(c) How many computer runs are necessary so we can be 99.9% sure that the number of people required is between $X^{(2)}$ and $X^{(n-1)}$?

(d) How many computer runs are necessary so that we can be 99.9% sure that the number of people required is no more than $X^{(n-4)}$?

PROBLEM

1. Use Table A3 to solve Exercise 3. Find the exact value of α.

3.4 THE SIGN TEST

After straying from hypothesis testing somewhat, at least in the previous section, we now return to discuss the oldest of all nonparametric tests, the sign test. Actually, the sign test is just the binomial test, with $p^* = 1/2$. But the sign test deserves special consideration because of its versatility, its age (dating back to 1710), and because $p^* = 1/2 = 1 - p^*$ makes it even simpler than the binomial test. The sign test is useful for testing whether one random variable in a pair (X, Y) tends to be larger than the other random variable in the pair. Also, as we will see in Section 3.5, it may be used to test for trend in a series of ordinal measurements or as a test for correlation. In many situations where the sign test may be used, more powerful nonparametric tests are available for the same model. However, the sign test is usually simpler and easier to use, and special tables to find the critical region are sometimes not needed.

▶ The Sign Test

Data The data consist of observations on a bivariate random sample (X_1, Y_1), (X_2, Y_2), . . . , $(X_{n'}, Y_{n'})$, where there are n' pairs of observations. There should be some natural basis for pairing the observations; otherwise the Xs and Ys are independent, and the more powerful Mann-Whitney test of Chapter 5 is more appropriate.

Within each pair (X_i, Y_i) a comparison is made, and the pair is classified as "+" or "plus" if $X_i < Y_i$, as "−" or "minus" if $X_i > Y_i$, or as "0" or "tie" if $X_i = Y_i$. Thus the measurement scale needs only to be ordinal.

Assumptions

1. The bivariate random variables (X_i, Y_i), $i = 1, 2, \ldots, n'$, are mutually independent.

T = total # of +'s
n = " " +'s and −'s
No tie's

3.4: The Sign Test

$n \leq 20$

$H_0: P(+) \geq P(-)$	$H_0: P(+) \leq P(-)$	$H_0: P(+) = P(-)$
$H_a: P(+) < P(-)$	$H_a: P(+) > P(-)$	$H_a: P(+) \neq P(-)$
or $P \geq .5$ & $P < .5$		
$C = \{T : T \leq t\}$	$C = \{T : T \geq n - t\}$	$C = \{T : T \leq t \text{ or } T \geq n - t\}$
where T = # of '+' and t: such that	where n = excluding # of ties	$P(y \leq t) = \alpha/2$
$P(y \leq t) = \alpha$	$P(y \leq n - t) = 1 - \alpha$	$P(y \leq n - t) = 1 - \alpha/2$

Using P.V.

$P.V = P(y \leq T)$	$P.V = P(y \geq T)$	$P.V = 2P(y \leq T)$ or $P.V = 2P(y \geq T)$ & smaller

3.4 Cont.

(2)

$n > 20$

$H_0: P(+) \geq P(-)$

$H_a: P(+) < P(-)$

$C = \{T : T \leq t\}$

where

$t = \frac{1}{2}(n + Z_\alpha \sqrt{n})$

$H_0: P(+) \leq P(-)$

$H_a: P(+) > P(-)$

$C = \{T : T \geq n - t\}$

where

$t = \frac{1}{2}(n + Z_\alpha \sqrt{n})$

$H_0: P(+) = P(-)$

$H_a: P(+) \neq P(-)$

$C = \{T : T \leq t \text{ or } T \geq n - t\}$

where

$t = \frac{1}{2}(n + Z_{\alpha/2} \sqrt{n})$

or using P.V

$P.V = P(y \leq T)$

$= P\left(Z \leq \frac{2T - n + 1}{\sqrt{n}}\right)$

(1)

$P.V = P(y \geq T)$

$= 1 - P\left(Z \leq \frac{2T - n - 1}{\sqrt{n}}\right)$

(2)

- use (1) & (2)
- Double them
- the Smaller