Sample Midterm Problems

Problem 1

List all the pure strategy Nash equilibria of this game:

	L	M	R
U	$20, 10$	$4, 5$	$2, 10$
M	$22, 3$	$5, 4$	$2, 3$
D	$15, 0$	$0, 10$	$0, 0$

Problem 2

Two firms decide simultaneously how much to invest in R&D to increase the quality of their product. They can choose a high or a low level of investment. If one invests more than the other one, consumers prefer its product so the payoff is 10 for the high investment firm and 5 for the other one. If they invest the same, consumers are indifferent between the two products. Each firm gets 8 if they both chose low investment, and 6 if they both choose high investment. Write down the payoffs matrix of the game and find the Nash equilibrium of the game. What type of a game is this?

Problem 3

Part a

Consider the following simultaneous-move game of a penalty kick. The kicker can shoot left or right. He is right-footed so he shoots more strongly when he shoots left. The goalie knows this. The goalie can throw himself left or right. If the goalie throws himself in the wrong direction, the kicker always scores. If the goalie throws himself to the left and the kicker shoots to the left, the kicker scores 60% of the time. If the goalie throws himself to the right and the kicker shoots to the right, the kicker scores 40% of the time. (These numbers reflect the fact that the kicker is right-footed – the goalie is more likely to defend successfully if the kicker shoots to his weaker side.)

If the kicker scores, the kicker gets a payoff of 1 and the goalie gets a payoff of 0.

If the kicker does not score, the kicker gets a payoff of 0 and the goalie gets a payoff of 1.

There is a unique mixed strategy equilibrium of this game. Find it.

Part b

Suppose you are the coach of the kicking team. You have access to two players. One is right-footed (as the kicker in Part a). The other one is ambidextrous. He shoots equally well to the right and to the left but his greater strength of one foot is compensated by the reduced strength of the other foot: a goalie who throws himself in the correct direction (whether left or right) defends his kicks 50% of the time. Which player should you have shoot the penalty kick? (Obviously, you want your team to score.) You should assume the goalie knows the players (so once you have chosen the player, the goalie knows whether it is the right-footed one or the ambidextrous one.)

Problem 4

Suppose two workers simultaneously choose to contribute their efforts s_1 and s_2 to a joint project. Suppose the joint revenue is determined by the two workers' efforts s_1 and s_2 according to the following production function:

$$R(s_1, s_2) = s_1 s_2 + s_1 + s_2$$

The workers split the revenue equally, each receiving $\frac{1}{2}R(s_1, s_2)$. Effort is privately costly; in particular, exerting effort s costs $c(s) = s^2$. Suppose the workers can choose any non-negative effort $s_i > 0$.

Solve for the unique (pure strategy) Nash equilibrium of this game.

What are the workers' equilibrium payoffs?

If they could sign a binding enforceable contract on their efforts, how hard would they work and what would their payoffs be?

Problem 5

Consider two firms sequentially competing in quantities. Firm 1 enters first and chooses a quantity Q_1. Firm 2 enters second, observes the quantity Firm 1 has produced, and then chooses its quantity Q_2. The price is then determined by the inverse demand $P = 20 - (Q_1 + Q_2)$. Both firms have a marginal cost of 4. Find the unique Subgame Perfect Equilibrium of the game.

Problem 6

Part a

You are considering buying a company. The company is either *good* or *bad*. Based on your initial information, the probability the company is good is 30%. You commission an audit of the company. The audit firm is either *corrupt* or *honest*. If the audit firm is corrupt, it produces a positive report regardless of whether the company is good or bad. If the audit firm is honest, it produces a positive report if and only if the company is good. The chance that the audit firm is corrupt is 20%. The audit report has come back positive. What should be your updated belief that the company is good?

Part b

Consider the exact same situation as above but now there are two audit firms. Each one is corrupt with probability 20%. Whether one is corrupt is independent of whether the other one is corrupt. You have commissioned a separate report from each one and **both** have come back with a positive report. What should be your updated belief that the company is good? (As before, your initial belief, before the reports, is that there is a 30% chance the company is good.)