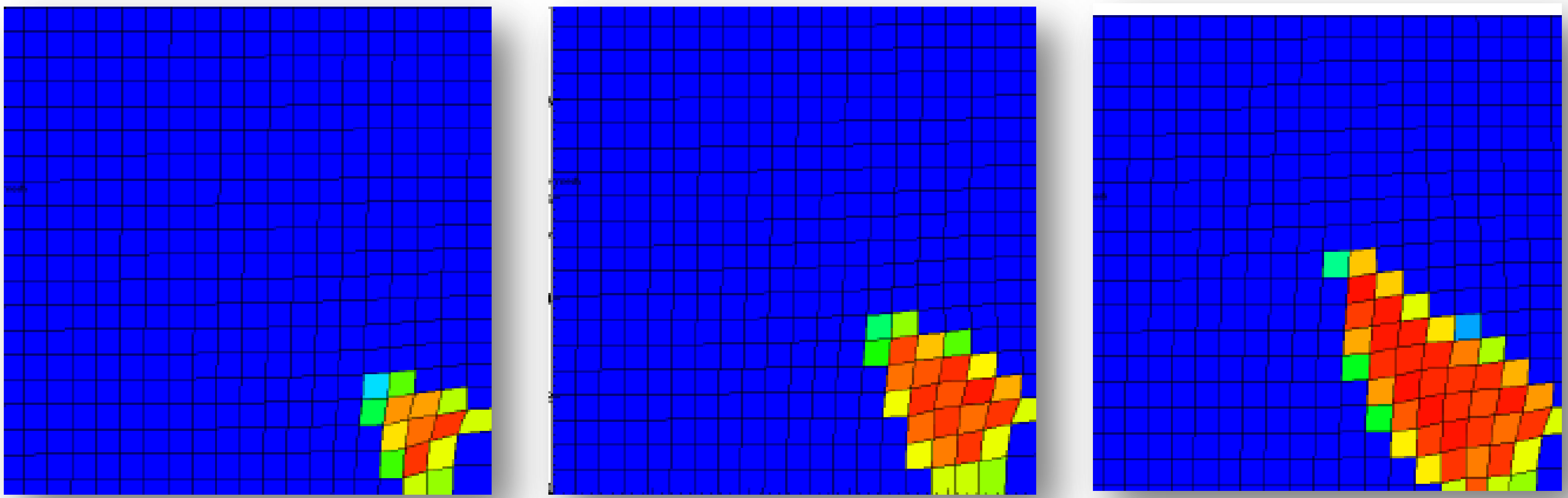

# A Smeared-Crack Approach to Modeling Hydraulic Fracture in GEOS

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# Outline

- Introduction & Approach
- Poromechanics
- Damage Mechanics
- GEOS Implementation
- Conclusions & Future Work

# Introduction

**Collaboration with LLNL**

- Interest in modeling hydraulic fracturing
- GEOS: suitable platform for such problems

**Main Goal**: Implement a new modeling approach within GEOS

# Approach

**Homogenized (``Smeared'') Approach for Modeling Hydraulic Fracture**

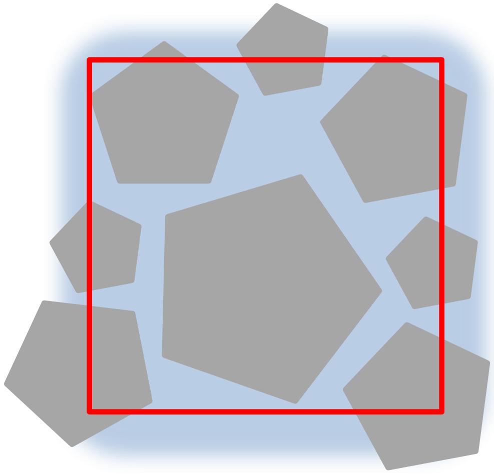
- Treat cracked rock mass as a homogenized medium
  - Variable and anisotropic stiffness and hydraulic conductivity
  - Evolve constitutive behavior via a damage parameter to simulate crack propagation

**Eliminates the need for:**

- Re-meshing the geometry as `cracks' evolve
- Contact enforcement between cracked faces
- The generation of a separate `flow mesh'

# Poromechanics

- Coupled solution of solid deformation and flow in a porous medium
  - Quasi-static deformation
  - Fully saturated, single phase, diffusive fluid flow
- Consider a RVE to define the poroelastic solid



# Governing Equations

- Equilibrium of the Poroelastic Solid

$$\sigma_{ij,j} + \rho g_i = 0 \qquad \forall x \in B$$

$\sigma_{ij}$ Total stress tensor
$\rho$ Overall mass density per unit of initial (undeformed) volume
$g_i$ Gravitational body force (vector)

- Compressible Fluid Flow in a Porous Medium

$$\frac{\partial}{\partial t}\phi + \mathcal{V}_{i,i} = 0 \qquad \forall x \in B$$

$\mathcal{V}_i$ Volumetric fluid flux
$\phi$ Lagrangian porosity

# Constitutive Relations

- For the Poroelastic Solid

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl} - b_{ij}p$$

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$$

$C_{ijkl}$ Tangent stiffness modulus of the “skeleton”

$\varepsilon_{ij}$ ``Skeleton” strain tensor

$b_{ij}$ Biot's modulus (symmetric rank 2 tensor; links total stress and fluid pressure increments)

$p$ Fluid (pore) pressure

$u_i$ Displacement of the ``skeleton”

# Constitutive Relations

- For the Compressible Fluid

$$\phi = \phi_0 + b_{ij}\varepsilon_{ij} + \frac{p}{M}$$

$$\mathcal{V}_i = -\frac{\kappa_{ij}}{\mu}(p_{,j} - \rho^f g_j)$$

$1/M$ Fluid capacity parameter (scalar valued quantity that links pore pressure and porosity variation)

$\kappa_{ij}$ Intrinsic permeability tensor

$\mu$ Fluid viscosity

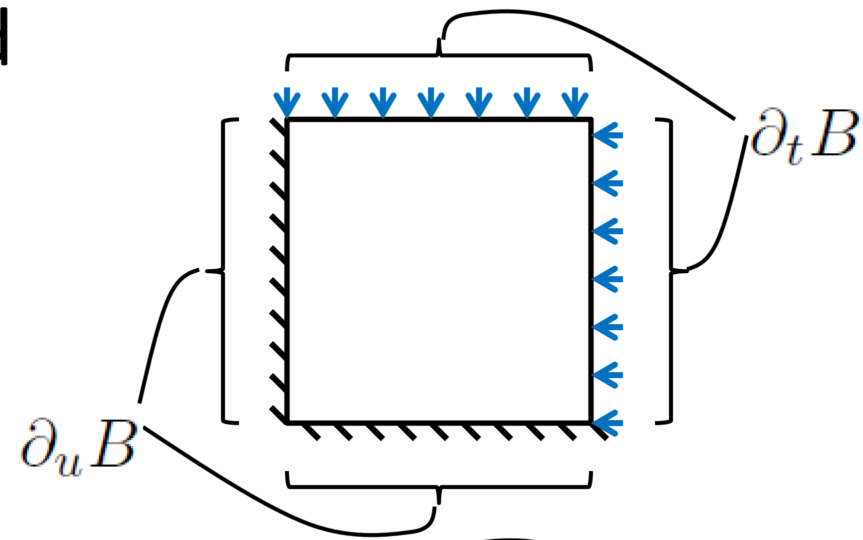
$\rho^f$ Intrinsic fluid mass density

# Boundary Conditions

- For the Poroelastic Solid

$$u_i = \bar{u}_i \quad on \quad \partial_u B$$

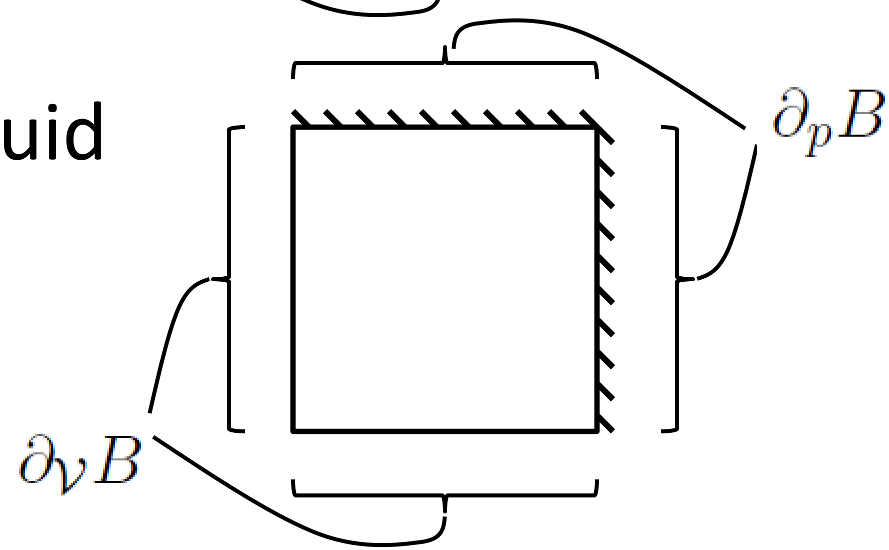
$$\sigma_{ij} n_j = \bar{t}_i \quad on \quad \partial_t B$$




- For the Compressible Fluid

$$p = \bar{p} \quad on \quad \partial_p B$$

$$\mathcal{V}_i n_i = \bar{\mathcal{V}} \quad on \quad \partial_{\mathcal{V}} B$$



# Weak Form

Find $u_i \in \mathcal{S}$ and $p \in \mathcal{T}$ such that:

$$\int_B \sigma_{ij} v_{i,j} dv = \int_{\partial_t B} \bar{t}_i v_i da + \int_B \rho g_i v_i dv \quad \forall v_i \in V$$

and

$$\int_B \mathcal{V}_i q_{,i} dv = \int_{\partial_{\mathcal{V}} B} \bar{\mathcal{V}} q da + \int_B \frac{\partial \phi}{\partial t} q dv \quad \forall q \in \mathcal{Q}$$

# Finite Element System of Equations

$$\begin{bmatrix} \mathbf{K_{uu}} & -\mathbf{K_{up}} \\ -\mathbf{K_{pu}} & -\left(\frac{\Delta t}{2}\mathbf{K_{pp}} + \mathbf{M_{pp}}\right) \end{bmatrix}^{(m+1)} \begin{bmatrix} \mathbf{u} \\ \mathbf{p} \end{bmatrix}^{(m+1)} \cdots$$

$$\cdots = \begin{bmatrix} -\mathbf{K_{uu}} & \mathbf{K_{up}} \\ -\mathbf{K_{pu}} & \left(\frac{\Delta t}{2}\mathbf{K_{pp}} - \mathbf{M_{pp}}\right) \end{bmatrix}^{(m)} \begin{bmatrix} \mathbf{u} \\ \mathbf{p} \end{bmatrix}^{(m)} \cdots$$

$$\cdots + \begin{bmatrix} \mathbf{F_u} \\ \left(\frac{\Delta t}{2}\mathbf{F_p^1} + \mathbf{F_p^2}\right) \end{bmatrix}^{(m+1)} + \begin{bmatrix} \mathbf{F_u} \\ \left(\frac{\Delta t}{2}\mathbf{F_p^1} - \mathbf{F_p^2}\right) \end{bmatrix}^{(m)}$$

# Damage Mechanics

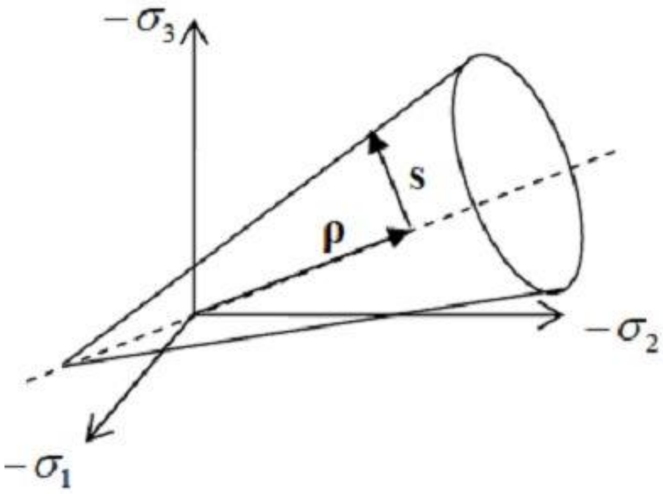
**Proof of concept using a simple isotropic material degradation scheme:**

- Consider a damage parameter: $D \in (0, 1]$

$$\lambda = \lambda(D); \qquad \mu = \mu(D); \qquad \kappa_{ij} = \kappa_{ij}(D)$$

- Evolve material properties using an appropriate failure criterion to emulate fracture
- Desired behavior:
  - $\sigma_t << \sigma_c$ (lower tensile rupture stress)
  - Pressure-dependent material failure
  - Damage linked to the deviatoric stress

# Failure Criterion




- Drucker-Prager failure criterion:

$$\sqrt{J_2} = A + BI_1$$

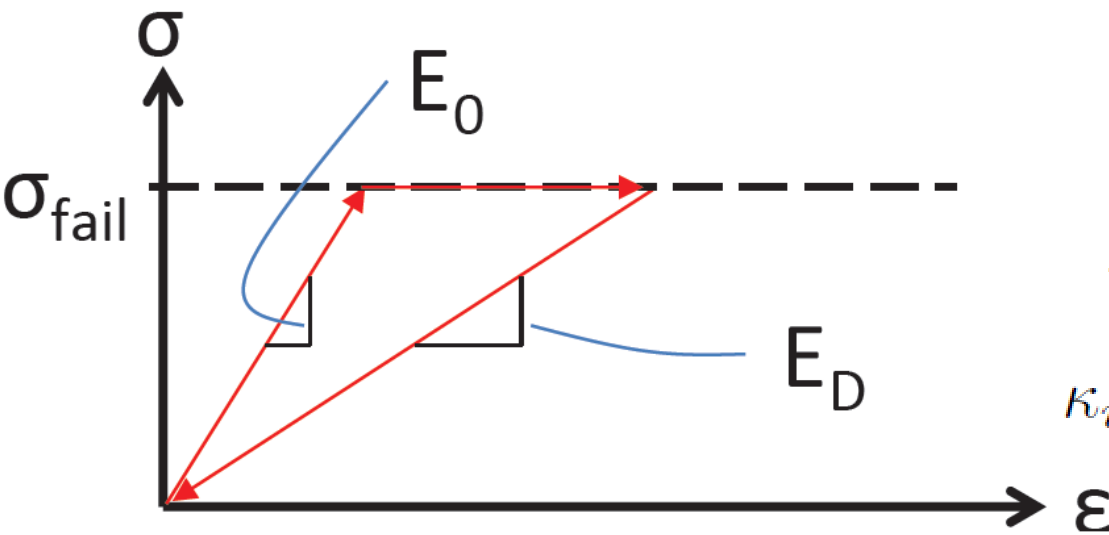
$J_2$ – second invariant of the deviatoric part of the *effective* skeleton stress

$I_1$ – first invariant of the *effective* skeleton stress

$A,\ B$ – constants that depend on $\sigma_t$ and $\sigma_c$

# Damage Evolution Law

- Permanent, irreversible degradation of the elastic properties and hydraulic conductivity:
  - Failure surface: $f(\sigma'_{ij}) = J_2 - (A + BI_1)^2 = 0$
  - Consistency condition: $df = 0$ during failure

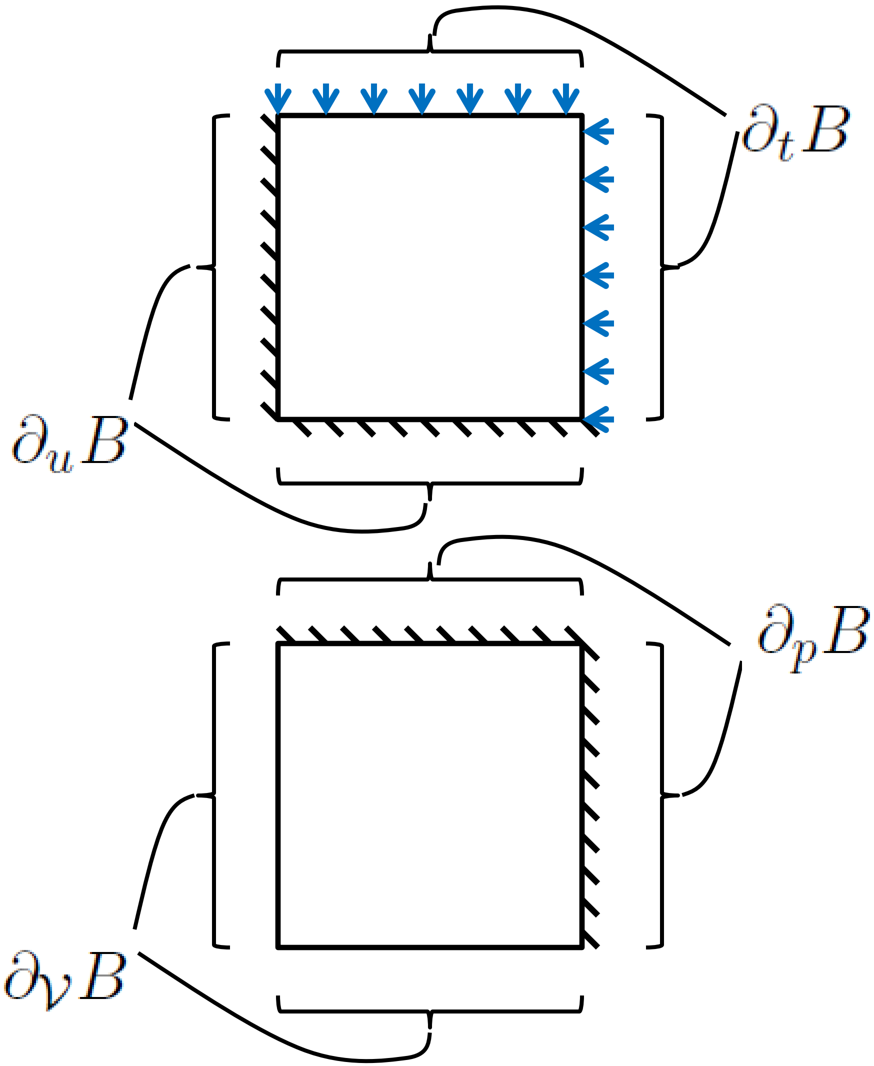



$$\mu(D) = \mu_0 D$$

$$\lambda(D) = \lambda_0 + \frac{2}{3}\mu_0(1 - D)$$

$$\kappa_{ij}(D) = \frac{\kappa}{D}\delta_{ij}$$

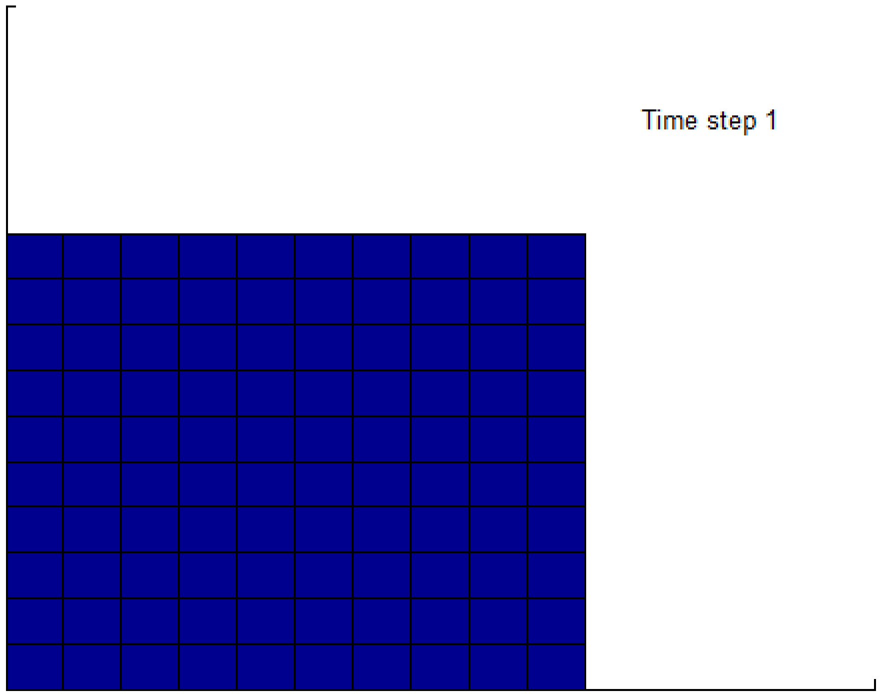
# Test Problem in MATLAB




- 2-D cube of poroelastic material subjected to an external pressure load
- $u_\perp = 0$ on $\partial_u B$
- $p = 0$ on $\partial_p B$
- $\mathcal{V}_i n_i = 0$ on $\partial_{\mathcal{V}} B$
- $\sigma_{\perp j} n_j = -P(t)$ on $\partial_t B$
- $\sigma_{\| j} n_j = 0$ on $\partial_t B$

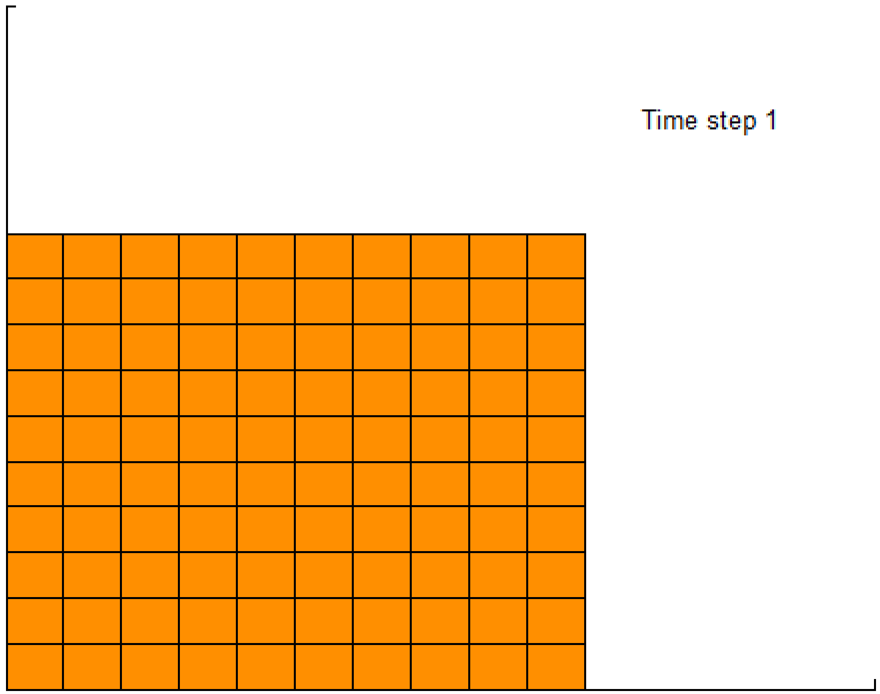
# Test Problem 1

Material: homogeneous, isotropic, time invariant




# Test Problem 2

P(t) = constant

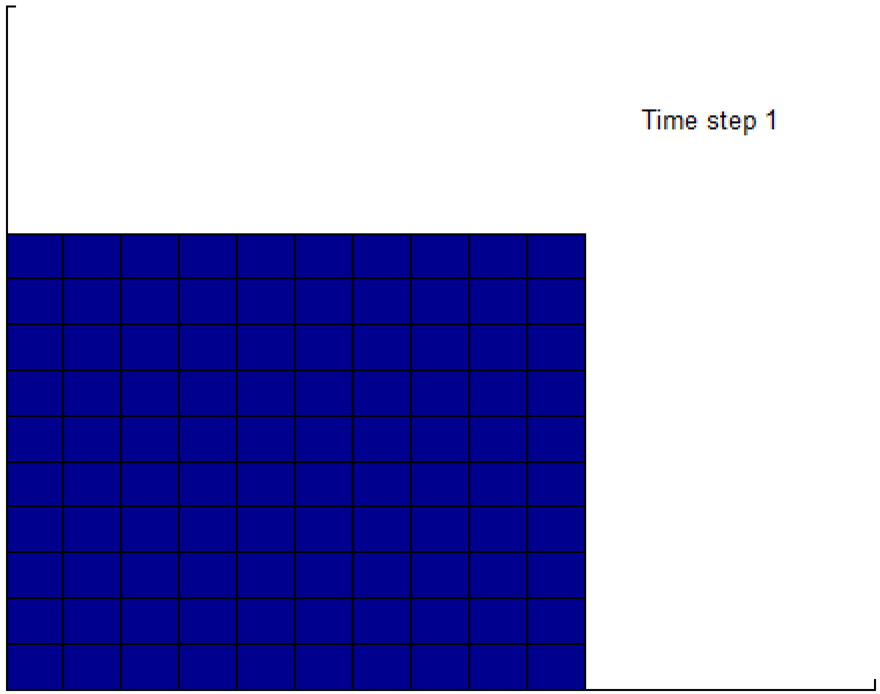



# Pressure Oscillations

- Known issue with coupled poromechanics
- Occurs for:
  - Incompressibility constraint
  - Violation of minimum time step criterion
- Can be overcome through mesh refinement
- There exist several remediation techniques
- Fluid pressure Laplacian stabilization (FPL)
  - Adds stabilization terms to the weak form

# Test Problem 3

Inhomogeneous material properties



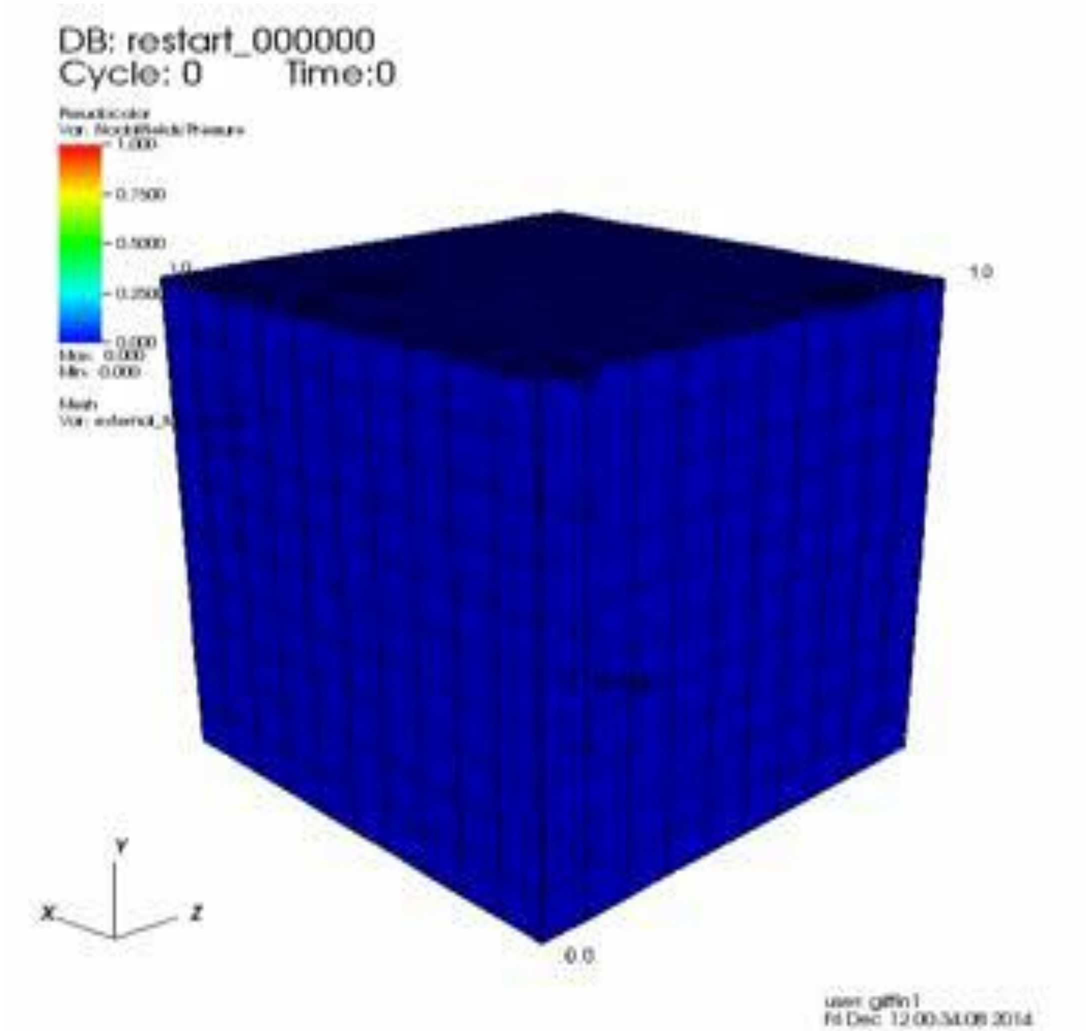

$$\kappa(\mathbf{x}) = \begin{cases} 100\kappa & 0 \leq x_2 \leq h_e \\ \kappa & h_e < x_2 \end{cases}$$

# Implementation in GEOS

- Code based around physics `solvers'
  - Different solvers model different physics
    - e.g. solid mechanics or fluid mechanics
- **Goal**: create a `Poromechanics Solver' that incorporates a rudimentary damage model

# 3D Test Problem

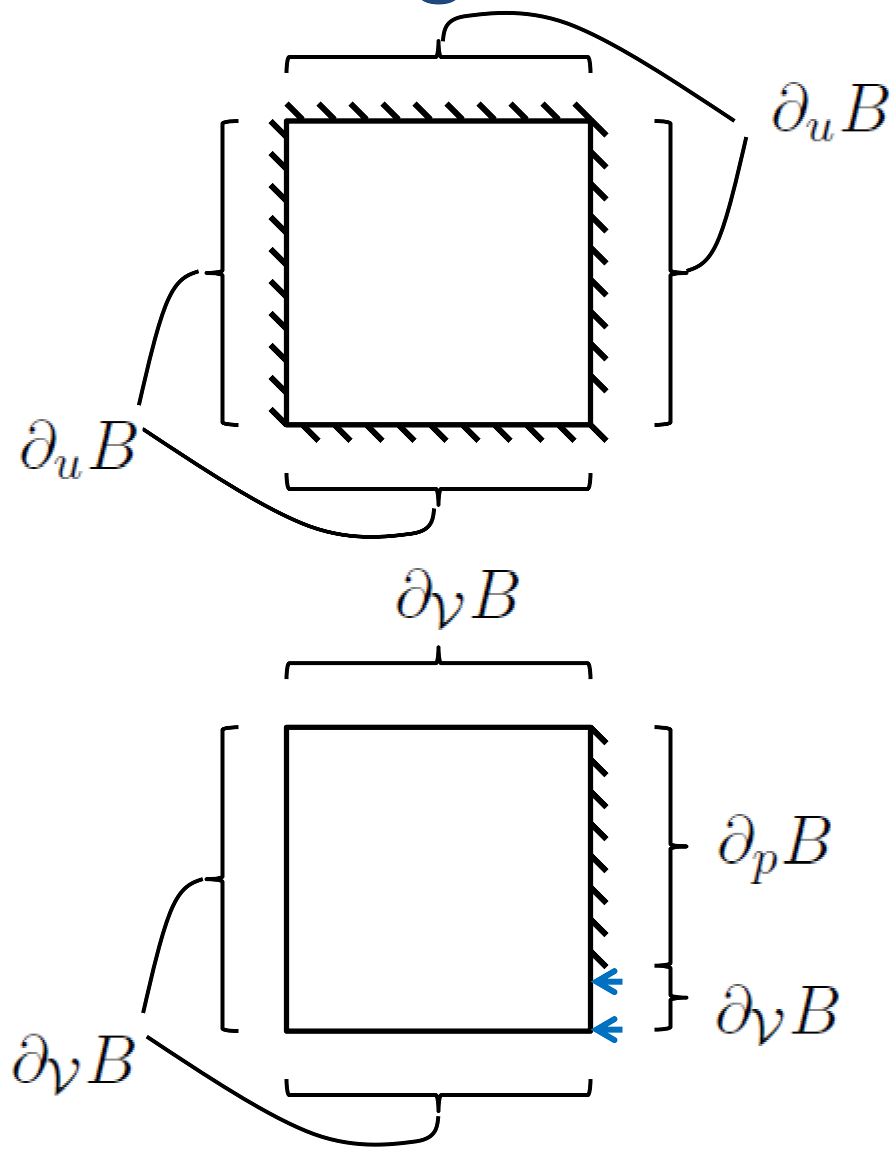
3D version of MATLAB Test Problem 1, using GEOS



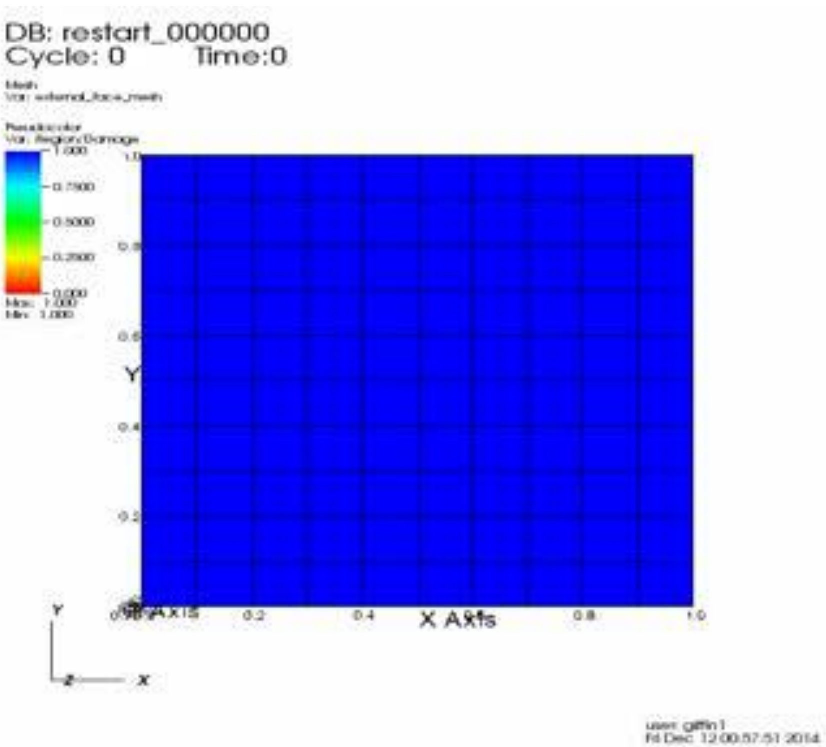
# Pressure-Driven Damage Test

- 2D block of initially undamaged poroelastic material
- $u_\perp = 0$ on $\partial_u B$
- $p = 0$ on $\partial_p B$
- $\mathcal{V}_j n_j = 0$ on top, left, bottom faces of $\partial_\mathcal{V} B$
- $\mathcal{V}_j n_j = F < 0$ on right face

  (F < 0, inward directed)



# Depiction of Damage Evolution

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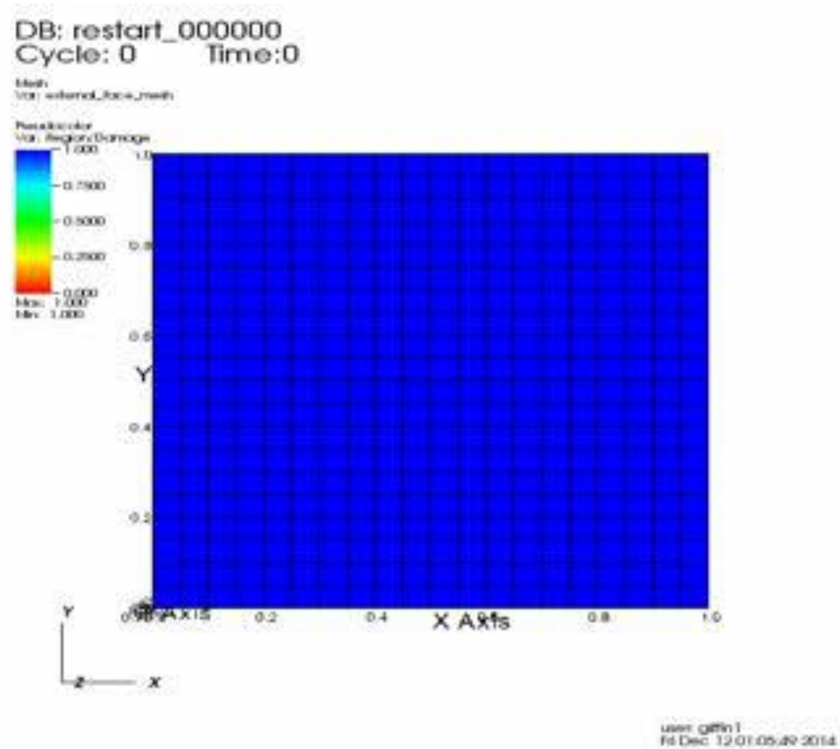


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# Opportunities for Future Work

- Differentiate between crack initiation and propagation
  - Would require proper identification of crack tips
- Investigate anisotropic material degradation
- Employ a more sophisticated damage evolution law
  - Look at plasticity models for granular materials
  - Nonlocal damage theories
- Pursue methods for abating spurious pressure oscillations (FPL)
- Implement dynamics and finite deformations
- Consider multi-phase fluid flow
- Investigate a double-porosity model to differentiate between crack porosity vs. rock porosity

Questions?