Final Exam Question 2

(a) Consider a random sample $y_1, y_2, \ldots, y_n$ taken from a Normal population with mean $= \mu$ and known variance $= \sigma^2$. Show that the likelihood is equivalent to the likelihood of a single observation of $\bar{y}$ taken from a Normal population with a mean of μ' and $\sigma^{2'}$ where $\bar{y}$ is the mean of the y's. Find the appropriate expressions for μ' and $\sigma^{2'}$.

(b) Suppose that y_i is Normally distributed with mean $= \mu$ and known variance $= \sigma_i^2$, for $i = 1, \ldots, n$. Show that if a uniform prior for μ is used then the posterior distribution of μ is Normal with mean $= \frac{\sum_{i=1}^{n} y_i/\sigma_i^2}{\sum_{i=1}^{n} 1/\sigma_i^2}$ and variance $= (\sum_{i=1}^{n} 1/\sigma_i^2)^{-1}$

Show all working.