

Assignment 10.2

Exercises 12 - 1 and 12 - 2

<http://thinkstats2.com>

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```
In [1]: # Imports
import numpy as np
import pandas as pd
import statsmodels.formula.api as smf

import thinkstats2
import thinkplot

In [2]: from IPython.core.display import HTML
table_css = 'table {align:left;display:block} '
HTML('<style>{}</style>'.format(table_css))
```

Out[2]:

Exercise 12 - 1

The linear model I used in this chapter has the obvious drawback that it is linear, and there is no way to expect prices to change linearly over time. We can add flexibility to the model by adding a quadratic term, as we did in Section 11.3.

Use a quadratic model to fit the time series of daily prices, and use the model to generate predictions. You will have to write a version of RunLinearModel that runs that quadratic model, but after that you should be able to reuse code from the chapter to generate predictions.

```
In [3]: # read mj.csv
transactions = pd.read_csv('mj-clean.csv', parse_dates = [5])
transactions.head()
```

Out[3]:

| | city | state | price | amount | quality | date | ppg | state.name | lat |
|---|------------|-------|-------|--------|---------|------------|-------|----------------|--------------|
| 0 | Annandale | VA | 100 | 7.075 | high | 2010-09-02 | 14.13 | Virginia | 38.830345 -7 |
| 1 | Auburn | AL | 60 | 28.300 | high | 2010-09-02 | 2.12 | Alabama | 32.578185 -8 |
| 2 | Austin | TX | 60 | 28.300 | medium | 2010-09-02 | 2.12 | Texas | 30.326374 -9 |
| 3 | Belleville | IL | 400 | 28.300 | high | 2010-09-02 | 14.13 | Illinois | 38.532311 -8 |
| 4 | Boone | NC | 55 | 3.540 | high | 2010-09-02 | 15.54 | North Carolina | 36.217052 -8 |

```

In [4]: def GroupByDay(transactions, func=np.mean):
        """
        Groups transactions by day and compute the daily mean ppg.

        args:
            transactions (DataFrame): transactions

        returns:
            daily (DataFrame): daily prices
        """
        grouped = transactions[["date", "ppg"]].groupby("date")
        daily = grouped.aggregate(func)

        daily["date"] = daily.index
        start = daily.date[0]
        one_year = np.timedelta64(1, "Y")
        daily["years"] = (daily.date - start) / one_year

        return daily

In [5]: def GroupByQualityAndDay(transactions):
        """
        Divides transactions by quality and computes mean daily price.

        args:
            transaction (DataFrame): transactions

        returns:
            dailies (map): quality to time series of ppg
        """
        groups = transactions.groupby("quality")
        dailies = {}
        for name, group in groups:
            dailies[name] = GroupByDay(group)

        return dailies

In [ ]: def RunLinearModel(daily):
        model = smf.ols('ppg ~ years', data=daily)
        results = model.fit()
        return model, results

In [38]: dailies = GroupByQualityAndDay(transactions)

        name = 'high'
        daily = dailies[name]

In [39]: def RunQuadraticModel(daily):
        daily['years2'] = daily.years**2
        model = smf.ols("ppg ~ years + years2", data=daily)
        results = model.fit()

        return model, results

In [40]: model, results = RunQuadraticModel(daily)
        display(results.summary())

```

OLS Regression Results

| | | | |
|--------------------------|------------------|----------------------------|-----------|
| Dep. Variable: | ppg | R-squared: | 0.455 |
| Model: | OLS | Adj. R-squared: | 0.454 |
| Method: | Least Squares | F-statistic: | 517.5 |
| Date: | Tue, 17 May 2022 | Prob (F-statistic): | 4.57e-164 |
| Time: | 17:10:26 | Log-Likelihood: | -1497.4 |
| No. Observations: | 1241 | AIC: | 3001. |
| Df Residuals: | 1238 | BIC: | 3016. |
| Df Model: | 2 | | |
| Covariance Type: | nonrobust | | |

| | coef | std err | t | P> t | [0.025 | 0.975] |
|------------------|---------|---------|---------|-------|--------|--------|
| Intercept | 13.6980 | 0.067 | 205.757 | 0.000 | 13.567 | 13.829 |
| years | -1.1171 | 0.084 | -13.326 | 0.000 | -1.282 | -0.953 |
| years2 | 0.1132 | 0.022 | 5.060 | 0.000 | 0.069 | 0.157 |

| | | | |
|-----------------------|--------|--------------------------|----------|
| Omnibus: | 49.112 | Durbin-Watson: | 1.885 |
| Prob(Omnibus): | 0.000 | Jarque-Bera (JB): | 113.885 |
| Skew: | 0.199 | Prob(JB): | 1.86e-25 |
| Kurtosis: | 4.430 | Cond. No. | 27.5 |

Notes:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

```
In [9]: def PlotFittedValues(model, results, label=""):
        """
        Plots original data and fitted values.

        args:
            model (object): StatsModel model object
            results (object): StatsModel results object

        returns:
            None
        """
        years = model.exog[:, 1]
        values = model.endog
        thinkplot.Scatter(years, values, s=15, label=label)
        thinkplot.Plot(years, results.fittedvalues, label="model", color="#ff0000")
```

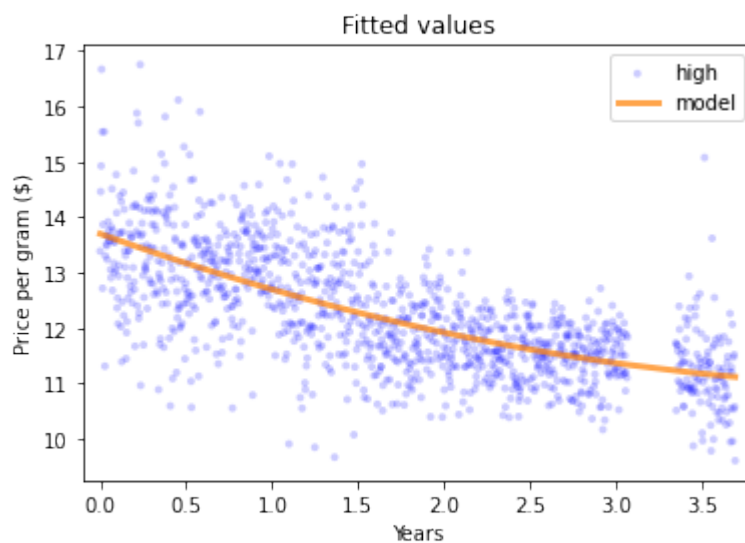
```
In [12]: def PlotLinearModel(daily, name):
        """
        Plots a linear fit to a sequence of prices, and the residuals.

        args:
            daily (DataFrame): daily prices
            name (string): label name

        returns:
            None
        """
        model, results = RunQuadraticModel(daily)
        PlotFittedValues(model, results, label=name)
        thinkplot.Config(
            title="Fitted values",
            xlabel="Years",
            xlim=[-0.1, 3.8],
            ylabel="Price per gram ($)",
        )
```

```
In [13]: name = "high"
        daily = dailies[name]
```

```
PlotLinearModel(daily, name)
```



```

In [15]: def SimulateResults(daily, iters=101, func = RunQuadraticModel):
    """
    Run simulations based on resampling residuals.

    args:
        daily: DataFrame of daily prices
        iters: number of simulations
        func: function that fits a model to the data

    returns:
        list of result objects
    """
    _, results = func(daily)
    fake = daily.copy()

    result_seq = []
    for _ in range(iters):
        fake.ppg = results.fittedvalues + thinkstats2.Resample(results.resid, n)
        _, fake_results = func(fake)
        result_seq.append(fake_results)

    return result_seq

In [16]: def GeneratePredictions(result_seq, years, add_resid=False):
    """
    Generates an array of predicted values from a list of model results.

    When add_resid is False, predictions represent sampling error only.

    When add_resid is True, they also include residual error (which is
    more relevant to prediction).

    args:
        result_seq: list of model results
        years: sequence of times (in years) to make predictions for
        add_resid: boolean, whether to add in resampled residuals

    returns:
        sequence of predictions
    """
    n = len(years)
    d = dict(Intercept=np.ones(n), years=years, years2=years**2)
    predict_df = pd.DataFrame(d)

    predict_seq = []
    for fake_results in result_seq:
        predict = fake_results.predict(predict_df)
        if add_resid:
            predict += thinkstats2.Resample(fake_results.resid, n)
        predict_seq.append(predict)

    return predict_seq

```

```
In [18]: def PlotPredictions(daily, years, iters=101, percent=90, func = RunQuadra
        """
        Plots predictions.

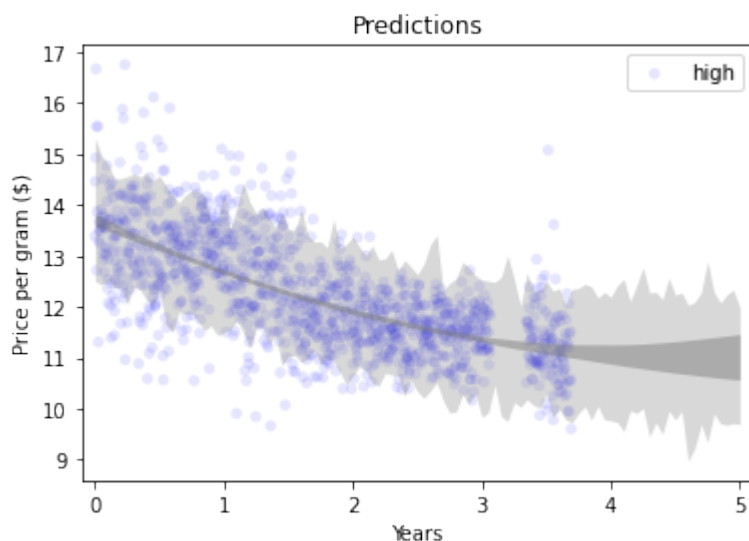
        args:
            daily: DataFrame of daily prices
            years: sequence of times (in years) to make predictions for
            iters: number of simulations
            percent: what percentile range to show
            func: function that fits a model to the data

        returns:
            None
        """
        result_seq = SimulateResults(daily, iters=iters, func=func)
        p = (100 - percent) / 2
        percents = p, 100 - p

        predict_seq = GeneratePredictions(result_seq, years, add_resid=True)
        low, high = thinkstats2.PercentileRows(predict_seq, percents)
        thinkplot.FillBetween(years, low, high, alpha=0.3, color="gray")

        predict_seq = GeneratePredictions(result_seq, years, add_resid=False)
        low, high = thinkstats2.PercentileRows(predict_seq, percents)
        thinkplot.FillBetween(years, low, high, alpha=0.5, color="gray")

In [19]: years = np.linspace(0, 5, 101)
        thinkplot.Scatter(daily.years, daily.ppg, alpha=0.1, label=name)
        PlotPredictions(daily, years)
        xlim = years[0] - 0.1, years[-1] + 0.1
        thinkplot.Config(
            title="Predictions", xlabel="Years", xlim=xlim, ylabel="Price per gram
        )
```



Exercise 12 - 2

Write a definition for a class named `SerialCorrelationTest` that extends `HypothesisTest` from Sect. 10.1. It should take a series and a lag as data, compute the serial correlation of the series with the given lag and then compute the p-value of the observed correlation.

Use this class to test whether the serial correlation in raw price data is statistically significant. Also test the residuals of the linear model and (if you did the previous exercise), the quadratic model.

```
In [26]: class HypothesisTest(object):

    def __init__(self, data):
        self.data = data
        self.MakeModel()
        self.actual = self.TestStatistic(data)

    def PValue(self, iters=1000):
        self.test_stats = [self.TestStatistic(self.RunModel())
                           for _ in range(iters)]

        count = sum(1 for x in self.test_stats if x >= self.actual)
        return count / iters

    def TestStatistic(self, data):
        raise NotImplementedError()

    def MakeModel(self):
        pass

    def RunModel(self):
        raise NotImplementedError()
```

```
In [29]: def SerialCorr(series, lag = 1):
    xs = series[lag:]
    ys = series.shift(lag)[lag:]

    corr = thinkstats2.Corr(xs, ys)

    return corr
```

```
In [30]: class SerialCorrelationTest(HypothesisTest):
    def TestStatistic(self, data):
        series, lag = data
        test_stat = abs(SerialCorr(series, lag))
        return test_stat

    def RunModel(self):
        series, lag = self.data
        permutation = series.reindex(np.random.permutation(series.index))

        return permutation, lag
```

```
In [41]: name = "high"
        daily = dailies[name]

        series = daily.ppg
        test = SerialCorrelationTest((series, 1))
        pvalue = test.PValue()
        print(f"Correlaton: {test.actual}")
        print(f"P-Value: {pvalue}")
```

Correlaton: 0.4852293761947381
P-Value: 0.0

There is a 0.49 correlation between consecutive prices and the p-value is close to 0 indicating th significant.

```
In [42]: # test for serial correlation in residuals of the linear model
```

```
_, results = RunLinearModel(daily)
series = results.resid
test = SerialCorrelationTest((series, 1))
pvalue = test.PValue()
print(f"Residuals Correlaton: {test.actual}")
print(f"P-Value: {pvalue}")
```

Correlaton: 0.07570473767506262
P-Value: 0.009

The residuals have a correlation of 0.08 and the p-value is 0.01 which is significant

```
In [43]: # test for serial correlation in residuals of the quadratic model
```

```
_, results = RunQuadraticModel(daily)
series = results.resid
test = SerialCorrelationTest((series, 1))
pvalue = test.PValue()
print(f"Correlaton: {test.actual}")
print(f"P-Value: {pvalue}")
```

Correlaton: 0.05607308161289913
P-Value: 0.051

The residuals in a quadratic model have a correlation of 0.06 and a p-value which is significant.