Another Quine-McCluskey Example*

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This is another example of how to use Quine-McCluskey to reduce Boolean expressions. This example includes "don't cares" in the truth table. Consider the following boolean equation:

$$F(A,B,C,D) = \sum m(4,5,6,8,9,10,13) + d(0,7,15)$$

This notation says the function is the sum ($\sum$) of the minterms (m) listed, and the value of the function for the terms 0, 7 and 15 are don't cares (d). The first step of the Quine-McCluskey (QM) method is to list all of the minterms grouped by the number of 1's in the minterm's binary representation. This step includes the don't-care terms, too. We compare each *implicant* in adjacent groups to find patterns that differ by only a single bit, which yields:

0000		0-00 -000
0100 1000		010- 01-0 100- 10-0
0101 0110 1001 1010	$\Longrightarrow$	01-1 -101 011- 1-01
0111 1101		-111 11-1
1111		

(The implicants in the second column cover all of the implicants in the first/left column, and so the checkmarks are not shown.)

Repeating the group-wise comparison on this new set of terms, we get:

*Example adopted from "Contemporary Logic Design" by Randy Katz.

0-00	01- -
-000	01- -
010- ✓	-1-1
01-0 ✓	-1-1
100-	
10-0	
01-1 ✓	
-101 ✓	
011- ✓	
1-01	
-111 ✓	
11-1 ✓	

The last column only has a single group, which ends this phase of the QM technique. The remaining (unchecked) 7 terms are: 0-00, -000, 100-, 10-0, 1-01, 01–, -1-1.

For the last phase, we build the table with the minterms in the top row, and the reduced terms (the prime implicants) in the left column. Note that the top row only includes the “real” minterms, and the don’t-cares are omitted. This is because we do not need to make sure that the don’t-care entries get covered.

		4	5	6	8	9	10	13
0,4	(0-00)	X						
0,8	(-000)				X			
8,9	(100-)				X	X		
8,10	(10-0)				X		X	
9,13	(1-01)					X		X
4,5,6,7	(01- -)	X	X	X				
5,7,13,15	(-1-1)		X					X

Scanning the columns for minterms that are covered by only a single unique prime implicant. In this example, minterms 6 and 10 are each covered by only one prime implicant. Therefore, (01–) must be included in the final expression to cover minterm-6, and (10-0) must be included to cover minterm-10. By including these two prime implicants, we also end up covering 4,5, and 8:

<table>
<tr><th></th><th></th><th>4</th><th>5</th><th>6</th><th>8</th><th>9</th><th>10</th><th>13</th><th></th></tr>
<tr><td>0,4</td><td>(0-00)</td><td>X</td><td>|</td><td>|</td><td>|</td><td></td><td>|</td><td></td><td></td></tr>
<tr><td>0,8</td><td>(-000)</td><td>|</td><td>|</td><td>|</td><td>X</td><td></td><td>|</td><td></td><td></td></tr>
<tr><td>8,9</td><td>(100-)</td><td>|</td><td>|</td><td>|</td><td>X</td><td>X</td><td>|</td><td></td><td></td></tr>
<tr><td>8,10</td><td>(10-0)</td><td>-|-</td><td>-|-</td><td>-|-</td><td>-X-</td><td>—</td><td>-X-</td><td>—</td><td>$A.\overline{B}.\overline{D}$</td></tr>
<tr><td>9,13</td><td>(1-01)</td><td>|</td><td>|</td><td>|</td><td>|</td><td>X</td><td>|</td><td>X</td><td>$A.\overline{C}.D$</td></tr>
<tr><td>4,5,6,7</td><td>(01- -)</td><td>-X-</td><td>-X-</td><td>-X-</td><td>-|-</td><td>—</td><td>-|-</td><td>—</td><td>$\overline{A}.B$</td></tr>
<tr><td>5,7,13,15</td><td>(-1-1)</td><td>|</td><td>X</td><td>|</td><td>|</td><td></td><td>|</td><td>X</td><td></td></tr>
</table>

The only two remaining minterms that need to be covered are 9 and 13. are three prime implicants to consider here. We could choose to include (100-) to cover minterm-9 and (-1-1) to cover 13. Instead, we could choose to use (1-01) which simultaneously covers both. This leads to a final equation of:

$$f(A,B,C,D) = \overline{A}.B + A.\overline{B}.\overline{D} + A.\overline{C}.d$$

Note also that in the end, the equation covered the don’t-care minterm 7, but it does not cover 0 or 15.