Homework Assignment 9

Deadline: April 24, 11:59 pm

1. Consider an AR(1) model with $\phi = -0.5$ and $\mu = 14$. If our last observation occurred at time 50 and the value observed was 12, calculate $\hat{Y}_{50}(1)$, $\hat{Y}_{50}(2)$, and $\hat{Y}_{50}(10)$.

2. Consider the AR(2) model $Y_t = 5 + 1.1Y_{t-1} - 0.5Y_{t-2} + e_t$ with $\sigma_e^2 = 2$.

   (a) If the last 3 observed values of the sequence are $Y_{90} = 9$, $Y_{91} = 11$, and $Y_{92} = 10$, find the forecasts $\hat{Y}_{92}(1)$, $\hat{Y}_{92}(2)$, and $\hat{Y}_{92}(3)$.

   (b) Find the variances of $e_{92}(1)$, $e_{92}(2)$, $e_{92}(3)$.

   (c) Find the 95% prediction limits for the forecasts $\hat{Y}_{92}(1)$, $\hat{Y}_{92}(2)$, and $\hat{Y}_{92}(3)$.

3. Use `arima.sim` with $n = 100$ to simulate an ARIMA(0,2,2) with $\theta_1 = 1$ and $\theta_2 = -0.75$ (Use your NUID in `set.seed`). Store the data as `y`. Look at the data, notice that there are 102 observations and the first two are both zero. Remove the first two (zero) observations.

   (a) Fit an ARIMA(0,2,2) model to `y[1:95]`, the first 95 observations of the simulated series, and find the maximum likelihood estimates of θ_1 and θ_2.

   (b) Construct a time series plot that shows observations 91 through 95, and the forecasts (with prediction limits) for observations 96 through 100.

   (c) Use `points(96:100, y[96:100], col="red")` to add the actual observations to the plot. Compare the forecasts with the actual observations.

4. We have previously fitted an AR(3) to the square-root of the `hare` data.

   (a) Fit the model and plot the time series along with the forecasts (with prediction limits) for the next 12 observations of the square root of hare abundance.

   (b) Use your answer to part (a) to find the numerical values of the forecasts for the hare abundance for the next 12 years.

   (c) How do we plot the values you found in (b) along with the corresponding intervals? The plot command has a `transform` option that allows the data to be transformed. Firstly, dene the square function as follows: `square=function(x){y=x^2}`. Then use `transform=square` inside your plot command. Compare the forecast plot to those values calculated in (b). Are they the same?

5. Consider the multiplicative seasonal ARIMA$(0, 0, 2) \times (0, 1, 0)_4$ model.

   (a) Write down the model: $Y_t =$ ________________________________.

   (b) Find the first four ψ-weights for this model.

   (c) Suppose that $\theta_1 = 0.5$, $\theta_2 = -0.25$ and $\sigma_e^2 = 1$, and that the last four observed values were: 25, 20, 25, 40, with corresponding residuals: 2, 1, 2, 3. Predict the next 4 values.

   (d) Construct prediction intervals for the predictions found in (b).

6. The dataset `JJ` contains the earnings per share for each quarter from 1960 to 1980 for Johnson and Johnson.

   (a) Plot the time series and also the logarithm of the series. Whether should we use the log transformation to model this series? Explain.

   (b) Based on your decision of part (a), find the most appropriate ARIMA$(p, d, q) \times (P, D, Q)_s$ to fit the data or transformed data.

   (c) Estimate the parameters of the chosen model.

   (d) Use your model to predict the next 4 values of the series.