To: All Executives and Hotel Managers

From: Decision Support Systems

Subject: Optimal Configuration for Luxary Suites

Date: October 12, 2017

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1 Modeling and Optimization

1.1 The fully booked rooms case

Let X be a random variable representing the number of suites requested for booking on a given day. Then X follows a binomial distribution, $X \sim Bin(n, \theta)$. We are given some data,

Suite Demand	0	1	2	3	4	5	6	7	8	9	10
Frequency	0	2	6	13	21	33	38	39	28	16	4

Visualizing this as a histogram as shown in Figure 1, we believe that suite demand can be modeled with a binomial distribution.

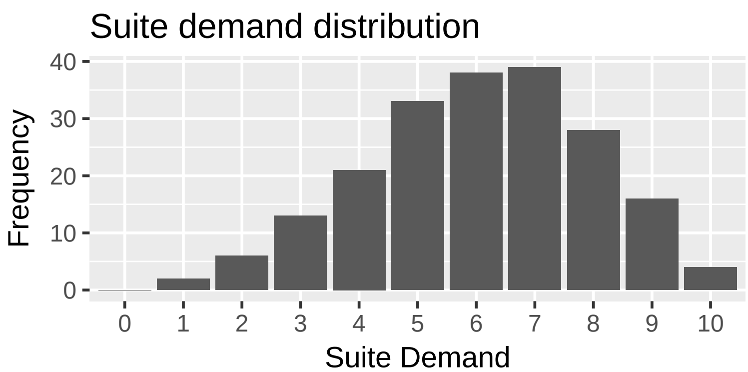

Figure 1: Distribution of suite demand.

We calculate the sample mean, $\overline{X}$ to be 6.055. Since $\overline{X}$ is an unbiased estimator of μ, and for the binomial distribution, $\mu = n\theta$, we find that $\theta = \frac{\mu}{n} = \frac{6.055}{10} = 0.6055$

For a hotel configuration based on number of suites, $S \in [0, 1, 2, \ldots, 10]$ and regular rooms, $20 - 2S$, we are given R_{suite}, the revenue for a booked suite, R_{room}, the revenue for a booked room, U_{suite}, the unbooked suite cost, and V_{suite}, the unavailable suite cost. However, we are not given the costs for an unbooked or unavailable regular room, so we will have to estimate it based on the information we have for suites.

$$
\begin{aligned}
R_{suite} &= 285.00 \\
R_{room} &= 85.00 \\
U_{suite} &= -150.00 \\
V_{suite} &= -190.00 \\
U_{room} &= R_{room}\left(\frac{U_{suite}}{R_{suite}}\right) \approx -45.00 \\
V_{room} &= R_{room}\left(\frac{V_{suite}}{R_{suite}}\right) \approx -57.00
\end{aligned} \tag{1}
$$

We now make the assumption that for any number of regular rooms, that is, $S < 10$, the regular rooms will always be fully booked.

Our revenue loss function, L, is parameterized by α, the number of suites available.

$$
L(x;\alpha) = \begin{cases} (x-\alpha)U_{suites} & \text{if } x \geq \alpha \\ |(x-\alpha)|V_{suites} & \text{if } x < \alpha \end{cases} \tag{2}
$$

Then our profit for any configuration of S is the profits of the suites and rooms, minus the cost of underbooked or overbooked suites.

$$
Y = min(X, S) \cdot R_{suite} + (20 - 2S) \cdot R_{room} + L_{suite}(X; S) \tag{3}
$$

To see how Y behaves under the various configurations, we simulate 100,000 draws from $Bin(10, 0.6055)$, and calculate Y under each configuration. We then take the mean, $\overline{Y}$ under each configuration and plot our results.

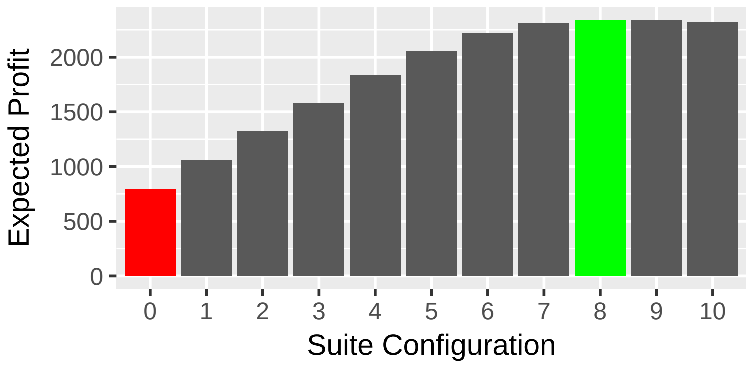

Figure 2: Expected profits with each configuration, and the naive assumption that all regular rooms are always filled. The red and green bars highlight the worst and best configurations, respectively.

With the assumption that regular rooms are always fully booked under any configuration $S < 10$, we see profits steadily rise the more suites we add until they peak at 8 suites. They slightly decline if we add a 9th or 10th suite.

2 Generalizing the model

While it would be nice if there was a constant demand for rooms, in reality this is not the case. To get a more accurate recommendation, we can relax the first assumption by generalizing the model.

Since we are missing some crucial data to make an optimal recommendation, we must make some further assumptions.

1. What is the demand distribution for regular rooms?
2. If a suite is not available but a room is, what is the probability that guests will downgrade to a regular room versus booking with a competitor?
3. If a regular room is not available but a suite is, what is the probability the guest will upgrade to a suite, versus booking with a competitor?

The number of those wishing to book suites remains X as defined before. Now, let W be the number wishing to book regular rooms, then we have

$$W \sim Bin(n, \theta_1) \tag{4}$$

We have no data with which to estimate θ_1, so instead, we'll fix n to be 20, the maximum number of rooms if we build no suites, and simulate with $\theta_1 \in [0, 0.1, 0.2, \ldots, 1]$.

We generalize our loss function, L with parameter ω, which represents either rooms or suites. α is the number of rooms or suites available, respective of ω.

$$L_\omega(x;\alpha) = \begin{cases} (x-\alpha)U_\omega & \text{if } x \geq \alpha \\ |(x-\alpha)|V_\omega & \text{if } x < \alpha \end{cases} \tag{5}$$

If we have more guests, X, than available suites, S, we define X' as the *overflow* - the number of suite guests we cannot accommodate. Likewise, for W regular room guests, we define W' as the overflow of regular room guests.

$$\begin{aligned} X' &= max(0, X - S) \\ W' &= max(0, W - (20 - 2S)) \end{aligned} \tag{6}$$

Another assumption that we can make is that the overflow population would be willing to downgrade to a regular room with probability θ_2, or upgrade to a suite with probability θ_3, for populations X' and W' respectively. We can then define new random variables X'' and W'' as those from the overflow willing to switch room types.

$$\begin{aligned} X'' &\sim Bin(X', \theta_2) \\ W'' &\sim Bin(W', \theta_3) \end{aligned} \tag{7}$$

Thus, we arrive at the model,

$$\begin{aligned} Y = {} & min\big((X + W''), S\big) \cdot R_{suite} + min\big((W + X''), (20 - 2S)\big) \cdot R_{room} \\ & + \big[L_{suite}\big(X + W''; S\big) + L_{room}\big(W + X''; (20 - 2S)\big)\big] \end{aligned} \tag{8}$$

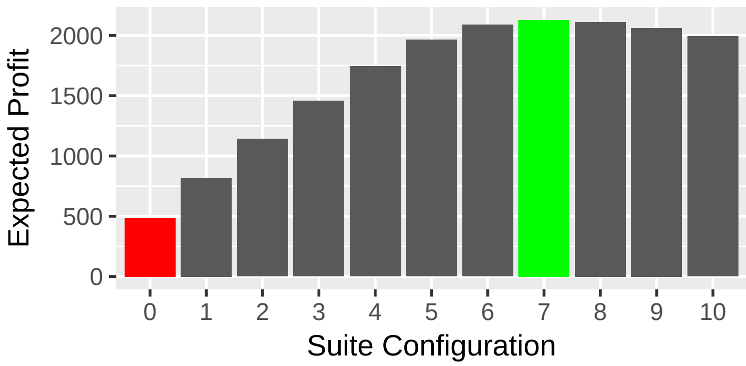

Figure 3: Expected profits with each suite configuration, under the general model with average room demand, $\theta_1 = 0.5, \theta_2 = 0.4, \theta_3 = 0.1$

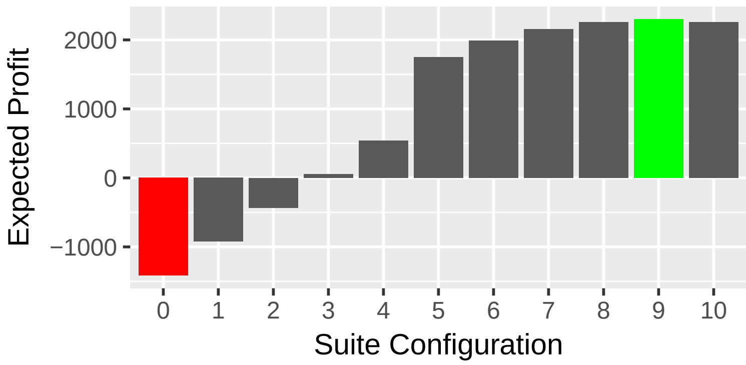

Figure 4: Expected profits with each suite configuration, under the general model with weak room demand $\theta_1 = 0.1, \theta_2 = 0.4, \theta_3 = 0.1$

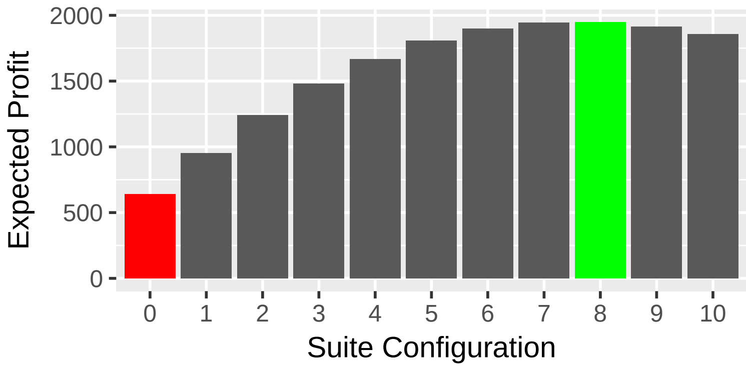

Figure 5: Expected profits with each suite configuration, under the general model with heavy room demand $\theta_1 = 0.7, \theta_2 = 0.4, \theta_3 = 0.1$