Construction of Hex20 shape functions

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Shape functions:

$$\varphi_a = \frac{1}{8}(1+\xi\xi_a)(1+\eta\eta_a)(1+\zeta\zeta_a)(-2+\xi\xi_a+\eta\eta_a+\zeta\zeta_a) \quad \forall\, a \in [1,\ldots,8] \tag{1}$$

$$\varphi_a = \frac{1}{4}(1+\xi\xi_a-(\xi\eta_a\zeta_a)^2)(1+\eta\eta_a-(\xi_a\eta\zeta_a)^2)(1+\zeta\zeta_a-(\xi_a\eta_a\zeta)^2) \quad \forall\, a \in [9,\ldots,20] \tag{2}$$

Derivatives:

$$\varphi_{a,\xi} = \frac{1}{8}\xi_a(1+\eta\eta_a)(1+\zeta\zeta_a)(2\xi\xi_a+\eta\eta_a+\zeta\zeta_a-1) \quad \forall\, a \in [1,\ldots,8] \tag{3}$$

$$\varphi_{a,\eta} = \frac{1}{8}(1+\xi\xi_a)\eta_a(1+\zeta\zeta_a)(2\eta\eta_a+\zeta\zeta_a+\xi\xi_a-1) \quad \forall\, a \in [1,\ldots,8] \tag{4}$$

$$\varphi_{a,\zeta} = \frac{1}{8}(1+\xi\xi_a)(1+\eta\eta_a)\zeta_a(2\zeta\zeta_a+\xi\xi_a+\eta\eta_a-1) \quad \forall\, a \in [1,\ldots,8] \tag{5}$$

$$\varphi_{a,\xi} = \frac{1}{4}(\xi_a-2\xi\eta_a^2\zeta_a^2)(1+\eta\eta_a-(\xi_a\eta\zeta_a)^2)(1+\zeta\zeta_a-(\xi_a\eta_a\zeta)^2) \quad \forall\, a \in [9,\ldots,20] \tag{6}$$

$$\varphi_{a,\eta} = \frac{1}{4}(1+\xi\xi_a-(\xi\eta_a\zeta_a)^2)(\eta_a-2\xi_a^2\eta\zeta_a^2)(1+\zeta\zeta_a-(\xi_a\eta_a\zeta)^2) \quad \forall\, a \in [9,\ldots,20] \tag{7}$$

$$\varphi_{a,\zeta} = \frac{1}{4}(1+\xi\xi_a-(\xi\eta_a\zeta_a)^2)(1+\eta\eta_a-(\xi_a\eta\zeta_a)^2)(\zeta_a-2\xi_a^2\eta_a^2\zeta) \quad \forall\, a \in [9,\ldots,20] \tag{8}$$