Homework Assignment 4

Deadline: February 22, 11:59 pm

1. Calculate and sketch the autocorrelation functions ρ_k for the following stationary processes.

   (a) $Y_t = -0.9Y_{t-1} + e_t$

   (b) $Y_t = 8 + e_t - 0.75e_{t-1} + 0.5e_{t-2} - 0.25e_{t-3}$

2. Verify that for an MA(1) process

$$\max_\theta \rho_1 = 0.5 \qquad \text{and} \qquad \min_\theta \rho_1 = -0.5$$

3. Consider the ARMA(1,2) model

$$Y_t = 0.7Y_{t-1} + e_t + 0.8e_{t-1} - 0.6e_{t-2}$$

   Assume that $\{e_t\}$ is a white noise process with zero mean and unit variance ($\sigma_e^2 = 1$). Find the numerical values of ρ_0, ρ_1 and ρ_2 by hand. Also find a recursive relationship between ρ_k and ρ_{k-1} for $k > 2$.

4. Consider a "AR(1)" process satisfying $Y_t = \phi Y_{t-1} + e_t$, where $t > 0$, ϕ can be any number and $\{e_t\}$ is a white noise process with zero mean and variance σ_e^2. Let Y_0 be a random variable with mean μ and variance σ_0^2. Show that for $t > 0$ we have

   (a) $Y_t = e_t + \phi e_{t-1} + \phi^2 e_{t-2} + \cdots + \phi^{t-1} e_1 + \phi^t Y_0$

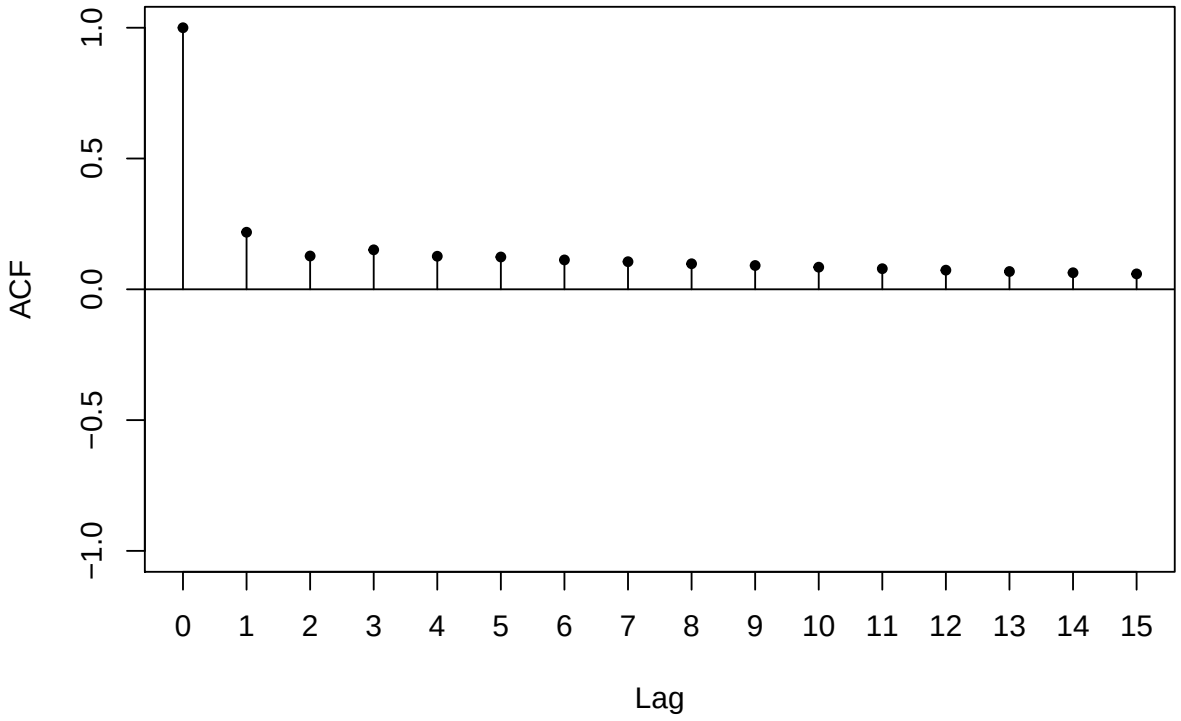
   (b) $E(Y_t) = \phi^t \mu$.

   (c) $Var(Y_t) = \begin{cases} \frac{1-\phi^{2t}}{1-\phi^2}\sigma_e^2 + \phi^{2t}\sigma_0^2 & \text{for } \phi \neq 1 \\ t\sigma_e^2 + \sigma_0^2 & \text{for } \phi = 1 \end{cases}$

   (d) Suppose $\mu = 0$. Show that if $\{Y_t\}$ is stationary, then $Var(Y_t) = \frac{\sigma_e^2}{1-\phi^2}$.

5. The following command in R will plot the **theoretical** autocorrelation function of an ARMA(2,2) model $Y_t = 0.5Y_{t-1} + 0.4Y_{t-2} + e_t - 0.7e_{t-1} - 0.6e_{t-2}$ for the first 15 lags:

```
n = 15
ACF = ARMAacf(ar=c(0.5,0.4), ma=c(-0.7,-0.6), lag.max=n)
plot(0:n, ACF, type='h', xlab='Lag', ylim=c(-1,1), xaxp=c(0,n,n))
points(0:n, ACF, pch=20)
abline(h=0)
```



Modify the code to generate the theoretical autocorrelation functions up to 20 lags of the following ARMA prcoesses:

(a) MA(1) with $\theta = 0.5$

(b) MA(1) with $\theta = -0.5$

(c) MA(2) with $\theta_1 = \theta_2 = 0.1$

(d) AR(1) with $\phi = 0.4$

(e) AR(1) with $\phi = -0.4$

(f) AR(2) with $\phi_1 = 0.5$ and $\phi_2 = -0.9$

(g) ARMA(1,1) with $\phi = 0.7$ and $\theta = 0.4$

(h) ARMA(1,2) given in Question 3