Homework Assignment 3

Deadline: February 12, 11:59 pm

1. For the cosine model on Page 34-35, it can be shown that the variance of the estimate for the trend in January can be given by Equation (3.4.6) on Page 38:

$$Var(\hat{\mu}_1) = Var(\hat{\beta}_0) + Var(\hat{\beta}_1)\left[\cos\left(\frac{2\pi}{12}\right)\right]^2 + Var(\hat{\beta}_2)\left[\sin\left(\frac{2\pi}{12}\right)\right]^2.$$

   Given the fact that

$$\begin{aligned} \hat{\beta}_0 &= \frac{1}{n}\sum_{t=1}^{n} Y_t \\ \hat{\beta}_1 &= \frac{2}{n}\sum_{t=1}^{n}\left[\cos\left(\frac{2\pi}{12}t\right)Y_t\right] \\ \hat{\beta}_2 &= \frac{2}{n}\sum_{t=1}^{n}\left[\sin\left(\frac{2\pi}{12}t\right)Y_t\right] \end{aligned}$$

   and

$$\begin{aligned} Y_t &= \mu_t + X_t \\ \{X_t\} &\text{ is } \text{white noise with mean 0 and variance } \sigma^2 \end{aligned}$$

   Show that $Var(\hat{\mu}_1) = \frac{3\sigma^2}{n}$. (Hint: $\sum_{t=1}^{n}\left[\cos\left(\frac{2\pi}{12}t\right)\right]^2 = \sum_{t=1}^{n}\left[\sin\left(\frac{2\pi}{12}t\right)\right]^2 = \frac{n}{2}$.)

2. Let μ be a constant, and $\{e_t\}$ be a white noise process with mean zero and variance σ_e^2. Consider the following three stochastic processes of $\{Y_t\}$:

   (a) $Y_t = \mu + e_t$
   (b) $Y_t = \mu + e_t - e_{t-1}$
   (c) $Y_t = \mu + e_t + e_{t-1}$

   For each of the three processes, find ρ_k for $k > 0$. Furthermore, for each $\{Y_t\}$, find $Var(\bar{Y})$ where $\bar{Y} = \frac{1}{n}\sum_{t=1}^{n} Y_t$.

3. The data file `wages` contains monthly values of the average hourly wages (in dollars) for workers in the U.S. apparel and textile products industry for July 1981 through July 1987.

   (a) Display and interpret the time series plot for these data.
   (b) Use least squares to fit a linear time trend to this time series. Interpret the regression output. Save the standardized residuals from the fit for further analysis.

(c) Construct and interpret the time series plot of the standardized residuals from part (b).

(d) Use least squares to fit a quadratic time trend to the wages time series. Interpret the regression output. Save the standardized residuals from the fit.

(e) Construct and interpret the time series plot of the standardized residuals from part (d).

(f) Perform a runs test on the standardized residuals from part (d) and interpret the results.

(g) Calculate and interpret the sample autocorrelations for the standardized residuals from part (d).

(h) Investigate the normality of the standardized residuals from part (d). Consider histograms and normal probability plots. Interpret the plots. Perform the Shapiro-Wilk test for Normality.

4. The data file `retail` lists total U.K. retail sales (in billions of pounds) from January 1986 through March 2007. Note that year 2000 = 100 is the base year.

   (a) Display and interpret the time series plot for these data. Do you see any seasonally trend from the data?

   (b) Use least squares to fit a seasonal-means plus linear time trend to this time series. Interpret the regression output. Save the standardized residuals from the fit for further analysis.

   (c) Construct and interpret the time series plot of the standardized residuals from part (b). Are there still any seasonally trend in the residuals?

   (d) Perform a runs test on the standardized residuals from part (b) and interpret the results.

   (e) Calculate and interpret the sample autocorrelations for the standardized residuals from part (b).

   (f) Investigate the normality of the standardized residuals from part (b). Consider histograms and normal probability plots. Interpret the plots. Perform the Shapiro-Wilk test for Normality.