Game Theory: Week 2

Emir Kamenica

University of Chicago Booth School of Business

Last week

- Solution concepts
  - dominant strategies
  - iterated elimination of dominated strategies
- Games
  - prisoner's dilemma
  - attack game
  - beauty contest
  - location game
- Take aways
  - if there is a dominant strategy: play it
  - if there is a dominated strategy: avoid it
  - put yourself in others' shoes
  - predict rationality of others
  - predict others' views of rationality of others
- Implications of strategic interaction
  - rational play does not imply efficient outcomes
  - catering to centrist tastes and excessive homogeneity

Nash equilibrium

Consider this game

	D	E	F
A	$15, 20$	$9, 5$	$2, 8$
B	$20, 5$	$3, 10$	$5, 6$
C	$18, 1$	$7, 2$	$12, 10$

- No dominant strategies
- No dominated strategies
- How to proceed?

Best-response

	D	E	F
A	$15, 20$	$9, 5$	$2, 8$
B	$\mathbf{20}, 5$	$3, 10$	$5, 6$
C	$18, 1$	$7, 2$	$12, 10$

Best-response

	D	E	F
A	$15, 20$	$\mathbf{9}, 5$	$2, 8$
B	$\mathbf{20}, 5$	$3, 10$	$5, 6$
C	$18, 1$	$7, 2$	$12, 10$

Best-response

	D	E	F
A	$15, 20$	$\mathbf{9}, 5$	$2, 8$
B	$\mathbf{20}, 5$	$3, 10$	$5, 6$
C	$18, 1$	$7, 2$	$\mathbf{12}, 10$

Best-response

	D	E	F
A	$15, \mathbf{20}$	$\mathbf{9}, 5$	$2, 8$
B	$\mathbf{20}, 5$	$3, 10$	$5, 6$
C	$18, 1$	$7, 2$	$\mathbf{12}, 10$

Best-response

	D	E	F
A	$15, \mathbf{20}$	$\mathbf{9}, 5$	$2, 8$
B	$\mathbf{20}, 5$	$3, \mathbf{10}$	$5, 6$
C	$18, 1$	$7, 2$	$\mathbf{12}, 10$

Best-response

	D	E	F
A	$15, \mathbf{20}$	$\mathbf{9}, 5$	$2, 8$
B	$\mathbf{20}, 5$	$3, \mathbf{10}$	$5, 6$
C	$18, 1$	$7, 2$	$\mathbf{12}, \mathbf{10}$

Nash equilibrium definition

Definition

A strategy profile $\boldsymbol{s}^*$ is a *Nash Equilibrium* if for each player i,
$u_i\left(s_i^*, \boldsymbol{s}_{-i}^*\right) \geq u_i\left(s_i', \boldsymbol{s}_{-i}^*\right)$ for all $s_i' \in S_i$.

- Each player
  - correctly anticipates what others will do
  - responds optimally

Interpretation of Nash equilibrium

- Internally consistent common prediction
- Only Nash equilibria are "stable"

Nash equilibrium

	D	E	F
A	$15, 20$	$9, 5$	$2, 8$
B	$20, 5$	$3, 10$	$5, 6$
C	$18, 1$	$7, 2$	$\mathbf{12}, \mathbf{10}$

Relationship between concepts

- If in some game, each player has a dominant strategy
  - this strategy profile is IEDS
  - this strategy profile is a Nash equilibrium
- If some game has IEDS
  - this strategy profile is a Nash equilibrium
- Dominant strategies $\Rightarrow$ IEDS $\Rightarrow$ Nash equilibrium
- Nash equilibrium $\not\Rightarrow$ IEDS $\not\Rightarrow$ Dominant strategies

Coordination games

Stag hunt

	stag	*hare*
stag	$5, 5$	$0, 2$
hare	$2, 0$	$2, 2$

Stag hunt

	A	B
A	$5, 5$	$0, 2$
B	$2, 0$	$2, 2$

Stag hunt

	stag	*hare*
stag	$\mathbf{5}, \mathbf{5}$	$0, 2$
hare	$2, 0$	$\mathbf{2}, \mathbf{2}$

- Multiple Nash equilibria
  - "trusting," efficient equilibrium
  - "safe" equilibrium
- Feature of many real-world interactions
  - e.g., complaining about a manager to his boss
  - recognizing you are in a stag-hunt game crucial
  - distinct from Prisoners' Dilemma
- Pre-play communication
  - Stag Hunt vs. Prisoners' Dilemma

Extreme stag hunt

- Everyone in class plays together
- $s_i \in \{0, 1, ..., 9\}$
- If everyone chooses the same s_i
  - everyone gets $\overline{\boldsymbol{s}} = s_i$
- If there is not *complete* agreement
  - player i gets $-100 \times s_i$
- Efficient outcome is an equilibrium but "strategically risky"
  - not about risk aversion

Simple coordination game

	A	B
A	$1, 1$	$0, 0$
B	$0, 0$	$1, 1$

Simple coordination game

	A	B
A	$\mathbf{1}, \mathbf{1}$	$0, 0$
B	$0, 0$	$\mathbf{1}, \mathbf{1}$

- Focal points
- Pick-a-time game

Battle of the sexes

	football	*opera*
football	$3, 1$	$0, 0$
opera	$0, 0$	$1, 3$

Battle of the sexes

$$\begin{array}{ccc} & A & B \\ A & 3,1 & 0,0 \\ B & 0,0 & 1,3 \end{array}$$

- last digit even: row player
- last digit odd: column player

Battle of the sexes

	football	*opera*
football	$\mathbf{3}, \mathbf{1}$	$0, 0$
opera	$0, 0$	$\mathbf{1}, \mathbf{3}$

- Both parties wish to coordinate (as before)
- Have conflicting preferences on what equilibrium to coordinate on

Hawk-dove

	hawk	*dove*
hawk	$-1, -1$	$2, 0$
dove	$0, 2$	$1, 1$

Hawk-dove

	A	B
A	$-1, -1$	$2, 0$
B	$0, 2$	$1, 1$

Hawk-dove

	hawk	*dove*
hawk	$-1, -1$	$\mathbf{2}, \mathbf{0}$
dove	$\mathbf{0}, \mathbf{2}$	$1, 1$

- Closely related to the Battles of the Sexes
- Aggression desirable but only if the other is not aggressive

Lessons about coordination

- Efficient outcomes can be difficult to attain because they are strategically risky
  - Stag hunt
- Focal points can aid coordination
  - Simple coordination games
- Asymmetries in payoffs make coordination more difficult
  - Battle of the sexes
- Benefits to being perceived as aggressive
  - Hawk-dove

Public good games

Partnership game

- Two partners choose effort $s_i \in \{0, 1, .., 9\}$
- Each player's share of joint profits $\Pi(\boldsymbol{s}) = s_1 + s_2 + \frac{s_1 s_2}{2}$
- Effort cost $c(s_i) = \frac{s_i^2}{2}$
- Player i's payoff: $u_i(\boldsymbol{s}) = \Pi(\boldsymbol{s}) - c(s_i)$

Best response

- Player i solves

$$max_{s_i} s_i + s_j + \frac{s_i s_j}{2} - \frac{s_i^2}{2}$$

- f.o.c:

$$1 + \frac{s_j}{2} - s_i = 0$$

$$s_i = 1 + \frac{s_j}{2}$$

Intersecting best response functions

$$
\begin{aligned}
s_i &= 1 + \frac{s_j}{2} \\
s_j &= 1 + \frac{s_i}{2}
\end{aligned}
$$

$$
\begin{aligned}
s_i &= 1 + \frac{1}{2}\left(1 + \frac{s_i}{2}\right) \\
s_i &= 1 + \frac{1}{2} + \frac{s_i}{4} \\
4s_i &= 4 + 2 + s_i \\
3s_i &= 6 \\
s_i &= 2
\end{aligned}
$$

- Unique Nash equilibrium: $s_1 = s_2 = 2$

Comparison with efficient outcome

- Efficient outcome:

$$max_s s + s + \frac{s^2}{2} - \frac{s^2}{2}$$

Comparison with efficient outcome

- Efficient outcome:

$$max_s 2s$$

- Overall payoff always increasing in effort
- Maximum payoffs attained with $s_1 = s_2 = 9$

Public good games / tragedy of the commons

- Public good games
  - social benefit $>$ private benefit
  - undersupply of costly inputs
  - e.g., teamwork
- Tragedy of the commons
  - social cost $>$ private cost
  - overuse of common resources
  - e.g., overfishing, global warming, socializing during a pandemic
- More generally: (social benefit - social cost) $\neq$ (private benefit - private cost)
  - If (social benefit - social cost) $>$ (private benefit - private cost)
    - we do not do it enough
  - If (social benefit - social cost) $<$ (private benefit - private cost)
    - we overdo it

Advertising

- If advertising is business stealing
  - firms advertise too much in equilibrium
- If advertising is industry expanding
  - firms advertise too little in equilibrium

- American Tobacco Company
  - dissolved under the Sherman Act in 1911
    - became four competing companies
    - total advertising expenditure more than doubled
- US Pharmaceutical industry today

Competition and cost curves

A competitive industry

- Two firms simultaneously choose prices
- Each consumer buys from cheaper firm if lower price $\leq$ WTP
  - WTP distributed uniformly on $[0, 1]$
- Hence, industry demand is $1 - p$ where p is the lower price

Competition and cost curves

- Examine two scenarios
  - each firm has zero costs
  - it costs each firm $2q^2$ to produce q units
- What are the firms' profits in each case?

The easy case

- Each firm has zero costs
- What is the Nash equilibrium?
  - $p_1 = p_2 = 0$
  - $\Pi_1 = \Pi_2 = 0$
- Are there other equilibria?
  - no

More difficult case

- It costs each firm $2q^2$ to produce q units
- What is the Nash equilibrium?
- Suppose firm 2 charges some p_2
- Firm 1 chooses p to maximize

$$\Pi_i(p, p_2) = \begin{cases} p(1-p) - 2(1-p)^2 & \text{if } p < p_2 \\ p\left(\frac{1-p}{2}\right) - 2\left(\frac{1-p}{2}\right)^2 & \text{if } p = p_2 \\ 0 & \text{if } p > p_2 \end{cases}$$

- How do we proceed?

Solving for equilibrium

- Suppose firm maximizes $p\left(\frac{1-p}{2}\right) - 2\left(\frac{1-p}{2}\right)^2$
  - optimal price is $p = \frac{3}{4}$, yields $\Pi = \frac{1}{16}$
- Now, suppose the other firm charges $p_2 = \frac{3}{4}$
  - what is my optimal price?
  - charge less: $p(1-p) - 2(1-p)^2 < \frac{1}{16}$ for all $p < \frac{3}{4}$
  - charge the same: my profit is $\frac{1}{16}$
  - charge more: my profit is zero
- Hence, $p_1 = p_2 = \frac{3}{4}$ is an equilibrium
- Profits are $\Pi_1 = \Pi_2 = \frac{1}{16}$

Competition and cost curves

- If each firm has zero costs:
  - $\Pi_1 = \Pi_2 = 0$
- If each firm has positive, increasing, convex costs:
  - $\Pi_1 = \Pi_2 > 0$
  - in fact, profits same as under collusion!
- An increase in firms' costs can **increase** profits!

- Suppose you could *choose* the cost curve for your industry?
- What type of a cost curve would give you highest profits?

The tobacco settlement

Tobacco settlement

- Cigarettes cause cancer
- State governments incur medical expenses due to smoking
- Major lawsuit to reclaim damages
- November 23, 1998 WSJ Headline: **Forty-Six States Agree to Accept $206B Tobacco Settlement**

Tobacco settlement

- How does the settlement change the cost curves?
- Suppose that the four companies must collectively write a check to the government
  - change in cost curve $c^{new}(q) = c^{old}(q) + \frac{\$206B}{4}$
  - change in profits $\Pi^{new} = \Pi^{old} - \frac{\$206B}{4}$
- This is not how the settlement changed the cost curve
  - \$206B collected from "a tax on sales exceeding a set level"
  - $c^{new}(x) = \begin{cases} c^{old}(q) & \text{if } q \leq q^* \\ c^{old}(q) + t(q - q^*) & \text{if } q > q^* \end{cases}$
- This happens to be a cost curve that maximizes profits
  - in fact, $\Pi^{new} > \Pi^{old}$
  - net of the \$206B payment!

Take aways

- Absence of dominant strategies $\rightarrow$ need to predict what others will do
- Absence of dominated strategies $\rightarrow$ hard to predict what others will do
- Nash equilibrium: an internally consistent outcome
- Multiplicity of Nash equilibria and the need for coordination
    - focal points (simple coordination game)
    - efficiency vs. risk (stag hunt)
    - conflict in coordination (battle of the sexes)
    - benefits and costs of aggression (hawk-dove)
- Public good games / tragedy of the commons
- Cost curves are a competitive firm's best friend
    - convex cost curves yield higher profits than constant marginal costs
    - self-designed cost curves mandated by the government are best of all

Thank you