Image Enhancement and Restoration

- In Frequency Domain

#1 Fourier Transform Application

Frequency Filtering

- **for noise reduction**
- **for contrast enhancement**
- **for texture analysis**
- **for shape analysis**

FOURIER TRANSFORM

For Filtering

Filtering in Frequency Domain

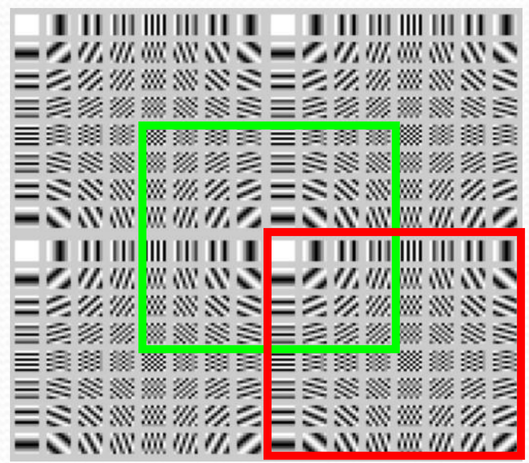
$$f(x,y) \longrightarrow \boxed{\text{Shift}} \xrightarrow{f'(x,y)} \boxed{\text{DFT}} \xrightarrow{F(u,v)} \boxed{H(u,v)} \xrightarrow{F(u,v)H(u,v)} \boxed{\text{IDFT}} \xrightarrow{g'(x,y)} \boxed{\text{Shift}} \xrightarrow{g(x,y)}$$

Shift: $(-1)^{x+y}$

$$F(u,v) = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)e^{-j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$

$e^{-j2\pi st} = \cos(2\pi st) - j\sin(2\pi st)$

$$\boxed{\begin{aligned} F(u,v) &= a - jb \\ &= \sqrt{a^2+b^2} \angle \tan^{-1}(b/a) \end{aligned}}$$



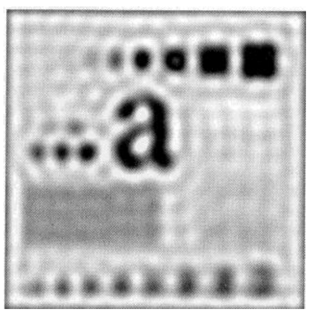
$$g(x,y) = d + \cancel{je}$$

Filter Categories

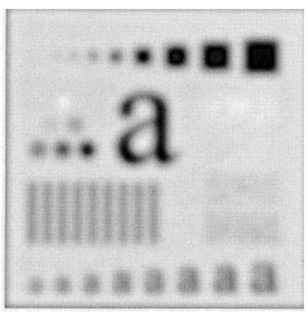
- Regular Filter
  - Low Pass Filter (LPF)
  - High Pass Filter (HPF)
  - Band Pass Filter (BPF)
  - Band Reject Filter (BRF)
  - High Frequency Emphasis Filter (HFE)
- Inverse Filter
- Wiener Filter (Minimum Mean Square Error Filter)

Matching image and filters

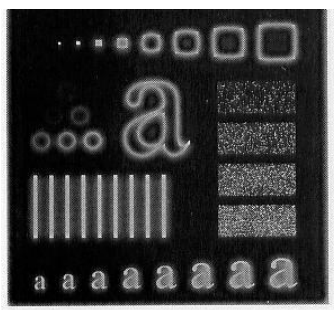
Images



(a)



(b)



(c)

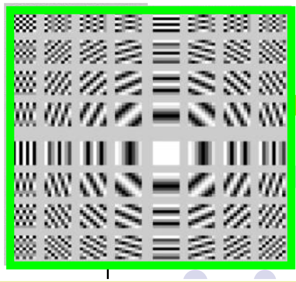
Filters

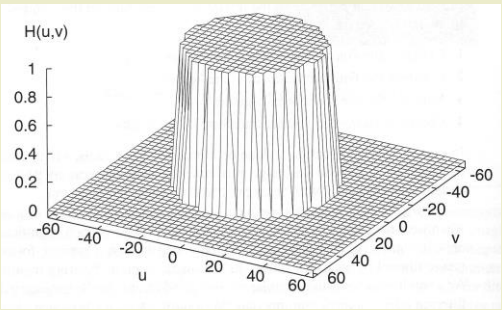
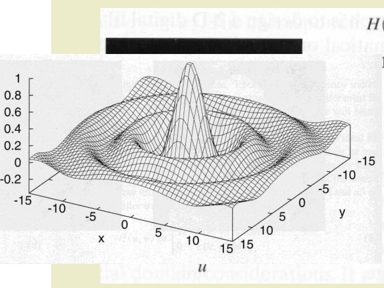
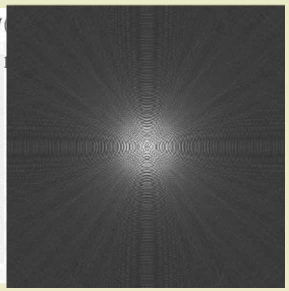
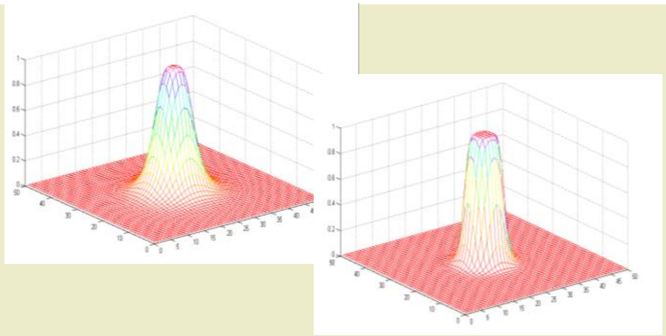
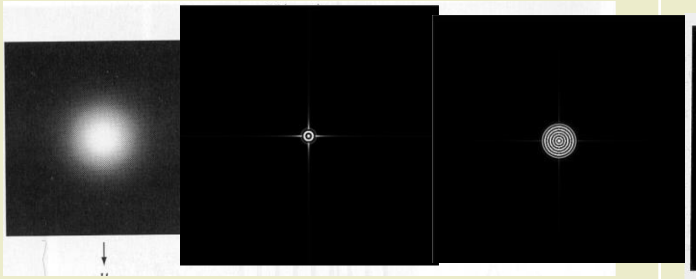
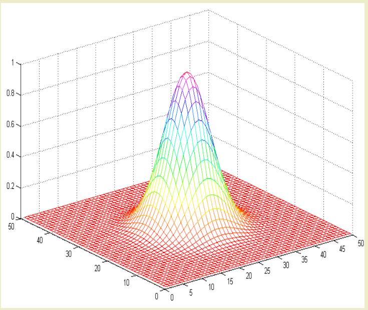
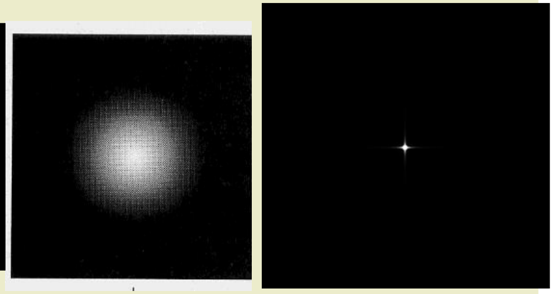
- Low Pass Filter
  - ผลลัพธ์น่าจะเป็นรูปใด
    (a), (b), (c)
- High Pass Filter
  - ผลลัพธ์น่าจะเป็นรูปใด
    (a), (b), (c)

Regular Filter

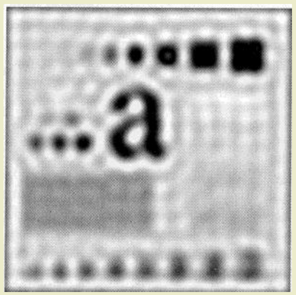
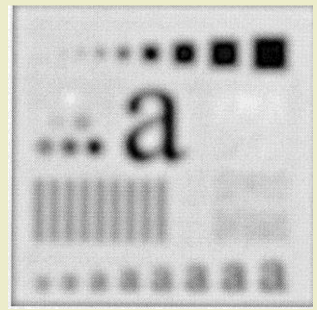
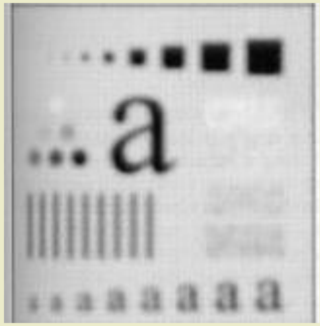
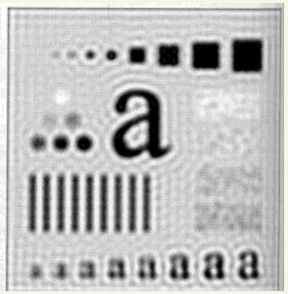
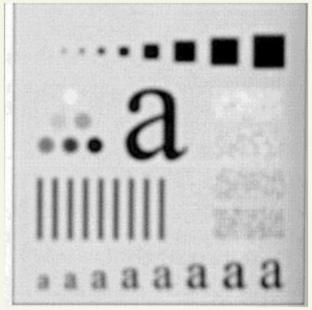
Low Pass Filter (LPF)

Low Pass Filtering (H(u,v))

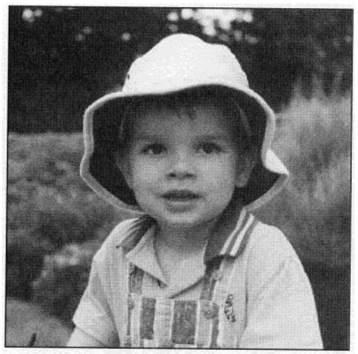


Ideal LPF	Butterworth LPF	Gaussian LPF
  	 	 
Frequency **Pixel**	**Frequency** **Pixel**	**Frequency** **Pixel**
$H(u,v)=\begin{cases}1 & r(u,v)\le r_0\\ 0 & r(u,v)>r_0\end{cases}$	$H(u,v)=\dfrac{1}{1+[r(u,v)/r_0]^{2n}}$	$H(u,v)=e^{-r^2(u,v)/2D_0^2}$
$r(u,v)=\sqrt{(u-M/2)^2+(v-N/2)^2}$	$r(u,v)=\sqrt{(u-M/2)^2+(v-N/2)^2}$	$r(u,v)=\sqrt{(u-M/2)^2+(v-N/2)^2}$ $D_0=r_0$

Low Pass Filtering

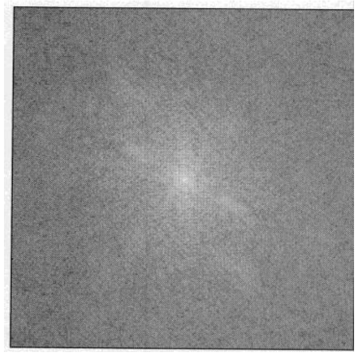
Ideal LPF	Butterworth LPF (N=2)	Gaussian LPF
 $r_0 = 15$	 $r_0 = 15$	 $r_0 = 15$
 $r_0 = 30$	 $r_0 = 30$	 $r_0 = 30$

ILPF ripple effects

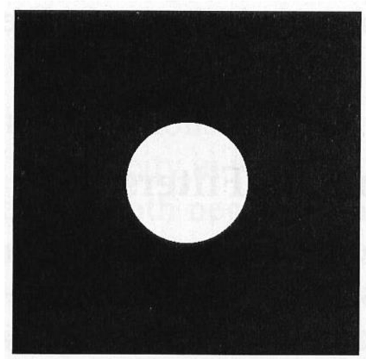


Original image [f(x,y)]

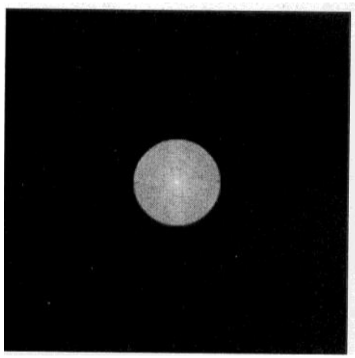
DFT



F(u,v)



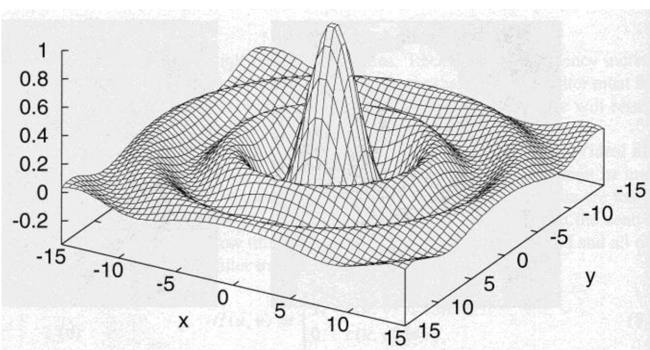
H(u,v)



G(u,v) = F(u,v) H(u,v)

$f * h$

Inverse DFT

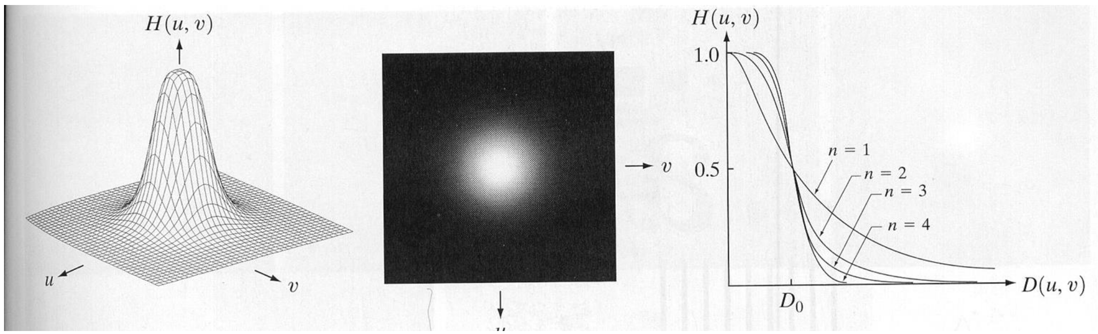


h(x,y)

$$f * h = inverse\ DFT[FH]$$

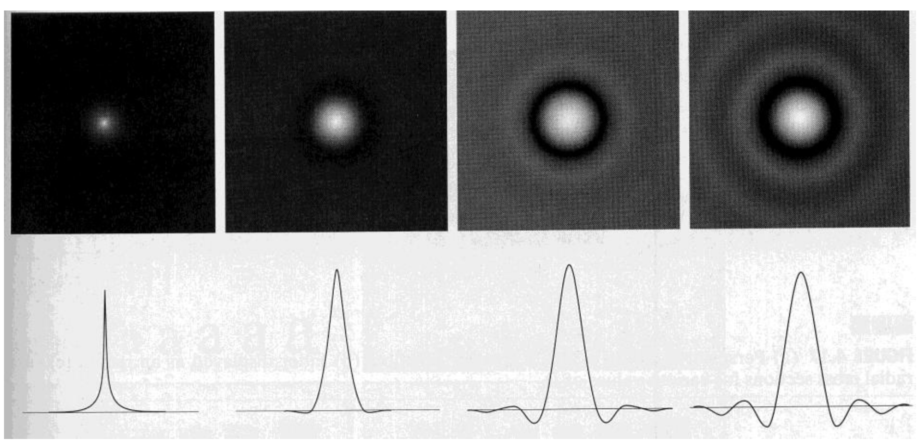
g(x,y)

Butterworth Low Pass Filter (BLPF)



$$H(u,v) = \frac{1}{1+[r(u,v)/r_0]^{2n}}$$

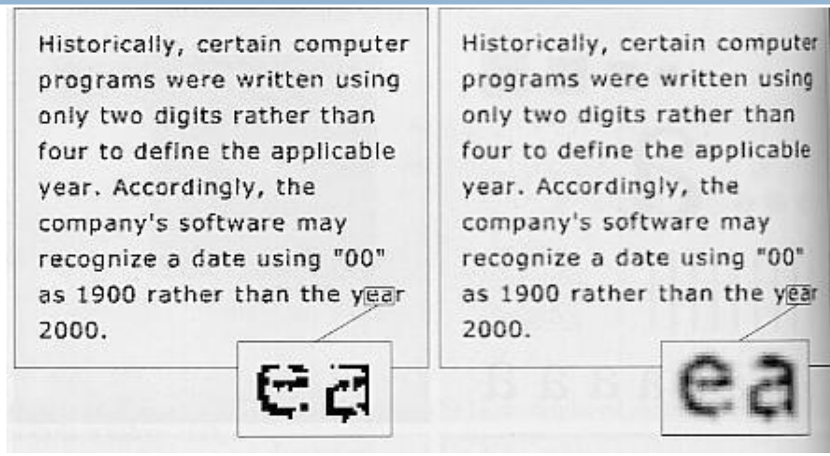
$$r(u,v) = \sqrt{(u-M/2)^2+(v-N/2)^2}$$



h(x,y)

n = 1 n = 2 n = 3 n = 4

Applications of Low Pass Filter



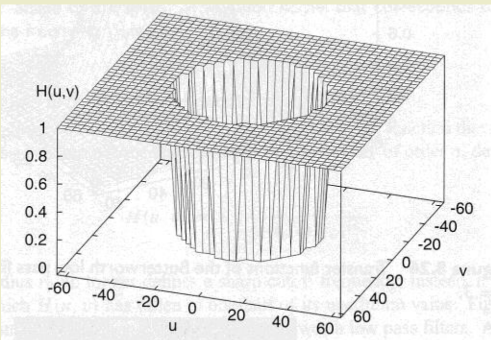
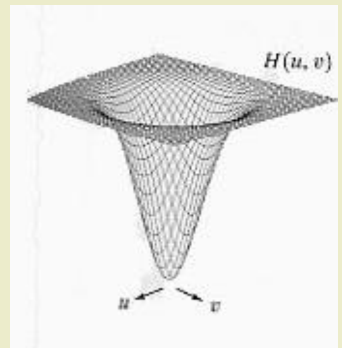
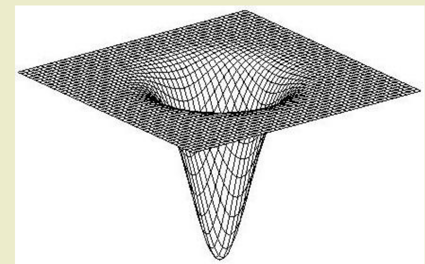
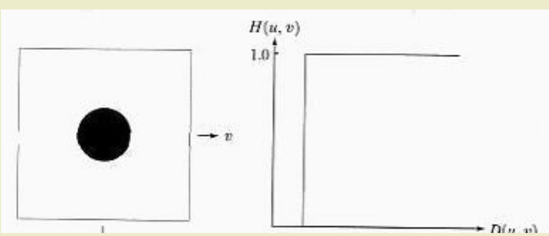
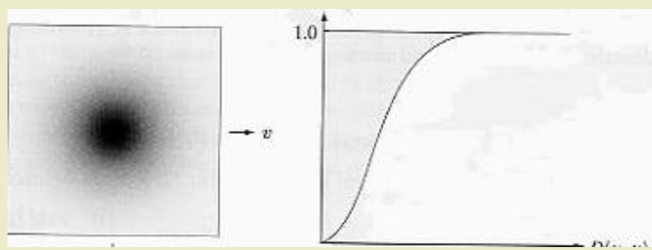
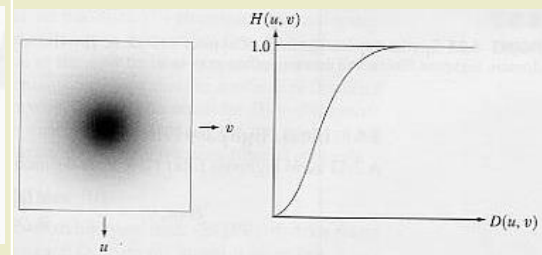
Character recognition

Picture Studio Decoration

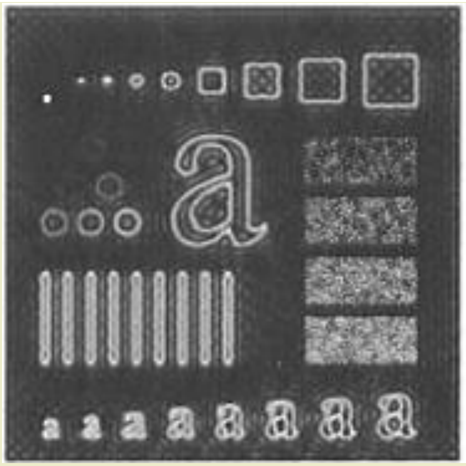
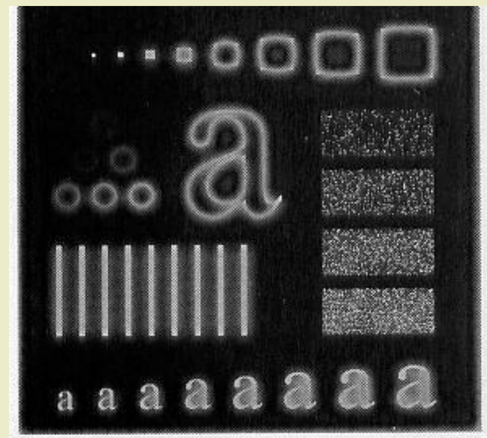
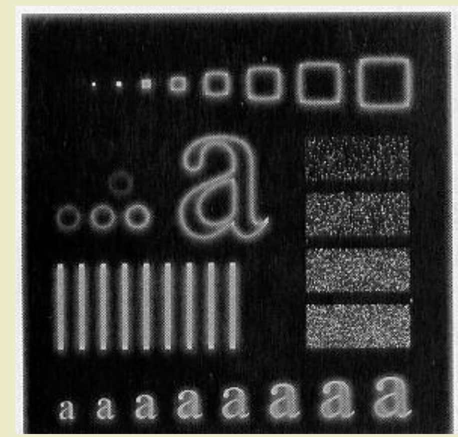
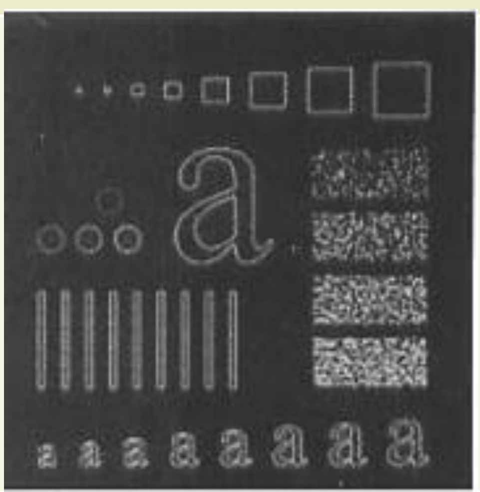
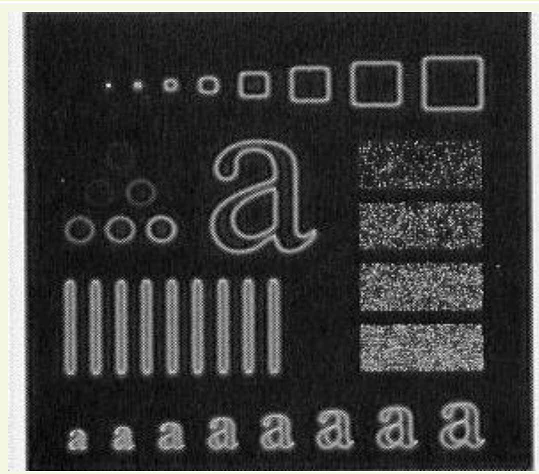
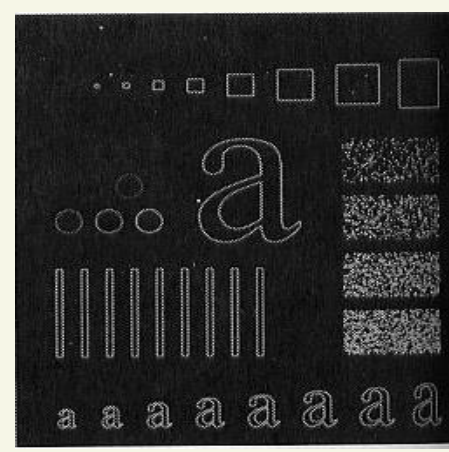
Regular Filter

High Pass Filter (HPF)

High Pass Filtering

Ideal HPF	Butterworth HPF	Gaussian HPF
		
		
$H(u,v)=\begin{cases}0 & r(u,v)\le r_0\\ 1 & r(u,v)>r_0\end{cases}$ $r(u,v)=\sqrt{(u-M/2)^2+(v-N/2)^2}$	$H(u,v)=\dfrac{1}{1+[r_0/r(u,v)]^{2n}}$ $r(u,v)=\sqrt{(u-M/2)^2+(v-N/2)^2}$	$H(u,v)=1-e^{-r^2(u,v)/2D_0^2}$ $r(u,v)=\sqrt{(u-M/2)^2+(v-N/2)^2}$ $D_0=r_0$

High Pass Filtering

	Ideal HPF	Butterworth HPF	Gaussian HPF
$r_0 = 15$			
$r_0 = 30$			

High Frequency Emphasis Filter

Original image

HPF

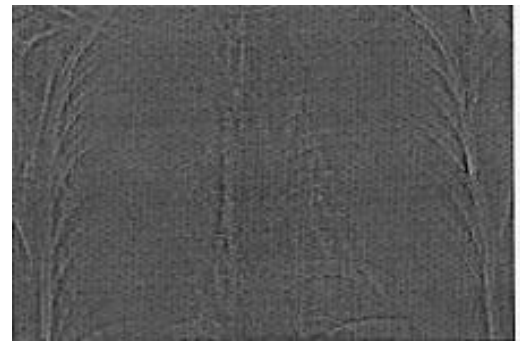
High Freq. filtered
H*hp*(u,v)
image

$$H_{hfe}(u,v) = a + bH_{hp}(u,v)$$

High Frequency Emphasis Filter

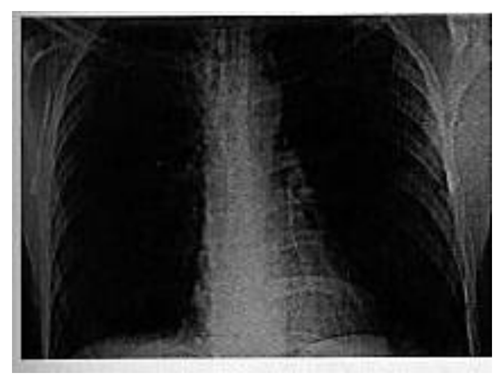
f(x,y)

$H_{hp}(u,v)$

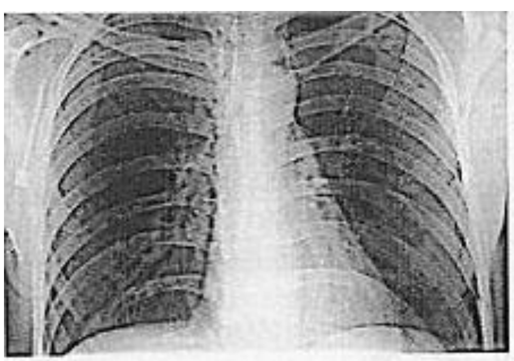


BHPF

$$H_{hfe}(u,v) = a + bH_{hp}(u,v)$$



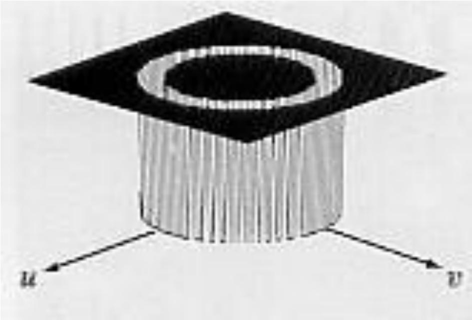
Histogram equalization



Regular Filter

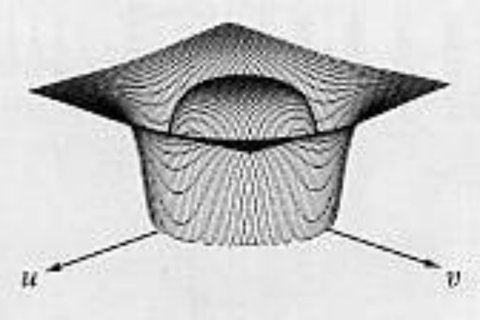
Band Pass Filter (BPF)
Vs
Band Reject Filter (BRF)

Band Reject Filter (BRF)



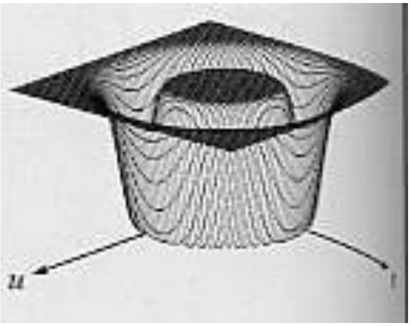
Ideal BRF

$$H(u,v)=\begin{cases}1 & ; \quad r(u,v) < r_0-\dfrac{BW}{2}\\ 0 & ; \quad r_0-\dfrac{BW}{2}\le r(u,v)\le r_0+\dfrac{BW}{2}\\ 1 & ; \quad r(u,v) > r_0+\dfrac{BW}{2}\end{cases}$$



Butterworth BRF

$$H(u,v)=\frac{1}{1+\left[\dfrac{r(u,v).BW}{r^2(u,v)-r_0^2}\right]}$$

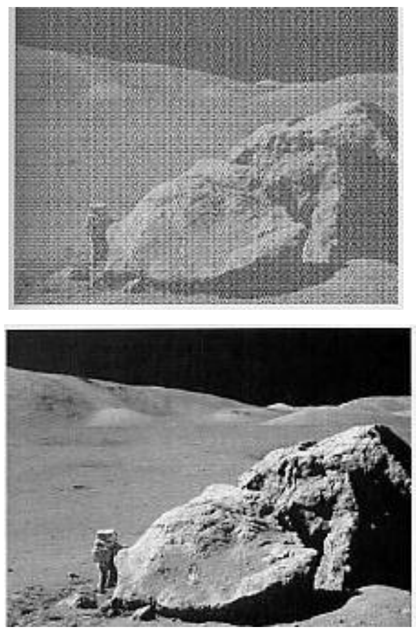


Gaussian BRF

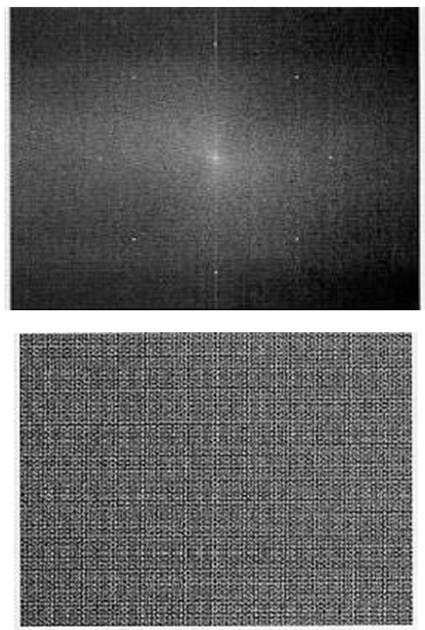
$$H(u,v)=1-e^{-\frac{1}{2}\left[\frac{r^2(u,v)-r_0^2}{r(u,v).BW}\right]}$$

Matching image and filters

Images



(a)



(b)

Filters

- Band Pass Filter
  - ผลลัพธ์น่าจะเป็นรูปใด

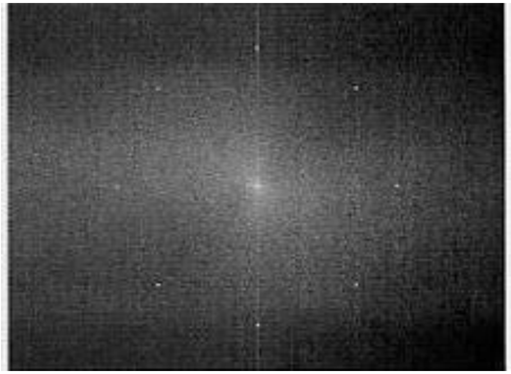
    (a), (b)
- Band Reject Pass Filter
  - ผลลัพธ์น่าจะเป็นรูปใด

    (a), (b)

BRF results

f(x,y)

DFT

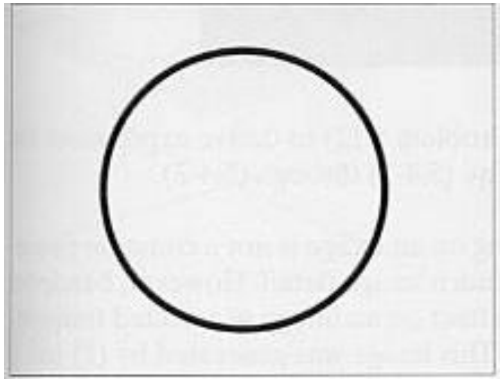


F(u,v)

g(x,y)

Inverse DFT

F(u,v)H(u,v)

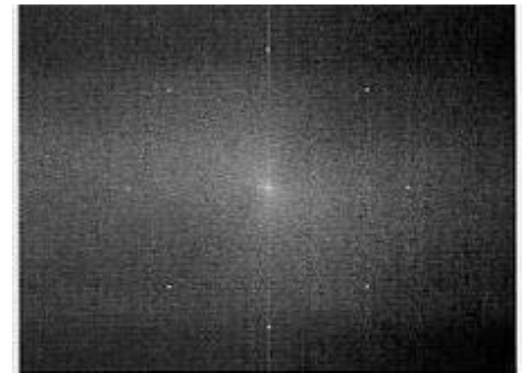


H(u,v)

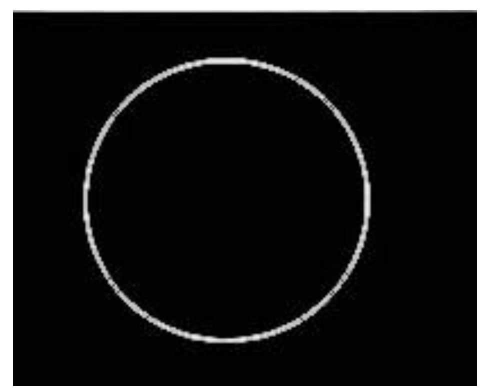
Band Pass Filter (BPF)

f(x,y)

DFT



F(u,v)

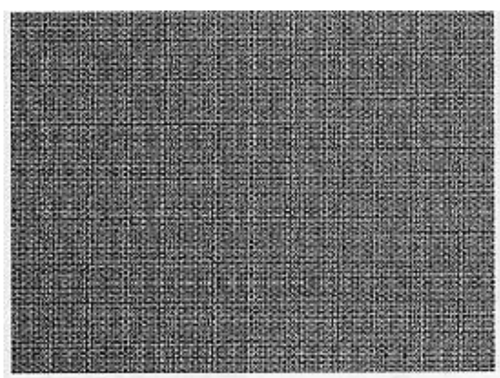


H(u,v)

$$H_{BP}(u,v) = 1 - H_{BR}(u,v)$$

Inverse DFT

F(u,v)H(u,v)



g(x,y)