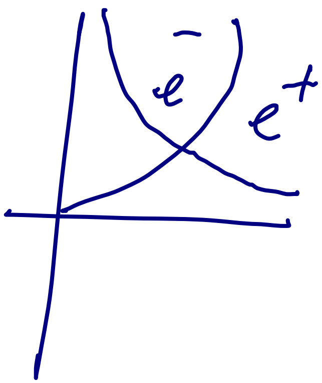
BJT small-signal Analysis

จากคุณสมบัติของ Diode สามารถเขียนเป็นสมการ สำหรับ Forward และ Reverse bias region ได้ดังนี้



$$I_D = I_S\left(e^{kV_D/T_k} - 1\right)$$

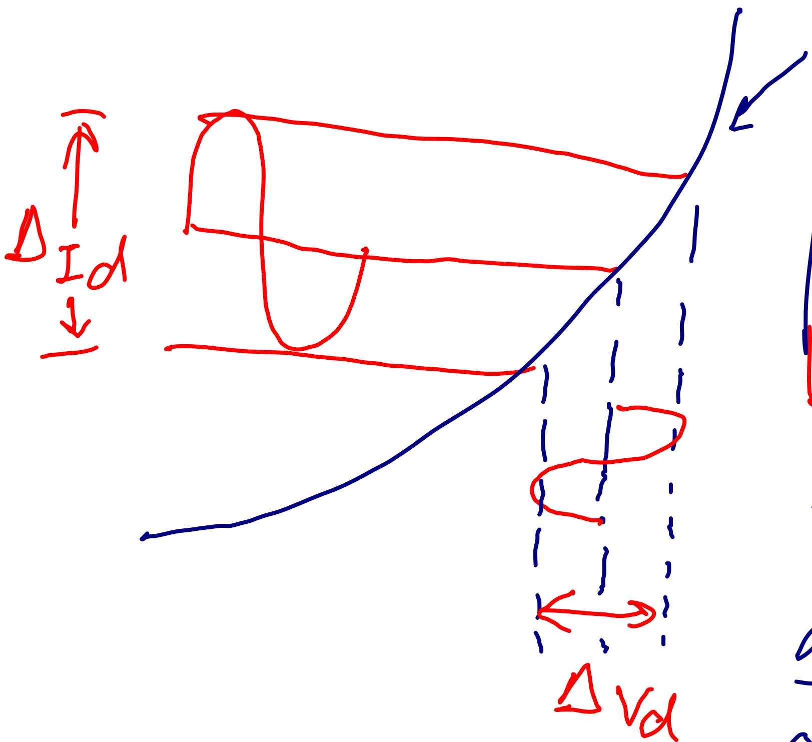
I_S = reverse saturation current

$k = 11,600/\eta \rightarrow \eta = \begin{cases} 1 & \text{เมื่อเป็น Ge} \\ 2 & \text{" Si} \end{cases}$

$T_k = T_C + 273°$ (kelvin)

V_D = แรงดันที่จ่ายให้กับ Diode

AC หรือ Dynamic Resistance



$$\boxed{r_d = \frac{\Delta V_d}{\Delta I_d}}$$

ถ้าต้องการหา

$$\frac{d(I_D)}{dV_D} = \frac{d\left(I_S\left(e^{kV_D/T_k} - 1\right)\right)}{dV_D}$$

และ $\frac{dI_D}{dV_D} = \frac{k}{T_k}(I_D + I_S)$ ถ้า $I_D \gg I_S$

$$\frac{dI_D}{dV_D} = \frac{k}{T_k} I_D$$

สมมติให้ Diode เป็น Ge จะได้

$$k = \frac{11,600}{\eta} = \frac{11,600}{1} = 11,600$$

ที่อุณหภูมิห้องปกติ 25°C จะได้

$$T_k = 25^\circ + 273^\circ = 298^\circ$$

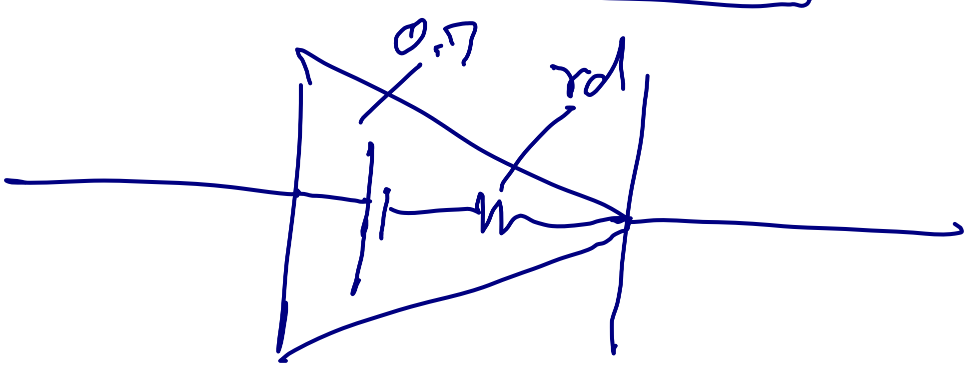
หา k/T_k จะได้

$$\frac{k}{T_k} = \frac{11,600}{298} = 38.93$$

$$\frac{dI_D}{dV_D} = 38.93\, I_D$$

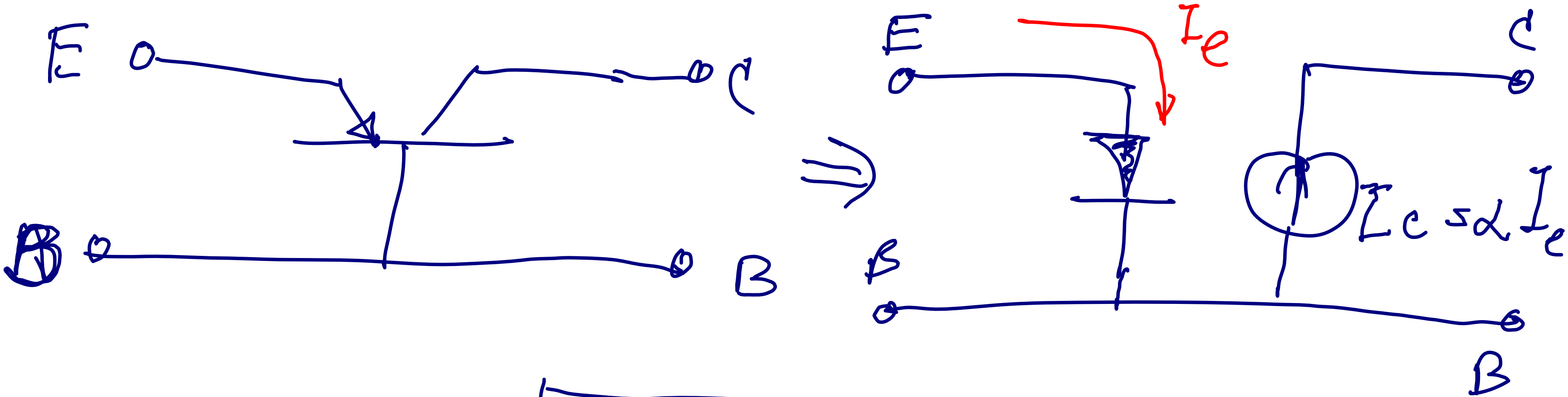
$$\frac{dV_D}{dI_D} = \frac{1}{38.93\, I_D} = \frac{0.026\,V}{I_D}$$

$$\boxed{\therefore r_d = \frac{26\,mV}{I_D}} \Longleftarrow \text{Diode}$$



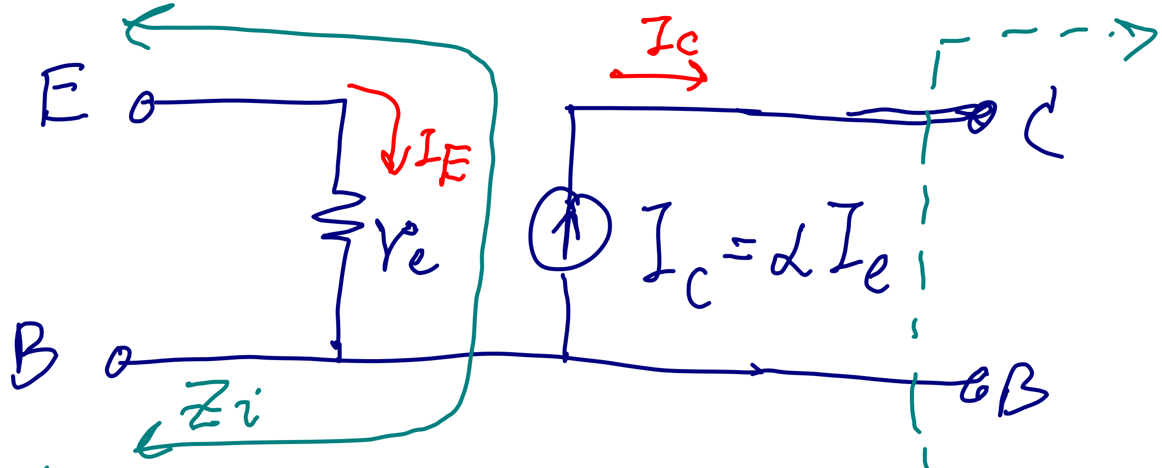
The r_e Transistor Model (Transistor)

- Common base



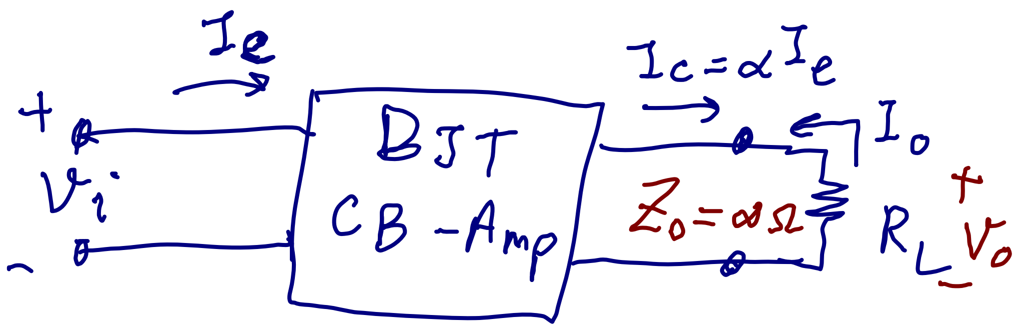
$$\boxed{r_e = \frac{26mV}{I_E}}$$

$\cancel{I_e}$ I_E



$$\boxed{Z_i = r_e \,,\; Z_o = \infty\ \Omega}$$

Z_o



$$V_o = -I_o R_L = -(-I_c R_L) = \alpha I_e R_L$$

$$V_i = I_e Z_i = I_e r_e$$

อัตราขยายแรงดัน $\rightarrow A_V = \dfrac{V_o}{V_i} = \dfrac{\alpha \cancel{I_e} R_L}{\cancel{I_e} r_e} = \alpha \dfrac{R_L}{r_e}$

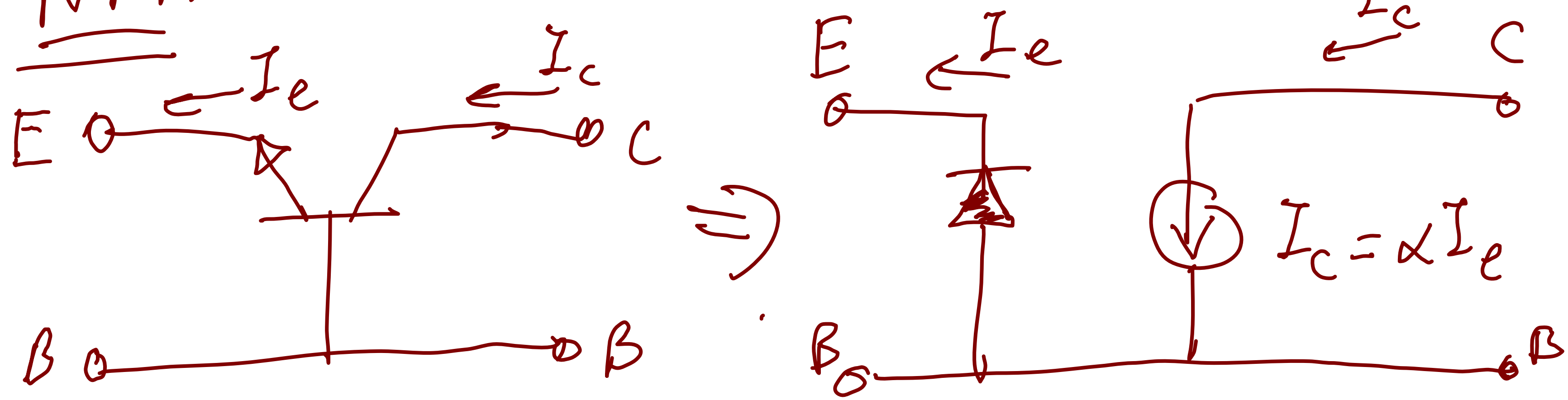
ถ้า $\alpha \simeq 1$

ดังนั้น $\boxed{A_V = \dfrac{R_L}{r_e}}$ #

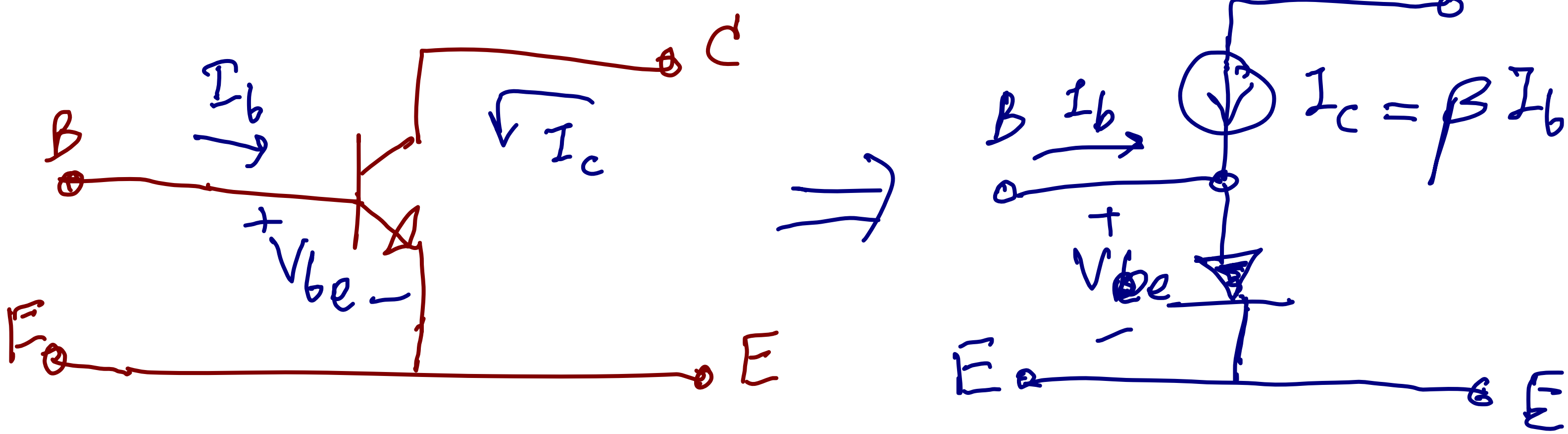
A_i (อัตราการขยายกระแส) $= \dfrac{I_o}{I_i}$

$A_{i\,(CB)} = -\dfrac{I_c}{I_e} = \dfrac{-\alpha \cancel{I_e}}{\cancel{I_e}} = -\alpha \simeq -1$

NPN



Common Emitter



$$I_c = \beta I_b$$

$$I_e = I_c + I_b = \beta I_b + I_b = (\beta+1) I_b$$

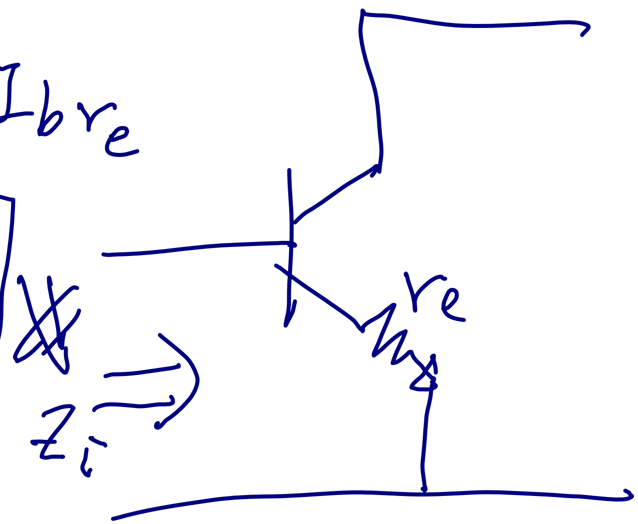
$$I_e \cong \beta I_b$$

חש Z_i

$$\boxed{Z_i = \frac{V_i}{I_i} = \frac{V_{be}}{I_b}}$$

חוב $V_i = V_{be} = I_e r_e \cong \beta I_b r_e$

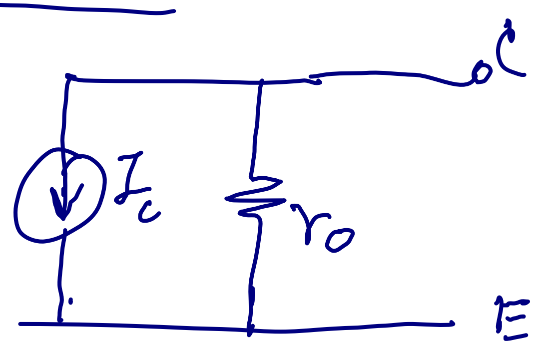
$$\boxed{Z_i = \frac{\beta \cancel{I_b} r_e}{\cancel{I_b}} = \beta r_e}$$



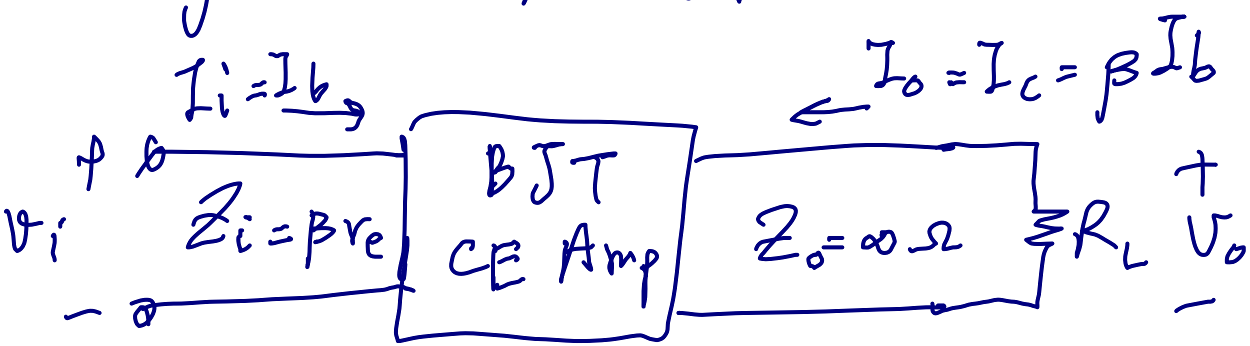
สำหรับ CE ค่าของ Z_i มีค่าประมาณ $100\,\Omega - 7\,k\Omega$

Z_o (output impedance)

$$\boxed{Z_o = r_o}$$



ถ้าสมมติให้ $Z_o = \infty\,\Omega$ เราสามารถหา Voltage gain ได้ดังนี้



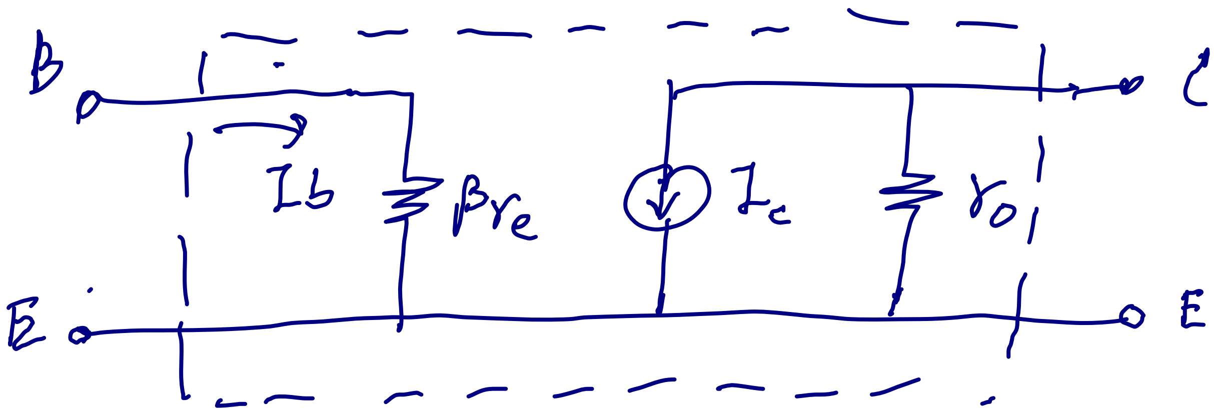
$V_o = I_c R_L = \beta I_b R_L$

และ $V_i = I_i Z_i = I_b \beta r_e$

$$\therefore \boxed{A_v = \frac{V_o}{V_i} = \frac{\beta I_b R_L}{\beta I_b r_e} = \frac{R_L}{r_e}}$$ เมื่อ $r_o = \infty\,\Omega$

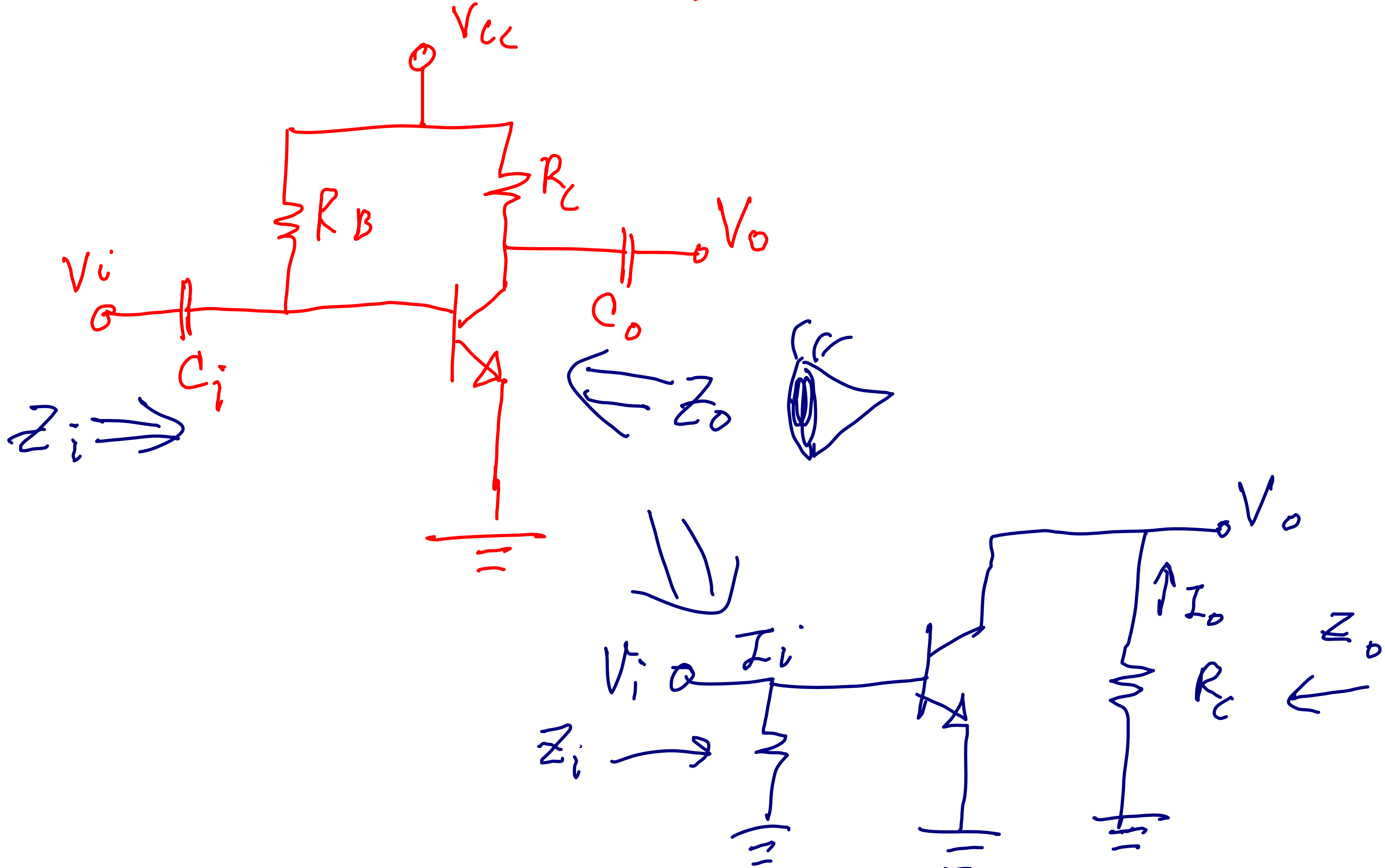
อัตราส่วนการขยายกระแส (A_i)

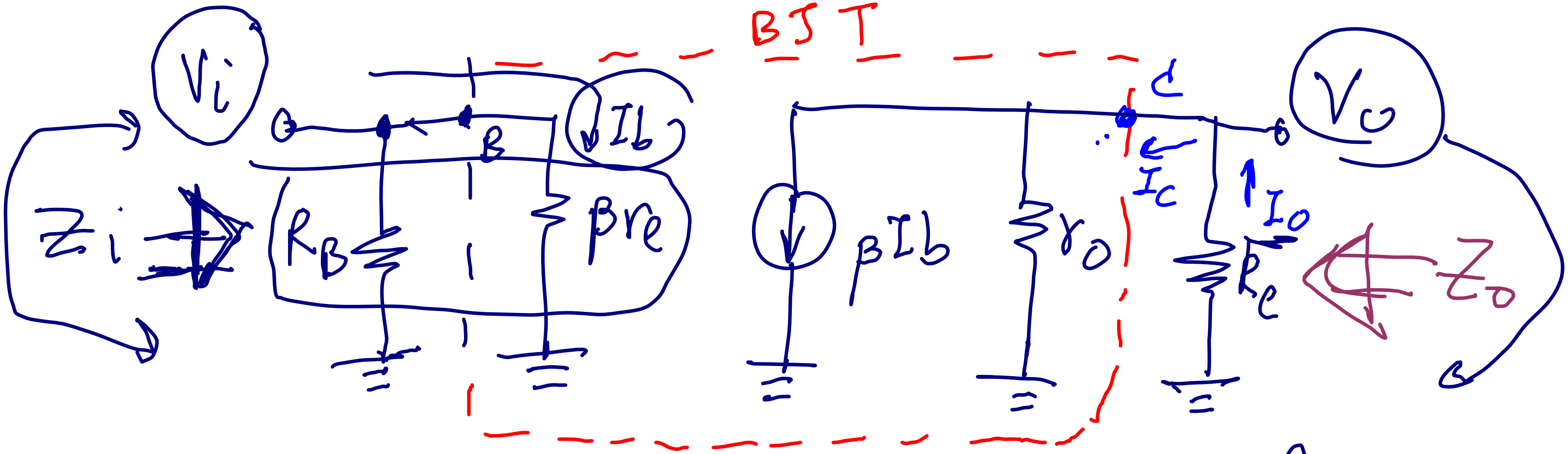
$$\boxed{A_i = \frac{I_o}{I_i} = \frac{I_c}{I_b} = \frac{\beta I_b}{I_b} = \beta}$$ เมื่อ $r_o = \infty\,\Omega$



COMMON EMITTER FIXED-BIAS

out send





หา Z_i

$$Z_{i(tot)} = R_B \,\|\, \beta r_e$$

$$\frac{R_B \times \beta r_e}{R_B + \beta r_e}$$

กรณี $R_B >> 10\beta r_e$ จะได้ว่า

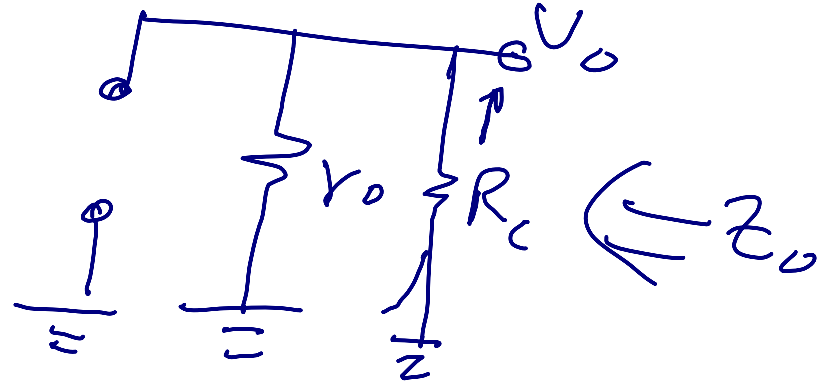
$$\boxed{Z_i = \beta r_e}$$

หา Z_o

$$\boxed{Z_o = R_C \,\|\, r_o}$$

ถ้า $r_o >> 10R_C$ จะได้

$$\boxed{Z_o = R_C}$$



<u>หา A_V</u>

$$V_o = -\overbrace{\beta I_b}^{I_c}(R_C \| r_o)$$

โดยที่ $I_b = \dfrac{V_i}{\beta r_e}$

แทน I_b ใน จะได้

$$V_o = -\beta\left(\frac{V_i}{\beta r_e}\right)(R_C \| r_o) =$$

$$A_V = \frac{V_o}{V_i} = -\frac{(R_C \| r_o)}{r_e}$$

เมื่อ $r_o >> 10R_C$ จะได้ว่า

$$\boxed{A_V = -\frac{R_C}{r_e}}$$ #

หา A_i

จากรูปจะได้

$$I_o = \frac{r_o \beta I_b}{r_o + R_C}$$ และ $$\frac{I_o}{I_b} = \frac{r_o \beta}{r_o + R_C}$$

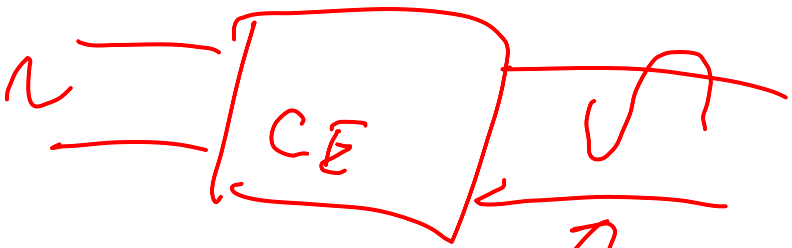
จากรูป I_b จะได้

$$I_b = \frac{R_B I_i}{\beta r_e + R_B}$$ โดยที่ $$I_i = \frac{(\beta r_e + R_B) I_b}{R_B}$$

ดังนั้น $$A_i = \frac{I_o}{I_i} = \frac{r_o \beta \cancel{I_b}}{r_o + R_C} \cdot \frac{R_B}{(\beta r_e + R_B)\cancel{I_b}} = \frac{R_B r_o \beta}{(r_o + R_C)(\beta r_e + R_B)}$$

ถ้า $r_o \gg 10R_C$ และ $R_B \gg 10\beta r_e$ จะได้ว่า

$$\boxed{A_i = \frac{\beta \cancel{R_B} \cancel{r_o}}{\cancel{r_o} \cancel{R_B}} = \beta}$$

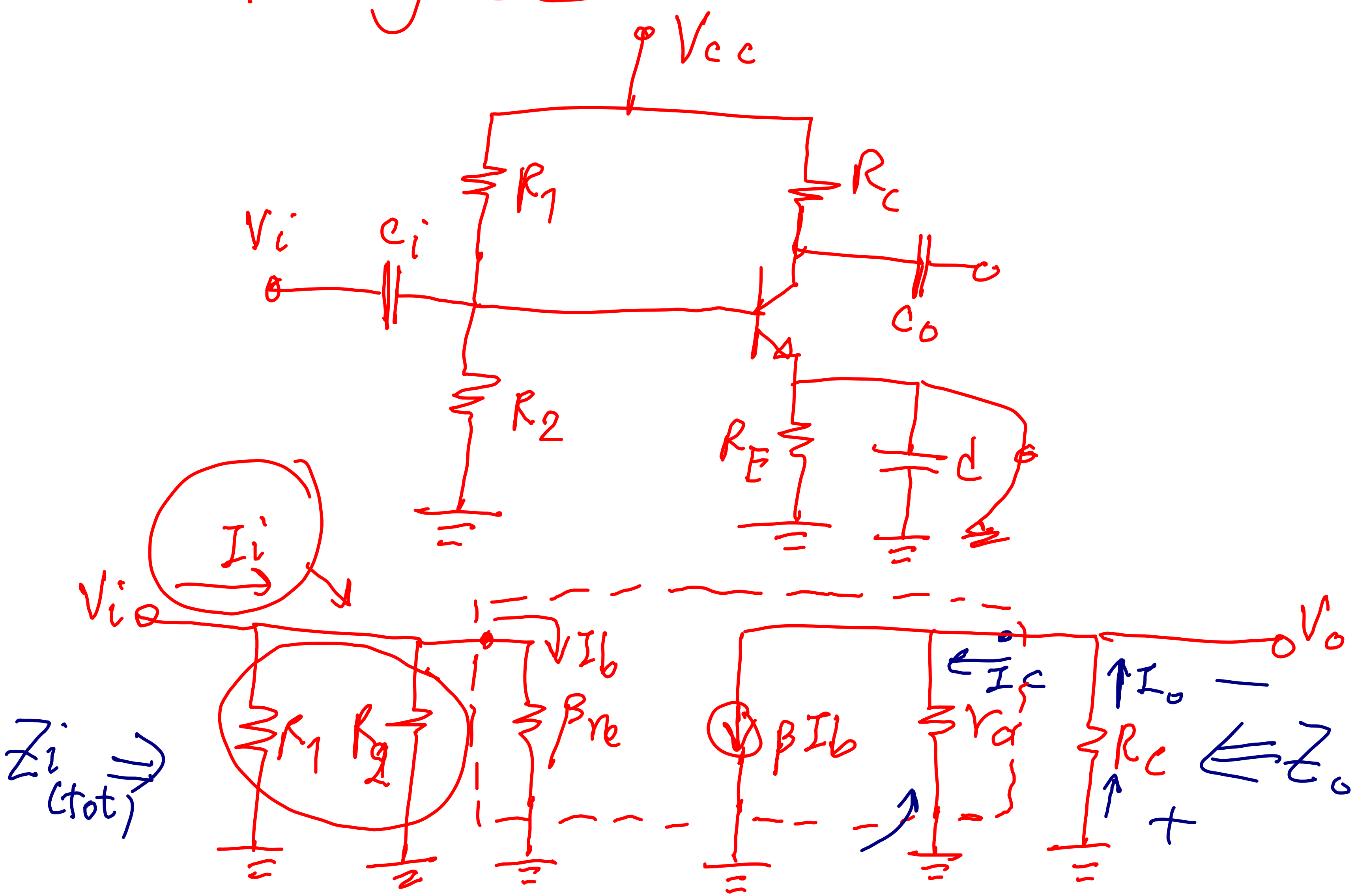


inverse

$A_i = \beta$

$A_v = -\frac{R_C}{r_e}$

Voltage-Divider bias



$$\boxed{Z_{i(tot)} = R_1 \| R_2 \| \beta r_e}$$ #

$$\boxed{Z_o = r_o \| R_C}$$

และเมื่อ $\boxed{r_o \gg 10 R_C}$ จะได้ $\boxed{Z_o = R_C}$

$$V_o = -(R_C \| r_o)\beta I_b$$

$I_b = \dfrac{V_i}{\beta r_e}$ เมื่อนำไปแทนในสมการ V_o จะได้

$$V_o = -(R_C \| r_o)\cancel{\beta}\left(\frac{V_i}{\cancel{\beta} r_e}\right) = \frac{-(R_C \| r_o) \cdot V_i}{r_e}$$

$A_v = \dfrac{V_o}{V_i} = -\dfrac{R_C \| r_o}{r_e}$ เมื่อ $r_o \gg 10 R_C \Rightarrow \boxed{A_v = -\dfrac{R_C}{r_e}}$ #

$$A_i = \frac{I_o}{I_i}$$

$$I_o = \frac{r_o \beta I_b}{r_o + R_c}$$

$$I_b = \frac{(R_1 \| R_2) I_i}{(R_1 \| R_2) + \beta r_e}$$

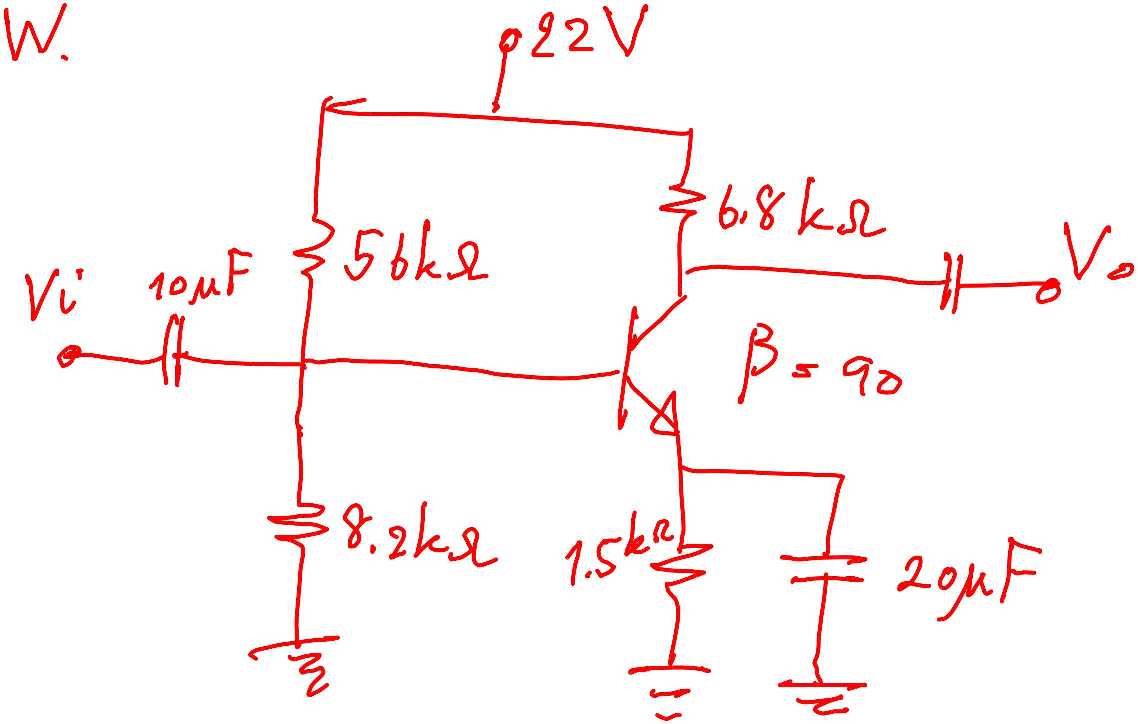
$$I_i = \frac{[(R_1 \| R_2) + \beta r_e] I_b}{(R_1 \| R_2)}$$

$$\therefore A_i = \frac{r_o \beta \cancel{I_b}}{r_o + R_c} \cdot \frac{(R_1 \| R_2)}{[(R_1 \| R_2) + \beta r_e] \cancel{I_b}} = \frac{r_o \beta (R_1 \| R_2)}{(r_o + R_c)[(R_1 \| R_2) + \beta r_e]}$$

אם $r_o \gg 10 R_c$ אזי

$$\boxed{A_i = \frac{\beta (R_1 \| R_2)}{(R_1 \| R_2) + \beta r_e}}$$

H.W.



Req.

a) r_e (7.15 kΩ)

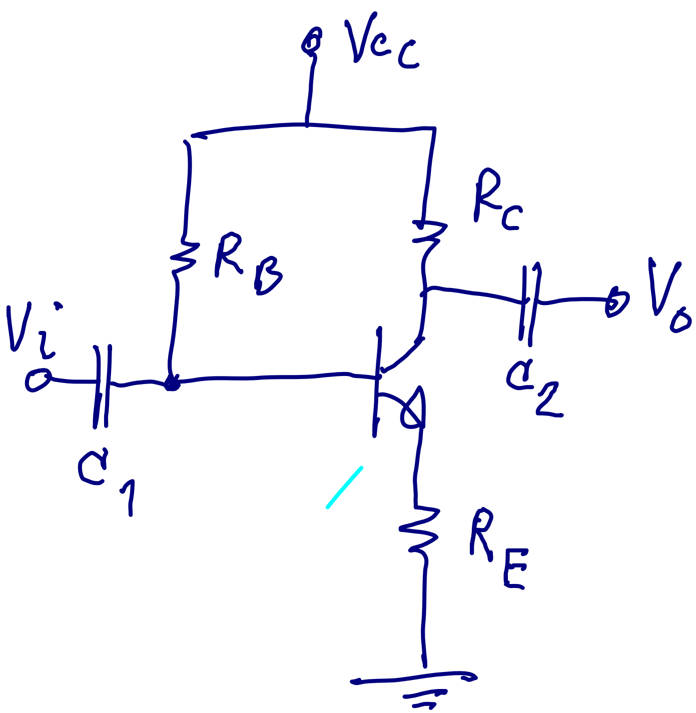
b) Z_i (1.35 kΩ)

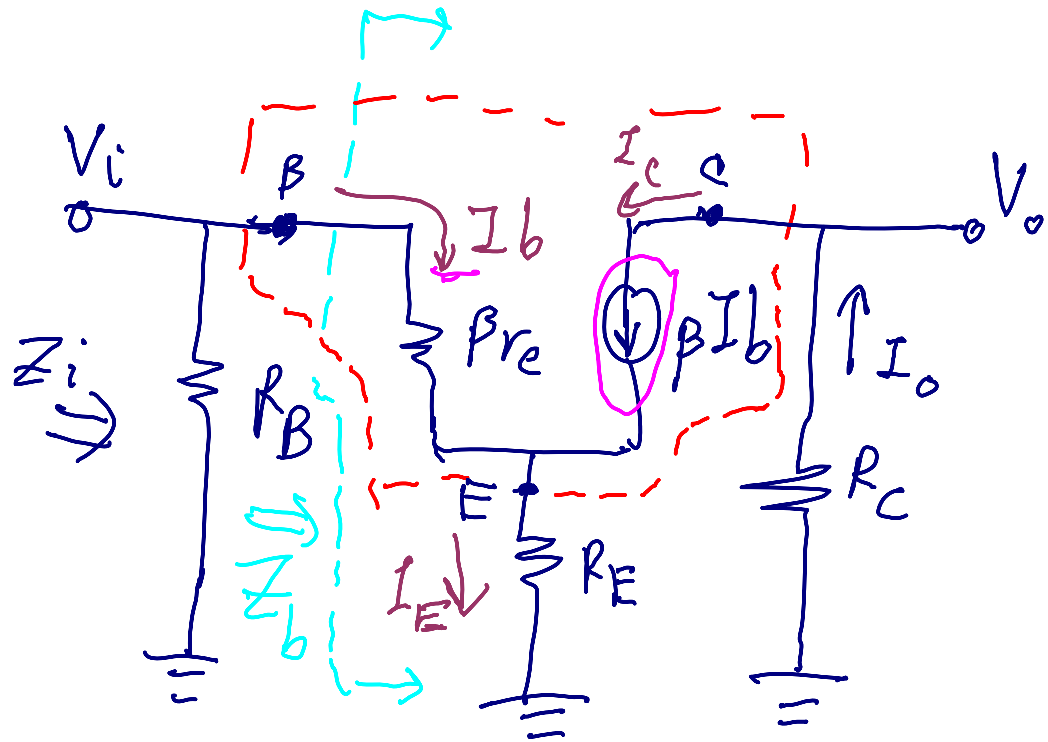
c) Z_o ($r_o = \infty\,\Omega$) (6.8 kΩ)

d) A_v ($r_o = \infty\,\Omega$) (−367.97)

e) A_i ($r_o = \infty\,\Omega$) (73.04)

CE EMITTER-BIAS





מה Z_b

לקחנו

$(\beta+1)\, I_b$

$$V_i = I_b \beta r_e + I_E R_E$$

$$V_i = I_b \beta r_e + (\beta+1) I_b R_E$$

$$v_i = I_b\left[\beta r_e + (\beta+1)R_E\right]$$

$$Z_b = \frac{v_i}{I_b} = \frac{\cancel{I_b}\left[\beta r_e + (\beta+1)R_E\right]}{\cancel{I_b}}$$

$$Z_b = \beta r_e + (\beta+1)R_E$$

ถ้า $(\beta+1) \approx \beta$

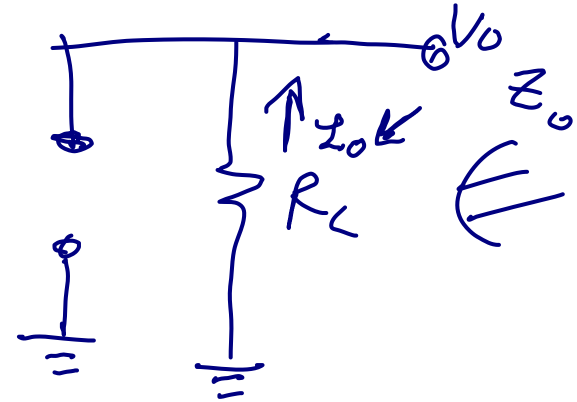
$$\therefore Z_b = \beta(r_e + R_E)$$

ถ้า $R_E \gg r_e$

$$\therefore \boxed{Z_b = \beta R_E}$$

$$\boxed{Z_i = Z_b \parallel R_B = \beta R_E \parallel R_B} \quad \#$$

$\therefore Z_o = R_C$ | open βI_b



หา A_V

$$V_o = -\beta I_b R_C$$

$$I_b = \frac{V_i}{Z_b} = \frac{V_i}{\beta R_E}$$

$$V_i = \beta R_E I_b$$

$$\boxed{\therefore A_V = \frac{V_o}{V_i} = \frac{-\cancel{\beta I_b} R_C}{\cancel{\beta} R_E \cancel{I_b}} = -\frac{R_C}{R_E}}$$ #

$$A_i = \frac{I_o}{I_i}$$

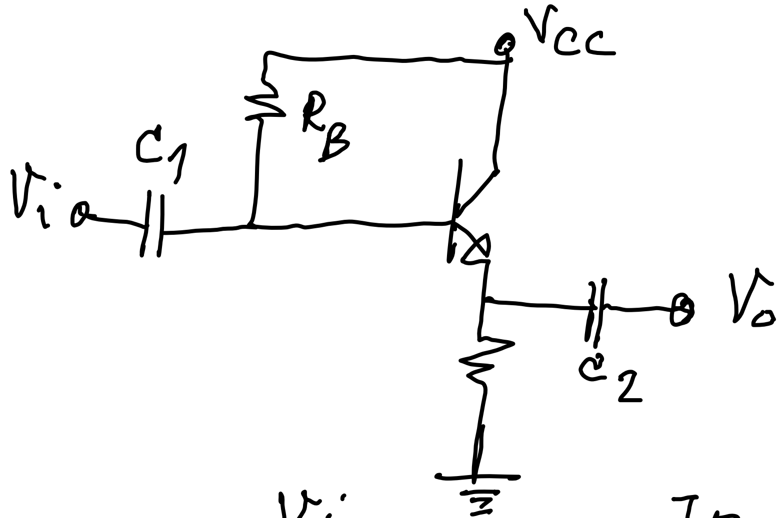
$$I_o = -\frac{V_o}{R_C} = -\frac{\beta I_b \cancel{R_C}}{\cancel{R_C}} = -\beta I_b = -I_c$$

חדק $$I_b = \frac{R_B I_i}{R_B + Z_b} \quad (\text{current divider})$$

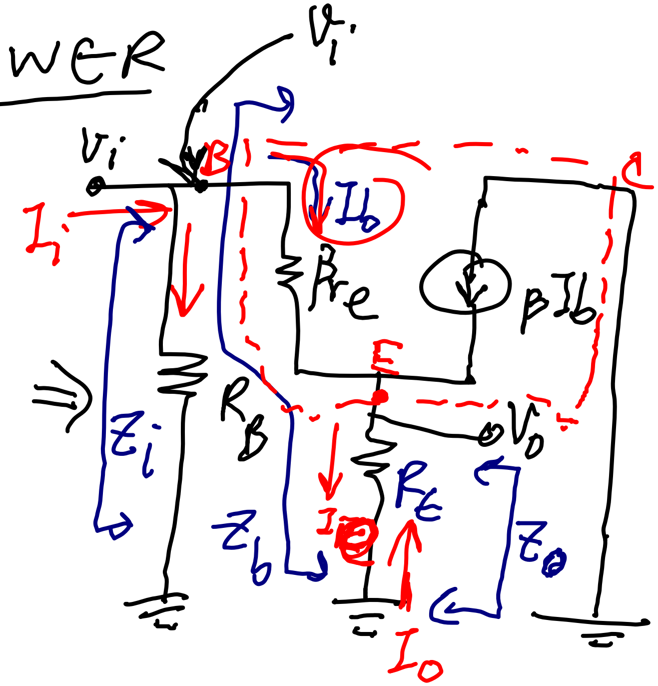
$$I_i = \frac{(R_B + Z_b) I_b}{R_B}$$

$$\therefore A_i = \frac{I_o}{I_i} = \frac{-\beta \cancel{I_b}}{(R_B + Z_b)\cancel{I_b}} \cdot R_B = \frac{-\beta R_B}{R_B + Z_b}$$

EMITTER-FOLLOWER



$\Rightarrow$



$$Z_b = \frac{V_i}{I_b} = \frac{\overbrace{\cancel{I_b}\beta r_e + \cancel{I_b}\overbrace{(\beta+1)}^{I_E} R_E}^{V_i'}}{\cancel{I_b}} = \boxed{\beta r_e + (\beta+1) R_E}$$

$$\approx \beta (r_e + R_E)$$

$$Z_i = R_B \| Z_b = R_B \| \beta (r_e + R_E)$$

$$Z_o = ?$$

จาก $I_b = V_i / z_b$ ——— (*)

นำ $(\beta+1)$ คูณทั้งสมการ (*) ทั้งสองข้าง

$$(\beta+1) I_b = (\beta+1) V_i / z_b$$

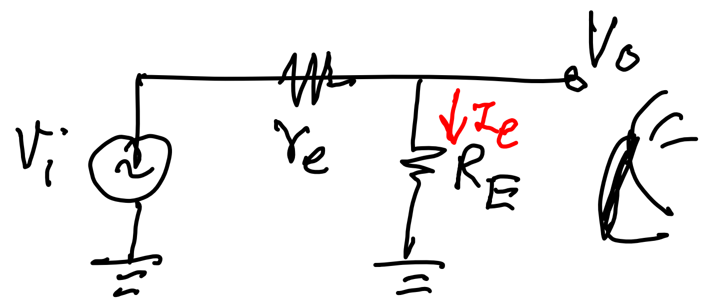
$$I_e = \frac{(\beta+1)V_i}{\beta r_e + (\beta+1)R_E}$$

ถ้า $R_E \gg 10 r_e$ จะได้

$$\boxed{Z_o = r_e}$$

ให้ $(\beta+1) \approx \beta$

$$\therefore I_e = \frac{V_i}{r_e + R_E}$$



$$Z_o = R_E \| r_e \Big|_{V_i = 0}$$

หา A_v

$$V_o = \frac{R_E V_i}{R_E + r_e}$$

$$A_v = \frac{V_o}{V_i} = \frac{R_E}{R_E + r_e}$$

ถ้า $R_E \gg 10 r_e$ จะได้

$$A_v = \frac{R_E}{R_E} = 1$$

หา A_i

$A_i = I_o / I_i$

จาก $I_b = \dfrac{R_B I_i}{R_B + Z_b}$

$$I_i = \frac{I_b (R_B + Z_b)}{R_B}$$

$$I_o = -I_e = -(\beta + 1) I_b$$

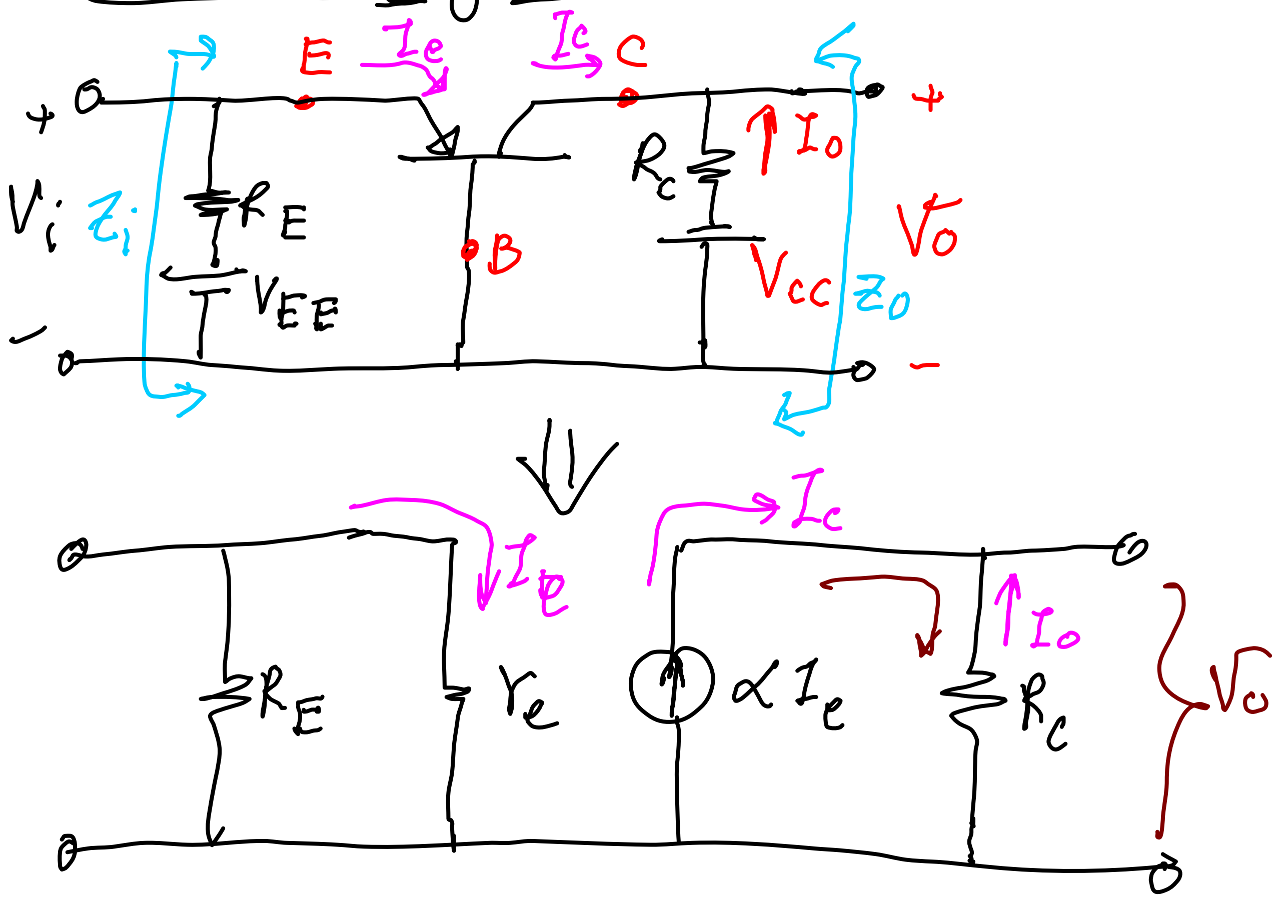
$$\therefore A_i = \frac{I_o}{I_i} = \frac{-(\beta+1)\cancel{I_b} \cdot R_B}{(R_B + Z_b)\cancel{I_b}}$$

$$= \frac{-(\beta+1) R_B}{R_B + Z_b}$$

ถ้าให้ $(\beta + 1) \approx \beta$

$$\therefore A_i = \frac{-\beta R_B}{R_B + Z_b}$$

C-B configuration



$$Z_i = R_E \,||\, r_e$$

$$Z_o = R_C$$

$$V_o = -I_o R_C = -(-I_c) R_C = \alpha I_e R_C$$

$$V_i = I_e r_e \rightarrow I_i$$

$$I_e = V_i / r_e$$

จาก $v_o = \alpha I_e R_C = \alpha \left(\frac{v_i}{r_e} \right) R_C$

$$A_v = \frac{v_o}{V_i} = \alpha R_C / r_e$$

ถ้า $\alpha \approx 1$

$\therefore \quad A_v = R_C / r_e \Big|_{\alpha \approx 1}$

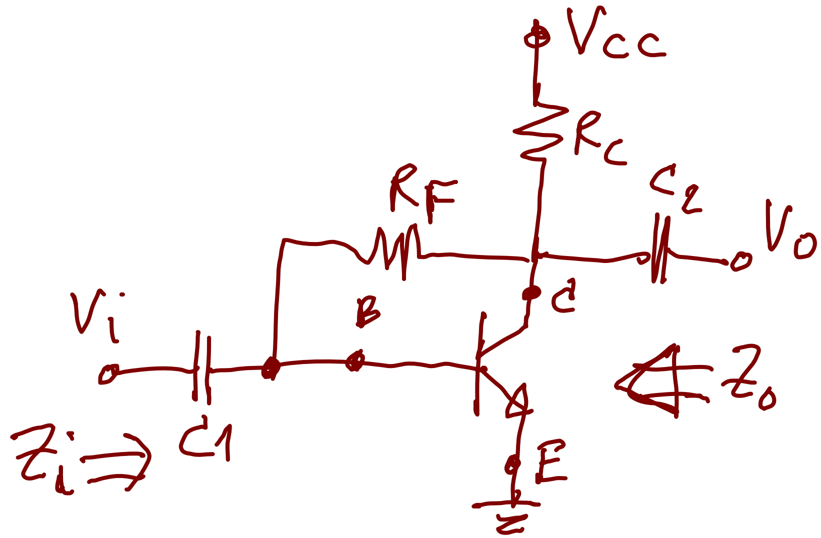
และ $A_i = \frac{I_o}{I_i}$

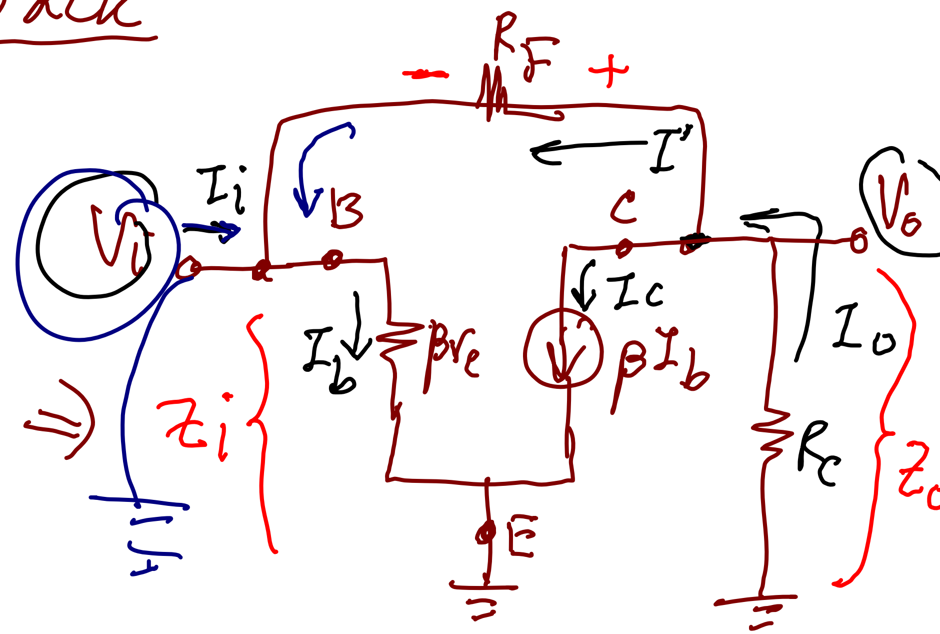
จาก $I_o = -\alpha I_e = -\alpha I_i$

$$A_i = \frac{I_o}{I_i} = -\alpha$$

$$A_i \approx -1$$

Collector Feedback





$$I' = \frac{V_o - V_i}{R_F} \quad \text{———} \quad (1)$$

เมื่อ $V_o = -I_o R_C$

$$I_o = I' + \beta I_b$$

โดยทั่วไปแล้ว βI_b จะมีค่ามากกว่า I'

$\therefore I_o \approx \beta I_b$

$V_o = -\beta I_b R_C$ —— (2)

เมื่อ $I_b = \dfrac{V_i}{\beta r_e}$

นำไปแทนใน (2)

$V_o = -\beta \dfrac{V_i}{\beta r_e} R_C$ —— (3)

$A_V = \dfrac{V_o}{V_i} = -\cancel{\beta}R_C/\cancel{\beta}r_e = -R_C/r_e$

นำสมการที่ (3) ไปแทนใน (1)

$I' = \left(-\dfrac{V_i R_C}{r_e} - V_i\right)/R_F = -\dfrac{V_i}{R_F}\left(\dfrac{R_C}{r_e}+1\right)$

מה $V_i = I_b \beta r_e$

בנוסף $I_b = I_i + I'$

סה"כ $V_i = (I_i + I')\beta r_e = \Big[I_i + \underbrace{\Big[-\frac{V_i}{R_F}\Big(\frac{R_c}{r_e}+1\Big)\Big]}_{I'}\Big]\beta r_e$

$$V_i = I_i \beta r_e - \frac{V_i \beta r_e}{R_F}\left(\frac{R_c}{r_e}+1\right)$$

$$I_i \beta r_e = V_i + \frac{V_i \beta r_e}{R_F}\left(\frac{R_c}{r_e}+1\right)$$

$$I_i \beta r_e = V_i\left[1 + \frac{\beta r_e}{R_F}\left(\frac{R_c}{r_e}+1\right)\right] = V_i\left[\frac{R_F + \beta r_e\left(\frac{R_c}{r_e}+1\right)}{R_F}\right]$$

$$Z_i = \frac{V_i}{I_i} = \frac{\beta r_e}{\left[1 + \left(\frac{\beta r_e}{R_F}\left(\frac{R_c}{r_e}+1\right)\right)\right]}$$

$$Z_i = \frac{V_i}{I_i} = \frac{\beta r_e R_F}{R_F + \beta(R_c + r_e)}$$

ถ้า $R_c \gg r_e$

$$\boxed{\therefore Z_i = \frac{\beta r_e R_F}{R_F + \beta R_c}}$$

หา Z_o

$$Z_o = R_C \| R_F \Big|_{V_i = 0}$$

หา A_i ให้ใช้ KVL ใน Loop ด้านนอกของรูป

$$V_i + V_{R_F} - V_o = 0$$

จาก $V_i = I_b \beta r_e$

$$V_{R_F} = I' R_F = (I_b - I_i) R_F$$

$$V_o = -I_o R_c \approx -\beta I_b R_c$$

แทน V_i, V_{R_F}, V_o จะได้

$$I_b \beta r_e + (I_b - I_i) R_F + \beta I_b R_c = 0$$

$$I_b (\beta r_e + R_F + \beta R_c) = I_i R_F$$

จาก $I_o = \beta I_b$ ดังนั้น $I_b = \frac{I_o}{\beta}$

$$\frac{I_o}{\beta}(\beta r_e + R_F + \beta R_C) = I_i R_F$$

นำ β คูณทั้ง 2 ข้าง

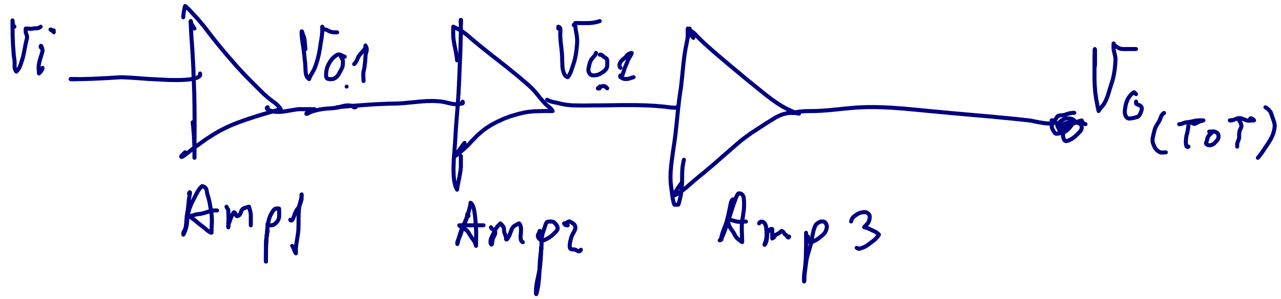
$$I_o(\beta r_e + R_F + \beta R_C) = I_i \beta R_F$$

$$A_i = \frac{I_o}{I_i} = \frac{\beta R_F}{\beta r_e + R_F + \beta R_C}$$

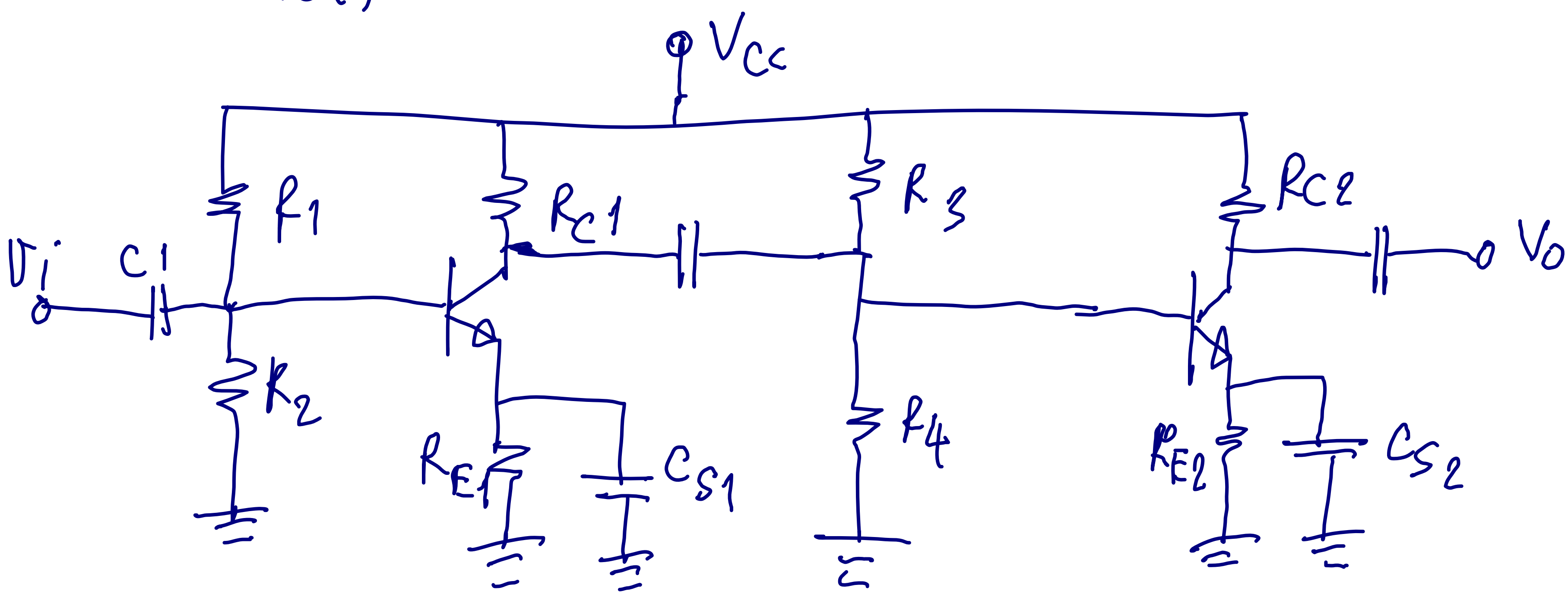
ถ้า $\beta r_e \leq R_F \leq \beta R_C$

$$\boxed{A_i = \beta R_F / \beta R_C = R_F / R_C}$$

BJT cascade Amplifier



$$A_{V(tot)} = A_{V1} \times A_{V2} \times A_{V3}$$



$$Z_i = R_1 \parallel R_2 \parallel \beta r_e$$

$$Z_o = r_{o_2} \parallel R_C$$

$$A_{v_1} = \frac{R_{C1} \parallel R_3 \parallel R_4 \parallel \beta r_{e2}}{r_{e1}}$$

$$A_{v_2} = \frac{R_{C2}}{r_{e2}}$$

$$\therefore A_v = A_{v1} \times A_{v2}$$

$$\cancel{A_i} \qquad v_o = A_v \times v_i$$