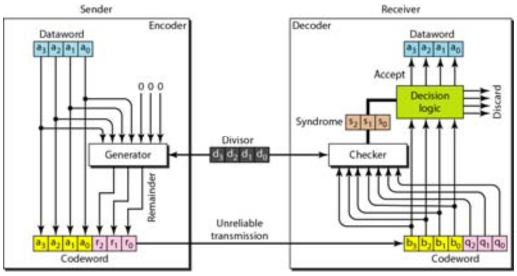
10.4 CYCLIC CODES

10.4.1 Cyclic Redundancy Check

Dataword	Codeword	Dataword	Codeword
0000	0000000	1000	1000101
0001	0001011	1001	1001110
0010	0010110	1010	1010011
0011	0011101	1011	1011000
0100	0100111	1100	1100010
0101	0101100	1101	1101001
0110	0110001	1110	1110100
0111	0111010	1111	1111111

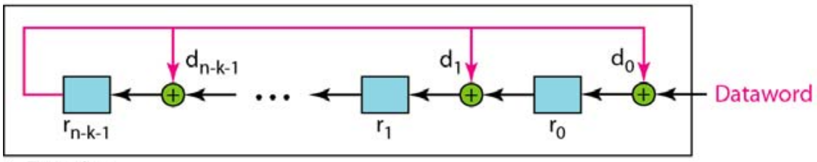


Division in CRC

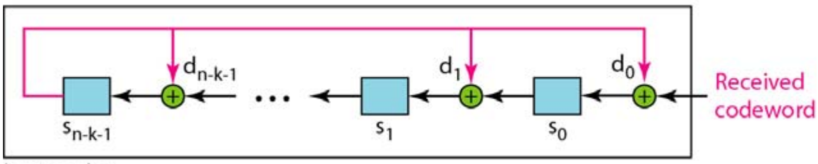
1) Encoder : Dataword 1001 / Divisor 1011

2) Decoder : Codeword 1001110 / Codeword 1000110

10.4.2 **Hardware Implementation** : Divisor, Augmented Dataword, Remainder



a. Encoder



b. Decoder

10.4.3 Polynomials

Degree of a Polynomial : $x^6 + x + 1$

Adding and Subtracting Polynomials : coefficients are only 0 and 1 => modulo-2

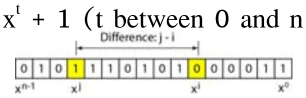
Multiplying or Dividing Terms / Shifting

Cyclic Code Encoder Using Polynomials *

10.4.4 Cyclic Code Analysis $\frac{x^g D(x)}{G(x)} = Q(x) \oplus \frac{R(x)}{G(x)}$ $C(x) = x^g D(x) \oplus R(x)$

Single-Bit Error : If the generator has more than one term and the coefficient of x^0 is 1, all single errors can be caught.

Two Isolated Single-Bit Errors : If a generator cannot divide $x^t + 1$ (t between 0 and n − 1), then all isolated double errors can be detected.



Odd Numbers of Errors : A generator that contains a factor of x + 1 can detect all odd-numbered errors.

Burst Errors : ❑ All burst errors with $L \leq r$ will be detected.

❑ All burst errors with L = r + 1 will be detected with probability $1 - (1/2)^{r-1}$.

❑ All burst errors with L > r + 1 will be detected with probability $1 - (1/2)^r$.

A good polynomial generator needs to have the following characteristics:

1) It should have at least two terms.

2) The coefficient of the term x^0 should be 1.

3) It should not divide $x^t + 1$, for t between 2 and n − 1.

4) It should have the factor x + 1.

Standard Polynomials

Name	*Polynomial*	*Application*
CRC-8	x^8+x^2+x+1	ATM header
CRC-10	$x^{10}+x^9+x^5+x^4+x^2+1$	ATM AAL
CRC-16	$x^{16}+x^{12}+x^5+1$	HDLC
CRC-32	$x^{32}+x^{26}+x^{23}+x^{22}+x^{16}+x^{12}+x^{11}+x^{10}+ x^8+x^7+x^5+x^4+x^2+x+1$	LANs

Advantages of Cyclic Codes

Other Cyclic Codes

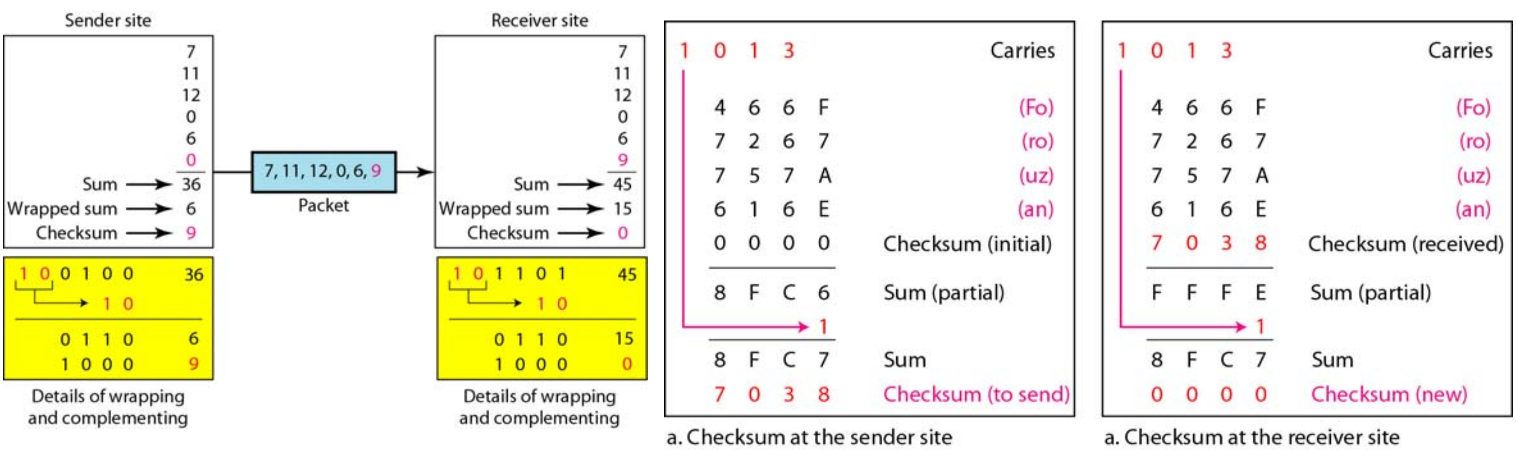
10.5 CHECKSUM

10.5.1 Idea

(7, 11, 12, 0, 6) => (7, 11, 12, 0, 6, 36)

(7, 11, 12, 0, 6) => (7, 11, 12, 0, 6, -36)

10.5.2 One's Complement



a. Checksum at the sender site

a. Checksum at the receiver site

10.5.3 Internet Checksum

Sender site:	**Receiver site:**
1. The message is divided into 16-bit words. 2. The value of the checksum word is set to 0. 3. All words including the checksum are added using one's complement addition. 4. The sum is complemented and becomes the checksum. 5. The checksum is sent with the data.	1. The message (including checksum) is divided into 16-bit words. 2. All words are added using one's complement addition. 3. The sum is complemented and becomes the new checksum. 4. If the value of checksum is 0, the message is accepted; otherwise, it is rejected.