where $\chi \sim$ chi-squar $(d.f=4)$

.999
18.47
1-.999 = .001 < .05

Ex.

Five coins with identical, but unknown value of $p = P(\text{head})$ are tossed together 100 times.

Define

$X = \#$ of heads Per toss (means 5 coins $n=5$)

$H_0: X \sim$ binomial $(n=5, p)$

$H_a: X$ is not H_0 ....

Since p is unknown, it must be estimated from the sample.

# of heads	0	1	2	3	4	5	
Frequency	3	16	36	32	11	2	100

$0(3) + 1(16) + \cdots 5(2) = 238$

$\frac{238}{100} = 2.38$: average # of heads Per five tosses.

$\hat{p} = \frac{2.38}{5} = .476$

$$E_i = 100 \cdot P_i = 100\binom{5}{i}(.476)^i(1-.476)^{5-i}, \quad i = 0, 1, \ldots, 5$$

	0	1	2	3	4	5	
E_i	4.0	17.9	32.6	29.6	13.5	2.4	100
	3	16	36	32	11	2	

Test statistic:

$$T = \frac{(3-4)^2}{4} + \cdots + \frac{(2-2.4)^2}{2.4} = \boxed{1.53}$$

$T \sim$ chi-square $(d.f = 6-1-1 = 4)$

d.f	.750
4	5.385

$\chi^2_{.9}(df=4) = 9.485$

Since $1.53 \not> 9.488$ accept H_0

$\hat{\alpha} = 1 - .750 = .250 > \alpha = .05$ accept

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$H_0: X \sim N(\mu, \sigma^2)$