Homework Assignment 2

Deadline: January 31, 11:59 pm

Note: $\{e_t\}$ always denotes a sequence of independent, identically distributed random variables with mean zero and constant variance σ_e^2.

1. If $Y_t = e_t - e_{t-7}$, show that $\{Y_t\}$ is (weakly) stationary and that, for $k > 0$, its autocorrelation function is nonzero only for lag $k = 7$.

2. Suppose that $\{Y_t\}$ is (weakly) stationary with autocovariance function γ_k.

   (a) Show that $W_t = \nabla Y_t = Y_t - Y_{t-1}$ is (weakly) stationary by finding the mean and autocovariance function for $\{W_t\}$.

   (b) Show that $U_t = \nabla^2 Y_t = \nabla[\nabla Y_t] = \nabla[Y_t - Y_{t-1}] = Y_t - 2Y_{t-1} + Y_{t-2}$ is (weakly) stationary.

3. For a fixed positive integer s and a constant α, consider the stochastic process defined by $Y_t = e_t + \alpha e_{t-1} + \alpha^2 e_{t-2} + \cdots + \alpha^s e_{t-s}$.

   (a) Show that the process is (weakly) stationary for any value of α.

   (b) Find the autocorrelation function.

4. Let $\{X_t\}$ be the time series of interest, however, due to measurement error we actually observe $Y_t = X_t + e_t$. We assume that $\{X_t\}$ and $\{e_t\}$ are independent processes. We call X_t the signal, and e_t the noise. If $\{X_t\}$ is stationary with autocorrelation function ρ_k, show that $\{Y_t\}$ is also stationary with $Corr(Y_t, Y_{t-k}) = \rho_k/(1 + \sigma_e^2/\sigma_X^2)$, where σ_e^2/σ_X^2 is called the signal-to-noise ratio.

5. Suppose
$$Y_t = \beta_0 + \sum_{i=1}^{k} [A_i \cos(2\pi f_i t) + B_i \sin(2\pi f_i t)]$$
where $\beta_0, f_1, f_2, \ldots, f_k$ are constants, and $A_1, A_2, \ldots, A_k, B_1, B_2, \ldots, B_k$ are independent random variables with zero means and variances $Var(A_i) = Var(B_i) = \sigma_i^2$. Show that $\{Y_t\}$ is (weakly) stationary by finding the mean and autocovariance function.