K-W-T

A		B		C	
73	6	84	13	82	12
64	2	80	10	79	9
67	3	81	11	71	5
62	1	77	8	75	7
70	4				

#1
p.297

H_0: they have the identical distribution
$(EX_A = EX_B = EX_C)$

H_a: at least two of them are different.

$N = 5+4+4 = 13$

$\boxed{R_A = 16 \quad , R_B = 42 \quad R_C = 33}$

the test statistic Equat (5) (No ties)

$$T = \left[\sum_{1}^{k} \frac{R_i^2}{n_i}\right] \cdot \frac{12}{N(N+1)} - 3(N+1)$$

$$= \left(\frac{R_A^2}{n_A} + \frac{R_B^2}{n_B} + \frac{R_C^2}{n_C}\right)\frac{12}{N(N+1)} - 3(N+1)$$

$$= \left(\frac{16^2}{5} + \frac{42^2}{4} + \frac{33^2}{4}\right)\left(\frac{12}{13(13+1)}\right) - 3(13+1)$$

$$= 50.4 - 42 = \boxed{8.4}$$

from table A-8

Sample Size	$W_{.90}$	$W_{.95}$	$W_{.99}$
5, 4, 4	4.6187	5.6174	7.744

P-Value $= P(T > 8.4) < .01$

We reject H_0 at $\alpha < .01$ they have different means.

Compare A and B $\quad S^2 = \frac{N(N+1)}{12}$

$$\left|\frac{R_i}{n_i} - \frac{R_j}{n_j}\right| > t_{1-\alpha/2\,(N-k)}\left(S^2 \frac{N-1-T}{N-K}\right)^{1/2}\left(\frac{1}{n_i}+\frac{1}{n_j}\right)^{1/2}$$

$$\left|\frac{R_A}{n_A} - \frac{R_B}{n_B}\right| = \left|\frac{16}{5} - \frac{45}{4}\right| = |3.2 - 10.5| = \underline{7.3}$$

at $\alpha = .05$ $\quad t_{1-\alpha/2}(df = N-k)$ $\quad$ table ~~A-2~~ ~~$\frac{559}{1000}$~~

$$t_{.975}(df = N-k = 13-3 = 10) = 2.228$$

$$t_{1-\alpha/2}\sqrt{\frac{1}{n_A}\cdot\frac{1}{n_B}}\sqrt{\frac{N(N+1)}{12}}\sqrt{\frac{N-1-T}{N-k}}$$

$$= (2.228)\sqrt{1/5+1/4}\sqrt{\frac{13(13+1)}{12}}\sqrt{\frac{13-1-8.4}{13-3}}$$

$$= (2.228)(.671)(3.89)(.6) = \boxed{3.49}$$

since $7.3 > 3.49$, we conclude $EX_A \neq EX_B$

Compare A and C

$$\left|\frac{R_A}{n_A} - \frac{R_C}{n_C}\right| = \left|\frac{16}{5} - \frac{33}{4}\right| = 5.05 > \boxed{3.49}$$

$$EX_A \neq EX_C$$

Compare ~~B and C~~

$$\left|\frac{R_B}{n_B} - \frac{R_C}{n_C}\right| = \left|\frac{42}{4} - \frac{33}{4}\right| = \underline{2.25}$$

$$RHS = (2.228)\sqrt{1/4+1/4}\,(3.89)(.6) = 3.69$$

Since $2.25 \not> 3.69$, $EX_B = EX_C$

Ch. 5.2

#4

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	Type A	B	C
Replaced	12	3	6
Repaired	10	8	7
Not Returned	82	96	58

$(t_1+1)/2$

$t_1+(t_2+1)/2$

$\vdots$

$\bar{R}_i = \sum_{i=1}^{c-1} t_i + (t_c+1/2)$

$i = 1 - c$

$j = 1 - k$

ordered categories

	1	2	3	Row t_i Total	$\bar{R}_i$
Replaced	12	3	6	21	$21+1/2 = 11$
Repaired	10	8	7	25	$21 + (25+1)/2 = 34$
Not Returned	82	96	58	236	$21+25+\frac{(236+1)}{2} = 164.5$
	104	107	71	282	
	n_1	n_2	n_3	N	

sum of the rank in Population (column)

Eq. 9

$$R_j = \sum_{i=1}^{c} o_{ij} \bar{R}_i$$

sum R $= \frac{N(N+1)}{2} = 39,903$

$$R_1 = (12)(11) + 10(34) + 82(164.5) = 13,961$$

$$R_2 = (3)(11) + 8(34) + 96(164.5) = 16,097$$

$$R_3 = (6)(11) + 7(34) + 58(164.5) = 9,845$$

Eq. 10

$$S^2 = \frac{1}{N-1}\left[\sum_{i=1}^{c} t_i \bar{R}_i^2 - \frac{N(N+1)^2}{4}\right]$$

$$= \frac{1}{282}\left[(27)(11)^2 + (25)(34)^2 + (236)(164.5)^2 - \frac{282(283)^2}{4}\right]$$

$$= \frac{1}{281}(2541 + 28{,}900 + 6{,}386{,}219 - 5\ldots)$$

$$= \frac{1}{281}(6{,}417{,}660 - 5{,}646{,}274.5)$$

$$S^2 = \frac{1}{281}(771{,}385.5) = 2{,}735.4096$$

$$= 2745.14$$

Equation 3

$$T = \frac{1}{S^2}\left(\sum_{i=1}^{n} \frac{R_i^2}{n_i} - \frac{N(N+1)^2}{4}\right)$$

$$T = \frac{1}{2{,}735.4096}\left[\frac{(13{,}961)^2}{104} + \frac{(16{,}097)^2}{107} + \frac{(9{,}845)^2}{71} - \ldots\right]$$

$$= \frac{1}{2{,}735.4096}(1{,}874{,}130.010 + 2{,}421{,}620.645 + 1{,}365{,}127.113 - 5{,}646{,}274.5)$$

$$= \frac{1}{2{,}735.4096}(5{,}660{,}877.768 - 5{,}646{,}274.5)$$

$$T = \frac{1}{11}(14{,}603.268) = \boxed{\cancel{5.3386}}\ 5.319$$

$5.3386 \not> 5.991$ $\qquad \chi^2_{.95(2)} = 5.991$

do not reject H_0

No significant difference among the reliability of radiation