Python & Numpy for Deep Learning

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Python

- Python is dynamic-type & multi-paradigm programming language.
- Its pseudo-like code allows you to express very powerful ideas in very few lines of code.

Example:

```
def quicksort(arr):
    if len(arr) <= 1:
        return arr
    pivot = arr[len(arr) // 2]
    left = [x for x in arr if x < pivot]
    middle = [x for x in arr if x == pivot]
    right = [x for x in arr if x > pivot]
    return quicksort(left) + middle + quicksort(right)

print(quicksort([3,6,8,10,1,2,1]))
# Prints "[1, 1, 2, 3, 6, 8, 10]"
```

Basic Data Types

- Numbers: Integers and floats

```
x = 3
print(type(x)) # Prints "<class 'int'>"
print(x)       # Prints "3"
print(x + 1)   # Addition; prints "4"
print(x - 1)   # Subtraction; prints "2"
print(x * 2)   # Multiplication; prints "6"
print(x ** 2)  # Exponentiation; prints "9"
x += 1         # python doesn't have x++, x--
print(x)  # Prints "4"
x *= 2
print(x)  # Prints "8"
y = 2.5
print(type(y)) # Prints "<class 'float'>"
print(y, y + 1, y * 2, y ** 2) # Prints "2.5 3.5 5.0 6.25"
```

Basic Data Types

- Booleans

```
t = True
f = False
print(type(t)) # Prints "<class 'bool'>"
print(t and f) # Logical AND; prints "False"
print(t or f)  # Logical OR; prints "True"
print(not t)   # Logical NOT; prints "False"
print(t != f)  # Logical XOR; prints "True"
```

Basic Data Types

- String

```
hello = 'hello'    # String literals can use single quotes
world = "world"    # or double quotes; it does not matter.
print(hello)       # Prints "hello"
print(len(hello))  # String length; prints "5"
hw = hello + ' ' + world  # String concatenation
print(hw)  # prints "hello world"
hw12 = '%s %s %d' % (hello, world, 12)  # sprintf style string formatting
print(hw12)  # prints "hello world 12"
```

Basic Data Types

```
s = "hello"
print(s.capitalize())  # Capitalize a string; prints "Hello"
print(s.upper())       # Convert a string to uppercase; prints "HELLO"
print(s.rjust(7))      # Right-justify a string, padding with spaces; prints "  hello"
print(s.center(7))     # Center a string, padding with spaces; prints " hello "
print(s.replace('l', '(ell)'))  # Replace all instances of one substring with another;
                                # prints "he(ell)(ell)o"
print('  world '.strip())  # Strip leading and trailing whitespace; prints "world"
```

Basic Data Types

- Containers
  1. Lists
  2. Dictionaries
  3. Sets
  4. Tuples

Basic Data Types

1. List : similar to array, but is resizable and can contain elements of different types.

```
xs = [3, 1, 2]    # Create a list
print(xs, xs[2])  # Prints "[3, 1, 2] 2"
print(xs[-1])     # Negative indices count from the end of the list; prints "2"
xs[2] = 'foo'     # Lists can contain elements of different types
print(xs)         # Prints "[3, 1, 'foo']"
xs.append('bar')  # Add a new element to the end of the list
print(xs)         # Prints "[3, 1, 'foo', 'bar']"
x = xs.pop()      # Remove and return the last element of the list
print(x, xs)      # Prints "bar [3, 1, 'foo']"
```

- List : Slicing

Python provides concise syntax to access sub lists; this is known as slicing.

```
nums = list(range(5))     # range is a built-in function that creates a list of integers
print(nums)               # Prints "[0, 1, 2, 3, 4]"
print(nums[2:4])          # Get a slice from index 2 to 4 (exclusive); prints "[2, 3]"
print(nums[2:])           # Get a slice from index 2 to the end; prints "[2, 3, 4]"
print(nums[:2])           # Get a slice from the start to index 2 (exclusive); prints "[0, 1]"
print(nums[:])            # Get a slice of the whole list; prints "[0, 1, 2, 3, 4]"
print(nums[:-1])          # Slice indices can be negative; prints "[0, 1, 2, 3]"
nums[2:4] = [8, 9]        # Assign a new sublist to a slice
print(nums)               # Prints "[0, 1, 8, 9, 4]"
```

○ List : Loop

```python
animals = ['cat', 'dog', 'monkey']
for animal in animals:
    print(animal)
# Prints "cat", "dog", "monkey", each on its own line.
```

If you want to access to the index of each element within the body of the loop, use the built-in enumerate function.

```python
animals = ['cat', 'dog', 'monkey']
for idx, animal in enumerate(animals):
    print('#%d: %s' % (idx + 1, animal))
# Prints "#1: cat", "#2: dog", "#3: monkey", each on its own line
```

- List : Comprehension

Instead of using a for loop, you can iterate through the list by using a list comprehension.

```
nums = [0, 1, 2, 3, 4]
squares = []
for x in nums:
    squares.append(x ** 2)
print(squares)   # Prints [0, 1, 4, 9, 16]
```

```
nums = [0, 1, 2, 3, 4]
squares = [x ** 2 for x in nums]
print(squares)   # Prints [0, 1, 4, 9, 16]
```

List comprehensions can also contain conditions:

```
nums = [0, 1, 2, 3, 4]
even_squares = [x ** 2 for x in nums if x % 2 == 0]
print(even_squares)  # Prints "[0, 4, 16]"
```

2. Dictionaries : A dictionary stores (key, value)

```python
d = {'cat': 'cute', 'dog': 'furry'}  # Create a new dictionary with some data
print(d['cat'])       # Get an entry from a dictionary; prints "cute"
print('cat' in d)     # Check if a dictionary has a given key; prints "True"
d['fish'] = 'wet'     # Set an entry in a dictionary
print(d['fish'])      # Prints "wet"
# print(d['monkey'])  # KeyError: 'monkey' not a key of d
print(d.get('monkey', 'N/A'))  # Get an element with a default; prints "N/A"
print(d.get('fish', 'N/A'))    # Get an element with a default; prints "wet"
del d['fish']         # Remove an element from a dictionary
print(d.get('fish', 'N/A')) # "fish" is no longer a key; prints "N/A"
```

- Dictionaries : Loop

  - You can iterate over the keys in a dictionary.

```
d = {'person': 2, 'cat': 4, 'spider': 8}
for animal in d:
    legs = d[animal]
    print('A %s has %d legs' % (animal, legs))
# Prints "A person has 2 legs", "A cat has 4 legs", "A spider has 8 legs"
```

  - To access to key and value pair, use the items method.

```
d = {'person': 2, 'cat': 4, 'spider': 8}
for animal, legs in d.items():
    print('A %s has %d legs' % (animal, legs))
# Prints "A person has 2 legs", "A cat has 4 legs", "A spider has 8 legs"
```

- Dictionaries : Comprehension

This is similar to list comprehension.

Create dictionary

```
nums = [0, 1, 2, 3, 4]
even_num_to_square = {x: x ** 2 for x in nums if x % 2 == 0}
print(even_num_to_square)  # Prints "{0: 0, 2: 4, 4: 16}"
```

3. Sets : An unordered collection of distinct elements.

```
animals = {'cat', 'dog'}
print('cat' in animals)   # Check if an element is in a set; prints "True"
print('fish' in animals)  # prints "False"
animals.add('fish')       # Add an element to a set
print('fish' in animals)  # Prints "True"
print(len(animals))       # Number of elements in a set; prints "3"
animals.add('cat')        # Adding an element that is already in the set does nothing
print(len(animals))       # Prints "3"
animals.remove('cat')     # Remove an element from a set
print(len(animals))       # Prints "2"
```

○ Set : Loop

Iterating over a set has the same syntax as iterating over a list, but the set is unordered.

```
animals = {'cat', 'dog', 'fish'}
for idx, animal in enumerate(animals):
    print('#%d: %s' % (idx + 1, animal))
    # Prints "#1: fish", "#2: dog", "#3: cat"
```

○ Set : Comprehension

Like lists and dictionaries, we can construct sets using set comprehension.

```
from math import sqrt
nums = {int(sqrt(x)) for x in range(30)}
print(nums)  # Prints "{0, 1, 2, 3, 4, 5}"
```

4. **Tuples :** An (immutable) ordered list of values. Tuple can be used as a key in dictionary and as elements of set, but list cannot.

Tuple

```
d = {(x, x + 1): x for x in range(10)}  # Create a dictionary with tuple keys
t = (5, 6)         # Create a tuple
print(type(t))     # Prints "<class 'tuple'>"
print(d[t])        # Prints "5"
print(d[(1, 2)])   # Prints "1"
```

```
{ (0,1):0
  (1,2):1
  (2,3):2
  (3,4):3
  (4,5):4
  (5,6):5
   ...
  (9,10):9 }
```

Function

- Python function is defined using ‘def’ keyword. For example:

```
def sign(x):
    if x > 0:
        return 'positive'
    elif x < 0:
        return 'negative'
    else:
        return 'zero'

for x in [-1, 0, 1]:
    print(sign(x))
# Prints "negative", "zero", "positive"
```

- We can define function to take optional keyword, like this:

```
def hello(name, loud=False):
    if loud:
        print('HELLO, %s!' % name.upper())
    else:
        print('Hello, %s' % name)

hello('Bob') # Prints "Hello, Bob"
hello('Fred', loud=True)  # Prints "HELLO, FRED!"
```

Class

- The following code is an example class in Python.

```
class Greeter(object):

    # Constructor
    def __init__(self, name):
        self.name = name  # Create an instance variable

    # Instance method
    def greet(self, loud=False):
        if loud:
            print('HELLO, %s!' % self.name.upper())
        else:
            print('Hello, %s' % self.name)

g = Greeter('Fred')  # Construct an instance of the Greeter class
g.greet()            # Call an instance method; prints "Hello, Fred"
g.greet(loud=True)   # Call an instance method; prints "HELLO, FRED!"
```

Numpy

- **Numpy** is a open-source library for the Python, adding support for large, multi-dimensional arrays and matrices, along with a large collection of high-level mathematical functions to operate on these arrays.

- **Array**
  - Python doesn't have an array. If you want to use it, you must import numpy.

```
import numpy as np

a = np.array([1, 2, 3])   # Create a rank 1 array
print(type(a))            # Prints "<class 'numpy.ndarray'>"
print(a.shape)            # Prints "(3,)"
print(a[0], a[1], a[2])   # Prints "1 2 3"
a[0] = 5                  # Change an element of the array
print(a)                  # Prints "[5, 2, 3]"

b = np.array([[1,2,3],[4,5,6]])    # Create a rank 2 array
print(b.shape)                     # Prints "(2, 3)"
print(b[0, 0], b[0, 1], b[1, 0])   # Prints "1 2 4"
```

Jokes of array subscription

Credit: Internet

– Numpy also provides many functions to create arrays.

```
import numpy as np

a = np.zeros((2,2))   # Create an array of all zeros
print(a)              # Prints "[[ 0.  0.]
                      #          [ 0.  0.]]"
b = np.ones((1,2))    # Create an array of all ones
print(b)              # Prints "[[ 1.  1.]]"

c = np.full((2,2), 7)  # Create a constant array
print(c)               # Prints "[[ 7.  7.]
                       #          [ 7.  7.]]"
d = np.eye(2)          # Create a 2x2 identity matrix
print(d)               # Prints "[[ 1.  0.]
                       #          [ 0.  1.]]"
e = np.random.random((2,2))  # Create an array filled with random values
print(e)                     # Might print "[[ 0.91940167  0.08143941]
                             #               [ 0.68744134  0.87236687]]"
```

- Array indexing: Slicing

```
import numpy as np
# Create the following rank 2 array with shape (3, 4)
# [[ 1  2  3  4]
#  [ 5  6  7  8]
#  [ 9 10 11 12]]
a = np.array([[1,2,3,4], [5,6,7,8], [9,10,11,12]])

# Use slicing to pull out the subarray consisting of the first 2 rows
# and columns 1 and 2; b is the following array of shape (2, 2):
# [[2 3]
#  [6 7]]
b = a[:2, 1:3]

# A slice of an array is a view into the same data, so modifying it
# will modify the original array.
print(a[0, 1])   # Prints "2"
b[0, 0] = 77     # b[0, 0] is the same piece of data as a[0, 1]
print(a[0, 1])   # Prints "77"
```

- You can also mix integer indexing with slice indexing.

```
import numpy as np

# Create the following rank 2 array with shape (3, 4)
# [[ 1  2  3  4]
#  [ 5  6  7  8]
#  [ 9 10 11 12]]
a = np.array([[1,2,3,4], [5,6,7,8], [9,10,11,12]])
                 1 will not increase rank (dimension)
row_r1 = a[1, :]    # Rank 1 view of the second row of a
row_r2 = a[1:2, :]  # Rank 2 view of the second row of a
print(row_r1, row_r1.shape)  # Prints "[5 6 7 8] (4,)"
print(row_r2, row_r2.shape)  # Prints "[[5 6 7 8]] (1, 4)"

# We can make the same distinction when accessing columns of an array:
col_r1 = a[:, 1]   Array of rank 1
col_r2 = a[:, 1:2]   Array of rank 2
print(col_r1, col_r1.shape)  # Prints "[ 2  6 10] (3,)"
print(col_r2, col_r2.shape)  # Prints "[[ 2]
                             #          [ 6]
                             #          [10]] (3, 1)"
```

- Integer array indexing: allows you to reference to array using integer array.

```
import numpy as np

a = np.array([[1,2], [3, 4], [5, 6]])

# An example of integer array indexing.
# The returned array will have shape (3,) and
print(a[[0, 1, 2], [0, 1, 0]])  # Prints "[1 4 5]"
```

Row indices   Column indices

```
# The above example of integer array indexing is equivalent to this:
print(np.array([a[0, 0], a[1, 1], a[2, 0]]))  # Prints "[1 4 5]"
```

1	2
3	4
5	6

- Integer array indexing can be used to mutate some elements of an array:

```
import numpy as np

# Create a new array from which we will select elements
a = np.array([[1,2,3], [4,5,6], [7,8,9], [10, 11, 12]])

# Create an array of indices
b = np.array([0, 2, 0, 1])

# Select one element from each row of a using the indices in b
print(a[np.arange(4), b])  # Prints "[ 1  6  7 11]"
```

[0,1,2,3] [0,2,0,1]

```
# Mutate one element from each row of a using the indices in b
a[np.arange(4), b] += 10

print(a)  # prints "array([[11,  2,  3],
          #                [ 4,  5, 16],
          #                [17,  8,  9],
          #                [10, 21, 12]])
```

1	2	3
4	5	6
7	8	9
10	11	12

1	2	3
4	5	6
7	8	9
10	11	12

11	2	3
4	5	16
17	8	9
10	21	12

- Boolean array indexing:

```python
import numpy as np

a = np.array([[1,2], [3, 4], [5, 6]])
```

1	2
3	4
5	6

```python
bool_idx = (a > 2)    # Find the elements of a that are bigger than 2;
                      # this returns a numpy array of Booleans of the same
                      # shape as a, where each slot of bool_idx tells
                      # whether that element of a is > 2.

print(bool_idx)       # Prints "[[False False]
                      #          [ True  True]
                      #          [ True  True]]"
print(a[bool_idx])    # Prints "[3 4 5 6]"
```

1	2
3	4
5	6

```python
# We can do all of the above in a single concise statement:
print(a[a > 2])       # Prints "[3 4 5 6]"
```

- Datatypes

```
import numpy as np
x = np.array([1,2])          # Let numpy chooses datatype
print(x.dtype)               # Print "int64"
y = np.array([1.0,2.0])      # Let numpy chooses datatype
print(y.dtype)               # Print "float64"

x=np.array([1,2],dtype=np.float64) # Force a float64 type
print(x.dtype)               # Print "float64"
```

- Array math

Elementwise operations

```
import numpy as np

x = np.array([[1,2],[3,4]], dtype=np.float64)
y = np.array([[5,6],[7,8]], dtype=np.float64)
```

1	2
3	4

5	6
7	8

```
# Elementwise sum,
# [[ 6.0  8.0]
#  [10.0 12.0]]
print(x + y)
print(np.add(x, y))
```

```
# Elementwise product;
# [[ 5.0 12.0]
#  [21.0 32.0]]
print(x * y)
print(np.multiply(x, y))
```

```
# Elementwise square root;
# [[ 1.          1.41421356]
#  [ 1.73205081  2.        ]]
print(np.sqrt(x))
```

```
# Elementwise difference
# [[-4.0 -4.0]
#  [-4.0 -4.0]]
print(x - y)
print(np.subtract(x, y))
```

```
# Elementwise division;
# [[ 0.2         0.33333333]
#  [ 0.42857143  0.5       ]]
print(x / y)
print(np.divide(x, y))
```

```
import numpy as np

x = np.array([[1,2],[3,4]])
y = np.array([[5,6],[7,8]])

v = np.array([9,10])
w = np.array([11, 12])
```

$$x = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad y = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} \quad v = \begin{bmatrix} 9 \\ 10 \end{bmatrix} \quad w = \begin{bmatrix} 11 \\ 12 \end{bmatrix}$$

```
# Inner product of vectors; both produce 219
print(v.dot(w))
print(np.dot(v, w))
```

→ (9*11+10*12)

Inner product of two vectors

```
# Matrix / vector product; both produce the rank 1 array [29 67]
print(x.dot(v))
print(np.dot(x, v))
```

$$\rightarrow \begin{bmatrix} (1*9)+(2*10) \\ (3*9)+(4*10) \end{bmatrix}$$

Matrix dot Vector product

```
# Matrix / matrix product; both produce the rank 2 array
# [[19 22]
#  [43 50]]
print(x.dot(y))
print(np.dot(x, y))
```

$$\begin{bmatrix} (1*5)+(2*7) & (1*6)+(2*8) \\ (3*5)+(4*7) & (3*6)+(4*8) \end{bmatrix}$$

Matrix dot Matrix product

Sum operation in array

```
import numpy as np

x = np.array([[1,2],[3,4]])
```

1	2
3	4

```
print(np.sum(x))  # Compute sum of all elements; prints "10"
print(np.sum(x, axis=0))  # Compute sum of each column; prints "[4 6]"
print(np.sum(x, axis=1))  # Compute sum of each row; prints "[3 7]"
```

Transpose in array

```
import numpy as np

x = np.array([[1,2], [3,4]])
```

1	2
3	4

```
print(x)     # Prints "[[1 2]
             #          [3 4]]"
print(x.T)   # Prints "[[1 3]
             #          [2 4]]"
```

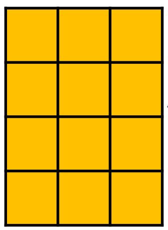
1	3
2	4

- **Broadcasting**: allows numpy to work with arrays of different shapes.

```
import numpy as np
# We will add the vector v to each row of the matrix x,
# storing the result in the matrix y
x = np.array([[1,2,3], [4,5,6], [7,8,9], [10, 11, 12]])
v = np.array([1, 0, 1])
y = np.empty_like(x)    # Create an empty matrix with the same shape as x
```

1	2	3
4	5	6
7	8	9
10	11	12

1	0	1



```
# Add the vector v to each row of the matrix x with an explicit loop
for i in range(4):
    y[i, :] = x[i, :] + v
```

This works but computing an explicit loop in Python could be slow !

```
# Now y is the following
# [[ 2  2  4]
#  [ 5  5  7]
#  [ 8  8 10]
#  [11 11 13]]
print(y)
```

2	2	4
5	5	7
8	8	10
11	11	13

A better solution.

```
import numpy as np

# We will add the vector v to each row of the matrix x,
# storing the result in the matrix y
x = np.array([[1,2,3], [4,5,6], [7,8,9], [10, 11, 12]])
v = np.array([1, 0, 1])
vv = np.tile(v, (4, 1))   # Stack 4 copies of v on top of each other
print(vv)                 # Prints "[[1 0 1]
                          #          [1 0 1]
                          #          [1 0 1]
                          #          [1 0 1]]"
y = x + vv  # Add x and vv elementwise
print(y)  # Prints "[[ 2  2  4
          #          [ 5  5  7]
          #          [ 8  8 10]
          #          [11 11 13]]"
```

1	2	3
4	5	6
7	8	9
10	11	12

1	0	1

1	0	1
1	0	1
1	0	1
1	0	1

But we need to create multiple copies of v!

2	2	4
5	5	7
8	8	10
11	11	13

“Broadcasting” allows us to perform this computation without actually creating multiple copies of v.

```
import numpy as np

# We will add the vector v to each row of the matrix x,
# storing the result in the matrix y
x = np.array([[1,2,3], [4,5,6], [7,8,9], [10, 11, 12]])
v = np.array([1, 0, 1])
y = x + v  # Add v to each row of x using broadcasting
print(y)  # Prints "[[ 2  2  4]
          #          [ 5  5  7]
          #          [ 8  8 10]
          #          [11 11 13]]"
```

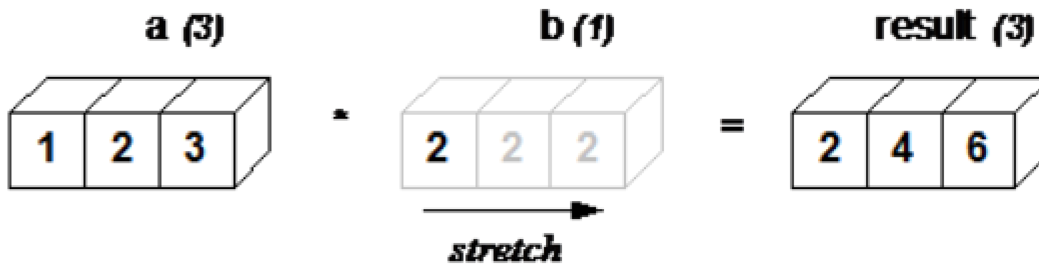
1	2	3
4	5	6
7	8	9
10	11	12

1	0	1

This works even though both x and v have shape (4,3) , where each row in v was a copy of v, and the sum was performed elementwise.

– The simplest broadcasting example occurs when an array and a scalar value are combined in an operation:

```
>>> from numpy import array
>>> a = array([1.0,2.0,3.0])
>>> b = 2.0
>>> a * b
array([ 2.,  4.,  6.])
```



– **Broadcasting Rule**: In order to broadcast, the size of the **trailing axes** for both arrays in an operation must either be the **same size** or **one of them must be one**.

- For example, if you have a 256x256x3 array of RGB values, and you want to scale each color in the image by 3 factors, you can multiply the image by a one-dimensional array with 3 values.

```
Image  (3d array): 256 x 256 x 3
Scale  (1d array):             3
Result (3d array): 256 x 256 x 3
```

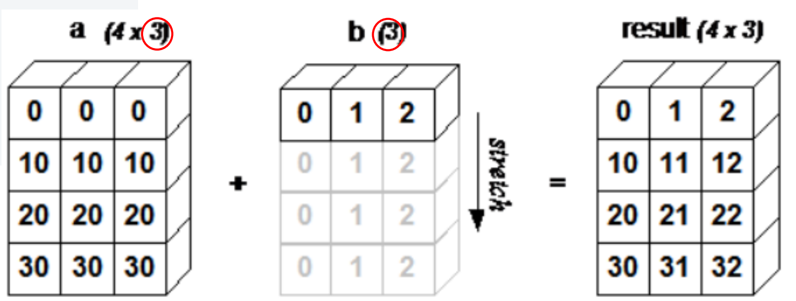
Same size of trailing axe

- Second example: the following A and B arrays have axes with length one that are expanded to a larger size in a broadcast operation.

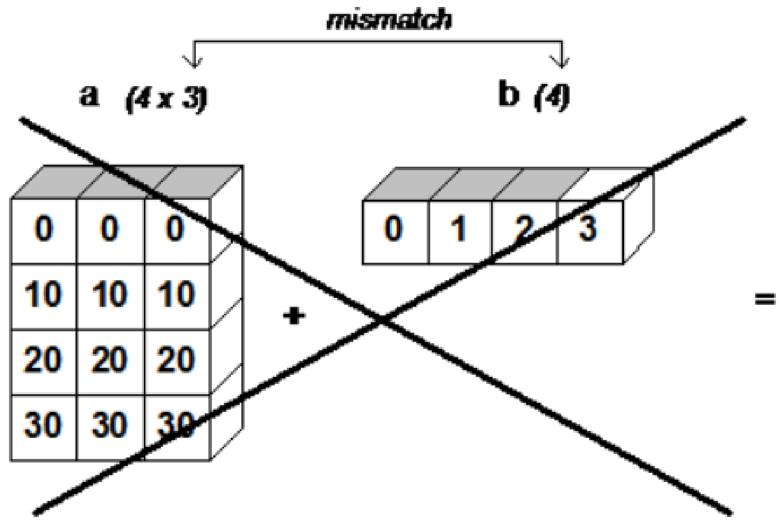
```
A      (4d array):  8 x 1 x 6 x 1
B      (3d array):      7 x 1 x 5
Result (4d array):  8 x 7 x 6 x 5
```

- Third example: adding a one-dimensional array to a two-dimensional array.

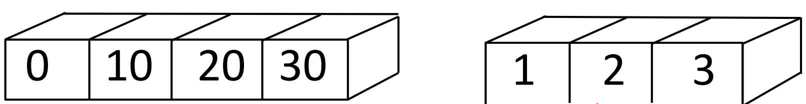
```
>>> from numpy import array
>>> a = array([[ 0.0, 0.0, 0.0],
...            [10.0,10.0,10.0],
...            [20.0,20.0,20.0],
...            [30.0,30.0,30.0]])
>>> b = array([1.0,2.0,3.0])
>>> a + b
array([[  1.,   2.,   3.],
       [ 11.,  12.,  13.],
       [ 21.,  22.,  23.],
       [ 31.,  32.,  33.]])
```



If the trailing dimensions of the arrays are unequal, broadcasting fails because it is impossible to align the values in the rows of the 1st array with the elements of the 2nd arrays for element-by-element addition.

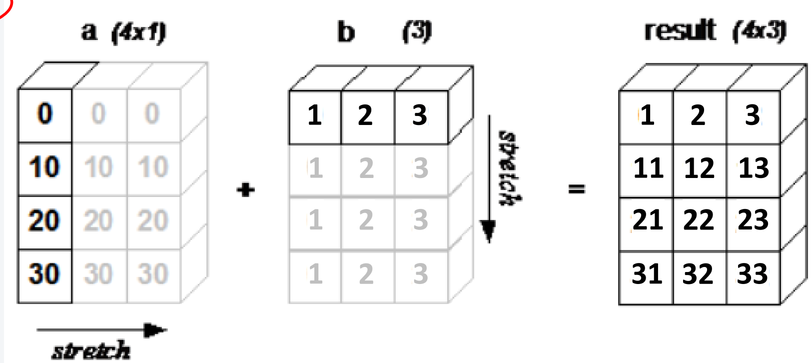


Broadcasting tries to do an operation on row basis.



- Fourth example: an outer addition operation of two 1-d arrays

```
>>> from numpy import array, newaxis
>>> a = array([0.0,10.0,20.0,30.0])
>>> b = array([1.0,2.0,3.0])
>>> a[:,newaxis] + b
array([[  1.,   2.,   3.],
       [ 11.,  12.,  13.],
       [ 21.,  22.,  23.],
       [ 31.,  32.,  33.]])
```



Here the **newaxis** index operator <u>inserts a new axis into a</u>, making it a two-dimensional 4x1 array.

More examples on broadcasting.

```
import numpy as np

# Compute outer product of vectors
v = np.array([1,2,3])  # v has shape (3,)
w = np.array([4,5])    # w has shape (2,)
# To compute an outer product, we first reshape v to be a column
# vector of shape (3, 1); we can then broadcast it against w to yield
# an output of shape (3, 2), which is the outer product of v and w:
# [[ 4  5]
#  [ 8 10]
#  [12 15]]
print(np.reshape(v, (3, 1)) * w)
```

$$\begin{bmatrix}1\\2\\3\end{bmatrix}.\begin{bmatrix}4 & 5\end{bmatrix} = \begin{bmatrix}4 & 5\\8 & 10\\12 & 15\end{bmatrix}$$

```python
# Add a vector to each row of a matrix
x = np.array([[1,2,3], [4,5,6]])
# x has shape (2, 3) and v has shape (3,) so they broadcast to (2, 3),
# giving the following matrix:
# [[2 4 6]
#  [5 7 9]]
print(x + v)
```

X

1	2	3
4	5	6

V

1	2	3

```python
# Add a vector to each column of a matrix
# x has shape (2, 3) and w has shape (2,).
# If we transpose x then it has shape (3, 2) and can be broadcast
# against w to yield a result of shape (3, 2); transposing this result
# yields the final result of shape (2, 3) which is the matrix x with
# the vector w added to each column. Gives the following matrix:
# [[ 5  6  7]
#  [ 9 10 11]]
print((x.T + w).T)
```

$$\mathbf{X} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \qquad \mathbf{w} = [4 \;\; 5]$$

$$\underbrace{\begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}}_{\mathbf{X}^{\mathbf{T}}} + \underbrace{[4 \;\; 5]}_{\mathbf{w}} = \begin{bmatrix} 5 & 9 \\ 6 & 10 \\ 7 & 11 \end{bmatrix}$$

```python
# Another solution is to reshape w to be a column vector of shape (2, 1);
# we can then broadcast it directly against x to produce the same
# output.
print(x + np.reshape(w, (2, 1)))
```

$$\mathbf{x} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \qquad \mathbf{w} = [4 \ \ 5]$$

```python
# Multiply a matrix by a constant:
# x has shape (2, 3). Numpy treats scalars as arrays of shape ();
# these can be broadcast together to shape (2, 3), producing the
# following array:
# [[ 2  4  6]
#  [ 8 10 12]]
print(x * 2)
```

$$\mathbf{x} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

Broadcasting typically makes your code more concise and faster, so you should strive to use it where possible.

- Next class
  - Regression and Classification with Linear models
  - Linear classification with perceptron