Game Theory: Problem Set 2 Solutions

1 Costly monitoring revisited

Consider the following costly monitoring game:

	work	shirk
monitor	$5-c, 5$	$0, 0$
trust	$5, 5$	$-5, 5+d$

Suppose $0 < c < 5$ and $0 < d$. Find all (pure and mixed strategy) Nash equilibria of this game.

Solution

There is no pure strategy equilibrium. To see this, consider each player's best response in bold below

	work	shirk
monitor	$5-c, \mathbf{5}$	$\mathbf{0}, 0$
trust	$\mathbf{5}, 5$	$-5, \mathbf{5+d}$

.

Since there is no pair of actions that are a best response to one another, there is no pure strategy equilibrium.

There is one mixed strategy equilibrium where the row player monitors with probability $\frac{d}{5+d}$ (and thus trusts with probability $\frac{5}{5+d}$) and the column player works with probability $\frac{5}{5+c}$ (and thus shirks with probability $\frac{c}{5+c}$).

To establish this, suppose the worker works with probability $p_2 \in (0,1)$, then the manager has the same expected utility from monitoring and trusting if

$$\begin{aligned} p_2 \cdot (5-c) + (1-p_2) \cdot 0 =& p_2 \cdot 5 + (1-p_2) \cdot (-5) \\ (5-c)p_2 =& 10p_2 - 5 \\ p_2 =& \frac{5}{5+c} \end{aligned}$$

Suppose the manager monitors with probability $p_1 \in (0,1)$, then the worker has the same expected utility from working and shirking if

$$\begin{aligned} p_1 \cdot 5 + (1-p_1) \cdot 5 =& p_1 \cdot 0 + (1-p_1) \cdot (5+d) \\ 5 =& 5 + d - (5+d)p_1 \\ p_1 =& \frac{d}{5+d}. \end{aligned}$$

2 Battle of the sexes revisited

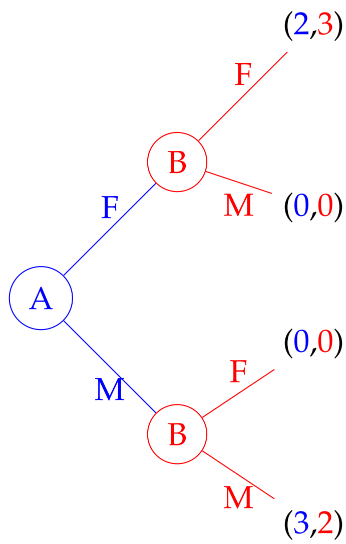
Alice and Bob are a couple and enjoy each other's company. Alice likes to watch Musicals (M) and Bob likes to watch Football (F) . If both choose M, Alice gets a payoff of 3 and Bob gets a payoff of 2. If both choose F, Alice gets a payoff of 2 and Bob gets a payoff of 3. If they choose different actions, they both get 0.

a. Suppose they *simultaneously* choose their actions and receive the respective payoffs as described above. Draw the matrix representation of the game with Alice as the row player and Bob as the column player. Find all (pure and mixed strategy) Nash equilibria of the game.
   Solution.
   Matrix form representation:

Alice\Bob	F	M
F	$2,3$	$0,0$
M	$0,0$	$3,2$

   There are two pure strategy equilibria you should be familiar with from class: (F, F) and (M, M). There is also, however, one mixed strategy equilibrium. Suppose that Anne plays F with probability p_A. Then, Bob is indifferent between playing F and M if $3p_A = 2(1-p_A)$ or $p_A = \frac{2}{5}$. Similarly, if Bob plays F with probability p_B, Anne is indifferent between F and M if $2p_B = 3(1-p_B)$, or $p_B = \frac{3}{5}$. So, we have a mixed strategy equilibrium if Anne plays F with probability $\frac{2}{5}$ and Bob plays F with probability $\frac{3}{5}$.

b. Suppose they move *sequentially.* First, Alice chooses F or M. Then, Bob observes her choice and chooses F or M. Draw the game tree. Find all (pure and mixed) Subgame Perfect Equilibria of the game. Describe these fully and clearly (recall that SPE specifies actions at each information node).
   Solution.
   Extensive form representation:



There is a unique Subgame perfect equilibrium. If Anne plays F, Bob will also play F. If Anne plays M, Bob will also play M. Thus, Anne will play M and the equilibrium payoffs will be 3 for Anne and 2 for Bob.

3 A hold-up problem

Two firms are engaged in a potential joint venture. Firm 1 first chooses whether to make a sunk, costly investment that increases the potential profitability of the venture. If it does not make the investment $(K = 0)$, the overall potential profitability will be small $(\Pi = 5)$. If Firm 1 does make an investment $(K = 5)$, the overall potential profitability will be large $(\Pi = 20)$. After Firm 1 chooses whether to invest or not, Firm 2 observes this choice. Then, the two firms bargain over how to split the profit. Consider two bargaining protocols.

Under protocol A, Firm 2 proposes a share of profits $s \in [0.20, 0.80]$ it gets to keep. If Firm 1 accepts, the project is implemented, and the payoffs are $u_1 = (1 - s) \times \Pi - K$ and $u_2 = s \times \Pi$. If Firm 1 rejects, the project is not implemented and the payoffs are $u_1 = -K$ and $u_2 = 0$.

Under protocol B, Firm 2 proposes a share of profits $s \in [0.40, 0.60]$ it gets to keep. Everything else is the same. If Firm 1 accepts, the project is implemented, and the payoffs are $u_1 = (1 - s) \times \Pi - K$ and $u_2 = s \times \Pi$. If Firm 1 rejects, the project is not implemented and the payoffs are $u_1 = -K$ and $u_2 = 0$.

Which protocol does Firm 1 prefer? Which protocol does Firm 2 prefer? Discuss.

Solution

Let us first consider Protocol A. We first note that, whatever share of profits Firm 2 proposes to keep, Firm 1 is better off accepting rather than rejecting the proposal since $(1-s)\Pi - K > -K$ regardless of $s \in [0.2, 0.8]$ and $K \in \{0, 5\}$. Hence, Firm 2 will propose $s = 0.8$. Knowing this, Firm 1 will be better off not making the investment (setting $K = 0$) rather than making the investment since $0.2 \times 20 - 5 = -1 < 1 = 0.2 \times 5 - 0$. Thus, in the unique SPE under Protocol A, Firm 1 does not make the investment, Firm 2 proposes to keep 80% of the profits, and the project is implemented. Firm 1's payoff is 1 and Firm 2's payoff is 4.

Under Protocol B, again, whatever share of profits Firm 2 proposes to keep, Firm 1 is better off accepting rather than rejecting the proposal since $(1-s)\Pi - K > -K$ regardless of $s \in [0.4, 0.6]$ and $K \in \{0, 5\}$. Hence, Firm 2 will propose $s = 0.6$. But now, even though Firm 1 knows that Firm 2 will demand the greatest share it can, Firm 1 still prefers to make the investment since $0.4 \times 20 - 5 = 3 > 2 = 0.4 \times 5 - 0$. Thus, in the unique SPE under Protocol B, Firm 1 makes the investment, Firm 2 proposes to keep 60% of the profits, and the project is implemented. Firm 1's payoff is 3 and Firm 2's payoff is 12.

Thus, both firms prefer Protocol B over Protocol A. The problem with Protocol A is that Firm 2 is "too strong" in the negotiating stage and will take too much of the surplus. Thus, Firm 1 is unwilling to make the privately costly sunk investment that increases the overall surplus. Protocol B allows Firm 2 to effectively commit to let Firm 1 keep at least 40% of the surplus, which in turn makes both firms better off. This is similar to Intel becoming credibly weak by granting second-source licensing to its competitors.