Assignment 1

1. Read Chapter 1, identify any areas of a confusion for further class discussion (Try to do this part as soon as possible, I will ask at the start of each class next week if there are any topics we'd like to go over.)

2. Consider an urn containing 9 balls, which can be either red or green. Let X be the number of red balls in the urn and before observing any balls we will assume that all possible values of X from 0 to 9 are equally likely.
   Suppose we plan to draw 3 balls from the urn, and let $Y_i = 1$ if the ith ball is red, and $Y_i = 0$ if the ith ball is green for $i = 1, 2, 3$.
   When we draw the 3 balls, we observe $Y_1 = 1$, $Y_2 = 1$, and $Y_3 = 0$. As per our examples in class, construct a table with columns X, *Prior*, *Likelihood*, *Likelihood* $\times$ *Prior*, and *Posterior* to obtain the Posterior Distribution of X.

3. Let Y_1 be the number of successes in $n = 10$ independent trials where each trial results in a success or failure, and θ, the probability of success in each trial is the same for each trial. Suppose we believe there are 4 possible values of θ: $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$, which we view as equally likely.
   Now suppose we observe $Y_1 = 7$, use a table similar to the previous question to find the posterior distribution of θ.

Questions 4, 5, and 6 on next page...

4. Following on from the previous question, suppose we observe another 5 independent trials and $Y_2 = 2$ successes are observed in those 5 trials. Use the posterior distribution for θ from the previous question as the new prior distribution of θ and use a table to find the new posterior distribution of θ based on the added trials.

5. Suppose we combine all 15 trials from questions 3 and 4 together and think of them as a single set of data in which we observed 9 successes. Starting with our initial uniform prior, use a table to find the posterior distribution of θ. Compare your answer to your answer at the end of question 4.

6. In R, install a package called `Bolstad`. This package includes a function called `binodp`, which stands for "Binomial Data, Discrete Prior", exactly like the situation described in questions 3, 4, and 5. The function requires 4 inputs, the number of successes, the number of trials, a vector containing the possible values of θ, and a vector containing the corresponding prior probabilities.
   Note: In R, a vector is specified by `c()`, so in this example, the vector containing the possible values of θ would be `c(1/5,2/5,3/5,4/5)`.
   Use this `binodp` function to calculate the posterior distribution based on the data in question 5. The function will generate several output tables and one graph. Copy/Paste (do not manually copy) the last output table (the posterior distribution) and the graph into your assignment.