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Designing State Machines Using State Diagrams

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- Designing state machines is probably the most creative task of a digital designer.
- Most people like to take a graphical approach to design.
- State diagrams are often used to design small- to medium-sized state machines.

One fundamental difference between a state diagram and a state table; a difference that makes state-diagram design simpler but also more error prone:

- A state table is an exhaustive listing of the next states for each state/input combination.
  - No ambiguity is possible.
- A state diagram contains a set of arcs labeled with transition expressions.
  - Even when there are many inputs, only one transition expression is required per arc.
  - However, when a state diagram is constructed, there is no guarantee that the transition expression written on the arcs leaving a particular state cover all the input combinations exactly once.

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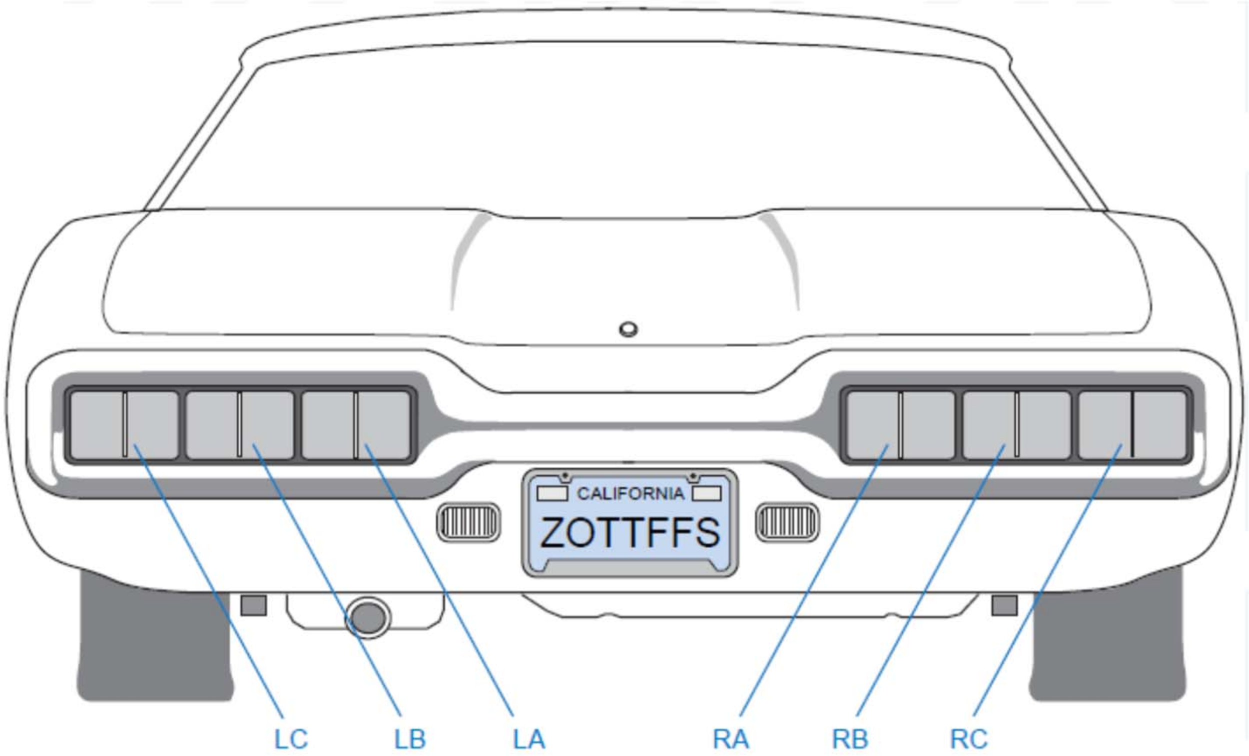
- Ambiguous state diagram
  - In an improperly constructed state diagram,
    - The next state for some input combinations may be unspecified, which is generally undesirable.
    - While multiple next states may be specified for others, which is just plain wrong.
  - Thus, considerable care must be taken in the design of state diagrams.



- The state machine has two input signals, LEFT and RIGHT.
- It also has an emergency flasher input, HAZ – all six lights flashing on and off in unison.
- Assume the existence of a free-running clock signal whose frequency equals the desired flashing rate for the lights.

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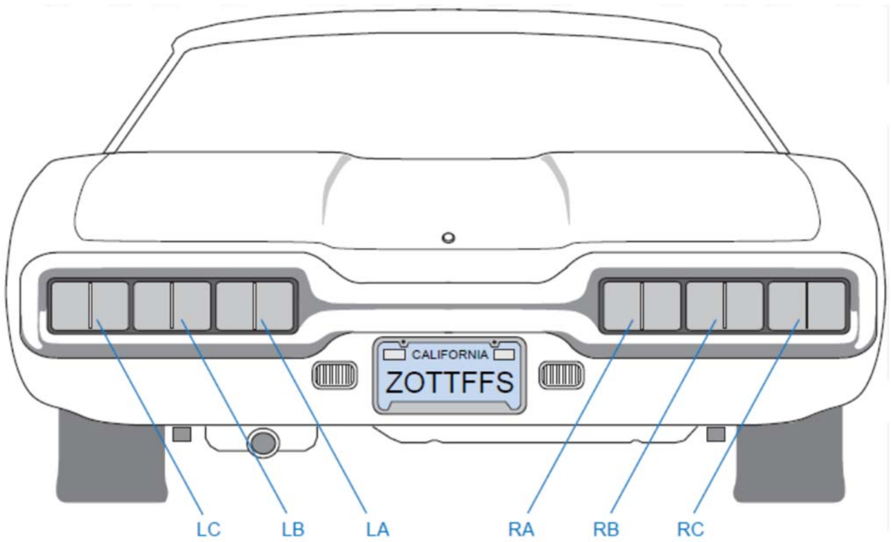

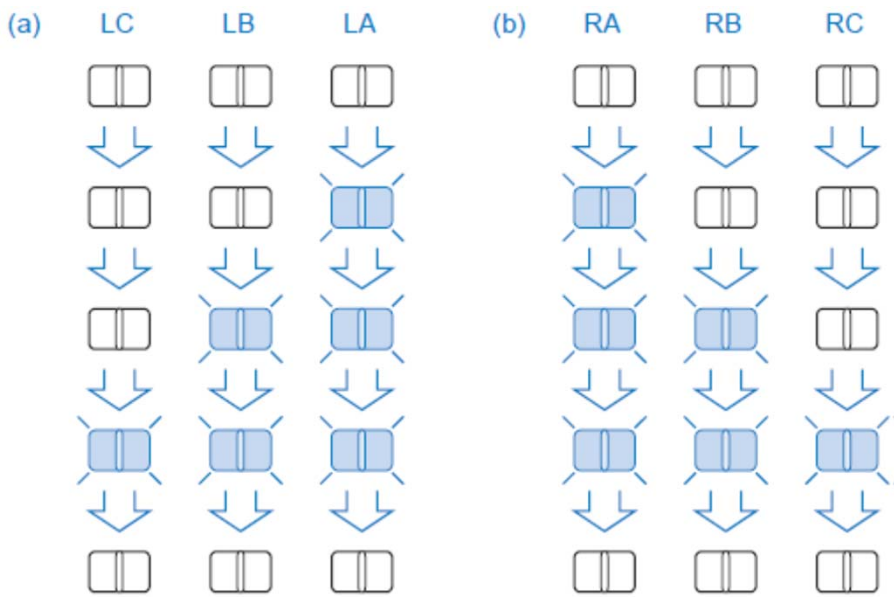

(a) Left turn (b) Right turn

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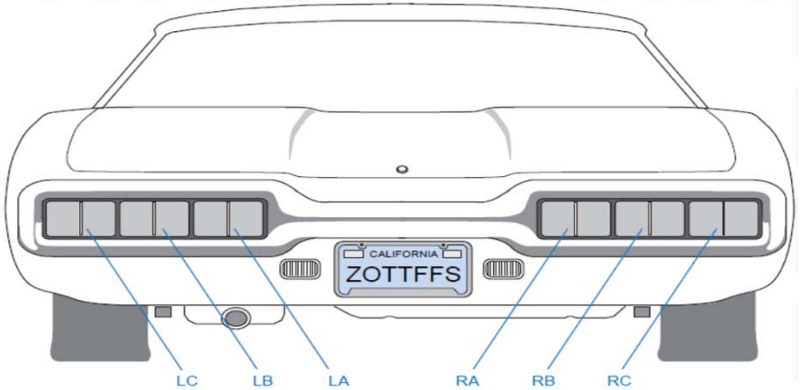
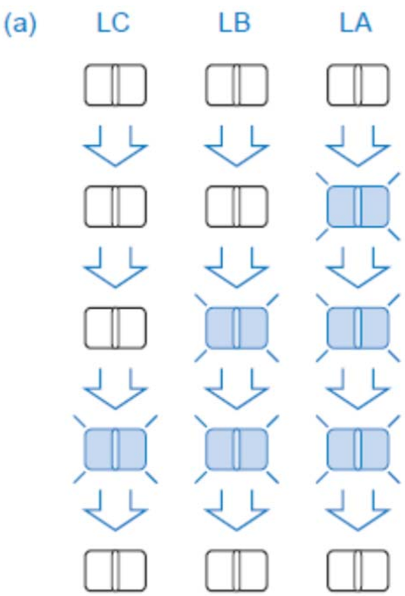
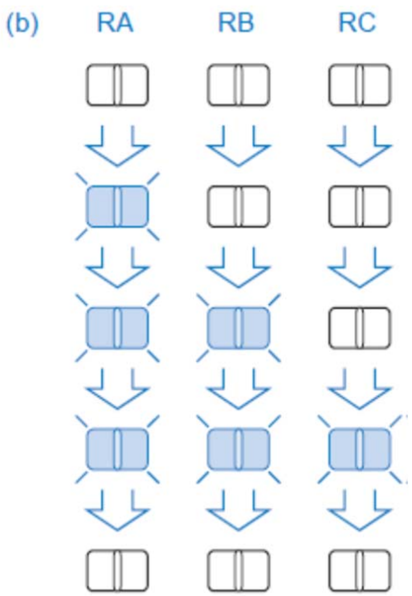
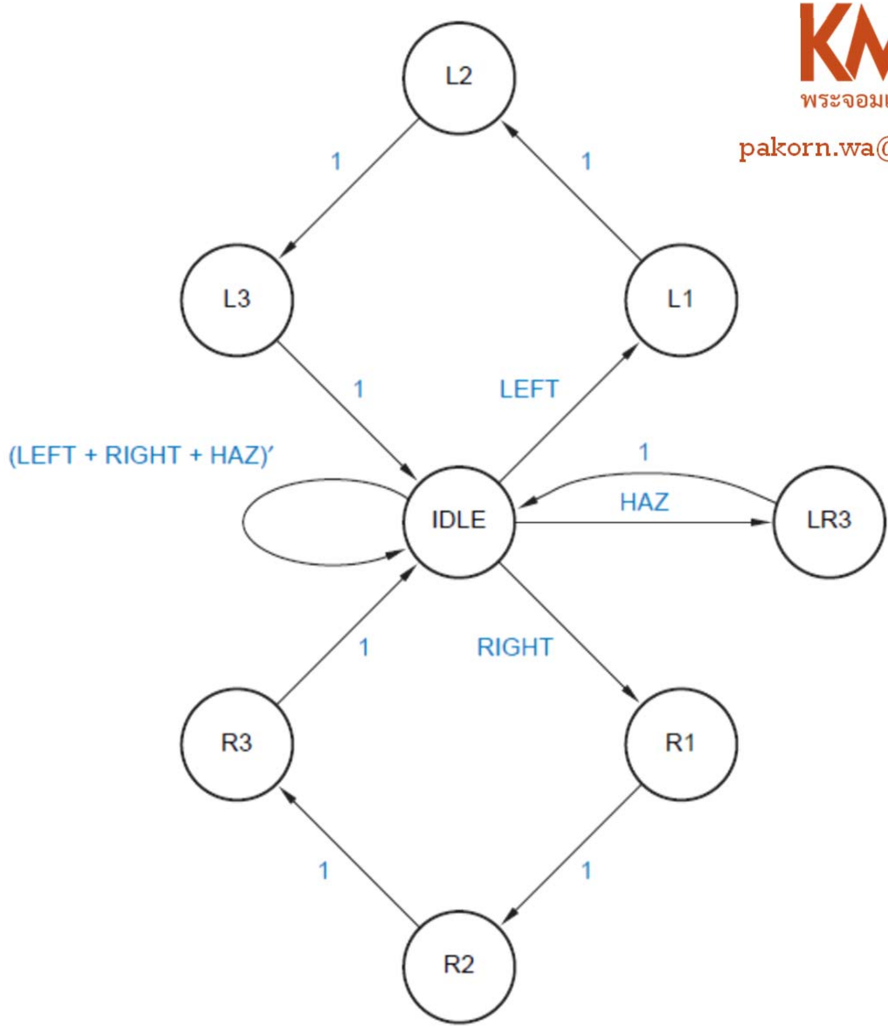




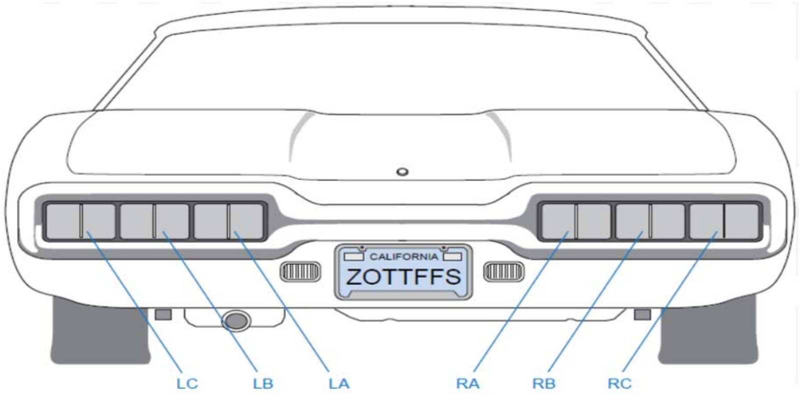
(a) Left turn

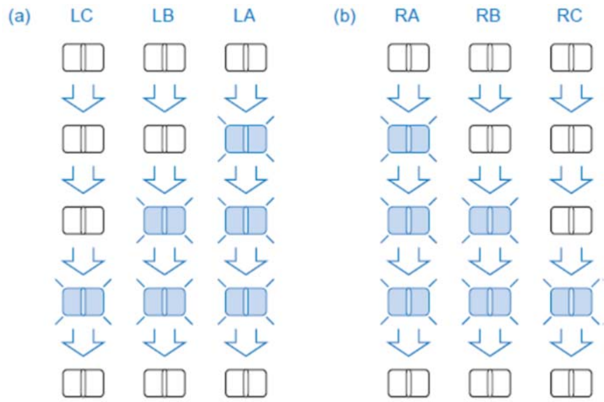


(b) Right turn

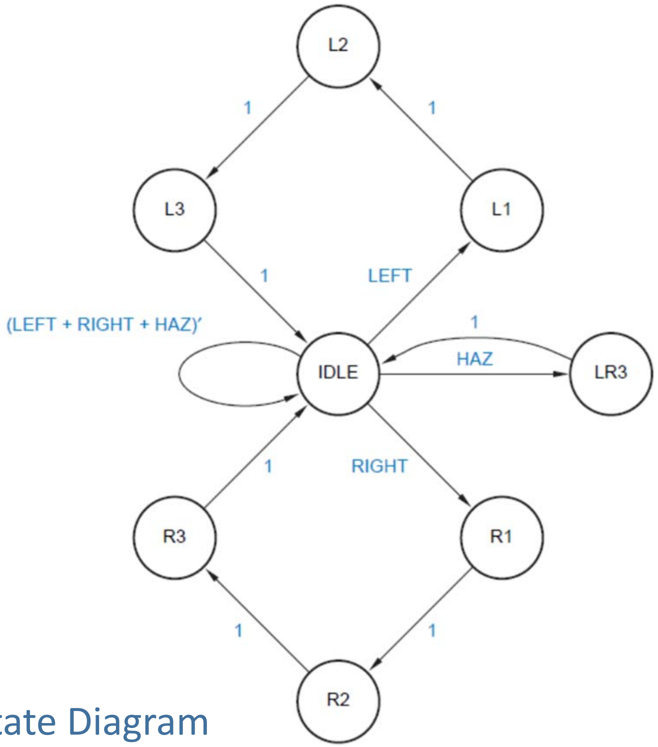


State Diagram





(a) Left turn (b) Right turn



State Diagram

State	LC	LB	LA	RA	RB	RC
IDLE	0	0	0	0	0	0
L1	0	0	1	0	0	0
L2	0	1	1	0	0	0
L3	1	1	1	0	0	0
R1	0	0	0	1	0	0
R2	0	0	0	1	1	0
R3	0	0	0	1	1	1
LR3	1	1	1	1	1	1

Output Table

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State	LC	LB	LA	RA	RB	RC
IDLE	0	0	0	0	0	0
L1	0	0	1	0	0	0
L2	0	1	1	0	0	0
L3	1	1	1	0	0	0
R1	0	0	0	1	0	0
R2	0	0	0	1	1	0
R3	0	0	0	1	1	1
LR3	1	1	1	1	1	1

Output Table

LA = L1 + L2 + L3 + LR3
LB = L2 + L3 + LR3
LC = L3 + LR3

RA = R1 + R2 + R3 + LR3
RB = R2 + R3 + LR3
RC = R3 + LR3

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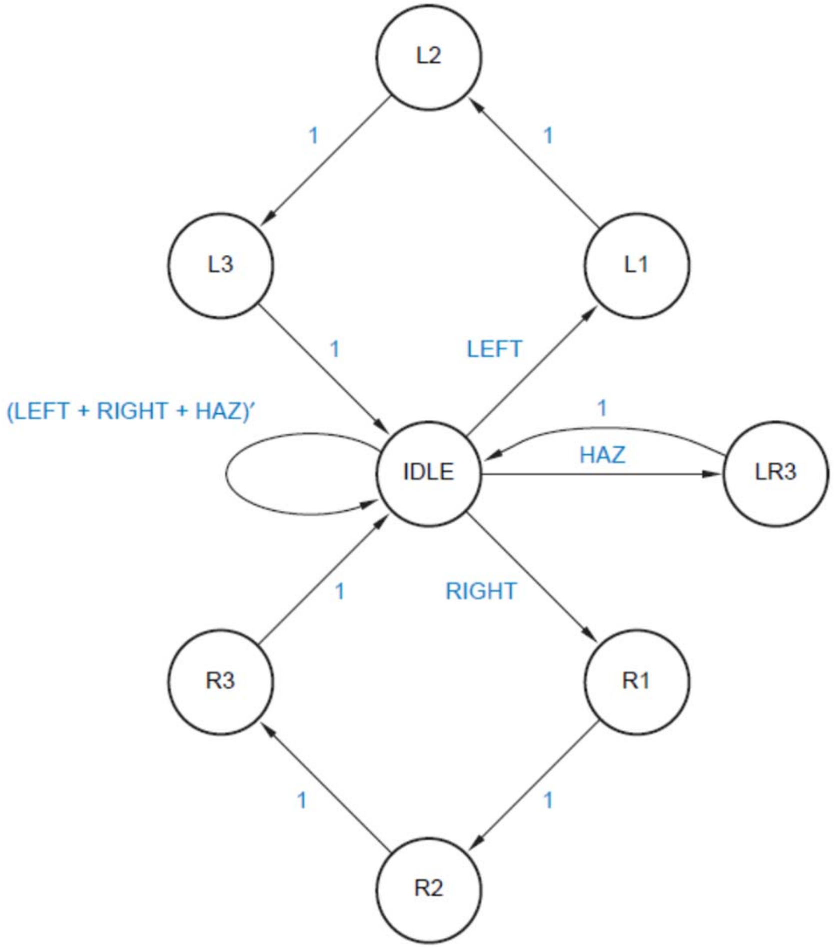
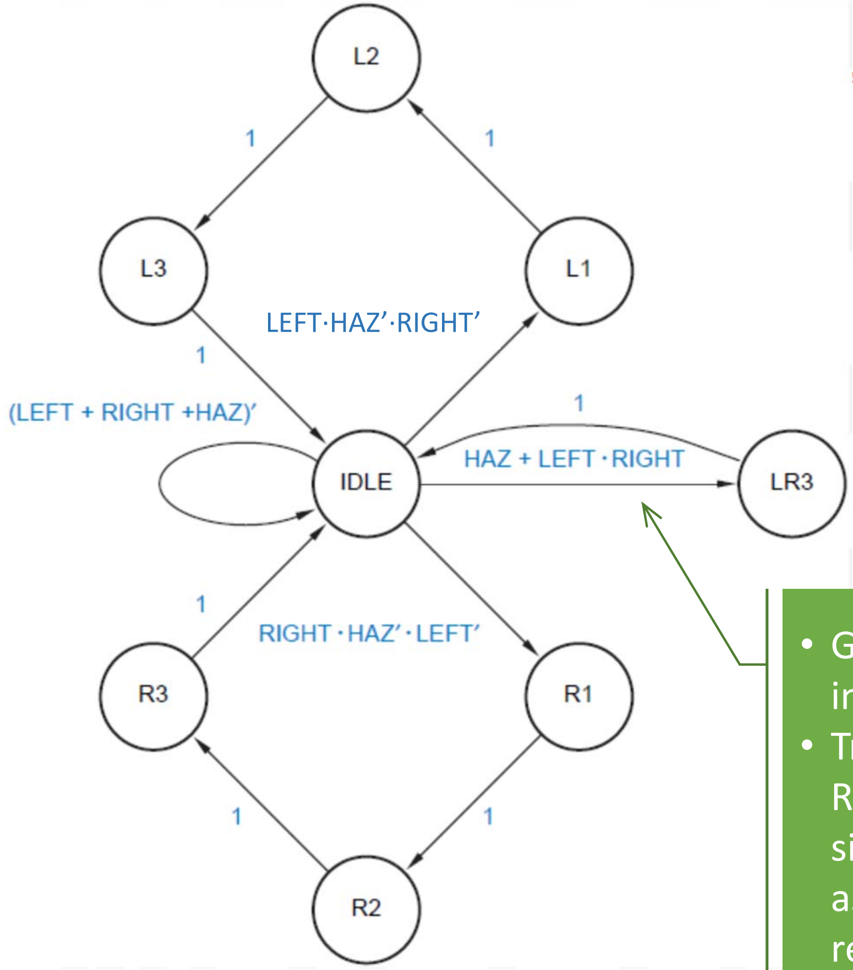
- What if ???
  - Both LEFT and HAZ are asserted simultaneously?
    - According to the state diagram, the machines goes to two states, L1 and LR3, which is impossible.
    - In reality, the machine would have only one next state, which could be L1, LR3 or a totally unrelated (and possible unused) third state, depending on details of the state machine's realization.



State Diagram



Corrected State Diagram

- Given the HAZ input priority
- Treat LEFT and RIGHT asserted simultaneously as a hazard request

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- The new state diagram is unambiguous because the transition expressions on the arcs leaving each state are mutually exclusive and all-inclusive.
- That is, for each state,
  - No two expression are 1 for the same input combination, and
  - Some expression is 1 for every input combination

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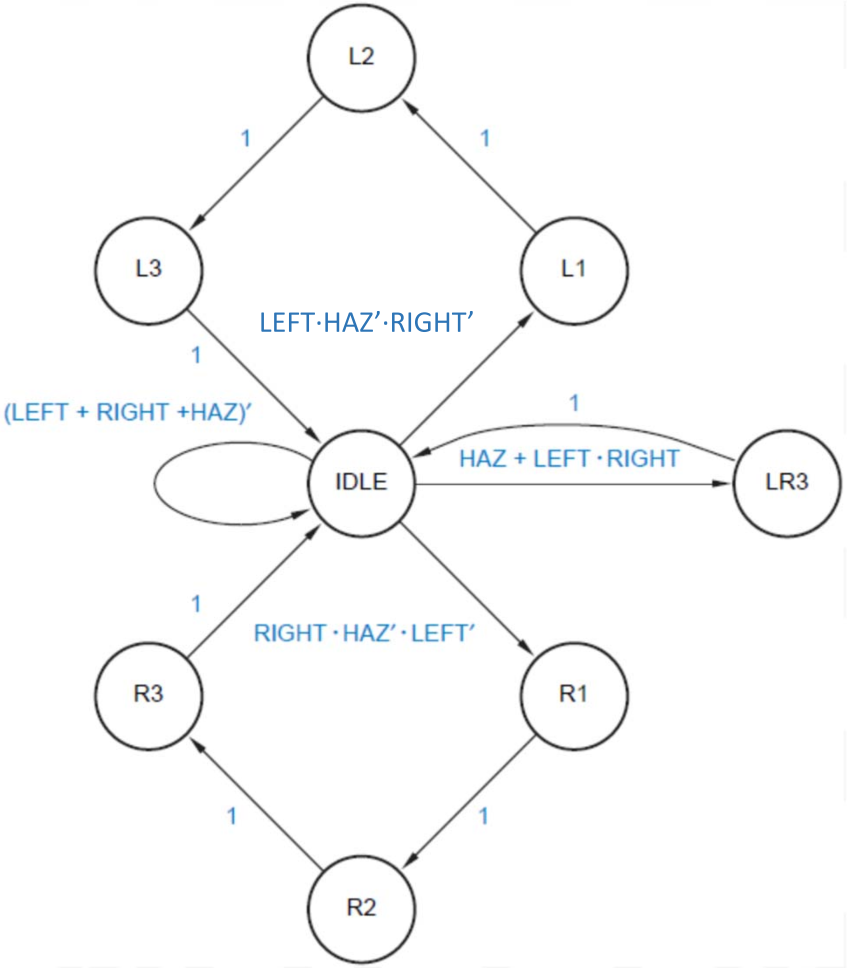
- This can be confirmed algebraically for this or any other state diagram by performing two steps:
  1. Mutual exclusion.
     - For each state, show that the logical product of each possible pair of transition expression on arcs leaving that state is 0.
     - If there are n arcs, then there are $n(n-1)/2$ logical products to evaluate.
  2. All inclusion.
     - For each state, show that the logical sum of the transition expressions on all arcs leaving that state is 1.



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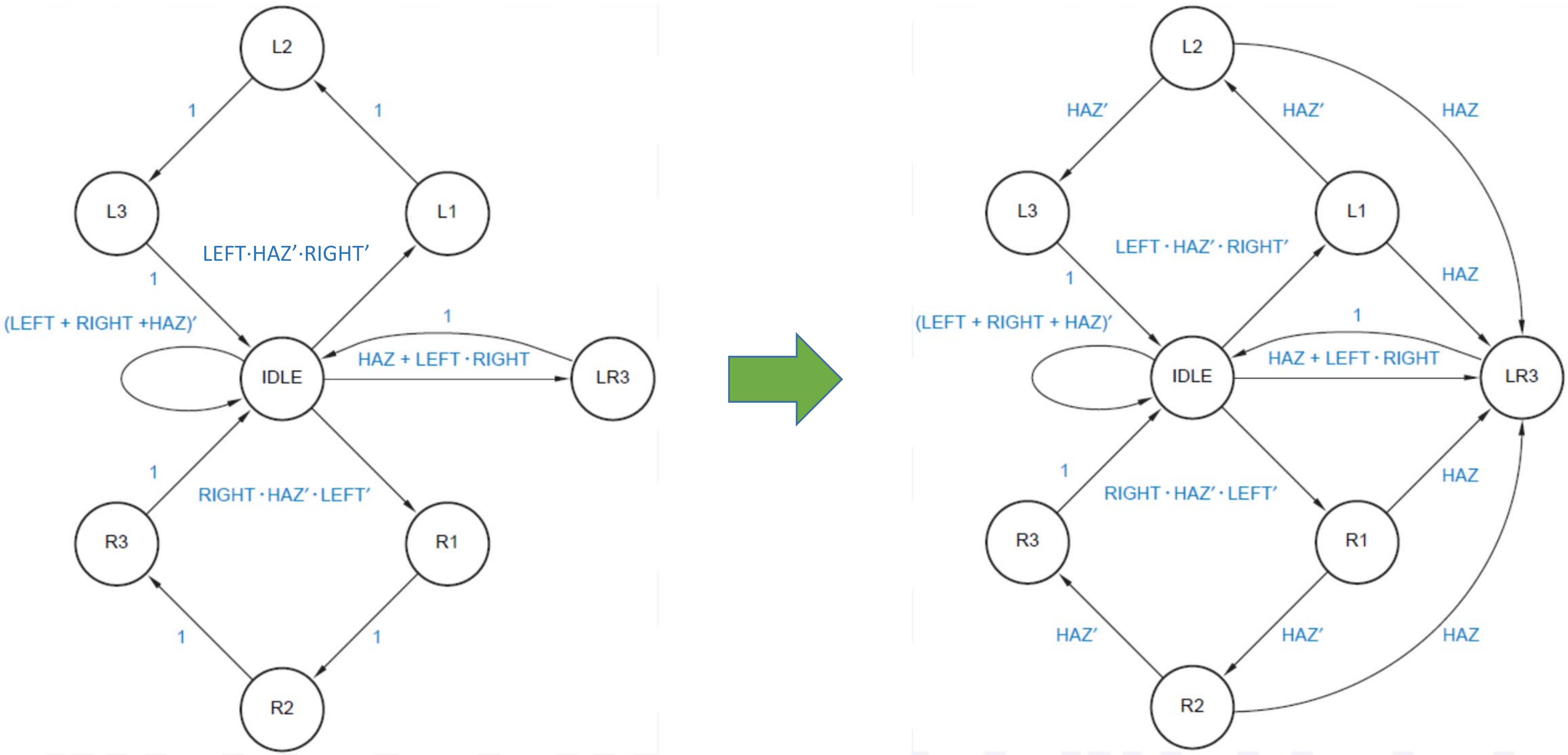
- What if ???
  - HAZ is asserted after a left- or right-turn cycles has begun.
  - While this may have a certain aesthetic appeal, it would be safer for the car’s occupants to have the machine go into hazard mode as soon as possible.


L2
1
1
L3
L1
LEFT·HAZ’·RIGHT’
1
(LEFT + RIGHT +HAZ)’
1
IDLE
HAZ + LEFT · RIGHT
LR3
1
RIGHT · HAZ’ · LEFT’
R3
R1
1
1
R2
L2
HAZ’
HAZ’
HAZ
L3
L1
LEFT · HAZ’ · RIGHT’
HAZ
1
(LEFT + RIGHT + HAZ)’
1
IDLE
HAZ + LEFT · RIGHT
LR3
1
RIGHT · HAZ’ · LEFT’
HAZ
R3
R1
HAZ’
HAZ’
HAZ
R2

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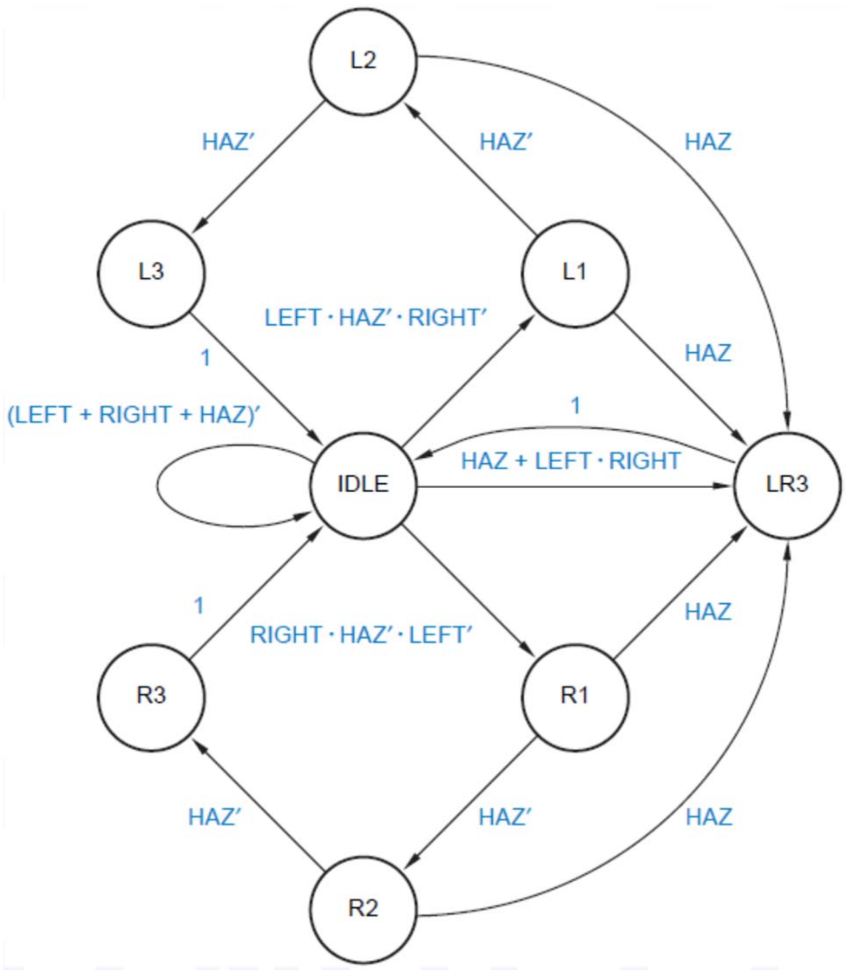

State	Q2	Q1	Q0
IDLE	0	0	0
L1	0	0	1
L2	0	1	1
L3	0	1	0
R1	1	0	1
R2	1	1	1
R3	1	1	0
LR3	1	0	0

State assignment

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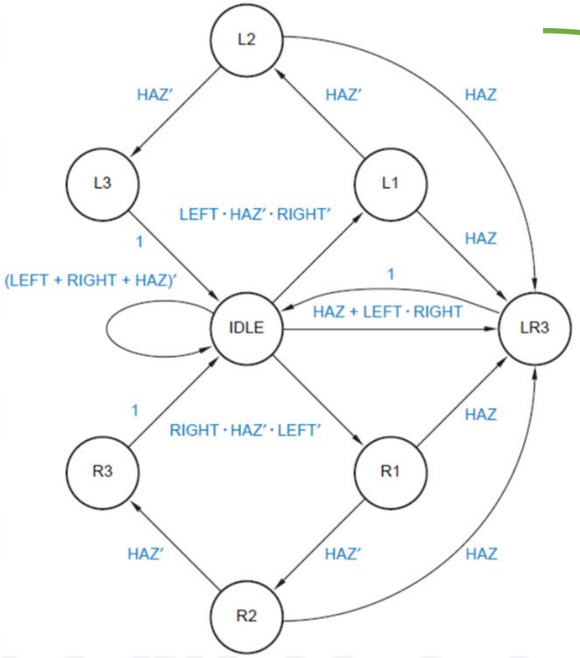
State	Q2	Q1	Q0
IDLE	0	0	0
L1	0	0	1
L2	0	1	1
L3	0	1	0
R1	1	0	1
R2	1	1	1
R3	1	1	0
LR3	1	0	0

- There are many state assignment possible.
- The table is used for the following reasons:
  1. An initial (idle) state of 000 is compatible with most flip-flops and registers, which are easily initialized to the 0 state.
  2. Two state variables, Q1 and Q0, are used to "count" in Gray-code sequence for the left-turn cycle (IDLE → L1 → L2 → L3 → IDLE). This minimizes the number of state-variable changes per state transition, which can **often** simplify the excitation logic.
  3. Because of the symmetry in the state diagram, the same sequence on Q1 and Q0 is used to "count" during a right-turn cycle, which Q2 is used to distinguish between left and right.
  4. The remaining state-variable combination is used for the LR3 state.



State	Q2	Q1	Q0
IDLE	0	0	0
L1	0	0	1
L2	0	1	1
L3	0	1	0
R1	1	0	1
R2	1	1	1
R3	1	1	0
LR3	1	0	0

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S	Q2	Q1	Q0	Transition expression	S*	Q2*	Q1*	Q0*
IDLE	0	0	0	(LEFT + RIGHT + HAZ)'	IDLE	0	0	0
IDLE	0	0	0	LEFT · HAZ' · RIGHT'	L1	0	0	1
IDLE	0	0	0	HAZ + LEFT · RIGHT	LR3	1	0	0
IDLE	0	0	0	RIGHT · HAZ' · LEFT'	R1	1	0	1
L1	0	0	1	HAZ'	L2	0	1	1
L1	0	0	1	HAZ	LR3	1	0	0
L2	0	1	1	HAZ'	L3	0	1	0
L2	0	1	1	HAZ	LR3	1	0	0
L3	0	1	0	1	IDLE	0	0	0
R1	1	0	1	HAZ'	R2	1	1	1
R1	1	0	1	HAZ	LR3	1	0	0
R2	1	1	1	HAZ'	R3	1	1	0
R2	1	1	1	HAZ	LR3	1	0	0
R3	1	1	0	1	IDLE	0	0	0
LR3	1	0	0	1	IDLE	0	0	0

Transition list

State-Machine Synthesis Using Transition Lists

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- The main purpose of this section is to help you understand the internal operation and the external quirks of CAD programs and languages that deal with state machines.

Transition Equations

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- The first step is to develop a set of equations that define each next-state variable V* in terms of the current state and input.
- The transition list can be viewed as a sort of hybrid truth table in which the state-variable combinations for current-state are listed explicitly and input combinations are listed algebraically.
- Reading down a V* column in a transition list, we find a sequence of 0s and 1s, indicating the value of V* for various (if we've done it right, all) state/input combinations.

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A transition equation for a next-state variable V* can be written using a sort of hybrid canonical sum:

$$V* = \sum_{\text{transition-list rows where } V*=1} (\text{transition p-term})$$

That is, the transition equation has one "transition p-term" for each row of the transition list that contains a 1 in the V* column. A row's transition p-term is the product of the current state's minterm and the transition expression.

S	Q2	Q1	Q0	Transition expression	S*	Q2*	Q1*	Q0*
IDLE	0	0	0	(LEFT + RIGHT + HAZ)’	IDLE	0	0	0
IDLE	0	0	0	LEFT · HAZ’ · RIGHT’	L1	0	0	1
IDLE	0	0	0	HAZ + LEFT · RIGHT	LR3	1	0	0
IDLE	0	0	0	RIGHT · HAZ’ · LEFT’	R1	1	0	1
L1	0	0	1	HAZ’	L2	0	1	1
L1	0	0	1	HAZ	LR3	1	0	0
L2	0	1	1	HAZ’	L3	0	1	0
L2	0	1	1	HAZ	LR3	1	0	0
L3	0	1	0	1	IDLE	0	0	0
R1	1	0	1	HAZ’	R2	1	1	1
R1	1	0	1	HAZ	LR3	1	0	0
R2	1	1	1	HAZ’	R3	1	1	0
R2	1	1	1	HAZ	LR3	1	0	0
R3	1	1	0	1	IDLE	0	0	0
LR3	1	0	0	1	IDLE	0	0	0

$$
\begin{aligned}
Q2^* = {} & Q2' \cdot Q1' \cdot Q0' \cdot (HAZ + LEFT \cdot RIGHT) \\
& + Q2' \cdot Q1' \cdot Q0' \cdot (RIGHT \cdot HAZ' \cdot LEFT') \\
& + Q2' \cdot Q1' \cdot Q0 \cdot (HAZ) \\
& + Q2' \cdot Q1 \cdot Q0 \cdot (HAZ) \\
& + Q2 \cdot Q1' \cdot Q0 \cdot (HAZ') \\
& + Q2 \cdot Q1' \cdot Q0 \cdot (HAZ) \\
& + Q2 \cdot Q1 \cdot Q0 \cdot (HAZ') \\
& + Q2 \cdot Q1 \cdot Q0 \cdot (HAZ)
\end{aligned}
$$

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S	Q2	Q1	Q0	Transition expression	S*	Q2*	Q1*	Q0*
IDLE	0	0	0	(LEFT + RIGHT + HAZ)’	IDLE	0	0	0
IDLE	0	0	0	LEFT · HAZ’ · RIGHT’	L1	0	0	1
IDLE	0	0	0	HAZ + LEFT · RIGHT	LR3	1	0	0
IDLE	0	0	0	RIGHT · HAZ’ · LEFT’	R1	1	0	1
L1	0	0	1	HAZ’	L2	0	1	1
L1	0	0	1	HAZ	LR3	1	0	0
L2	0	1	1	HAZ’	L3	0	1	0
L2	0	1	1	HAZ	LR3	1	0	0
L3	0	1	0	1	IDLE	0	0	0
R1	1	0	1	HAZ’	R2	1	1	1
R1	1	0	1	HAZ	LR3	1	0	0
R2	1	1	1	HAZ’	R3	1	1	0
R2	1	1	1	HAZ	LR3	1	0	0
R3	1	1	0	1	IDLE	0	0	0
LR3	1	0	0	1	IDLE	0	0	0

$$
\begin{aligned}
Q2^* = {} & Q2' \cdot Q1' \cdot Q0' \cdot (HAZ + LEFT \cdot RIGHT) \\
& + Q2' \cdot Q1' \cdot Q0' \cdot (RIGHT \cdot HAZ' \cdot LEFT') \\
& + Q2' \cdot Q1' \cdot Q0 \cdot (HAZ) \\
& + Q2' \cdot Q1 \cdot Q0 \cdot (HAZ) \\
& + Q2 \cdot Q1' \cdot Q0 \cdot (HAZ') \\
& + Q2 \cdot Q1' \cdot Q0 \cdot (HAZ) \\
& + Q2 \cdot Q1 \cdot Q0 \cdot (HAZ') \\
& + Q2 \cdot Q1 \cdot Q0 \cdot (HAZ)
\end{aligned}
$$

$$
\begin{aligned}
Q2^* = {} & Q2' \cdot Q1' \cdot Q0' \cdot (HAZ + RIGHT) \\
& + Q2' \cdot Q0 \cdot (HAZ) \\
& + Q2 \cdot Q0
\end{aligned}
$$

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S	Q2	Q1	Q0	Transition expression	S*	Q2*	Q1*	Q0*
IDLE	0	0	0	(LEFT + RIGHT + HAZ)’	IDLE	0	0	0
IDLE	0	0	0	LEFT · HAZ’ · RIGHT’	L1	0	0	1
IDLE	0	0	0	HAZ + LEFT · RIGHT	LR3	1	0	0
IDLE	0	0	0	RIGHT · HAZ’ · LEFT’	R1	1	0	1
L1	0	0	1	HAZ’	L2	0	1	1
L1	0	0	1	HAZ	LR3	1	0	0
L2	0	1	1	HAZ’	L3	0	1	0
L2	0	1	1	HAZ	LR3	1	0	0
L3	0	1	0	1	IDLE	0	0	0
R1	1	0	1	HAZ’	R2	1	1	1
R1	1	0	1	HAZ	LR3	1	0	0
R2	1	1	1	HAZ’	R3	1	1	0
R2	1	1	1	HAZ	LR3	1	0	0
R3	1	1	0	1	IDLE	0	0	0
LR3	1	0	0	1	IDLE	0	0	0

$$
\begin{aligned}
Q2^* = {} & Q2' \cdot Q1' \cdot Q0' \cdot (HAZ + RIGHT) \\
& + Q2' \cdot Q0 \cdot (HAZ) \\
& + Q2 \cdot Q0
\end{aligned}
$$

$$
\begin{aligned}
Q1^* = {} & Q2' \cdot Q1' \cdot Q0 \cdot (HAZ') \\
& + Q2' \cdot Q1 \cdot Q0 \cdot (HAZ') \\
& + Q2 \cdot Q1' \cdot Q0 \cdot (HAZ') \\
& + Q2 \cdot Q1 \cdot Q0 \cdot (HAZ') \\
= {} & Q0 \cdot HAZ'
\end{aligned}
$$

$$
\begin{aligned}
Q0^* = {} & Q2' \cdot Q1' \cdot Q0' \cdot (LEFT \cdot HAZ' \cdot RIGHT') \\
& + Q2' \cdot Q1' \cdot Q0' \cdot (RIGHT \cdot HAZ' \cdot LEFT') \\
& + Q2' \cdot Q1' \cdot Q0 \cdot (HAZ') \\
& + Q2 \cdot Q1' \cdot Q0 \cdot (HAZ') \\
= {} & Q2' \cdot Q1' \cdot Q0' \cdot HAZ' \cdot (LEFT \oplus RIGHT) \\
& + Q1' \cdot Q0 \cdot HAZ'
\end{aligned}
$$

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Excitation Equations

- So far we have derived only transition equations, not excitation equations.
- If we use D flip-flops as the memory elements in our state machines, then the excitation equations are trivial to derive from the transition equations, since the characteristic equation of a D flip-flop is $Q^* = D$.

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Therefore, if the transition equation for a state variable Q* is

$$Qi^* = \text{expression}$$

then the excitation equation for the corresponding D flip-flop input is

$$Di = \text{expression}$$

Variations on the Scheme

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If the column for a particular next-state variable contains fewer 0s than 1s, it may be advantages to write that variable's transition equation in terms of the 0s in its column.

$$V* = \prod_{\text{transition-list rows where } V^*=0} (\text{transition s-term})$$

Here, a row's transition s-term is the sum of the maxterm for the current state and the complement of the transition expression. If the transition expression is a simple product term, then its compliment is a sum, and the transition equation express V* in product-of-sums form.

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Q & A