

Homework 5

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1. For each of the following, identify it as an ARIMA model. That is, find the values of p, d , and q and the values of the parameters (ϕ 's and θ 's). Recall that by definition ARMA(p, q) models must be stationary and invertible.

(a) $Y_t = 0.6Y_{t-1} + 0.4Y_{t-2} + e_t - 0.5e_{t-1} + 0.25e_{t-2}$

This appears to be an ARMA(2, 2), with $\phi_1 = 0.6$ and $\phi_2 = 0.4$, $\theta_1 = -0.5$ and $\theta_2 = 0.25$.

We must verify the assumptions that it is stationary and invertible.

We have

$$\begin{aligned}\phi_1 + \phi_2 &= 0.6 + 0.4 = 1.0 \not< 1.0 \\ \phi_2 - \phi_1 &= 0.4 - 0.6 = -0.2 < 1 \\ |\phi_2| &= 0.4 < 1\end{aligned}$$

Here the first constraint is violated, so

(b) $Y_t = 2Y_{t-1} - Y_{t-2} + e_t$

(c) $Y_t = 0.5Y_{t-1} - 0.5Y_{t-2} + e_t - 0.1e_{t-1}$

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2. For each ARIMA model described in Question 1, find the numerical values of $\psi_0, \psi_1, \psi_2, \psi_3, \psi_4$ and a recurrence relation for $\psi_k, k > 4$.

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3. Consider a stationary process $\{Y_t\}$. Show that if $\rho_1 < 0.5$ then ∇Y_t has a larger variance than Y_t .

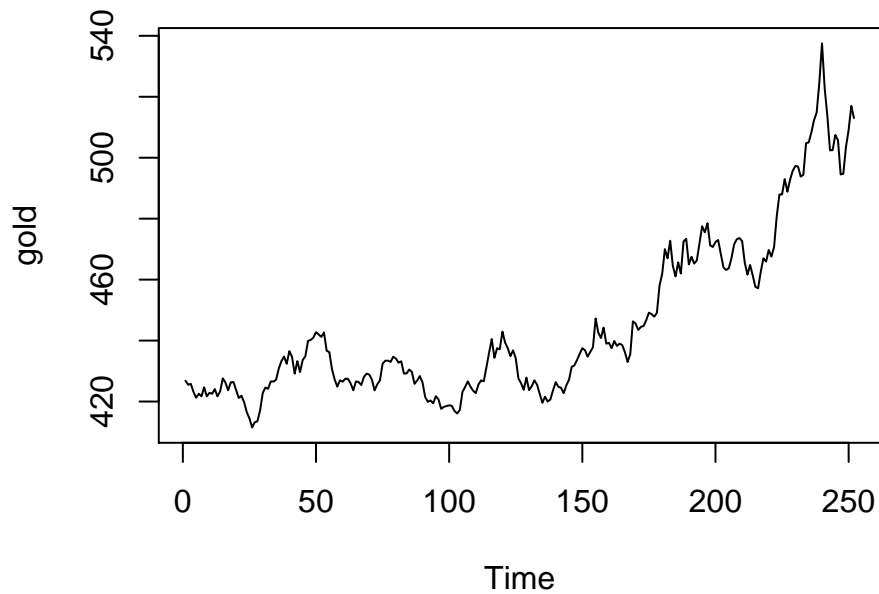
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4. The data set `gold` from the TSA library contains the daily price of gold for 252 trading days in 2005.

```
data(gold)
```

(a) Construct a time series plot of the price of gold Y_t . What are the interesting features of this process?

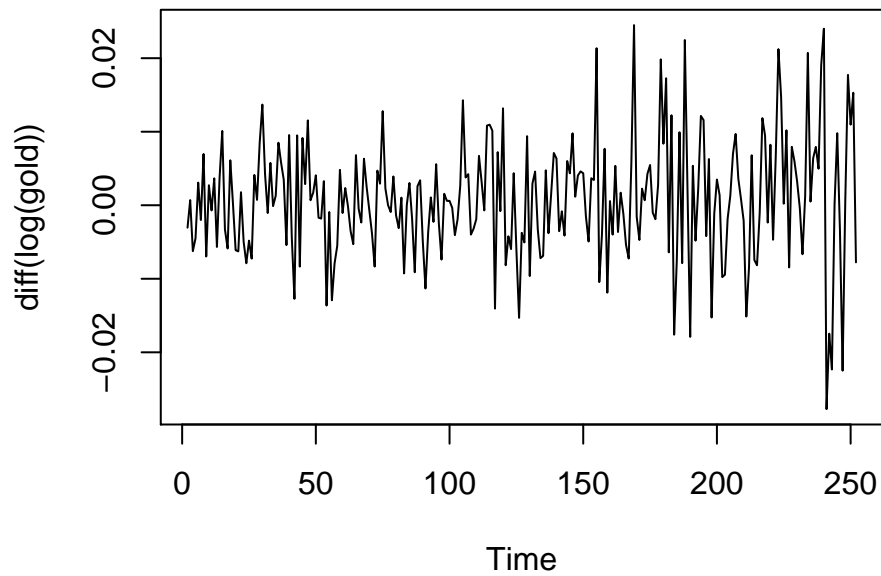
```
plot(gold)
```



The price of gold does not seem to be based on a deterministic trend, as we can see it begins to increase in variance after 150 days.

(b) Let $W_t = \nabla(\ln Y_t)$, the differences of the logarithms. Construct a time series plot of W_t . Does it look stationary?

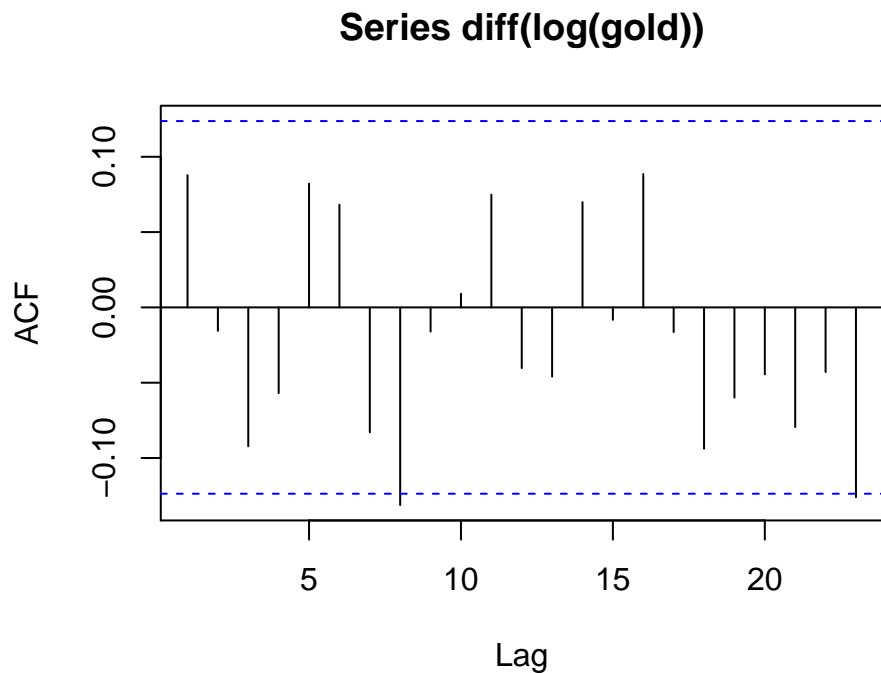
```
plot(diff(log(gold)))
```



This looks heteroskedastic, and therefore not stationary, however, the variance is actually quite small, between -0.0277298 and 0.0244966.

(c) Use the sample ACF to investigate whether W_t is a white noise process.

```
acf(diff(log(gold)))
```



(d) Investigate whether W_t is a normal white noise process.

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