Assignment 4 (2 pages)

1. Consider data from a Normal population with unknown mean μ and variance σ^2. A random sample of 100 observations is taken from this population, and the sample mean and variance were calculated to be 50 and 25 respectively.

   (a) If we choose to use a $N - Inv - \chi^2(40, 0.64, 1, 16)$ prior distribution, write down the corresponding posterior distribution.

   (b) Either analytically or via simulation, construct 95% credible intervals for σ^2 and μ.

2. Two random variables are said to have a bivariate normal distribution with parameters $\mu_U, \mu_V, \sigma_U^2, \sigma_V^2$, and ρ if they have the following density function:

$$f(u, v) = \frac{1}{2\pi\sigma_U\sigma_V\sqrt{1-\rho^2}} e^{-\frac{1}{2(1-\rho^2)}\left[\frac{(u-\mu_U)^2}{\sigma_U^2} + \frac{(v-\mu_V)^2}{\sigma_V^2} - \frac{2\rho(u-\mu_U)(v-\mu_V)}{\sigma_U\sigma_V}\right]}$$

   where μ_U and σ_U^2 are the mean and variance of U, μ_V and σ_V^2 are the mean and variance of V, and ρ is the correlation between U and V.

   Replace the uniform prior on α and β in the analysis of the bioassay by a bivariate normal prior with $\alpha \sim Normal(0, 4)$, $\beta \sim Normal(10, 100)$, and $corr(\alpha, \beta) = 0.5$.

   Repeat all the computations and plots discussed in section 3.7 and in class.

3. Consider the airline fatalities data discussed in the previous exercise. Let us suppose that we now assume that the number of fatal accidents in year t follows a Poisson distribution with mean $\alpha + \beta t$.

   (a) If we let y_t represent the number of fatal accidents in year t, write down $p(y_t|\alpha, \beta)$ the likelihood for year t in terms of the parameters α, and β.

   (b) If we assume uniform priors on α and β, write the posterior density for (α, β)

   (c) Following the same idea as the bioassay example (and the previous question) create a grid of possible α and β values on which to evaluate the joint posterior and plot the contours. Start with large ranges for α and β and refine based on the contour plot. Include all your iterations in your answer, not just your final grid and contour plot.

(d) Simulate 100,000 values of α and β from the joint posterior and plot the histogram of the posterior density of the expected number of fatal accidents in 1986, $\alpha + 1986\beta$.

(e) Use your simulated values of α and β to simulate the number of fatal accidents in 1986. Use your simulations to construct a 95% predictive (credible) interval.

(f) Returning to your simulated values of β, calculate (well, estimate) $P(\beta < 0)$, that is, the probability that the number of fatal accidents per year is decreasing.