Game Theory: Problem Set 2

1 Costly monitoring revisited

Consider the following costly monitoring game:

	work	shirk
monitor	$5-c, 5$	$0, 0$
trust	$5, 5$	$-5, 5+d$

Suppose $0 < c < 5$ and $0 < d$. Find all (pure and mixed strategy) Nash equilibria of this game.

2 Battle of the sexes revisited

Alice and Bob are a couple and enjoy each other's company. Alice likes to watch Musicals (M) and Bob likes to watch Football (F) . If both choose M, Alice gets a payoff of 3 and Bob gets a payoff of 2. If both choose F, Alice gets a payoff of 2 and Bob gets a payoff of 3. If they choose different actions, they both get 0.

1. Suppose they *simultaneously* choose their actions and receive the respective payoffs as described above. Draw the matrix representation of the game with Alice as the row player and Bob as the column player. Find all (pure and mixed strategy) Nash equilibria of the game.

2. Suppose they move *sequentially.* First, Alice chooses F or M. Then, Bob observes her choice and chooses F or M. Draw the game tree. Find all (pure and mixed) Subgame Perfect Equilibria of the game. Describe these fully and clearly (recall that SPE specifies actions at each information node).

3 A hold-up problem

Two firms are engaged in a potential joint venture. Firm 1 first chooses whether to make a sunk, costly investment that increases the potential profitability of the venture. If it does

not make the investment ($K = 0$), the overall potential profitability will be small ($\Pi = 5$). If Firm 1 does make an investment ($K = 5$), the overall potential profitability will be large ($\Pi = 20$). After Firm 1 chooses whether to invest or not, Firm 2 observes this choice. Then, the two firms bargain over how to split the profit. Consider two bargaining protocols.

Under protocol A, Firm 2 proposes a share of profits $s \in [0.20, 0.80]$ it gets to keep. If Firm 1 accepts, the project is implemented, and the payoffs are $u_1 = (1 - s) \times \Pi - K$ and $u_2 = s \times \Pi$. If Firm 1 rejects, the project is not implemented and the payoffs are $u_1 = -K$ and $u_2 = 0$.

Under protocol B, Firm 2 proposes a share of profits $s \in [0.40, 0.60]$ it gets to keep. Everything else is the same. If Firm 1 accepts, the project is implemented, and the payoffs are $u_1 = (1 - s) \times \Pi - K$ and $u_2 = s \times \Pi$. If Firm 1 rejects, the project is not implemented and the payoffs are $u_1 = -K$ and $u_2 = 0$.

Which protocol does Firm 1 prefer? Which protocol does Firm 2 prefer? Discuss.