ISQA 8160 Exam II

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Chapter 3.1

3.1 Problem 6.) A civic group reported to the town council that at least 60% of the town residents were in favor of a particular bond issue. Forty-eight said yes. Is the report of the civic group reasonable?

We have the following:

$$
\begin{aligned}
p &= 0.60 \\
p^* &= 0.48 \\
n &= 100 \\
\alpha &= 0.05
\end{aligned}
$$

The null hypothesis is that the sample probability is representative of a population probability of 60%. The alternative hypothesis is that they are not equal.

$$
\begin{aligned}
H_0 &: p = p^* \\
H_a &: p \neq p^*
\end{aligned}
$$

```r
library(binom)
binom.test(x=48, n=100, p=0.60, alternative=c('two.sided'), conf.level=0.95)
```

```
##
##  Exact binomial test
##
## data:  48 and 100
## number of successes = 48, number of trials = 100, p-value =
## 0.01844
## alternative hypothesis: true probability of success is not equal to 0.6
## 95 percent confidence interval:
##  0.3790055 0.5822102
## sample estimates:
## probability of success
##                   0.48
```

And thus we reject the null hypothesis with a p-value of 0.01844 and a 95% confidence interval of (0.3790055, 0.5822102). The population mean appears to be less than 60%.

■

3.1 Problem 7.) Out of 20 recent takeover attempts, 5 were successfully resisted byt the companies being taken over. Assume these are independent events, and estimate the probability of a takeover attempt being successfully resisted. That is, find a 95% confidence interval.

Setting up this problem we have the following values:

$$n = 20$$
$$x = 5$$

We can find the confidence interval using the binom.test like before:

```
binom.test(x=5, n=20, alternative=c('two.sided'), conf.level=0.95)
```

```
##
##  Exact binomial test
##
## data:  5 and 20
## number of successes = 5, number of trials = 20, p-value = 0.04139
## alternative hypothesis: true probability of success is not equal to 0.5
## 95 percent confidence interval:
##  0.08657147 0.49104587
## sample estimates:
## probability of success
##                   0.25
```

(a) Use Table A4.

Table A4 confirms the 95% confidence interval as (0.087, 0.491).

(b) Use Table A1.

Although $n = 20$ is a smaller sample size than we generally require for using the normal distribution, we can nevertheless use it to obtain a slightly less accurate answer:

$$L = \frac{Y}{n} - z_{1-\frac{\alpha}{2}}\sqrt{\frac{Y(n-Y)}{n^3}}$$
$$U = \frac{Y}{n} + z_{1-\frac{\alpha}{2}}\sqrt{\frac{Y(n-Y)}{n^3}}$$

The z-value at $z_{1-\frac{0.05}{2}} = z_{0.975}$ is 1.96. Plugging in our values, we get

```
L <- (5/20) - 1.96 * sqrt((20 * (20 - 5)) / (20^3))
U <- (5/20) + 1.96 * sqrt((20 * (20 - 5)) / (20^3))
print(paste(paste(paste(paste('(', L),  ', '), U), ')'))
```

```
## [1] "( -0.129552367928327 ,  0.629552367928327 )"
```

Clearly, with such a small sample size, the normal approximation will not return accurate results.

■

Chapter 3.2

3.2 Problem 1.) A random sample of tenth-grade boys resulted int he following 20 observed weights.

142	134	98	119	131
103	154	122	93	137
86	119	161	144	158
165	81	117	128	103

Test the hypothesis that the median weight is 103.

Let's first order the data.

```
weights <- sort(c(142, 134, 98,  119, 131,
                  103, 154, 122, 93,  137,
                  86,  119, 161, 144, 158,
                  165, 81,  117, 128, 103))
weights
```

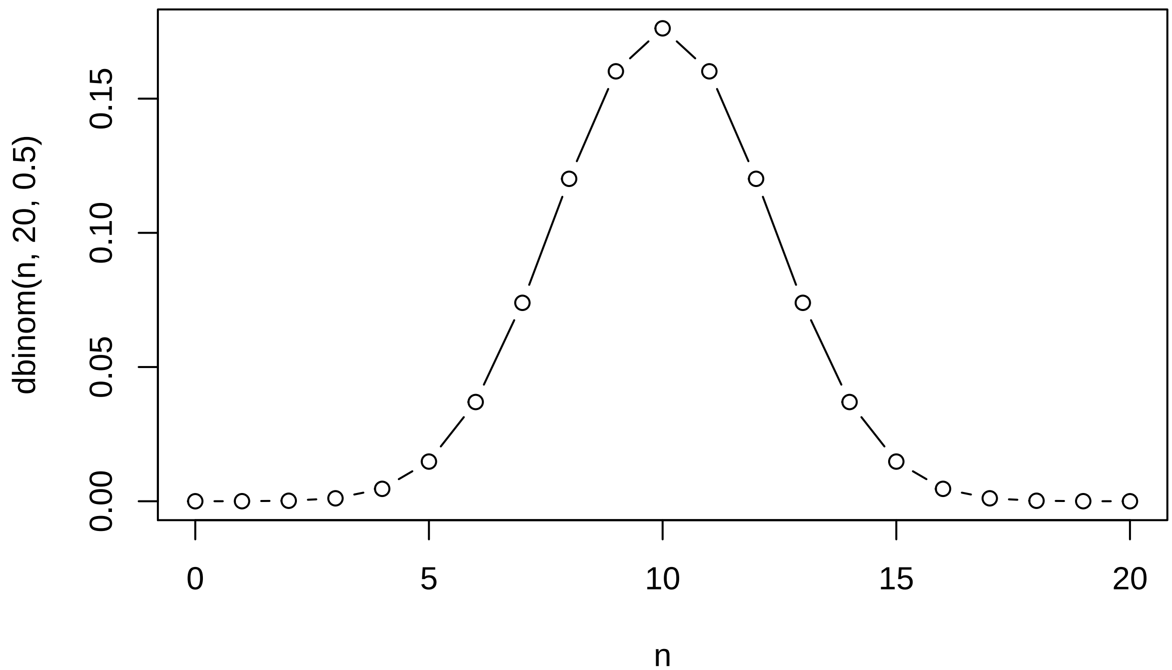
```
##  [1]  81  86  93  98 103 103 117 119 119 122 128 131 134 137 142 144 154
## [18] 158 161 165
```

We will test the hypothesis that $x_{0.50} = 103$ with $\alpha = 0.05$.

$$H_0 : p = p^*$$
$$H_a : p \neq p^*$$

Since $n \leq 20$, the critical region is $c = \{T_1 \leq t_1, T_2 \geq t_2\}$, such that $P(y \leq t_1) = \alpha$ and $P(y \leq t_2) = 1 - \alpha$.

Finding t_1 and t_2 can be done with a table, or with visual inspection and programmatic verification.



It is clear that t_1 must be less than or equal to 6 and t_2 must be greter than or equal to 14. We can do some quick checking to find the exact values

```
t_1 <- sum(dbinom(x=c(0:5), size=20, prob=0.5))
t_2 <- 1 - sum(dbinom(x=c(0:14), size=20, prob=0.5))
t_1
```

```
## [1] 0.02069473
```

```
t_2
```

```
## [1] 0.02069473
```

Now it is clear, $t_1 = 5$ and $t_2 = 14$. But since $x^{(5)} = 103$, this falls in the critical region. The median does not appear to be 103. We reject the null hypothesis with a p-value of

```
min(2*t_1, 2*t_2)
```

```
## [1] 0.04138947
```

■