

Applied Statistics and Probability for Engineers

Sixth Edition

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Chapter 1

The Role of Statistics in Engineering

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The Role of Statistics in Engineering

CHAPTER OUTLINE

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| 1-1 The Engineering Method and Statistical Thinking | 1-3 Mechanistic & Empirical Models |
| 1-2 Collecting Engineering Data | 1-4 Probability & Probability Models |
| 1-2.1 Basic Principles | |
| 1-2.2 Retrospective Study | |
| 1-2.3 Observational Study | |
| 1-2.4 Designed Experiments | |
| 1-2.5 Observed Processes Over Time | |

Learning Objectives for Chapter 1

After careful study of this chapter, you should be able to do the following:

1. Identify the role of statistics in engineering problem-solving process.
2. How variability affects the data collected and used for engineering decisions.
3. Differentiate between enumerative and analytical studies.
4. Discuss the different methods that engineers use to collect data.
5. Identify the advantages that designed experiments have in comparison to the other methods of collecting engineering data.
6. Explain the differences between mechanistic models & empirical models.
7. Discuss how probability and probability models are used in engineering and science.

What Engineers Do?

An **engineer** is someone who solves problems of interest to society with the efficient application of scientific principles by:

- Refining existing products
- Designing new products or processes

The Creative Process

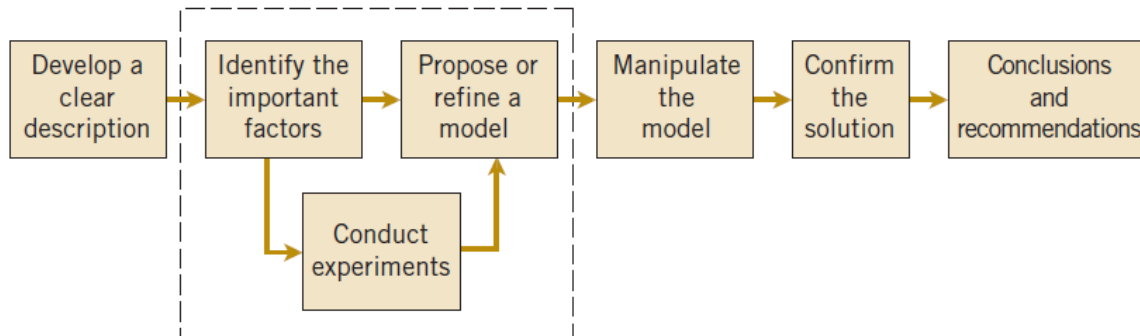


Figure 1-1 The engineering method

Statistics Supports The Creative Process

The field of **statistics** deals with the collection, presentation, analysis, and use of data to:

- Make decisions
- Solve problems
- Design products and processes

It is the **science of data**.

Variability

- Statistical techniques are useful to describe and understand **variability**.
- By variability, we mean successive observations of a system or phenomenon do *not* produce exactly the same result.
- Statistics gives us a framework for describing this variability and for learning about potential **sources of variability**.

An Engineering Example of Variability

Eight prototype units are produced and their pull-off forces are measured (in pounds): 12.6, 12.9, 13.4, 12.3, 13.6, 13.5, 12.6, 13.1.

All of the prototypes does not have the same pull-off force. We can see the variability in the above measurements as they exhibit variability.

The **dot diagram** is a very useful plot for displaying a small body of data - say up to about 20 observations.

This plot allows us to see easily two features of the data; the **location**, or the middle, and the **scatter** or **variability**.



Figure 1-2 Dot diagram of the pull-off force data.

Basic Methods of Collecting Data

Three basic methods of collecting data:

- A **retrospective** study using historical data
 - Data collected in the past for other purposes.
- An **observational** study
 - Data, presently collected, by a passive observer.
- A **designed experiment**
 - Data collected in response to process input changes.

Hypothesis Tests

Hypothesis Test

- A statement about a process behavior value.
- Compared to a claim about another process value.
- Data is gathered to support or refuse the claim.

One-sample hypothesis test:

- Example: Ford avg mpg = 30
vs
Ford avg mpg < 30

Two-sample hypothesis test:

- Example: Ford avg mpg – Chevy avg mpg = 0
vs
Ford avg mpg – Chevy avg mpg > 0

Factorial Experiment & Example

An experiment design which uses every possible combination of the factor levels to form a basic experiment with “k” different settings for the process. This type of experiment is called a **factorial experiment**.

Example:

Consider a petroleum distillation column:

- Output is acetone concentration
- Inputs (factors) are:
 1. Reboil temperature
 2. Condensate temperature
 3. Reflux rate
- Output changes as the inputs are changed by experimenter.

Factorial Experiment Example

- Each factor is set at 2 reasonable levels (-1 and +1)
- 8 (2^3) runs are made, at every combination of factors, to observe acetone output.
- Resultant data is used to create a mathematical model of the process representing cause and effect.

Reboil Temp.	Condensate Temp.	Reflux Rate
-1	-1	-1
+1	-1	-1
-1	+1	-1
+1	+1	-1
-1	-1	+1
+1	-1	+1
-1	+1	+1
+1	+1	+1

Table 1-1 The Designed Experiment (Factorial Design) for the Distillation Column

Fractional Factorial Experiment

- Factor experiments can get too large. For example, 8 factors will require $2^8 = 256$ experimental runs of the distillation column.
- Certain combinations of factor levels can be deleted from the experiments without degrading the resultant model.
- The result is called a **fractional factorial experiment**.

Fractional Factorial Experiment - Example

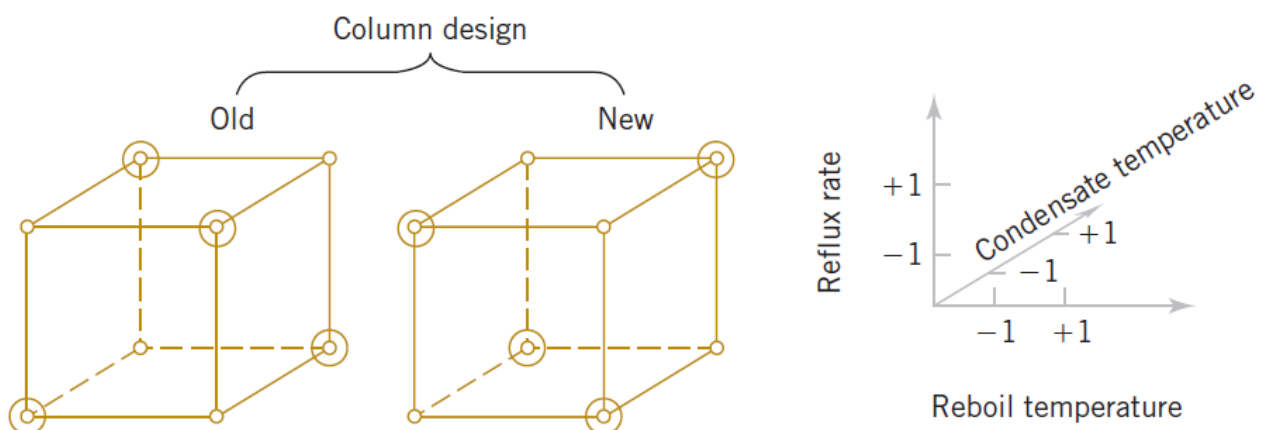


Figure 1-3 A fractional factorial experiment for the distillation column (one-half fraction) $2^4 / 2 = 8$ circled settings.

An Experiment in Variation

W. Edwards Deming, a famous industrial statistician & contributor to the Japanese quality revolution, conducted a illustrative experiment on process **over-control** or **tampering**.

Let's look at his apparatus and experimental procedure.

Deming's Experimental Set-up

Marbles were dropped through a funnel onto a target and the location where the marble struck the target was recorded.

Variation was caused by several factors:

Marble placement in funnel & release dynamics, vibration, air currents, measurement errors.

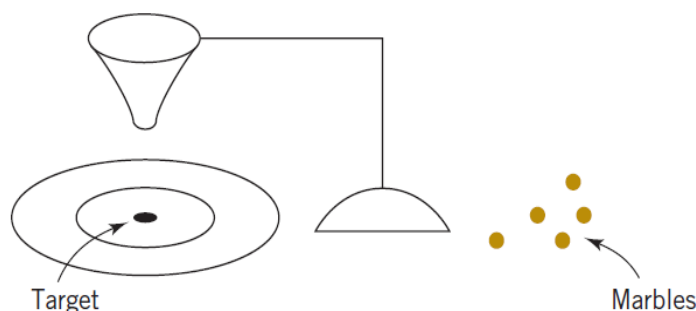


Figure 1-4 Deming's Funnel experiment

Deming's Experimental Procedure

- The funnel was aligned with the center of the target. Marbles were dropped. The distance from the strike point to the target center was measured and recorded.
- Strategy 1: The funnel was not moved. Then the process was repeated.
- Strategy 2: The funnel was moved an equal distance in the opposite direction to compensate for the error. He continued to make this type of adjustment after each marble was dropped. Then the process was repeated.

Deming's Experimental Procedure

- When both strategies were completed, he noticed the variability of the distance from the target for strategy 2 was approximately twice as large than for strategy 1.
- The deviations from the target is increased due to the adjustments to the funnel . Hence, adjustments to the funnel do not decrease future errors. Instead, they tend to move the funnel farther from the target.
- This experiment explains that the adjustments to a process based on random disturbances can actually increase the variation of the process. This is referred to as **overcontrol** or **tampering**.

Conclusions from the Deming Experiment

The lesson of the Deming experiment is that a process should not be adjusted in response to random variation, but only when a clear shift in the process value becomes apparent.

Then a process adjustment should be made to return the process outputs to their normal values.

To identify when the shift occurs, a **control chart** is used. Output values, plotted over time along with the outer limits of normal variation, pinpoint when the process leaves normal values and should be adjusted.

How Is the Change Detected?

- A **control chart** is used. Its characteristics are:
 - Time-oriented horizontal axis, e.g., hours.
 - Variable-of-interest vertical axis, e.g., % acetone.
- Long-term average is plotted as the center-line.
- Long-term usual variability is plotted as an upper and lower control limit around the long-term average.
- A sample of size n is taken and the averages are plotted over time. If the plotted points are between the control limits, then the process is normal; if not, it needs to be adjusted.

How Is the Change Detected Graphically?

The center line on the control chart is just the average of the concentration measurements for the first 20 samples

$$\bar{X} = 91.5 \text{ g/l}$$

when the process is stable. The upper control limit and the lower control limit are located 3 standard deviations of the concentration values above and below the center line.

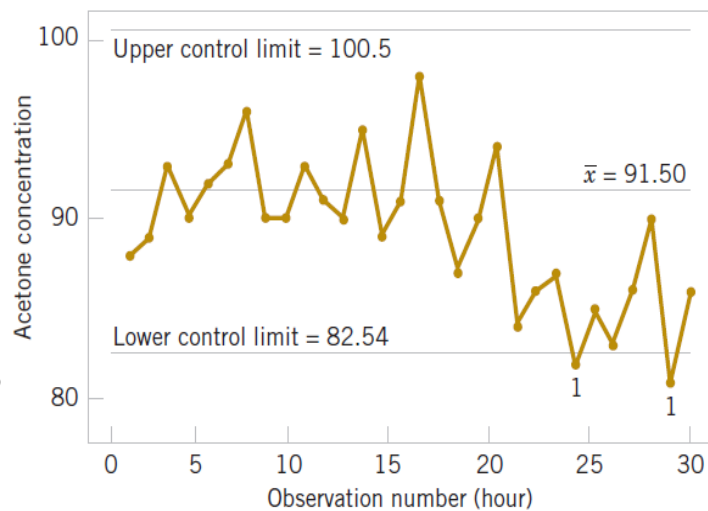


Figure 1-5 A control chart for the chemical process concentration data. Process steps out at hour 24 & 29. Shut down & adjust process.

Use of Control Charts

Deming contrasted two purposes of control charts:

- 1. Enumerative studies:** Control chart of past production lots. Used for lot-by-lot acceptance sampling.
- 2. Analytic studies:** Real-time control of a production process.

Mechanistic and Empirical Models

A **mechanistic model** is built from our underlying knowledge of the basic physical mechanism that relates several variables.

Example: Ohm's Law

$$\text{Current} = V/R$$

$$I = E/R$$

$$I = E/R + \varepsilon$$

where ε is a term added to the model to account for the fact that the observed values of current flow do not perfectly conform to the mechanistic model.

- The form of the function is known.

Mechanistic and Empirical Models

An **empirical model** is built from our engineering and scientific knowledge of the phenomenon, but is not directly developed from our theoretical or first-principles understanding of the underlying mechanism.

The form of the function is not known *a priori*.

An Example of an Empirical Model

- In a semiconductor manufacturing plant, the finished semiconductor is wire-bonded to a frame. In an observational study, the variables recorded were:
 - Pull strength to break the bond (y)
 - Wire length (x_1)
 - Die height (x_2)
- The data recorded are shown on the next slide.

An Example of an Empirical Model

Table 1-2 Wire Bond Pull Strength Data

Observation Number	Pull Strength y	Wire Length x_1	Die Height x_2
1	9.95	2	50
2	24.45	8	110
3	31.75	11	120
4	35.00	10	550
5	25.02	8	295
6	16.86	4	200
7	14.38	2	375
8	9.60	2	52
9	24.35	9	100
10	27.50	8	300
11	17.08	4	412
12	37.00	11	400
13	41.95	12	500
14	11.66	2	360
15	21.65	4	205
16	17.89	4	400
17	69.00	20	600
18	10.30	1	585
19	34.93	10	540
20	46.59	15	250
21	44.88	15	290
22	54.12	16	510
23	56.63	17	590
24	22.13	6	100
25	21.15	5	400

An Example of an Empirical Model

$$\widehat{\text{Pull strength}} = \beta_0 + \beta_1(\text{wire length}) + \beta_2(\text{die height}) + \epsilon$$

where the “hat,” or circumflex, over pull strength indicates that this is an estimated or predicted quality.

In general, this type of empirical model is called a **regression model**.

The **estimated** regression relationship is given by:

$$\widehat{\text{Pull strength}} = 2.26 + 2.74(\text{wire length}) + 0.0125(\text{die height})$$

Visualizing the Data

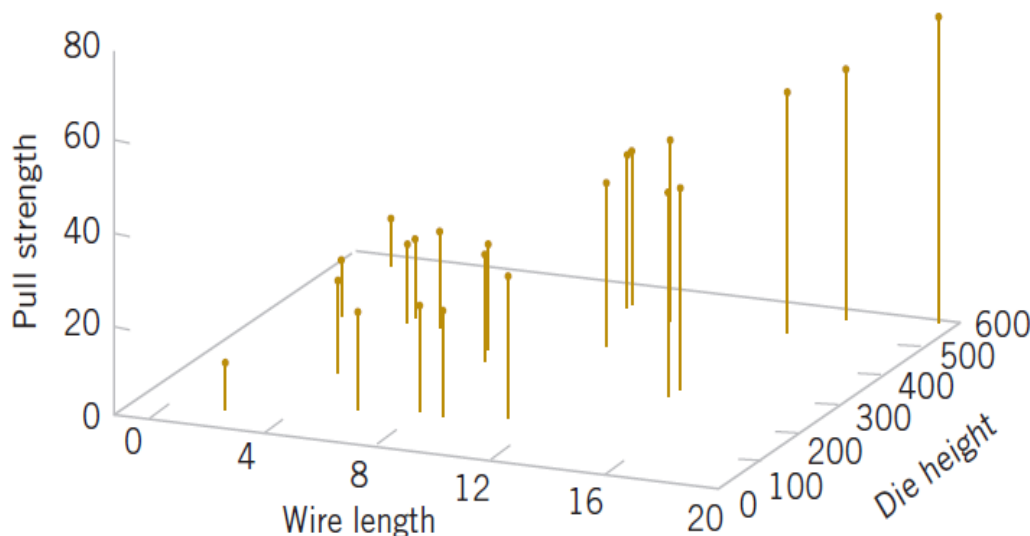


Figure 1-6 Three-dimensional plot of the pull strength (y), wire length (x_1) and die height (x_2) data.

Visualizing the Resultant Model Using Regression Analysis

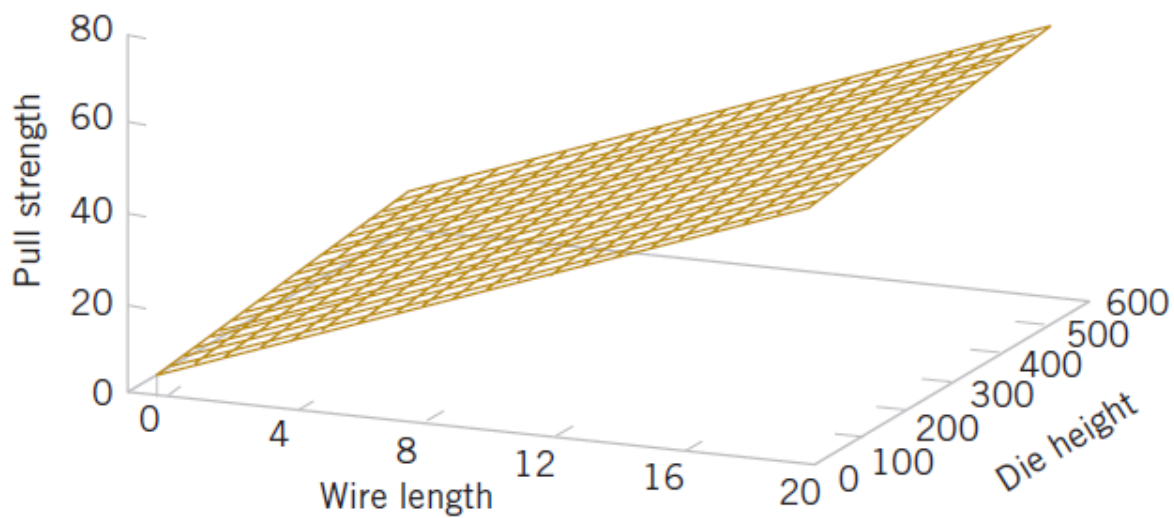


Figure 1-7 Plot of the predicted values (a plane) of pull strength from the empirical regression model.

Models Can Also Reflect Uncertainty

- **Probability models** help quantify the risks involved in statistical inference, that is, risks involved in decisions made every day.
- Probability provides the **framework** for the study and application of statistics.
- Probability concepts will be introduced in the next lecture.