# Game Theory: Week 1

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# Logistics

- Syllabus is required reading
- Canvas
- TAs
- Textbook
- Participation crucial
  - clickers

# Structure

- Across weeks
  - static games of complete information
  - dynamic games of complete information
  - static games of incomplete information
  - dynamic games of incomplete information
- Within each week
  - play a game
  - solution concept
  - analyze play
  - design of games

# Goal of the class

- How to *play* games
  - predict what others will do and respond appropriately
  - wide set of common applications
    - e.g., buying a house
- How to *design* games
  - structure the strategic environment
  - applications of great importance
    - medical match
    - FCC spectrum auction
  - tournament pay?

# Static games of complete information

# Set-up

- $n$ players simultaneously make a choice
- player $i$ chooses $s_i \in S_i$
- outcome of the game $\boldsymbol{s} = (s_1, ..., s_n)$
- player $i$'s payoff: $u_i(\boldsymbol{s})$
  - notation $\boldsymbol{s} = (s_i, \boldsymbol{s}_{-i})$
- assume everything commonly known
  - (except what others will do)

# Notation

- With 2 players we can describe the game with a matrix

$$\begin{array}{ccc} & L & R \\ U & 5,7 & 1,4 \\ M & 8,2 & 0,4 \\ D & 1,7 & 9,4 \end{array}$$

# Expected utility

- Suppose player $i$ expects
  - $L$ with probability $p$
  - $R$ with probability $1 - p$
- Player $i$ should choose $s_i$ to maximize

$$pu_i(s_i, L) + (1 - p)\, u_i(s_i, R)$$

- What if she is risk-averse?

# Example

| | $L$ | $R$ |
|---|---|---|
| $U$ | $\$0, \$3$ | $\$11, \$0$ |
| $D$ | $\$5, \$0$ | $\$5, \$2$ |

- Suppose $P1$ expects $L$ with 50% chance
- Should she play $U$ or $D$?
- Depends on risk-aversion

| | $L$ | $R$ |
|---|---|---|
| $U$ | $u(\$0), \$3$ | $u(\$11), \$0$ |
| $D$ | $u(\$5), \$0$ | $u(\$5), \$2$ |

# Example

| | $L$ | $R$ |
|---|---|---|
| $U$ | $\$0, \$3$ | $\$11, \$0$ |
| $D$ | $\$5, \$0$ | $\$5, \$2$ |

- Suppose $P1$ expects $L$ with 50% chance
- Should she play $U$ or $D$?
- Depends on risk-aversion and social preferences

| | $L$ | $R$ |
|---|---|---|
| $U$ | $u(\$0, \$3), \$3$ | $u(\$11, \$0), \$0$ |
| $D$ | $u(\$5, \$0), \$0$ | $u(\$5, \$2), \$2$ |

# Example

| | $L$ | $R$ |
|---|---|---|
| $U$ | \$0, \$3 | \$11, \$0 |
| $D$ | \$5, \$0 | \$5, \$2 |

- Suppose $P1$ expects $L$ with 50% chance
- Should she play $U$ or $D$?
- Suppose risk-averse and selfish

| | $L$ | $R$ |
|---|---|---|
| $U$ | $0, \cdot$ | $7, \cdot$ |
| $D$ | $4, \cdot$ | $4, \cdot$ |

# Example

| | $L$ | $R$ |
|---|---|---|
| $U$ | \$0, \$3 | \$11, \$0 |
| $D$ | \$5, \$0 | \$5, \$2 |

- Suppose $P1$ expects $L$ with 50% chance
- Should she play $U$ or $D$?
- Suppose risk-averse and selfish

| | $L$ | $R$ |
|---|---|---|
| $U$ | $0, \cdot$ | $7, \cdot$ |
| $D$ | $4, \cdot$ | $4, \cdot$ |

- Maximize expected utility $\rightarrow$ play $D$
- Outcome matrix vs. payoff matrix

# The games we'll play

# Game of dax

| | $A$ | $B$ | $C$ | $D$ | $E$ |
|---|---|---|---|---|---|
| $A$ | $7, 3$ | $6, 5$ | $6, 8$ | $6, 3$ | $5, 1$ |
| $B$ | $7, 8$ | $8, 1$ | $3, 0$ | $4, 4$ | $9, 6$ |
| $C$ | $6, 5$ | $2, 9$ | $7, 6$ | $3, 3$ | $5, 1$ |
| $D$ | $9, 2$ | $3, 8$ | $7, 3$ | $3, 9$ | $2, 0$ |
| $E$ | $1, 2$ | $9, 2$ | $5, 3$ | $8, 4$ | $1, 1$ |

# Games we will study

- Parables
  - games that distill key aspects of a strategic situation
  - e.g., prisoner's dilemma
- Paradigms
  - representative examples of real-world games
  - e.g., first-price auction

# Prisoner's dilemma

| | $A$ | $B$ |
|---|---|---|
| $A$ | $5, 5$ | $1, 9$ |
| $B$ | $9, 1$ | $3, 3$ |

# Dominance

- Strategy $s_i$ *dominates* strategy $s_i'$ if $u_i\left(s_i, \boldsymbol{s}_{-i}\right) > u_i\left(s_i', \boldsymbol{s}_{-i}\right)\ \forall \boldsymbol{s}_{-i}$
- A strategy is *dominant* if it dominates all other strategies
- A strategy is *dominated* if there is a strategy that dominates it

# Dominance

- If there is a dominant strategy: play it
  - it does better than any other other strategy *no matter what others do*
- If there is a dominated strategy: avoid it
  - there is something better to do *no matter what others do*

# Prisoner's dilemma

| | $A$ | $B$ |
|---|---|---|
| $A$ | $5, 5$ | $1, 9$ |
| $B$ | $9, 1$ | $3, 3$ |

- $A$ is a dominated strategy
  - no justification for playing it
- $B$ is a dominant strategy
  - unambiguously the best action

# Parable of prisoner's dilemma

- Key features:
  - each player has one dominant, one dominated strategy
  - rational play induces an inefficient outcome
- Stark contrast with the most fundamental idea in classical economics

  *Every individual... neither intends to promote the public interest, nor knows how much he is promoting it... he intends only his own security; and by directing that industry in such a manner ... he is ... led by an invisible hand to promote an end which was no part of his intention.*

  *It is not from the benevolence of the butcher, the brewer, or the baker, that we expect our dinner, but from their regard to their own interest. We address ourselves, not to their humanity but to their self-love...*
- First Fundamental Theorem of Welfare Economics:
  - selfish rational behavior leads to efficient outcomes

# Parable of prisoner's dilemma

- Key features:
  - each player has one dominant, one dominated strategy
  - rational play induces an inefficient outcome
- Many situations have this payoff structure
- We say any such situation is "a prisoner's dilemma"
- Recognizing that your situation fits the parable first step in analysis

# Information provision

- Two pharmaceutical companies, $i = 1, 2$
- $\omega = (\omega_1, \omega_2)$; $\omega_i \in \{l, h\}$ indicates quality of $i$'s drug
- Qualities are independent and $Pr(\omega_i = h) = \frac{1}{5}$
- Unit mass of consumers, all prefer high to low quality
  - $\frac{1}{2}$ always buy the drug with higher expected quality
  - $\frac{1}{2}$ buy the drug with higher expected quality if $Pr(\omega_i = h) > \frac{1}{2}$
  - randomize if indifferent
- Each firm chooses one of two clinical trials:
  - *null*: uninformative
  - *reveal*: fully reveals the quality of own drug

| | *null* | *reveal* |
|---|---|---|
| *null* | 0.25, 0.25 | 0.40, 0.20 |
| *reveal* | 0.20, 0.40 | 0.34, 0.34 |

# Information provision

| | *null* | *reveal* |
|---|---|---|
| *null* | $0.25, 0.25$ | $0.40, 0.20$ |
| *reveal* | $0.20, 0.40$ | $0.34, 0.34$ |

- Prisoner's dilemma
  - revealing information beneficial for the firm's joint profits
  - revealing information unilaterally unattractive
- Recognizing these key features of the environment
  - helps firms understand their interaction
  - helps regulators understand implications of a merger

# Iterated elimination of dominated strategies

# Attack game

| | *sea* | *mountain* |
|---|---|---|
| *sea* | $1,4$ | $2,3$ |
| *mountain* | $0,5$ | $3,2$ |

- Row player: defender
- Column player: attacker
- Defender wants to match
- Attacker wants to mismatch
- Mountain good for defense / bad for attack

# Attack game

| | $A$ | $B$ |
|---|---|---|
| $A$ | $1, 4$ | $2, 3$ |
| $B$ | $0, 5$ | $3, 2$ |

- Last digit even: defender (row player)
- Last digit odd: attacker (column player)

# Elimination of dominated strategies

- Procedure for predicting the outcome of the game:
  - do not play dominated strategies
  - assume others will not play dominated strategies
  - best respond

# Attack game

| | *sea* | *mountain* |
|---|---|---|
| *sea* | $1,4$ | $2,3$ |
| *mountain* | $0,5$ | $3,2$ |

- *mountain* is dominated for the attacker
- defender can assume attacker plays *sea*
- defender best responds with *sea*
- predicted outcome: $(sea, sea)$

# Beauty contest

- each player chooses a number $s_i \in \{0, 1, ..., 9\}$
- let $\bar{\boldsymbol{s}}$ be the average of the numbers chosen
- player(s) closest to $\frac{\bar{\boldsymbol{s}}}{3}$ earn 10
- other player(s) earn 0

# Iteration

- Play $\leq 3$ if
  - rational
- Play $\leq 1$ if
  - rational
  - think others are rational
- Play 0 if
  - rational
  - think others are rational
  - think that others think that others are rational
- iterated elimination of dominated strategies

# Way people play

- IEDS implies all play 0

| Group | mean |
|---|---|
| Portfolio managers | 24.31 |
| Economics Ph.D.'s | 27.44 |
| Caltech Board of Trustees | |
| All | 42.62 |
| CEOs only | 37.81 |
| College students | |
| Caltech | 21.88 |
| Germany | 36.73 |
| Singapore | 46.07 |
| UCLA | 42.26 |
| Wharton | 37.92 |
| U.S. High School Students | 32.45 |

# Way people play

- Departures due to:
  - lack of rationality
  - pessimistic views of others' rationality
    - chess players vs. chess players or students
  - view that others are pessimistic about others' rationality
  - etc.
- Repeated play of game consistently converges to 0

# Location game

- Two ice cream vendors operate along a beach 9 blocks long
- On every block 1-9, there are 10 sunbathing people who love ice cream
- Each customer buys an ice cream from the closest vendor
    - equidistant randomize
- Vendors simultaneously choose locations
- Profit from each sale is $1; each vendors wants to maximize sales

- you will be randomly paired with another player
- choose location $s_i \in \{1, 2, ..., 9\}$

# IEDS in the Location Game

- Is there a dominant strategy?
- Are any strategies dominated?

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 45 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 |
| 2 | 80 | 45 | 20 | 25 | 30 | 35 | 40 | 45 | 50 |

- 1 and 9 are dominated
- Once we eliminate 1 and 9
  - 2 and 8 are dominated;
- etc.
- IEDS implies both vendors locate at 5

# Location game as a parable

- Known as the "median voter theorem"
- Hotelling (1929):

  *"Our cities become uneconomically large and the business districts within them are too concentrated. Methodist and Presbyterian churches are too much alike; cider is too homogeneous."*

# Take aways

- Games can be "parables"
  - distill the key aspect of a strategic situation
- Prisoner's dilemma
  - notion of a dominant strategy
  - rational play does not always lead to efficiency
- Attack game
  - elimination of dominated strategies
  - opponent's action sometimes easy to predict
- Beauty contest
  - iterated elimination of dominated strategies
  - rationality vs. beliefs about rationality
  - stock market and fashion
- Location game
  - median voter theorem
  - socially inefficient homogeneity

Thank you