STAT 4440/8446 Final Exam

Deadline: May 4, 11:59 pm

1. Let $\{X_t\}$ be a sequence of iid random variables with mean 0 and variance σ_X^2, and ω a finite constant. Decide whether $Y_t = X_t \cos(\omega t) + X_{t-1} \sin(\omega t)$ is stationary or not.

2. For each of the following, identify it as an ARIMA model or a multiplicative seasonal ARIMA model. That is, find the values of p, d, q, P, D, Q, period s, and the values of the parameters (ϕ's, θ's, Φ's, and Θ's).

   (a) $Y_t = 2.8Y_{t-1} - 2.6Y_{t-2} + 0.8Y_{t-3} + e_t - 0.76e_{t-1} - 0.2e_{t-2}$

   (b) $Y_t = -0.4Y_{t-1} + Y_{t-4} + 0.4Y_{t-5} + e_t - 0.6e_{t-1} - 0.5e_{t-4} + 0.3e_{t-5}$

3. Consider an $ARMA(4,3)$ model:

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \phi_4 Y_{t-4} + e_t - \theta_1 e_{t-1} - \theta_3 e_{t-3}$$

   where $\{e_t\}$ is a sequence of iid random variables with zero mean and unit variance.

   (a) Find expressions for the ψ-weights: ψ_1, ψ_2, ψ_3, ψ_4, and ψ_5 in terms of the ϕ's and θ's. Find a recursive equation for ψ_k for $k > 5$.

   (b) Find a system of 5 equations involving $\gamma_0, \gamma_1, \gamma_2, \gamma_3, \gamma_4$ (and perhaps the ϕ's, θ's, and ψ's). You do not need to solve these equations.

   (c) Find a recursive equation for ρ_k for $k \geq 5$.

4. Consider the following model:

$$Y_t = 0.9Y_{t-4} + e_t - 0.5e_{t-1}$$

   (a) Find the autocorrelation function, and evaluate ρ_1, ρ_2, ρ_3, and ρ_4.

   (b) Find the partial autocorrelation function, and evaluate $\phi_{11}, \phi_{22}, \phi_{33}$, and ϕ_{44}. Show all working.

   (c) Suppose that we have 100 observations generated by this process. If the last 5 observations were 24, 17, 18, 20, and 25 with corresponding residuals 0, 1, 0, -1, and 1, compute the forecasts for the next 5 observations.

(d) If $\sigma_e^2 = 1$, calculate the variance of each of the forecasts made in (4c).

(e) Calculate 95% prediction limits for each of your forecasts.

5. The file `"FinalExam.RData"` contains three datasets: `y1`, `y2`, and `y3`. You can load the data into `R` using the command `load("FinalExam.RData")`. Note: If you fit a model and later decide that the model is unsuitable, do not remove the model from your answer. Include it, discuss why it was unsuitable, and then move on to another model.

   (a) Fit an appropriate model to `y1`. Use the third and fourth Bootstrap resampling methods to get the 95% confidence intervals for the parameters in your model.

   (b) Fit an appropriate model to `y2`. Predict the next 10 observations with 95% prediction limits.

   (c) `y3` is a data frame containing two time series `y3a` and `y3b`.

      i. Ignore `y3a` and fit an appropriate model to `y3b`. Forecast the next 4 observations with 95% prediction limits.

      ii. Are `y3a` and `y3b` correlated? If so, at what lags?

      iii. (Extra bonus) Fit an appropriate model to `y3b` using `y3a`. Forecast the next 4 observations with 95% prediction limits.