Game Theory: Week 3

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Mixed strategies

Matching pennies game

	H	T
H	$1, -1$	$-1, 1$
T	$-1, 1$	$1, -1$

- last digit even: row player
- last digit odd: column player

What is going on here?

- No equilibrium
- First reaction: equilibrium sucks as a prediction tool
- Instead: "lack of equilibrium" is a feature not a bug
- When no $(s_1, s_2, ...s_n)$ is an equilibrium...
  - players must be **unpredictable**
  - there is an equilibrium
  - but it involves acting in unpredictable ways
  - cannot be described by $(s_1, s_2, ...s_n)$

Mixed strategies

- Given a strategy space S_i
  - let $\tilde{S}_i$ be the set of all probability distributions on S_i
- Payoffs $u_i(\boldsymbol{s})$ imply payoffs $u_i(\tilde{\boldsymbol{s}})$

Definition

A strategy profile $\tilde{\boldsymbol{s}}^*$ is a *mixed strategy Nash equilibrium* if for each player i, $u_i\left(\tilde{s}_i^*, \tilde{\boldsymbol{s}}_{-i}^*\right) \geq u_i\left(\tilde{s}_i', \tilde{\boldsymbol{s}}_{-i}^*\right)$ for all $\tilde{s}_i' \in \tilde{S}_i$.

- What we studied last week, we will call *pure strategy* Nash equilibrium

Matching pennies

$$\begin{array}{ccc} & H & T \\ H & 1,-1 & -1,1 \\ T & -1,1 & 1,-1 \end{array}$$

- Unique equilibrium: each player plays H with probability $\frac{1}{2}$
- Column player wants to be as unpredictable as possible
  - hence plays two strategies with equal probability
- A best response for the row player:
  - play two strategies with equal probability

Matching pennies in the world

- Shooting penalty kicks
- Hide-and-seek
- Zara vs. high-end design
- Costly monitoring
  - checking up on an employee
  - auditing a tax payer

Costly monitoring

- Tax payer pays or cheats
- IRS audits or not
- Tax payer wants to cheat iff IRS does not audit
- IRS wants to audit iff taxpayer cheats

	audit	*not*
pay	$1, -1$	$-1, 1$
cheat	$-1, 1$	$1, -1$

- Symmetry not really plausible

Costly monitoring

- Tax payer pays or cheats
- IRS audits or not
- Tax payer wants to cheat iff IRS does not audit
- IRS wants to audit iff taxpayer cheats

	audit	*not*
pay	$0, -1$	$0, 0$
cheat	$-x, 1$	$1, -1$

Costly monitoring = generalized matching pennies

	audit	*not*
pay	$\mathbf{0}, -1$	$0, \mathbf{0}$
cheat	$-x, \mathbf{1}$	$\mathbf{1}, -1$

- x is the prison term for the cheating tax payer
  - lowers tax payer's utility
  - does not give direct benefits to the IRS
- Like matching pennies
  - IRS wants to match actions (audit/cheat, not/pay)
  - tax payer wants to mismatch (pay/audit, cheat/not)
- No pure strategy equilibrium
  - if IRS audits, tax payer pays$\rightarrow$ IRS shouldn't audit
  - if IRS doesn't audit, tax payer cheats $\rightarrow$ IRS should audit

Costly monitoring, version 1

	audit	*not*
pay	$0, -1$	$0, 0$
cheat	$-2, 1$	$1, -1$

- last digit even: row player
- last digit odd: column player

Costly monitoring, version 2

	audit	*not*
pay	$0, -1$	$0, 0$
cheat	$-3, 1$	$1, -1$

- last digit even: row player
- last digit odd: column player

Solving for a mixed strategy equilibrium

- Players must be playing both strategies
- Hence they must be **indifferent** between the two strategies

Solving for the mixed strategy equilibrium

- Let m be the probability of auditing
- Let p be the probability of paying
- Tax payer indifferent:

$$0 = -xm + (1 - m)$$

- IRS indifferent:

$$-p + (1 - p) = -(1 - p)$$

- Equilibrium:

$$\begin{aligned} m &= \frac{1}{1+x} \\ p &= \frac{2}{3} \end{aligned}$$

Penalties and behavior

- Equilibrium:

$$
\begin{aligned}
m &= \frac{1}{1+x} \\
p &= \frac{2}{3}
\end{aligned}
$$

- Remarkable feature:
  - x affects only the tax payer's payoff
  - cheating prevalence independent of x
  - affects only IRS' behavior
  - **increasing the penalty for a crime does not reduce its prevalence**

Entry games

Entry game, version 1

- 10 players choose whether to *enter* or *not*
- Payoff to each player
  - if enter: 5 - # firms who entered
  - if not: 0
- In other words,
  - market can support up to 5 firms
  - if 5 firms enter, profits are 0
  - if fewer enter, profits are positive
  - if more enter, profits are negative

Entry game, version 2

- 10 players choose whether to *enter* or *not*
- Payoff to each player
  - if enter: 3 - # firms who entered
  - if not: 0
- In other words,
  - market can support up to 3 firms
  - if 3 firms enter, profits are 0
  - if fewer enter, profits are positive
  - if more enter, profits are negative

Revisiting Hawk-Dove

Recall Hawk-Dove

	hawk	*dove*
hawk	$-1, -1$	$\mathbf{2}, \mathbf{0}$
dove	$\mathbf{0}, \mathbf{2}$	$1, 1$

- There are two pure strategy equilibria
- Is there a mixed strategy equilibrium?
- Yes: everyone plays hawk with probability $\frac{1}{2}$
- Let h be the probability of hawk
- Indifference:

$$\begin{aligned} -h + 2(1-h) &= 1-h \\ h &= \frac{1}{2} \end{aligned}$$

Zero-sum games

Two player zero-sum games

- Many games are zero sum: $u_i = -u_j$
  - attack game, location game, matching pennies, etc.
- There is an easy way to solve zero-sum games

A pessimistic approach

- Suppose each player believes in Murphy's law:
  - whatever he does, the worst possible thing will happen
- Say s_i^m is a MaxMin strategy if it maximizes $min_{\boldsymbol{s}_{-i}} u_i\left(s_i, \boldsymbol{s}_{-i}\right)$

MaxMin vs. Nash

- MaxMin well defined in any game, zero sum or not
- Generally distinct from Nash
    - players do not pay attention to other players' payoffs
- Main result on two player zero-sum games:

Theorem

In any two player zero-sum game, a strategy profile is a Nash equilibrium if and only if both players play MaxMin strategies.

MaxMin Theorem

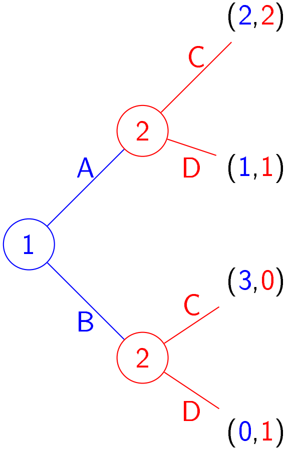
- MaxMin Theorem makes it easy to find equilibria of zero-sum games
- We can determine what will happen **player-by-player**
  - usually only possible with dominant strategies
- Think about the location game

Taking stock

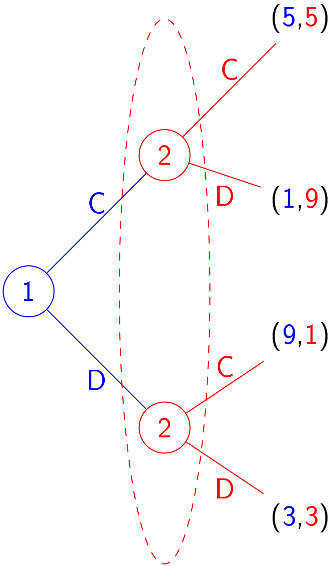
- Mixed strategies
  - unpredictability can be the only internally consistent behavior
  - does not mean we cannot make any predictions
  - mixed strategy equilibria often match the data well
  - counterintuitive comparative statics (costly monitoring)
- Zero-sum games and the MaxMin strategy
  - can treat a game as independent decision problems of pessimists

Dynamic games of complete information

A game tree



Information sets



Static games as game trees

	C	D
C	5,5	1,9
D	9,1	3,3

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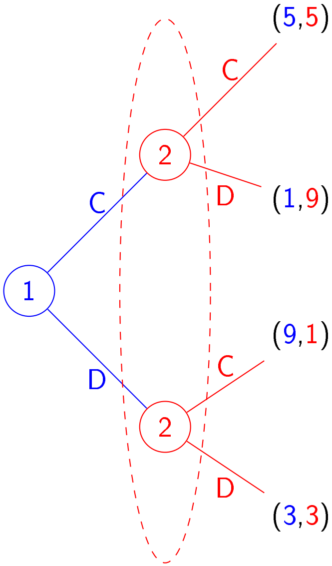
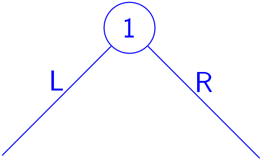


Choosing a game



Stag hunt

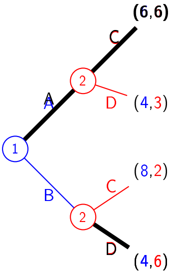
	S	R
S	5,5	0,2
R	2,0	2,2

Battle of the sexes

	F	O
F	3,1	0,0
O	0,0	1,3

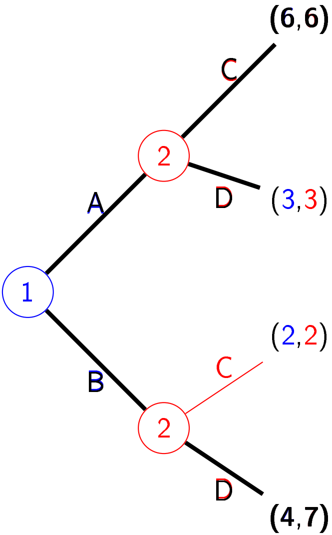
Subgame perfect equilibrium

Subgame perfect equilibrium



- Not enough to specify what happens in equilibrium
- Must specify what each player does at each potential node
- Only this way sure i is best-responding to j
- Related to, but distinct from, backward induction
  - A node can lead to a matrix; we assume NE in that matrix

Nash vs. SPE



Nash vs. SPE

	CC	CD	DC	DD
A	6,6	**6,6**	3,3	3,3
B	2,2	4,7	2,2	**4,7**

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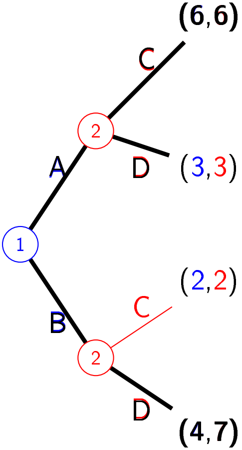
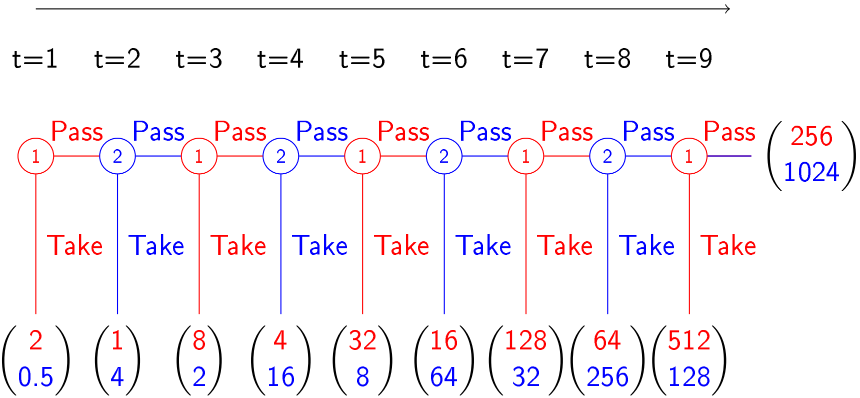
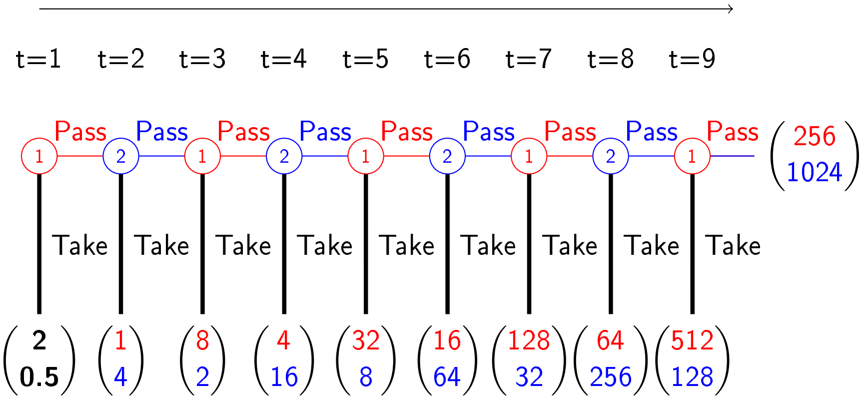


Centipede game



- If always Pass, select 0
- If Take at some node(s), select *first* node where you Take

Centipede game



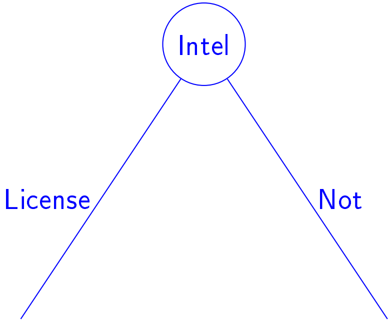
- As with IEDS, Subgame Perfect equilibrium requires common knowledge of rationality

Credible weakness

Second source licensing

- 1978: Intel develops the 8086 microprocessor
  - grants second-source licensing to IBM, AMD, and 10 foreign firms
- 1987: Intel has only a 30% market share of 8086
- Intel could have been a monopolist
  - why did it voluntarily give away its market power?

Credible weakness

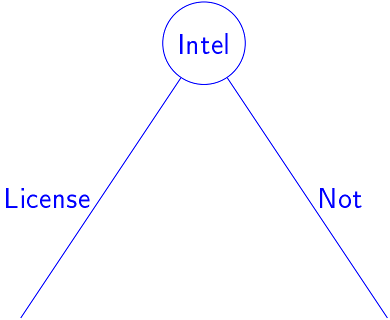


	Adopt	Not
Price High	5,8	1,2
Price Low	7,10	1,2

	Adopt	Not
Price High	20,1	2,2
Price Low	10,10	2,2

- If it licenses, Intel's payoff strictly lower for *every* possible outcome
- Hence, it was a mistake to license?
    - No

Credible weakness



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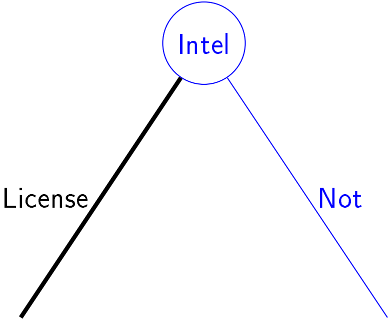
Credible weakness



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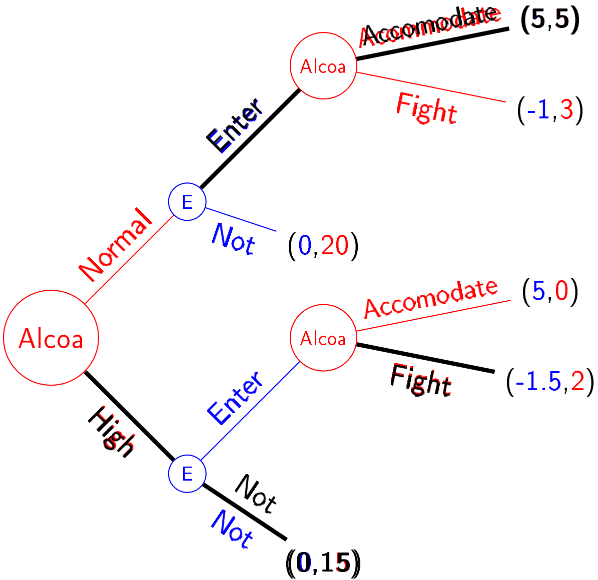
Credible belligerence

Excess capacity

- 1930s-40s Alcoa controlled the virgin aluminum market
- Built capacity well above the demand
- Building unused capacity is expensive
- Was it a mistake to have so much capacity?

Capacity game



Dynamic pricing

Dynamic pricing

- A monopolist sells a durable widget with zero marginal cost
- Consumers are patient and have unit demand
  - $\frac{1}{2}$ high valuation consumers willing to pay \$100
  - $\frac{1}{2}$ low valuation consumers willing to pay \$30
- What price should the monopolist charge?
  - Suppose he charges \$100
  - What price should he charge tomorrow?
  - But high valuation consumers will expect that
- No price greater than \$30 possible in a subgame perfect equilibrium!
- Monopolist's future self is a nasty competitor

Imperfectly patient consumers

- Suppose consumers have a discount rate of 10%
- Monopolist still cannot charge more than $33.33

Solving the problem

- Blow up the company after selling to the first batch of consumers
- Price protection plans
  - GE in sales of electric-turbine generators
- Lease-only policy
  - United Shoe Machinery
- Planned obsolescence

Take aways

- Mixed strategies
  - unpredictability can be the only internally consistent behavior
  - does not mean we cannot make any predictions
  - mixed strategy equilibria often match the data well
  - counterintuitive comparative statics (costly monitoring)
- Zero-sum games and the MaxMin strategy
  - can treat a game as independent decision problems of pessimists

Take aways, continued

- We use game trees to analyze dynamic games
- Subgame perfection captures the notion of credibility
- Lowering your payoffs can increase your equilibrium payoff
- Becoming weaker can make it easier to be trusted
  - Intel and the 8086 microprocessor
- Becoming more irritable can make it easier to be feared
  - Alcoa and excess capacity
- Dynamic pricing
  - monopolist's tomorrow self can be his worst competitor
  - variety of strategies for alleviating the problem

Thank you