Hyper-dimensional gap finite elements for the enforcement of frictionless contact constraints

Brian Doran Giffin[1],
[1]*Department of Civil and Environmental Engineering,*
Oklahoma State University, United States of America
E-mail: brian.giffin@okstate.edu

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A new methodology for the enforcement of frictionless contact constraints in the context of finite element analyses is proposed. The method entails the definition of a family of intermediate contact surfaces parameterized by $\alpha \in [0, 1]$ between two contacting bodies, as illustrated in Figure 1. The continuous collection of all such intermediate surfaces may be interpreted as an interstitial – albeit degenerate gap volume between the two surfaces in contact, bearing some similarities to a contact domain method [1]. However, a notable distinction of the proposed method concerns the treatment of the interpolation parameter α as an auxiliary spatial coordinate, such that the degenerate interstitial gap volume is properly regarded as a hyper-dimensional surface. Over this intermediate domain, the derivative of a given continuous field with respect to α furnishes the definition of a local oriented gap function, which may be used in the formulation of contact inequality constraints. The method further poses mortar contact surface integrals by integrating over the entire parameterized family of intermediate surfaces. Discretization of the ensuing hyper-dimensional gap volume into finite elements is explored, offering several advantages over existing contact discretization methods: the proposed method does not require the computation of geometric intersections or projections; it exploits conventional Gaussian quadrature schemes to integrate the hyper-dimensional gap integrals with a sufficient degree of accuracy; and may be naturally and efficiently extended to represent contact between higher-order surfaces. Building upon the initial development of the proposed method for the enforcement of tied kinematic constraints [2], the present work extends the proposed approach for the treatment of frictionless contact constraints.

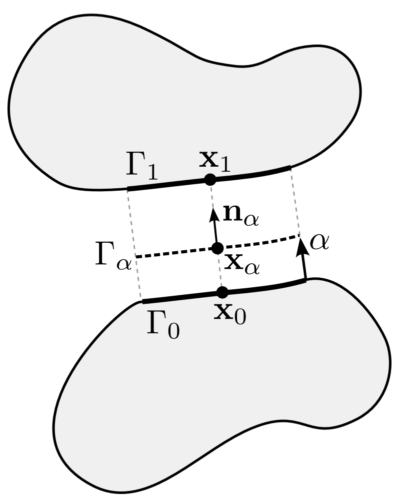

Figure 1: Illustration of a continuous collection of intermediate surfaces Γ_α parameterized by $\alpha \in [0, 1]$.

The central idea is summarized as follows: given two continuum bodies $\Omega_\alpha \, \forall\alpha = 0, 1$, the model problem under consideration supposes these two bodies share a common interface $\Gamma = \partial\Omega_0 \cap \partial\Omega_1$ defined over a subset of the their boundaries $\Gamma = \Gamma_\alpha \subset \partial\Omega_\alpha \, \forall\alpha = 0, 1$ which are in contact with one another, such that the following gap inequality constraint is satisfied:

$$[\mathbf{x}_1 - \mathbf{x}_0] \cdot \mathbf{n}_0 \geq 0 \quad \forall \mathbf{x}_\alpha \in \Gamma_\alpha, \, \alpha = 0, 1. \tag{1}$$

In the above, $\mathbf{n}_0$ denotes the outward unit vector normal to Γ_0, and $\mathbf{x}_1 \in \Gamma_1$ is conventionally defined as the closest point projection of $\mathbf{x}_0 \in \Gamma_0$ onto Γ_1. In a mortar method [3], the above inequality constraint may be equivalently enforced by introducing a Lagrange multiplier field $\lambda \in \mathcal{F}(\Gamma)$ with units of surface pressure (force per unit area) defined over Γ such that

$$\int_\Gamma \lambda \, [\mathbf{x}_1 - \mathbf{x}_0] \cdot \mathbf{n}_0 \, \mathrm{d}\Gamma = 0 \quad \forall \lambda \in \mathcal{F}(\Gamma). \tag{2}$$

In the mathematical formulation of the model problem, there is no ambiguity regarding the definition of Γ since both Γ_0 and Γ_1 are coincident. However, in the numerical setting, each Γ_α may be discretized (e.g. using finite elements) in a non-conforming manner, and it is not immediately clear how Γ should be defined.

By defining a continuous family of intermediate interfaces Γ_α parameterized by $\alpha \in [0, 1]$, the continuous collection of all such intermediate surfaces spanning $\alpha \in [0, 1]$ may be viewed as a hyper-dimensional intermediate domain $\Sigma = \{(\mathbf{x}_\alpha, \alpha) \,|\, \mathbf{x}_\alpha \in \Gamma_\alpha, \alpha \in [0, 1]\}$ spanning the void between Γ_0 and Γ_1. Further interpreting $\frac{\partial \mathbf{x}}{\partial \alpha}$ as a local measure of the infinitessimal jump in the position field $\mathbf{x}$, the mortar integral posed by Equation 2 is reformulated in terms of the following integral defined over the entire family of intermediate surfaces spanning $\alpha \in [0, 1]$:

$$\int_{\alpha=0}^{\alpha=1} \int_{\Gamma_\alpha} \lambda \frac{\partial \mathbf{x}}{\partial \alpha} \cdot \mathbf{n}_\alpha \, \mathrm{d}\Gamma_\alpha \mathrm{d}\alpha = 0 \quad \forall \lambda \in \mathcal{F}(\Sigma). \tag{3}$$

If the hyper-surface Σ is given a parametric representation, then the ensuing differential form for $\mathbf{n}_\alpha \, \mathrm{d}\Gamma_\alpha \mathrm{d}\alpha$ may be determined through a change of variables. In particular, finite element discretizations of Σ are explored further in the presented work, along with a number of alternative choices for the discretization of the multiplier field.

References

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