Convolutional Neural Networks

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(Slides are adapted from cs231n @Stanford University)

Convolutional Neural Networks

- Why multilayer neural networks do not satisfy our needs?
  - Fully-connected architecture is too complex for computations.
  - There are a lot of weights to update. Very long training time.
  - Having too many weights tends to encounter a poor generalization.

- The convolutional neural networks can vastly reduce the amount of parameters in the network.

History

A bit of history...

The **Mark I Perceptron** machine was the first implementation of the perceptron algorithm.

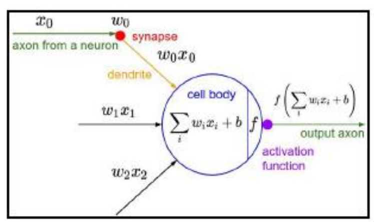
The machine was connected to a camera that used 20×20 cadmium sulfide photocells to produce a 400-pixel image.

recognized
letters of the alphabet

$$f(x) = \begin{cases} 1 & \text{if } w \cdot x + b > 0 \\ 0 & \text{otherwise} \end{cases}$$

update rule:

$$w_i(t+1) = w_i(t) + \alpha(d_j - y_j(t))x_{j,i},$$



Frank Rosenblatt, ~1957: Perceptron

Multilayer neural network

$$\frac{\partial E_p}{\partial w_{ji}} = \frac{\partial E_p}{\partial o_{pj}} \frac{\partial o_{pj}}{\partial w_{ji}}$$

recognizable math

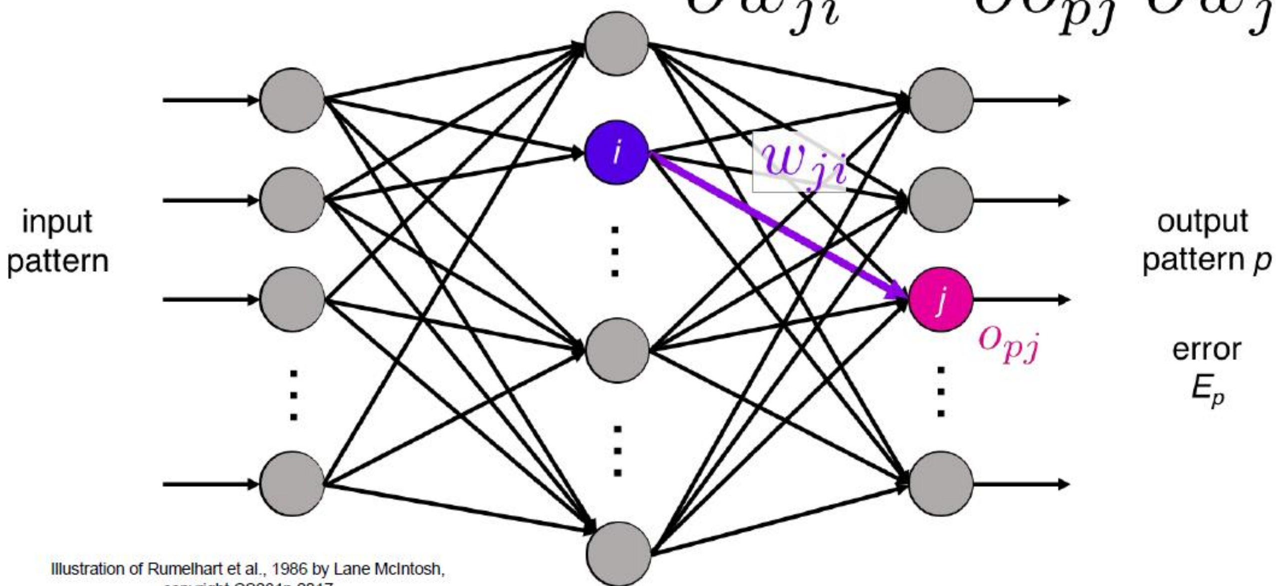
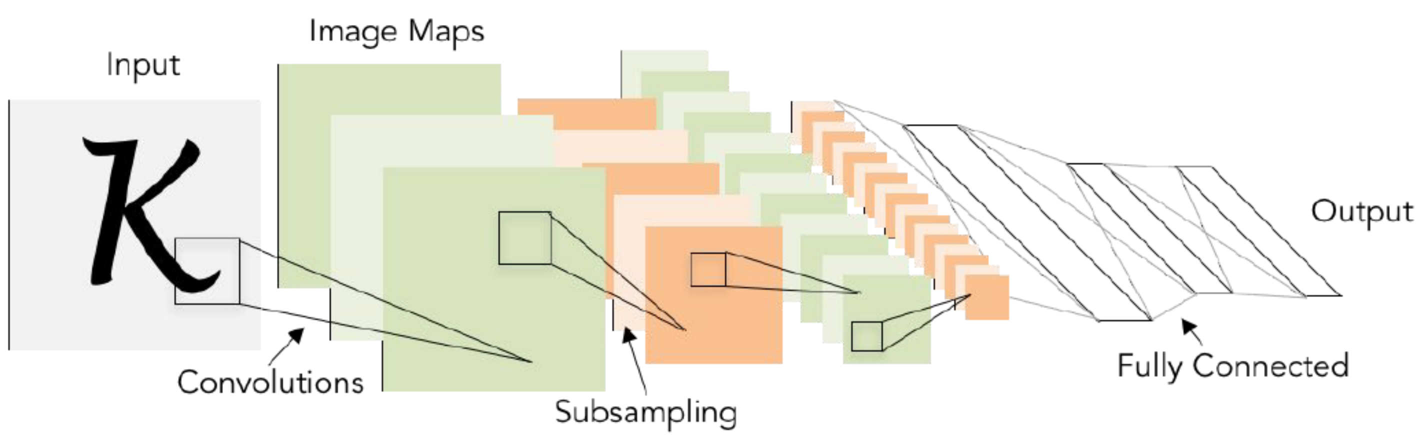


Illustration of Rumelhart et al., 1986 by Lane McIntosh, copyright CS231n 2017

Rumelhart et al., 1986: First time back-propagation became popular

Gradient-based learning applied to document recognition
[LeCun, Bottou, Bengio, Haffner 1998]

LeNet



LeNet-5

First strong results

Acoustic Modeling using Deep Belief Networks
Abdel-rahman Mohamed, George Dahl, Geoffrey Hinton, 2010
Context-Dependent Pre-trained Deep Neural Networks for Large Vocabulary Speech Recognition
George Dahl, Dong Yu, Li Deng, Alex Acero, 2012

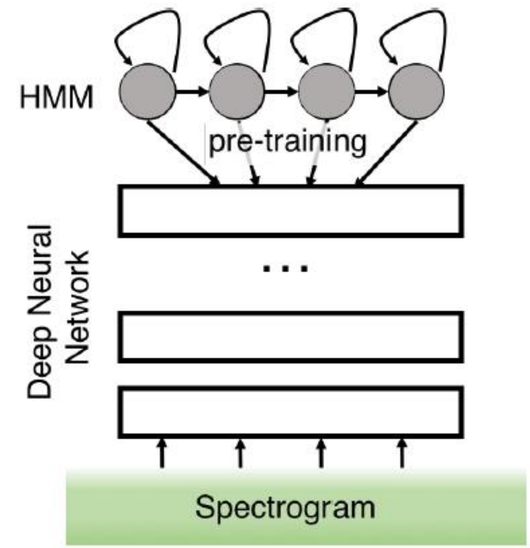
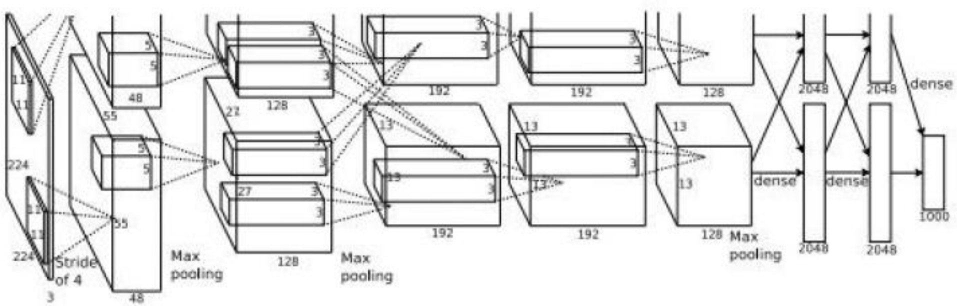
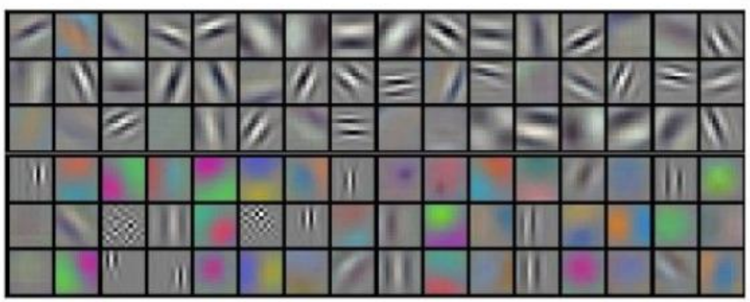


Illustration of Dahl et al. 2012 by Lane McIntosh, copyright CS231n 2017

Imagenet classification with deep convolutional neural networks
Alex Krizhevsky, Ilya Sutskever, Geoffrey E Hinton, 2012





AlexNet

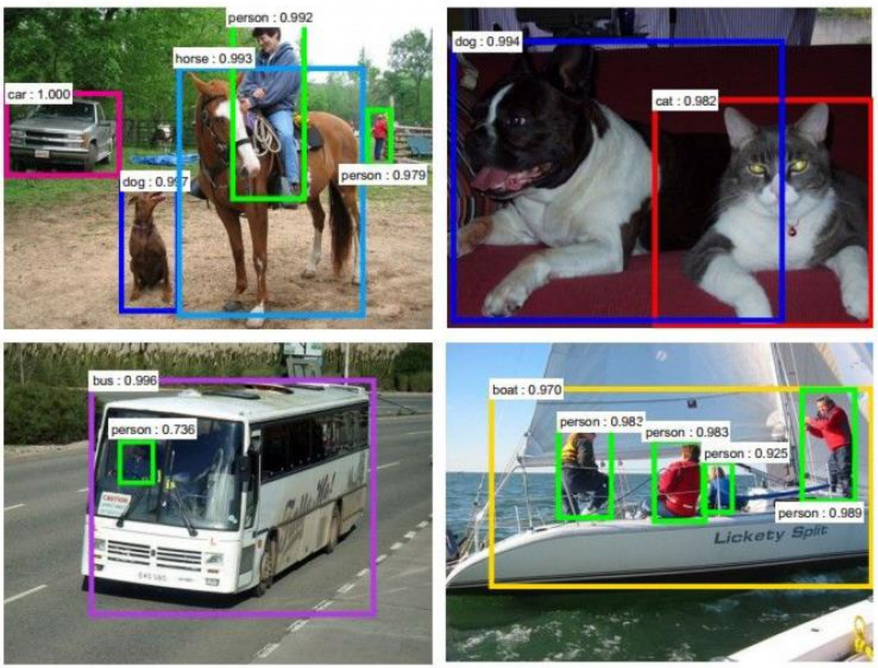
Fast-forward to today: ConvNets are everywhere

Classification

Retrieval

Fast-forward to today: ConvNets are everywhere

Detection



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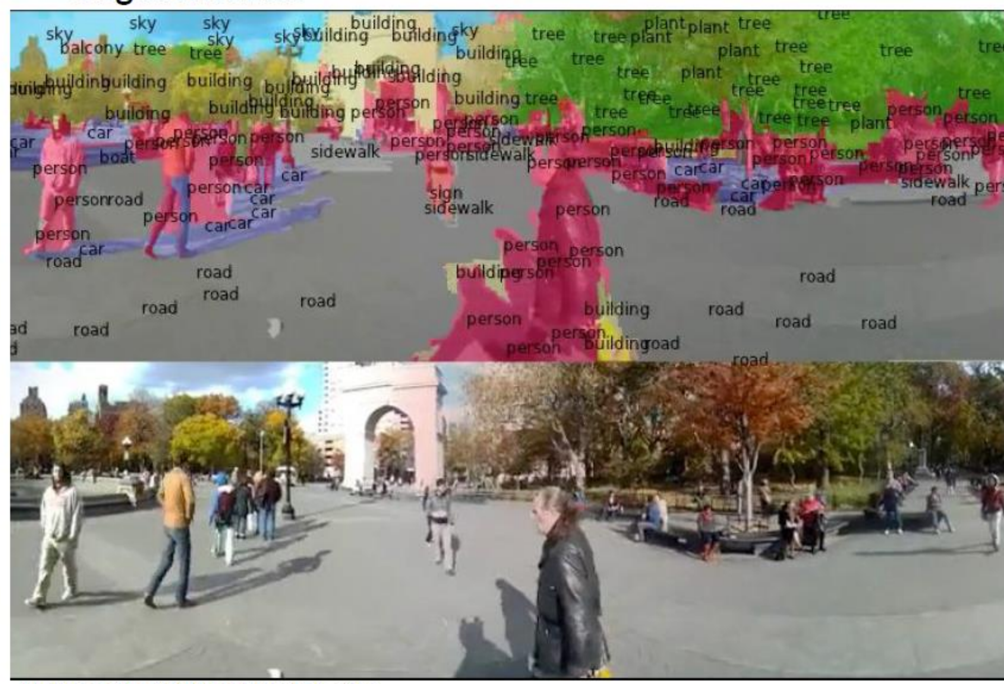
Segmentation



Figures copyright Clement Farabet, 2012. Reproduced with permission.

[Farabet et al., 2012]

Fast-forward to today: ConvNets are everywhere

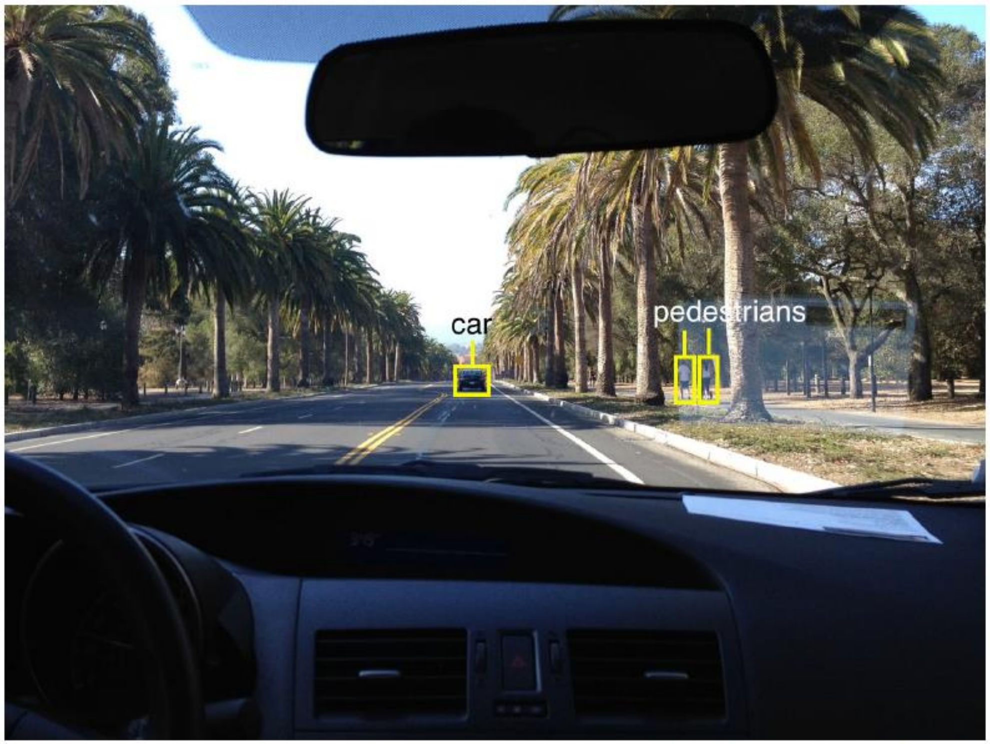


Photo by Lane McIntosh. Copyright CS231n 2017.

self-driving cars

NVIDIA Tesla line
(these are the GPUs on rye01.stanford.edu)

Note that for embedded systems a typical setup would involve NVIDIA Tegras, with integrated GPU and ARM-based CPU cores.

Image Captioning

[Vinyals et al., 2015]
[Karpathy and Fei-Fei, 2015]

No errors	Minor errors	Somewhat related

No errors

A white teddy bear sitting in the grass

Minor errors

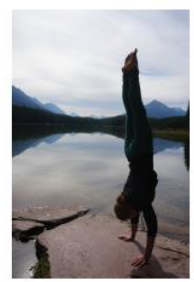
A man in a baseball uniform throwing a ball

Somewhat related

A woman is holding a cat in her hand

A man riding a wave on top of a surfboard

A cat sitting on a suitcase on the floor



A woman standing on a beach holding a surfboard

Captions generated by Justin Johnson using Neuraltalk2

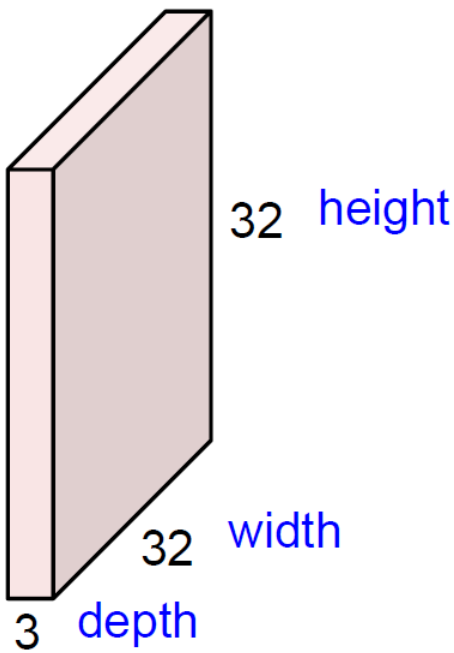
Layers in Convolutional Neural Networks

- CNN : Convolutional Neural Network

- There are usually 4 types of layers in CNN.
  1. Convolution layer
  2. Pooling layer
  3. ReLU (Rectified Linear Unit) layer
  4. Fully connected layer

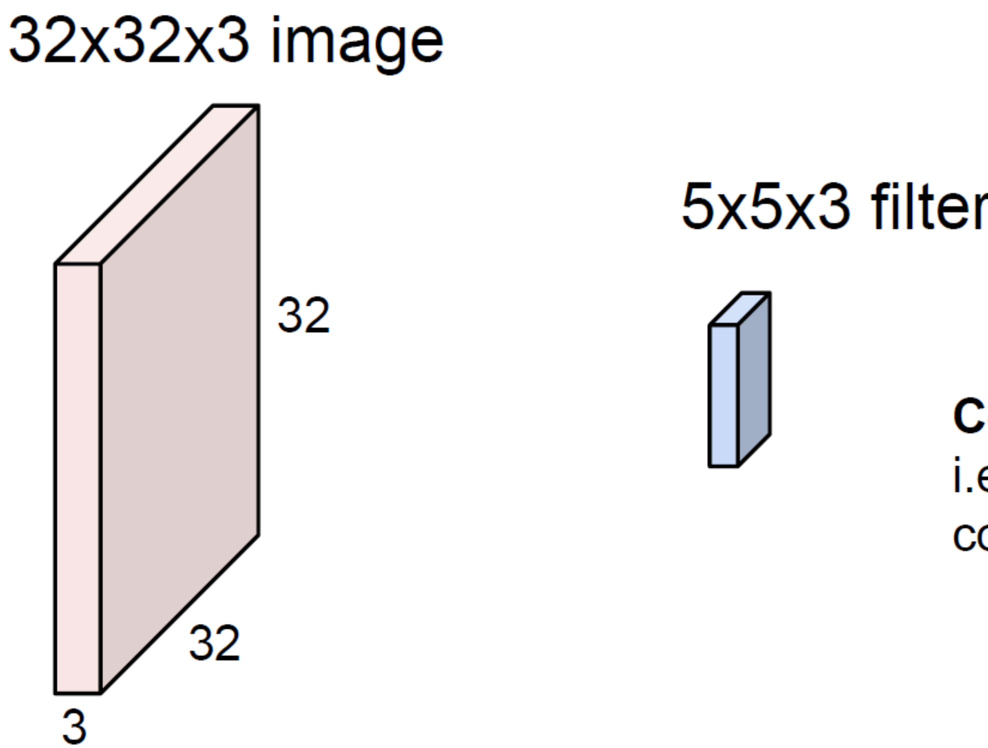
Convolution Layer

32x32x3 image -> preserve spatial structure



Input

Convolution Layer

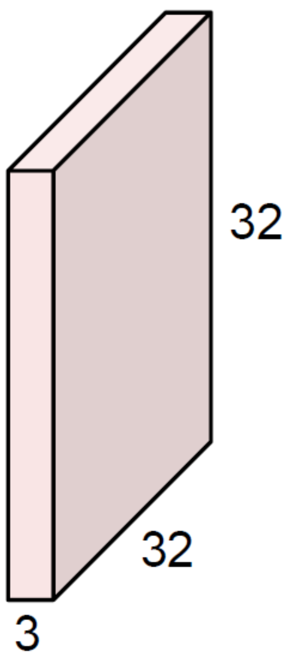


Convolve the filter with the image
i.e. “slide over the image spatially, computing dot products”

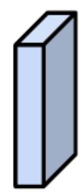
Convolution Layer

Filters always extend the full depth of the input volume

32x32x3 image

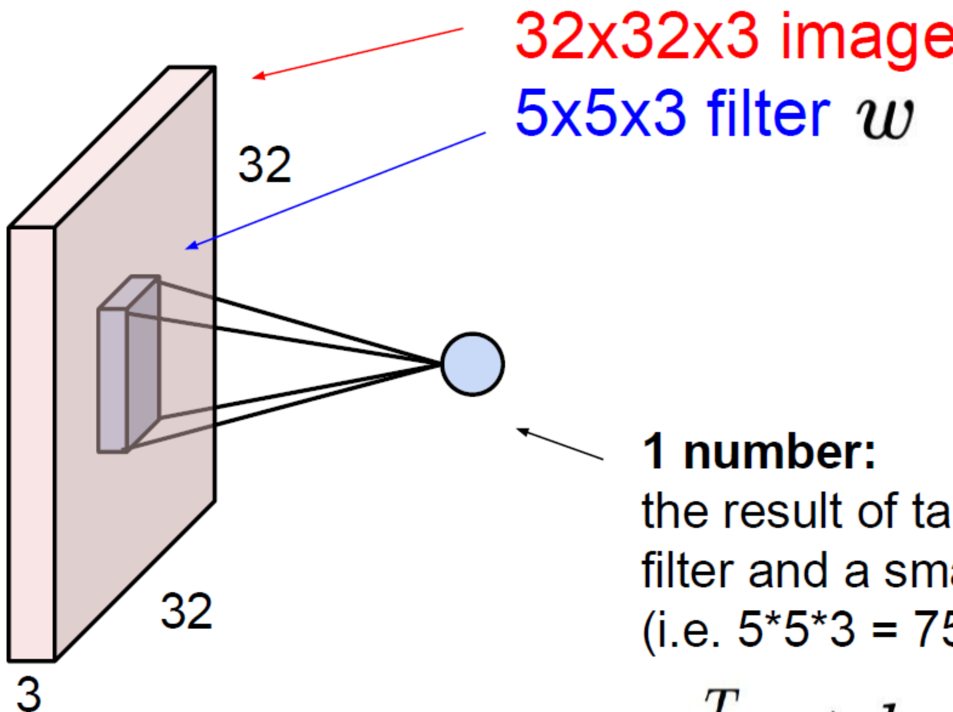


5x5x3 filter



Convolve the filter with the image i.e. "slide over the image spatially, computing dot products"

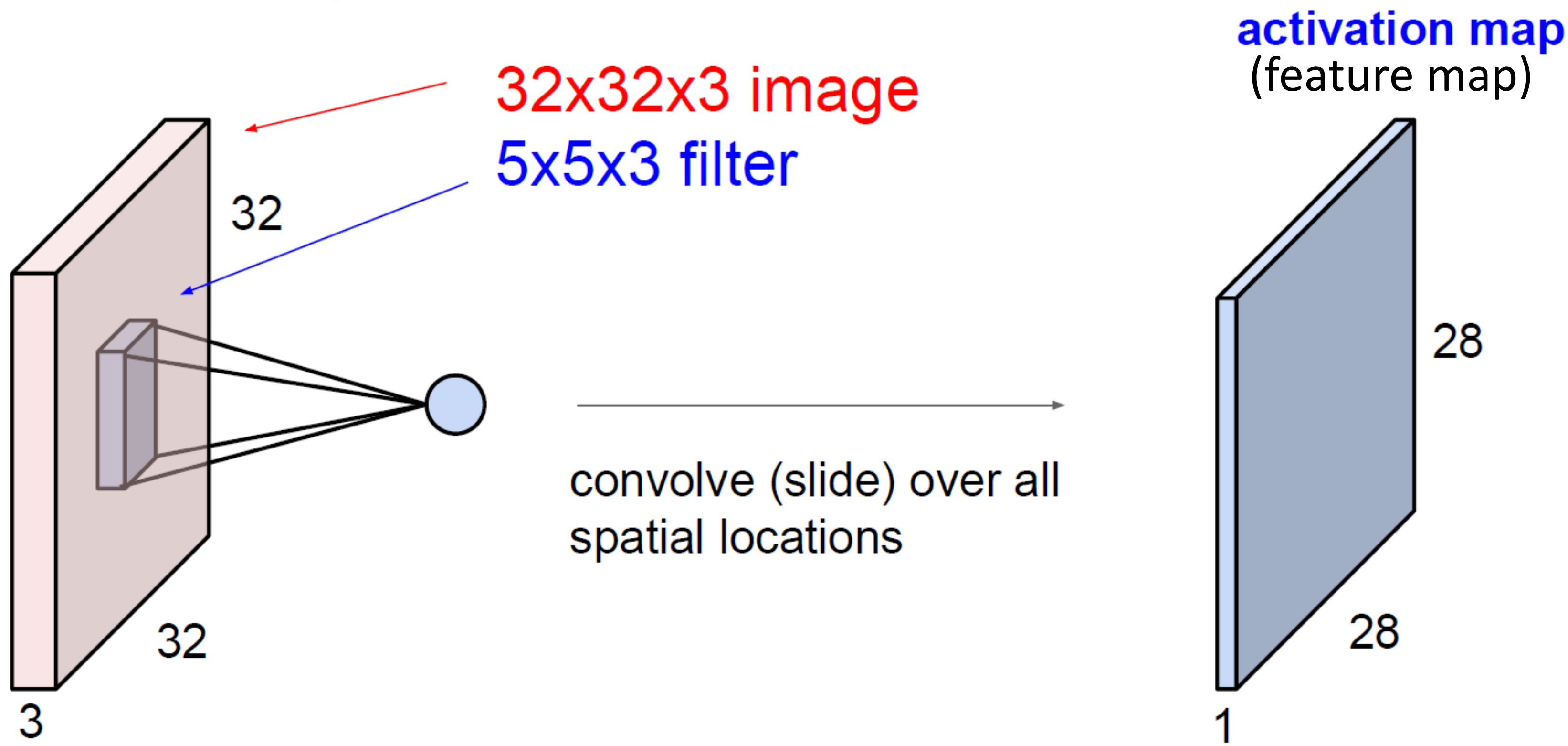
Convolution Layer



1 number:
the result of taking a dot product between the filter and a small 5x5x3 chunk of the image
(i.e. 5*5*3 = 75-dimensional dot product + bias)

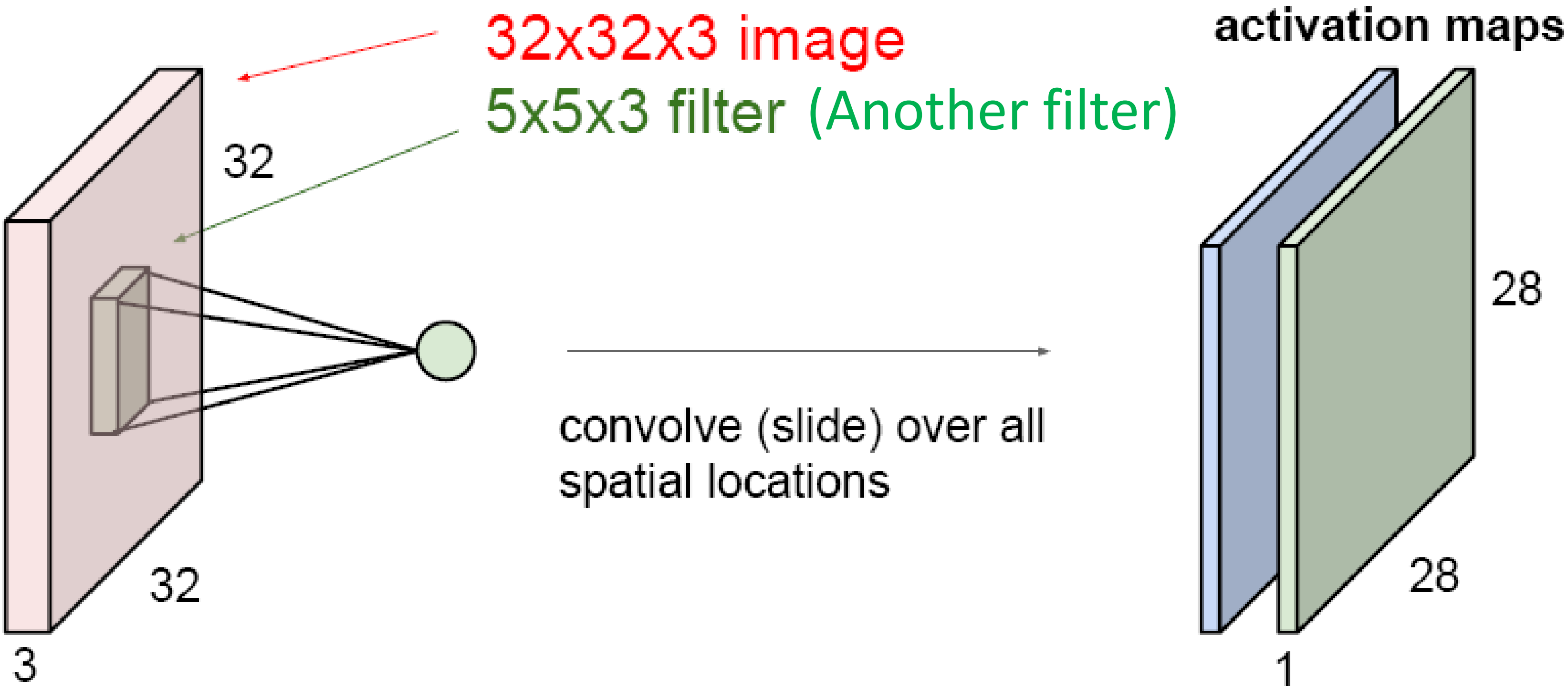
$$w^T x + b$$

Convolution Layer

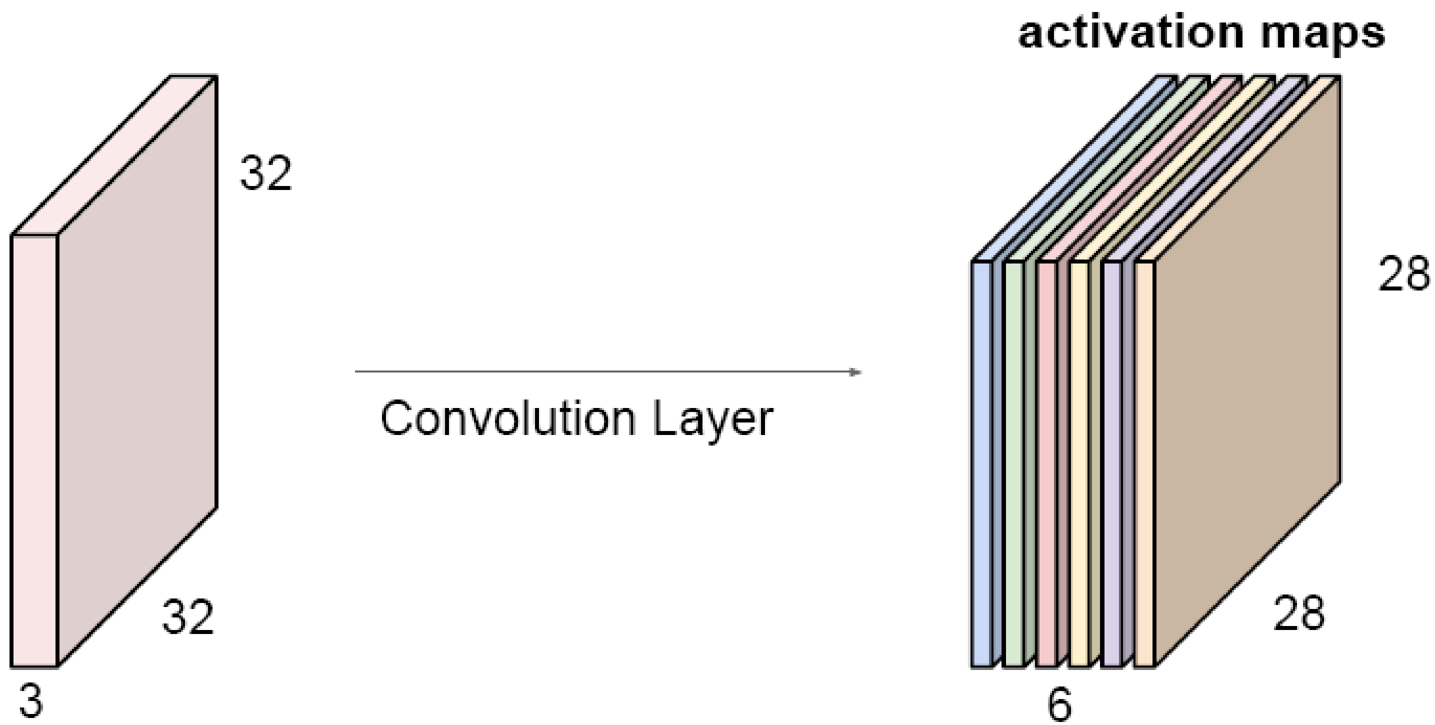


Convolution Layer

consider a second, green filter

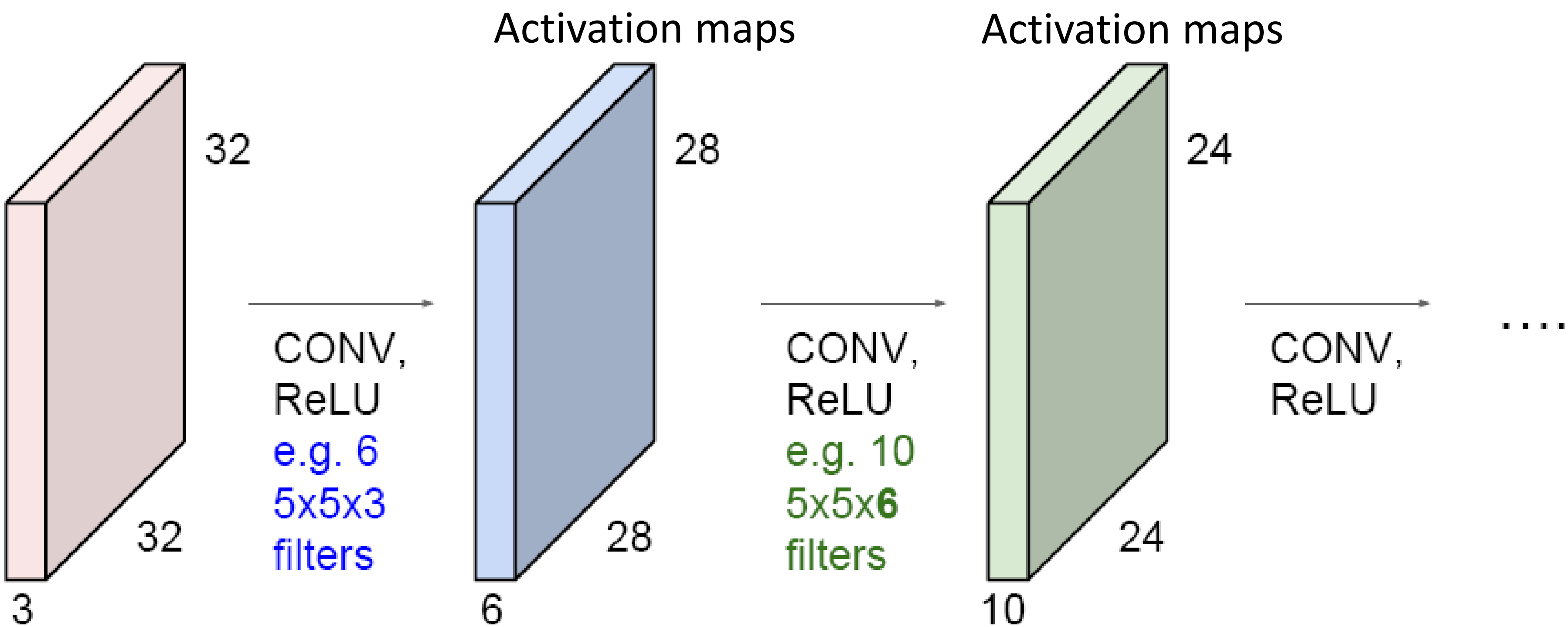


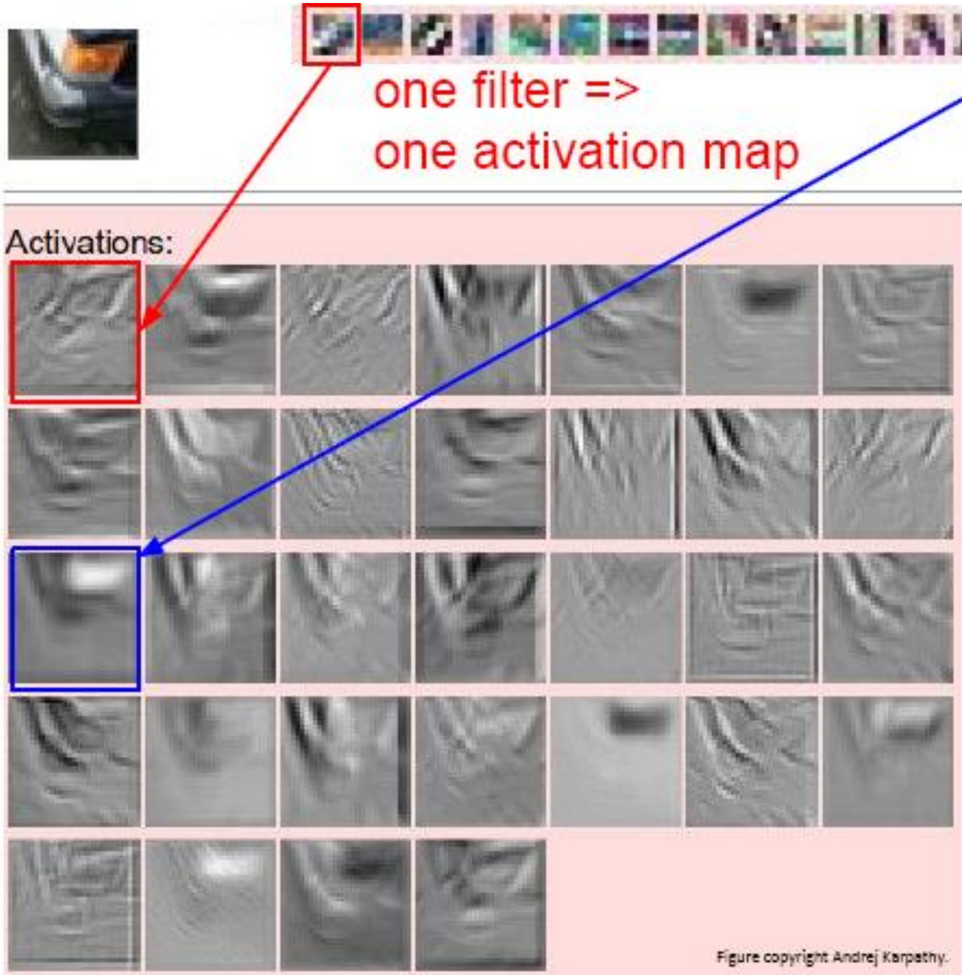
For example, if we had 6 5x5 filters, we'll get 6 separate activation maps:



We stack these up to get a "new image" of size 28x28x6!

Preview: ConvNet is a sequence of Convolution Layers



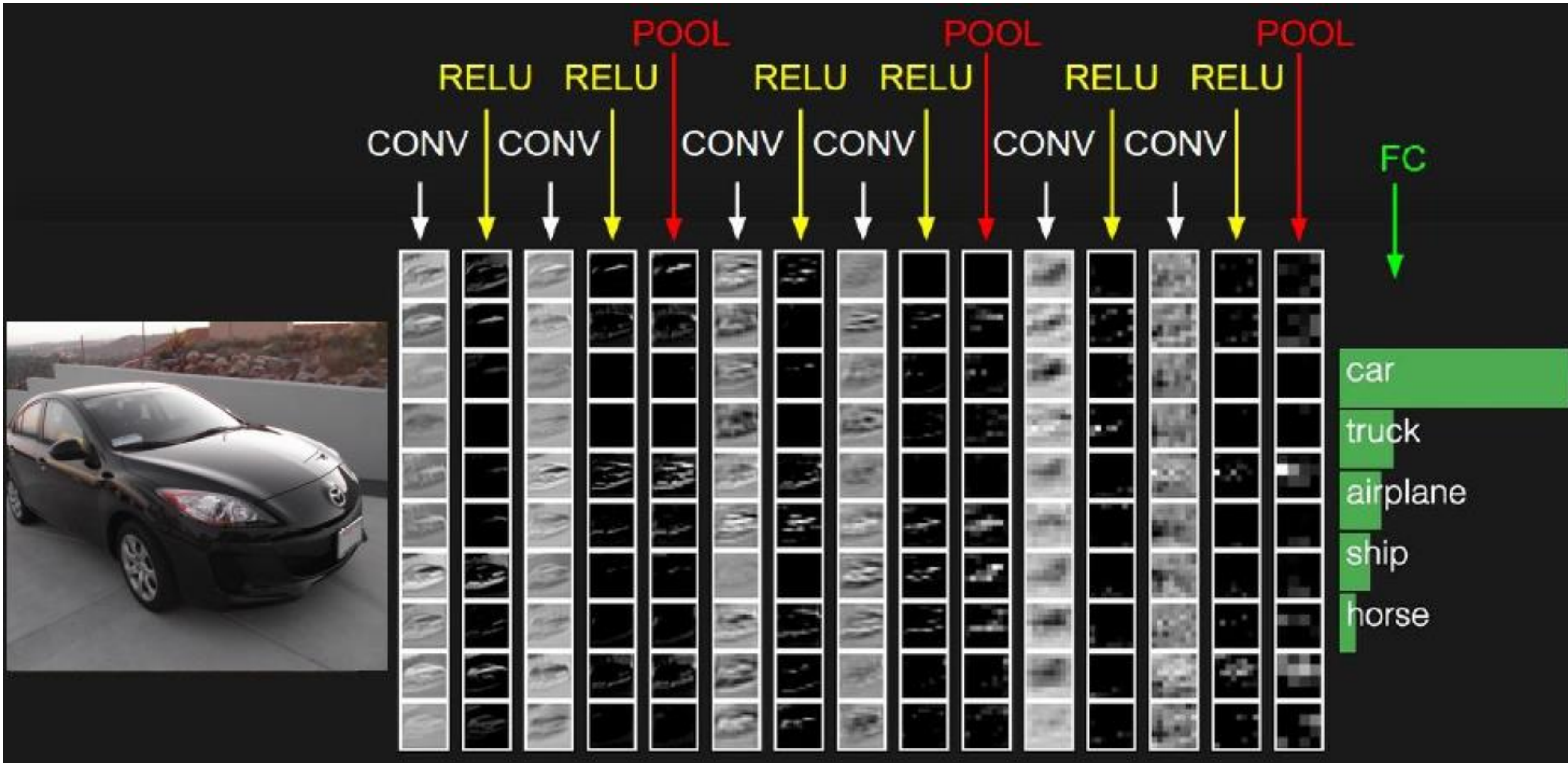


example 5x5 filters
(32 total)

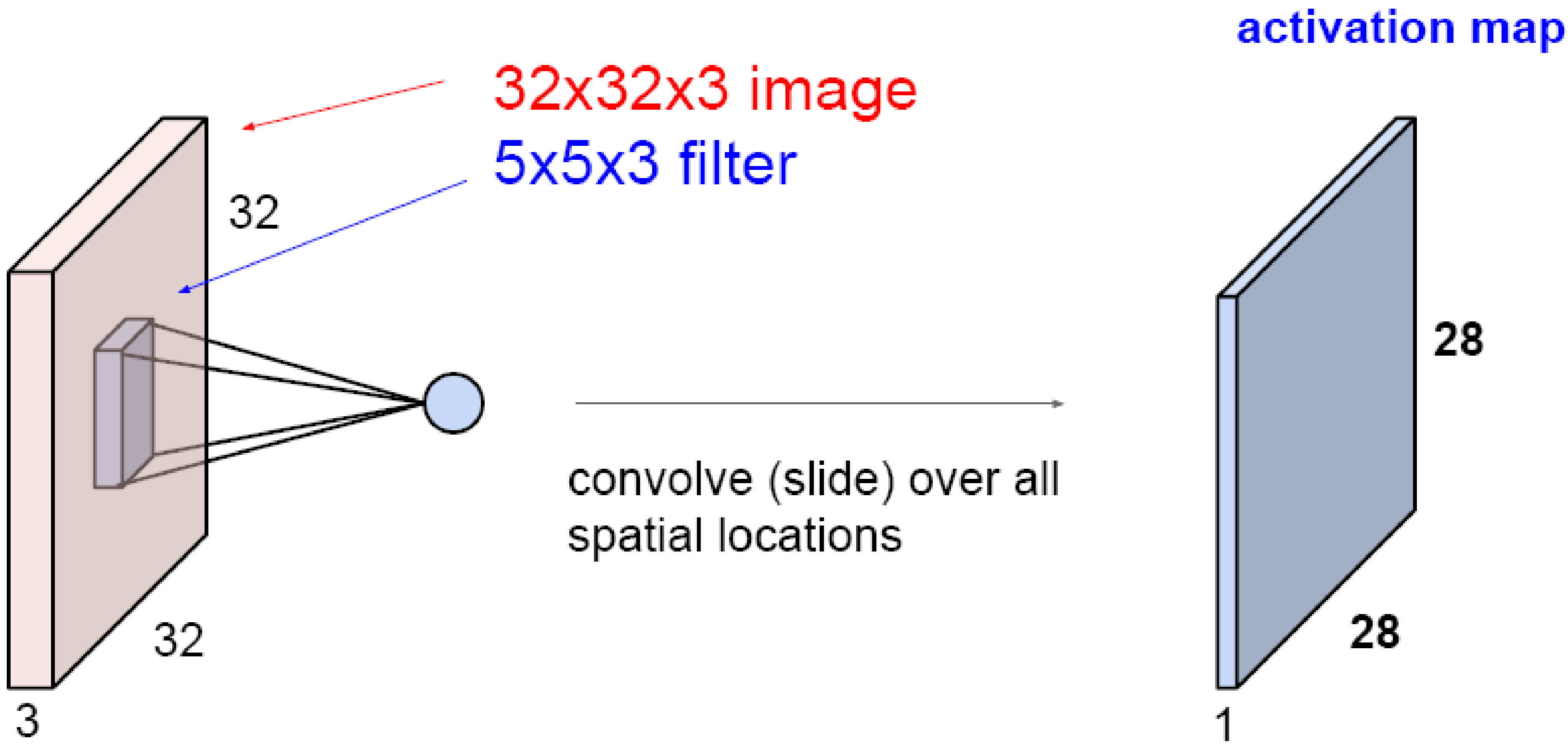
We call the layer convolutional because it is related to convolution of two signals:

$$f[x,y] * g[x,y] = \sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} f[n_1,n_2] \cdot g[x-n_1, y-n_2]$$

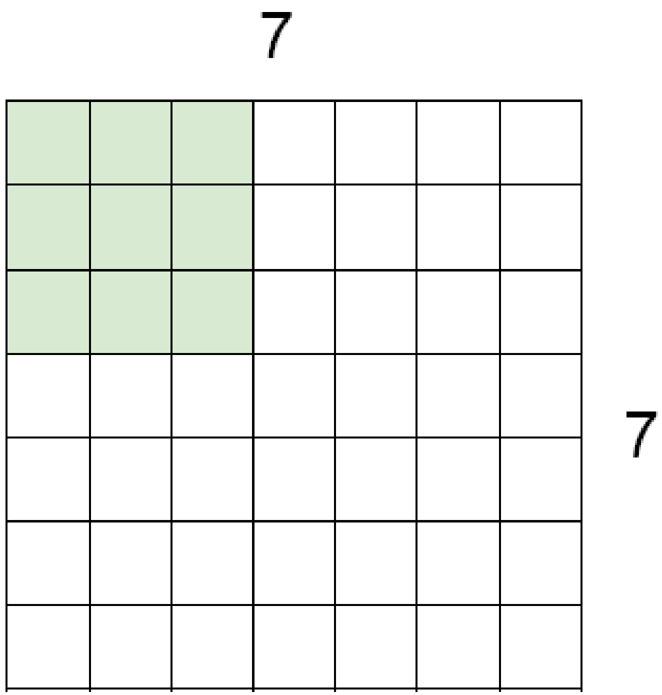
elementwise multiplication and sum of a filter and the signal (image)


POOL
POOL
POOL
RELU
RELU
RELU
RELU
RELU
RELU
CONV
CONV
CONV
CONV
CONV
CONV
FC
car
truck
airplane
ship
horse

A closer look at spatial dimensions:

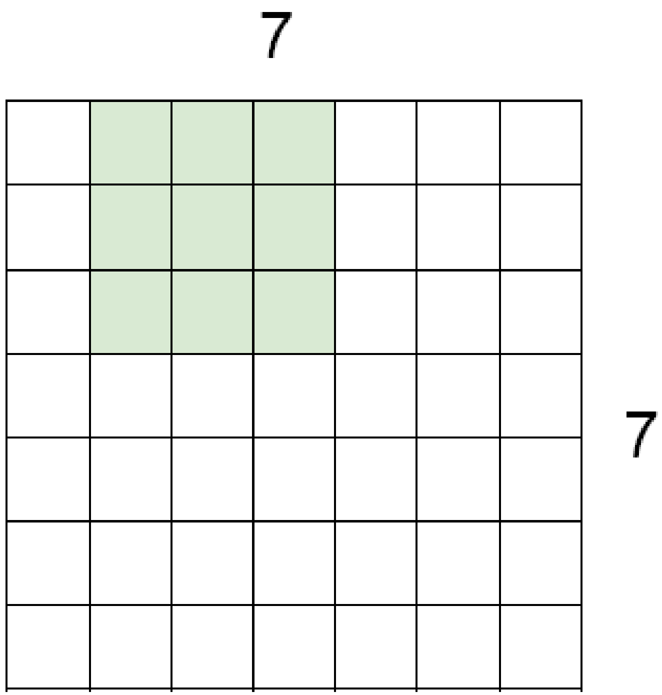


A closer look at spatial dimensions:



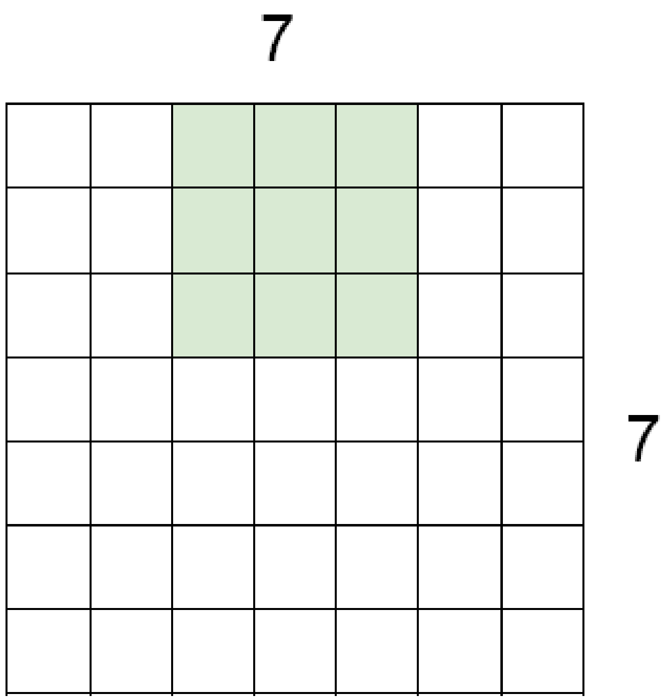
7x7 input (spatially)
assume 3x3 filter

A closer look at spatial dimensions:



7x7 input (spatially)
assume 3x3 filter

A closer look at spatial dimensions:



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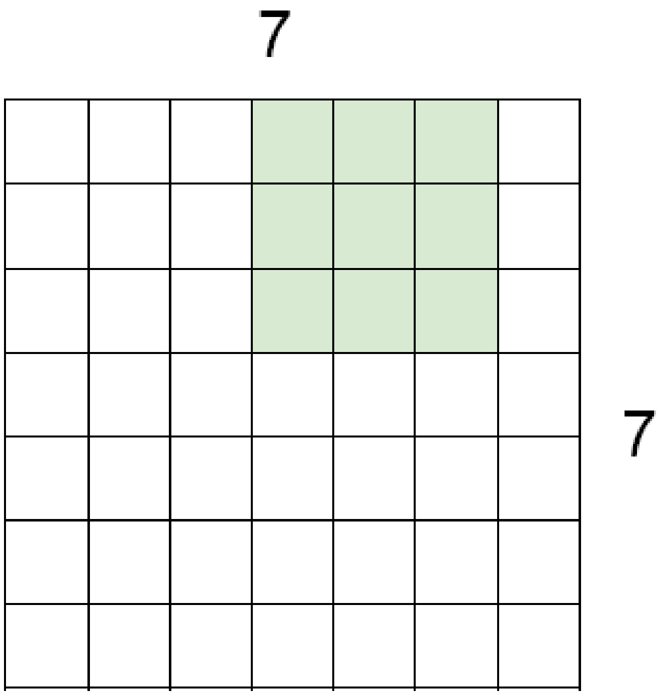
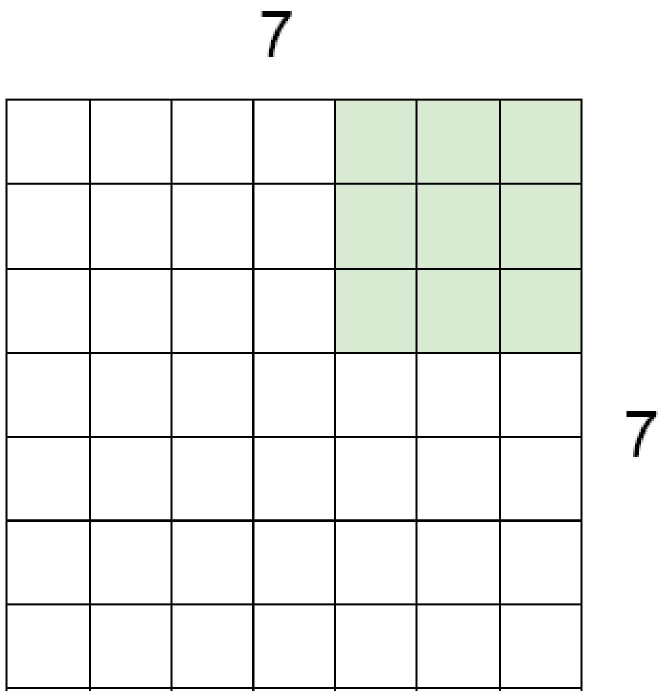
A closer look at spatial dimensions:



7x7 input (spatially)
assume 3x3 filter

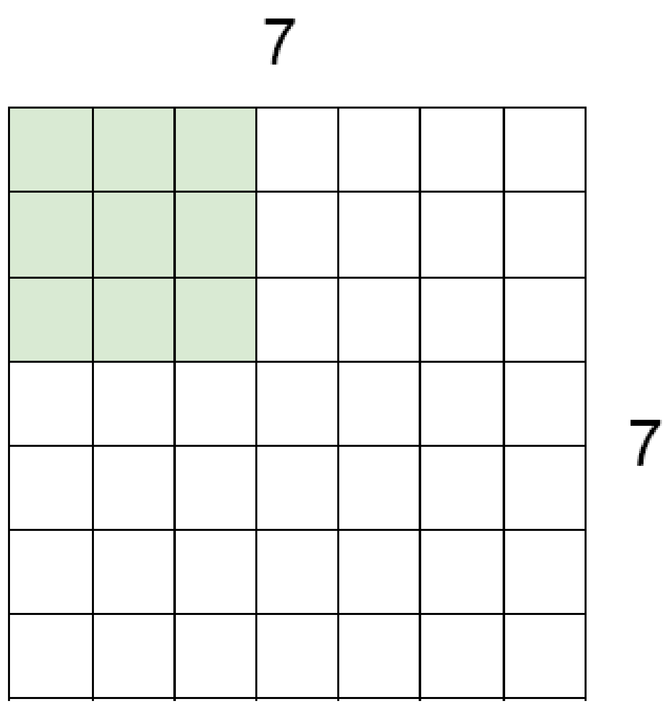
A closer look at spatial dimensions:



7x7 input (spatially)
assume 3x3 filter

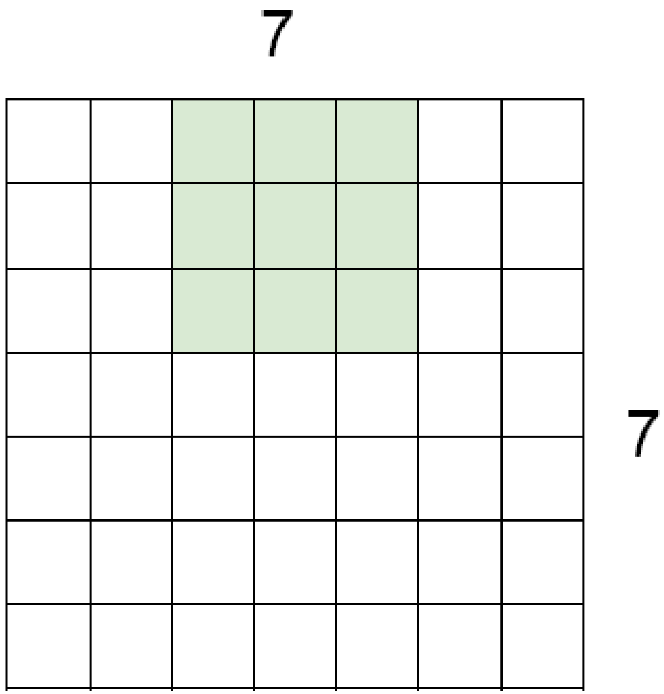
=> 5x5 output

A closer look at spatial dimensions:



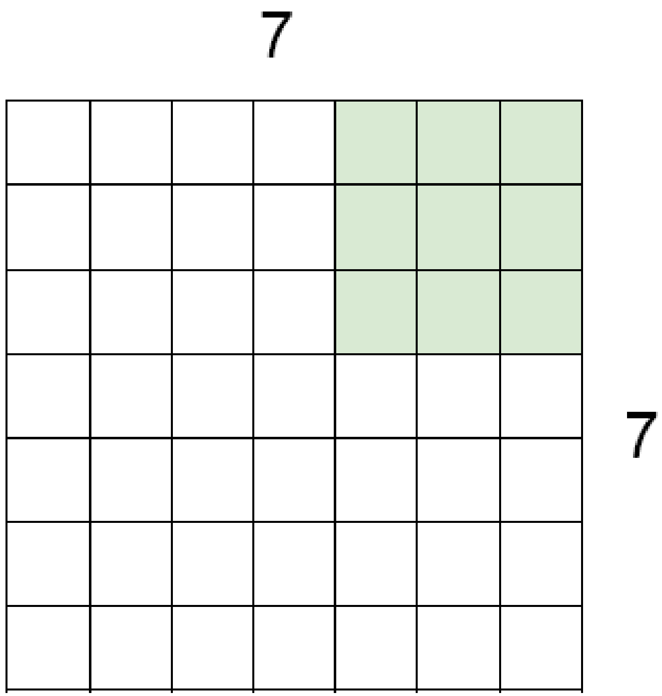
7x7 input (spatially)
assume 3x3 filter
applied **with stride 2**

A closer look at spatial dimensions:



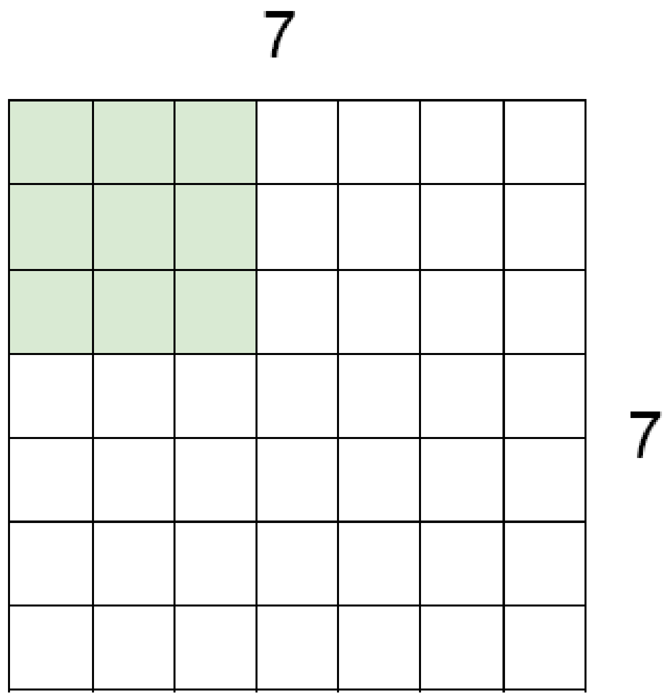
7x7 input (spatially)
assume 3x3 filter
applied **with stride 2**

A closer look at spatial dimensions:



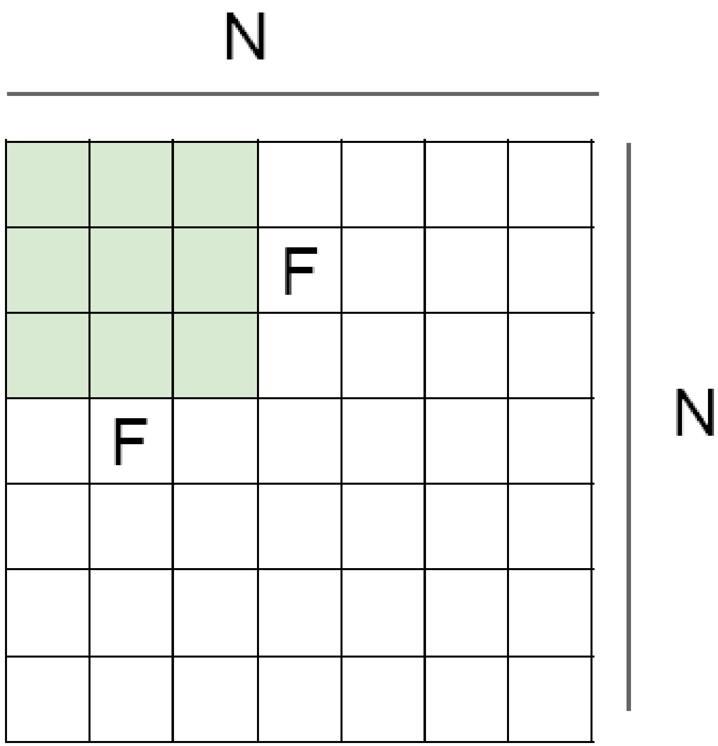
7x7 input (spatially)
assume 3x3 filter
applied **with stride 2**
=> 3x3 output!

A closer look at spatial dimensions:



7x7 input (spatially)
assume 3x3 filter
applied **with stride 3?**

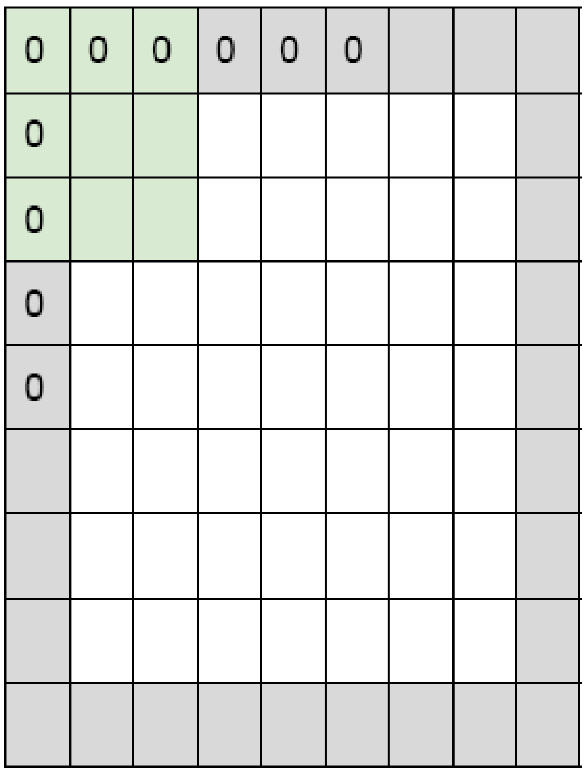
doesn't fit!
cannot apply 3x3 filter on
7x7 input with stride 3.



Output size:
(N - F) / stride + 1

e.g. N = 7, F = 3:
stride 1 => (7 - 3)/1 + 1 = 5
stride 2 => (7 - 3)/2 + 1 = 3
stride 3 => (7 - 3)/3 + 1 = 2.33

In practice: Common to zero pad the border



e.g. input 7x7
3x3 filter, applied with **stride 1**
pad with 1 pixel border => what is the output?

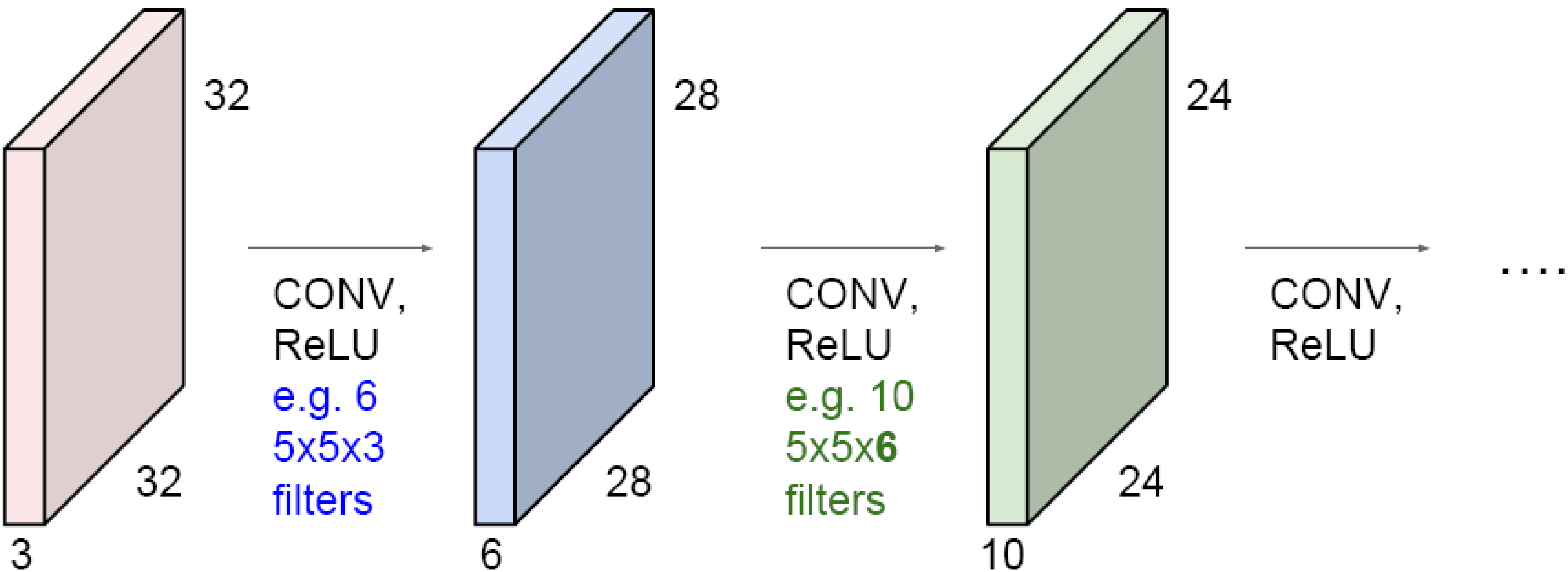
7x7 output!
in general, common to see CONV layers with stride 1, filters of size FxF, and zero-padding with (F-1)/2. (will preserve size spatially)

e.g. F = 3 => zero pad with 1
F = 5 => zero pad with 2
F = 7 => zero pad with 3

All of these produce 7x7 output.

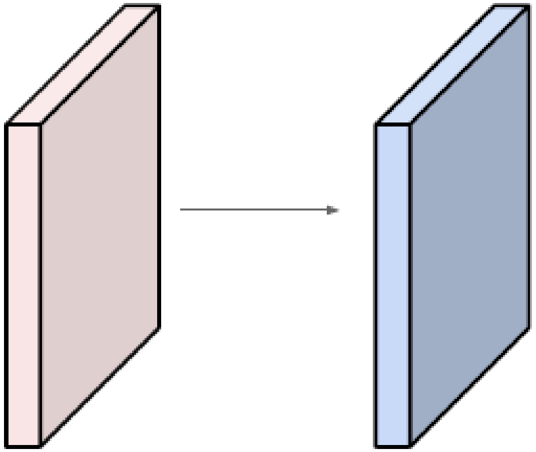
Remember back to…

E.g. 32x32 input convolved repeatedly with 5x5 filters shrinks volumes spatially! (32 -> 28 -> 24 ...). Shrinking too fast is not good, doesn't work well.



Examples time:

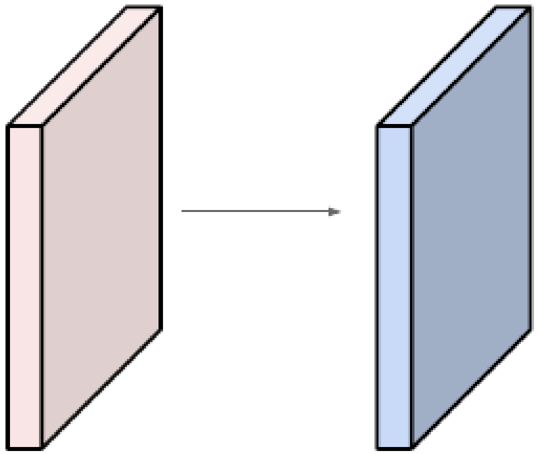
Input volume: **32x32x3**
10 5x5 filters with stride 1, pad 2



Output volume size: ? (N-F+2P)/S + 1

(32-5+2*2)/1+1 = 32 spatially, so
32x32x10

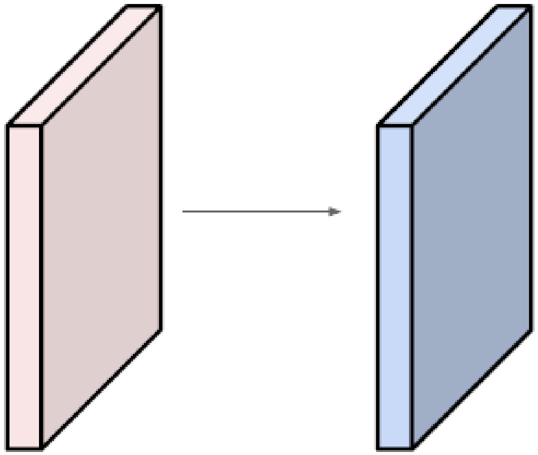
Examples time:



Input volume: **32x32x3**
10 5x5 filters with stride 1, pad 2

Number of parameters in this layer?

Examples time:



Input volume: **32x32x3**

10 5x5 filters with stride 1, pad 2

Number of parameters in this layer?

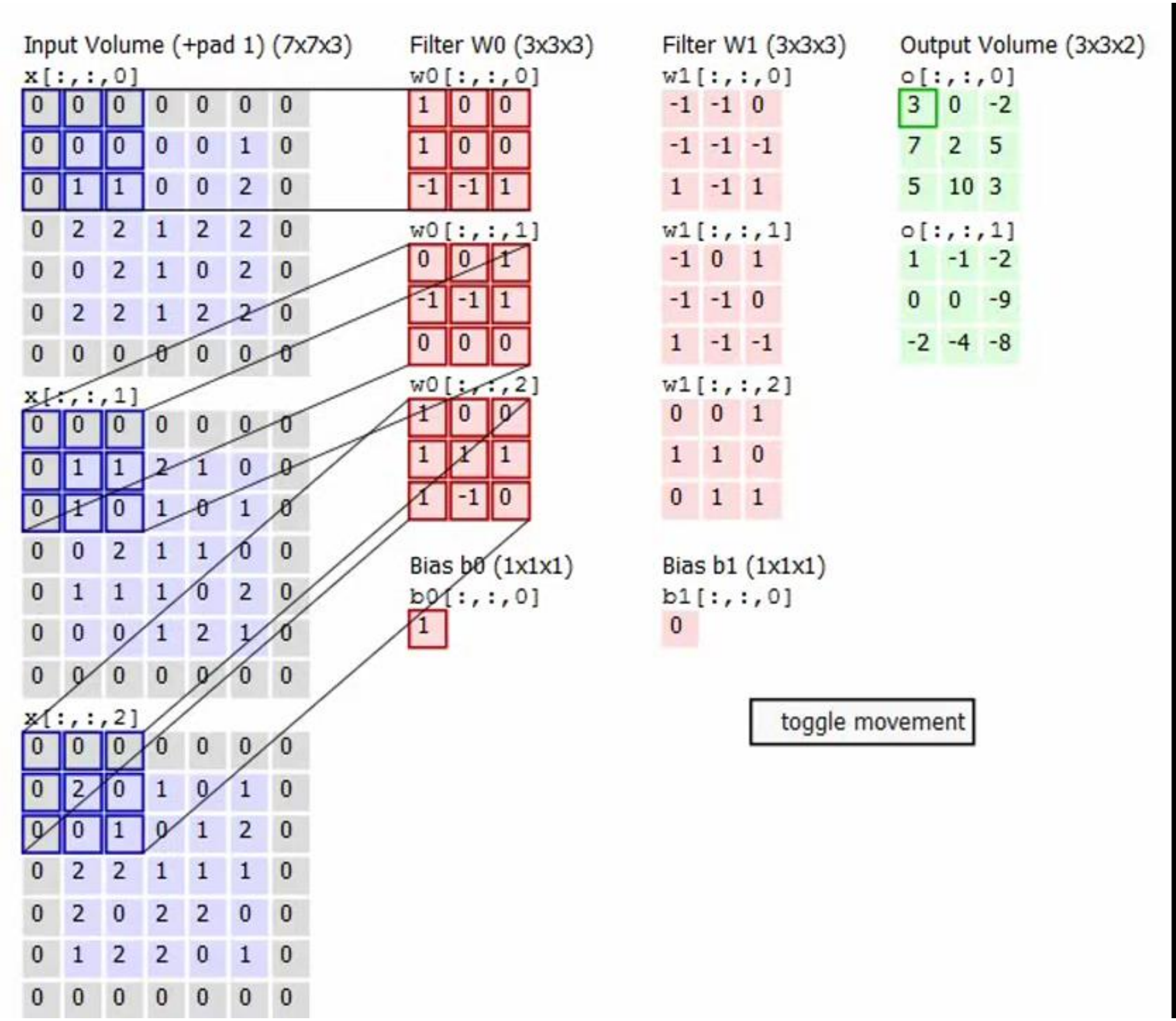
each filter has 5*5*3 + 1 = 76 params (+1 for bias)

=> 76*10 = **760**

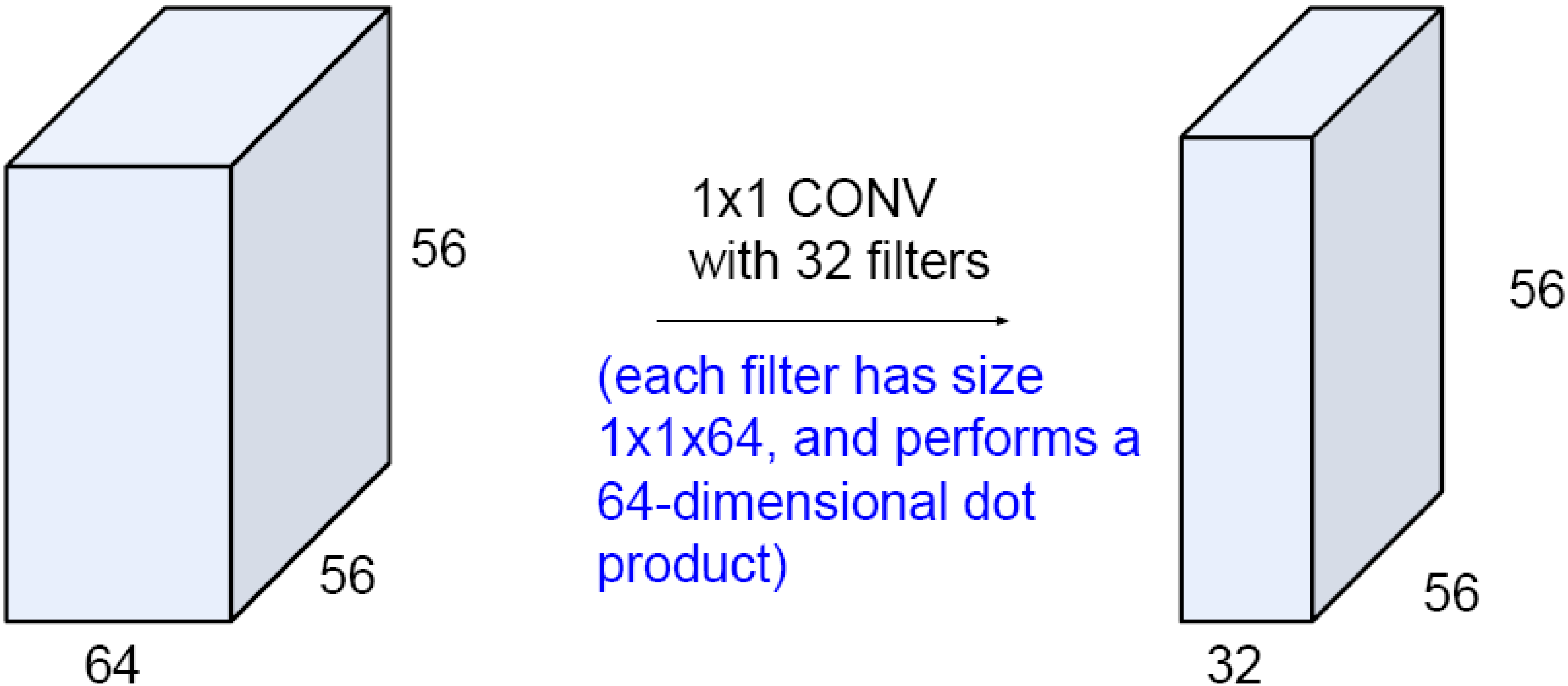
Summary. To summarize, the Conv Layer:

- Accepts a volume of size $W_1 \times H_1 \times D_1$
- Requires four hyperparameters:
  - Number of filters K,
  - their spatial extent F,
  - the stride S,
  - the amount of zero padding P.
- Produces a volume of size $W_2 \times H_2 \times D_2$ where:
  - $W_2 = (W_1 - F + 2P)/S + 1$
  - $H_2 = (H_1 - F + 2P)/S + 1$ (i.e. width and height are computed equally by symmetry)
  - $D_2 = K$
- With parameter sharing, it introduces $F \cdot F \cdot D_1$ weights per filter, for a total of $(F \cdot F \cdot D_1) \cdot K$ weights and K biases.
- In the output volume, the d-th depth slice (of size $W_2 \times H_2$) is the result of performing a valid convolution of the d-th filter over the input volume with a stride of S, and then offset by d-th bias.

Watch a simulation at http://cs231n.github.io/convolutional-networks/



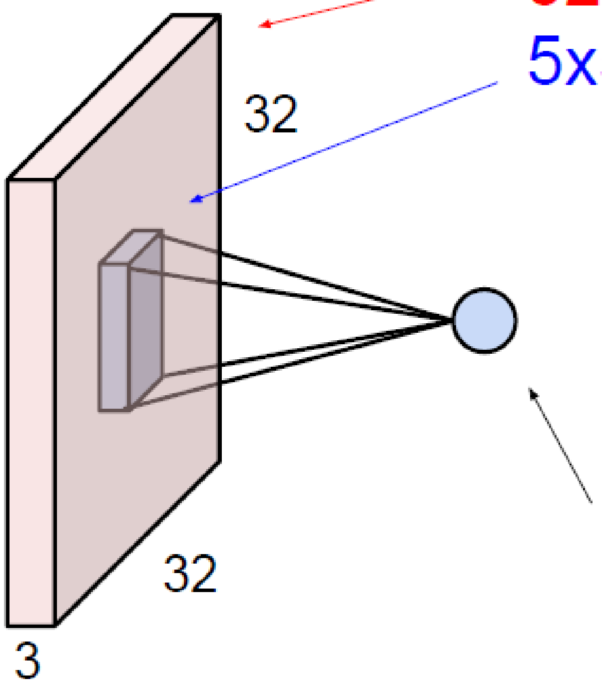
(btw, 1x1 convolution layers make perfect sense)

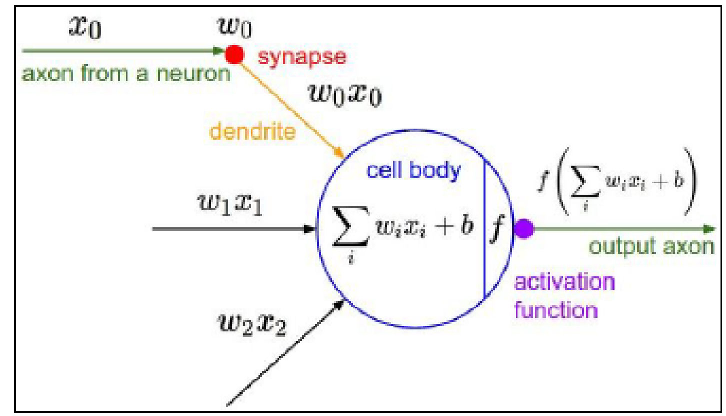


The brain/neuron view of CONV Layer

32x32x3 image

5x5x3 filter

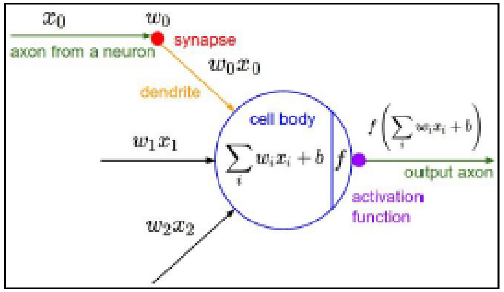


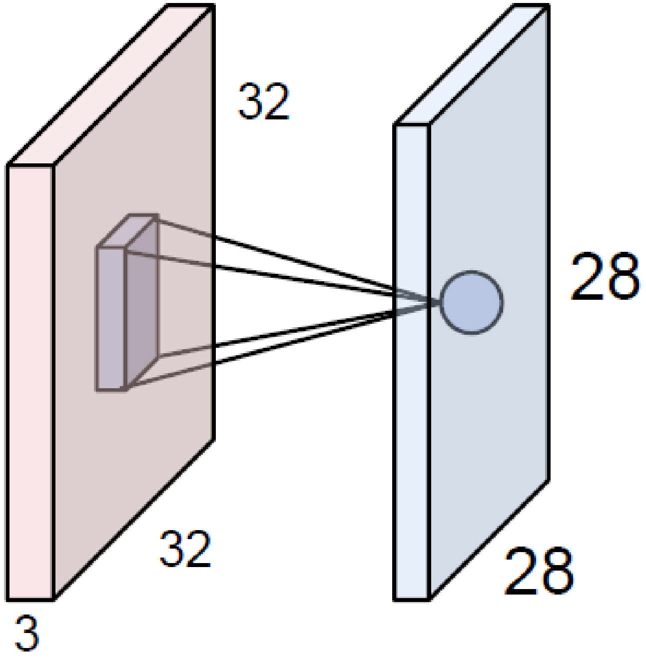


It's just a neuron with local connectivity...

1 number:
the result of taking a dot product between the filter and this part of the image
(i.e. 5*5*3 = 75-dimensional dot product)

The brain/neuron view of CONV Layer



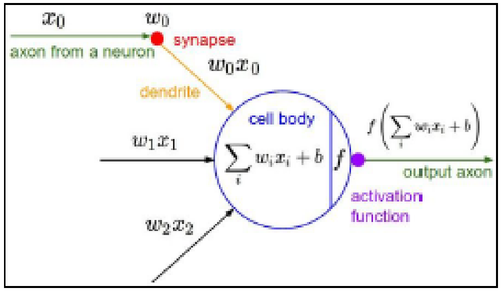


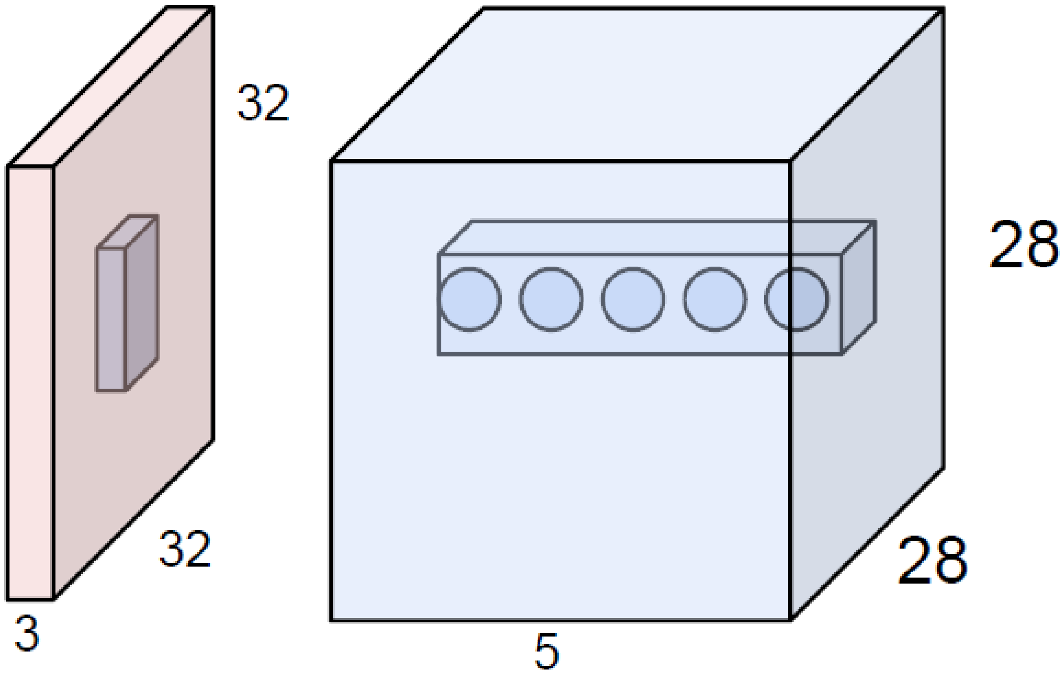
An activation map is a 28x28 sheet of neuron outputs:

1. Each is connected to a small region in the input
2. All of them share parameters

"5x5 filter" -> "5x5 receptive field for each neuron"

The brain/neuron view of CONV Layer





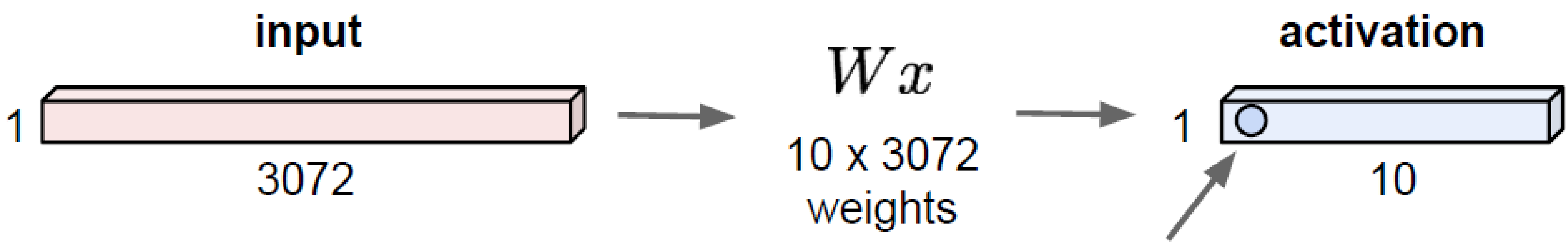
E.g. with 5 filters,
CONV layer consists of
neurons arranged in a 3D grid
(28x28x5)

There will be 5 different
neurons all looking at the same
region in the input volume

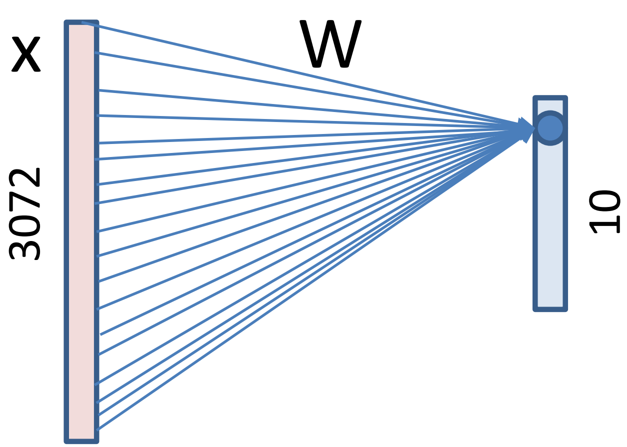
Reminder: Fully Connected Layer

32x32x3 image -> stretch to 3072 x 1

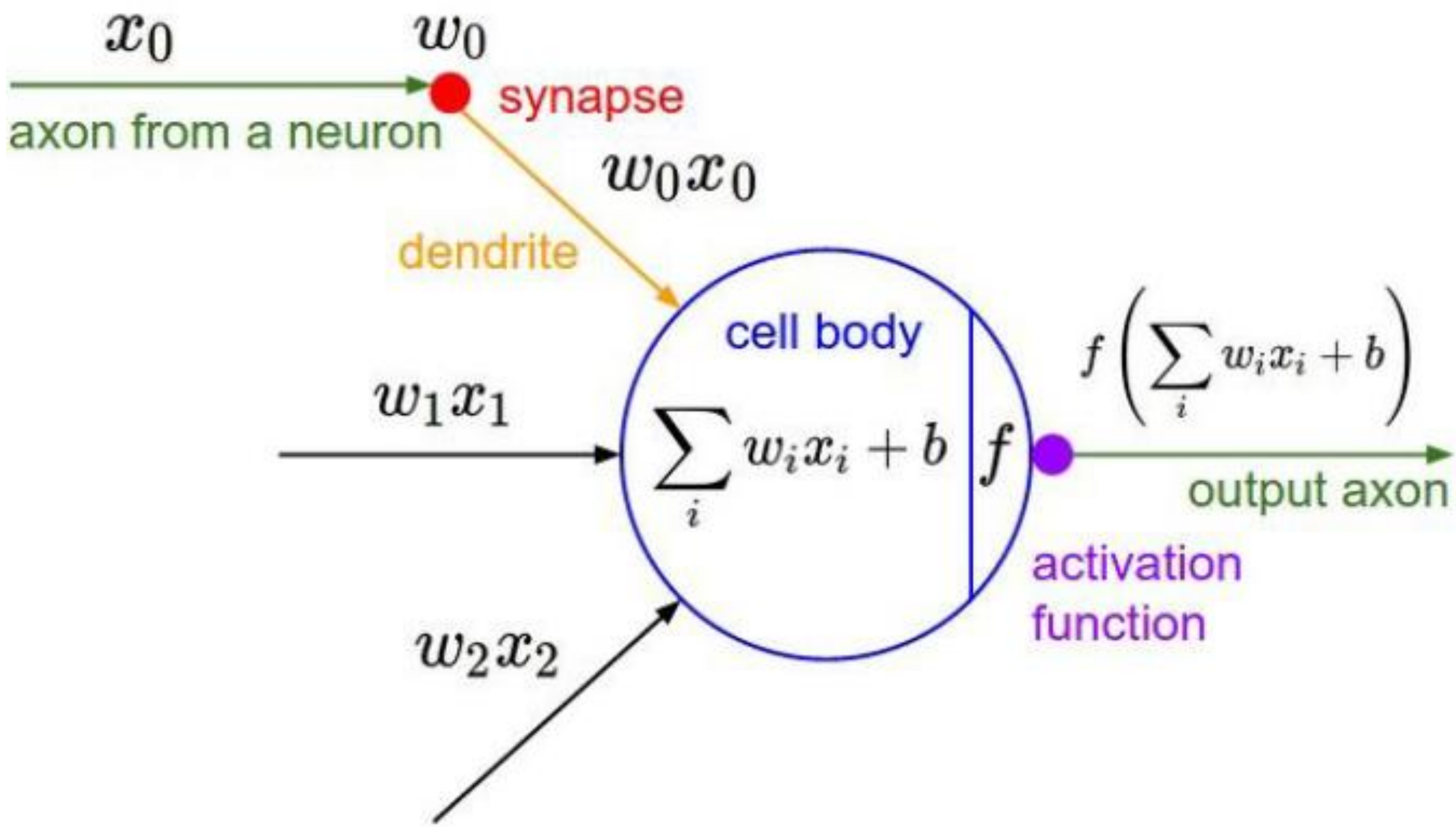
Each neuron looks at the full input volume



1 number:
the result of taking a dot product between a row of W and the input (a 3072-dimensional dot product)

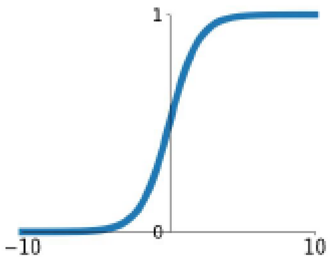


Activation of Convolution Layer



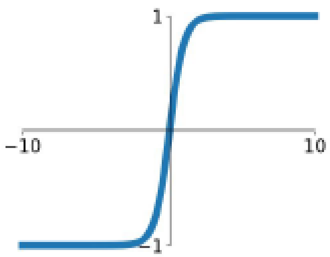
Sigmoid

$\sigma(x) = \frac{1}{1+e^{-x}}$



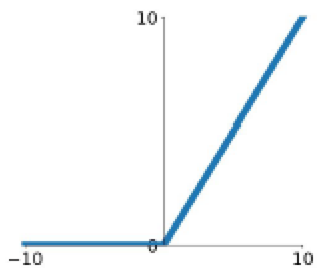
tanh

$\tanh(x)$



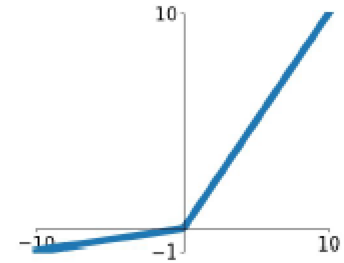
ReLU

$\max(0, x)$



Leaky ReLU

$\max(0.1x, x)$

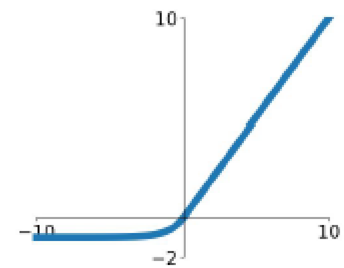


Maxout

$\max(w_1^T x + b_1, w_2^T x + b_2)$

ELU

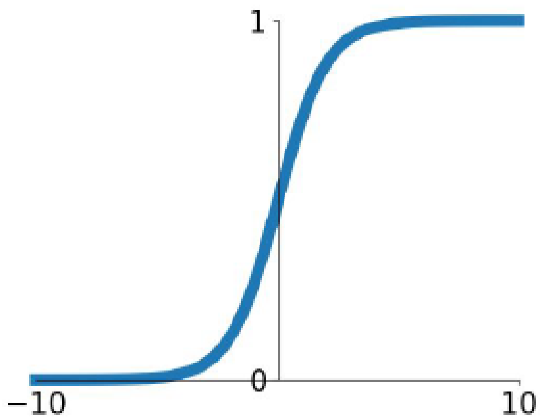
$$\begin{cases} x & x \geq 0 \\ \alpha(e^x - 1) & x < 0 \end{cases}$$



Why not Sigmoid?

Activation Functions

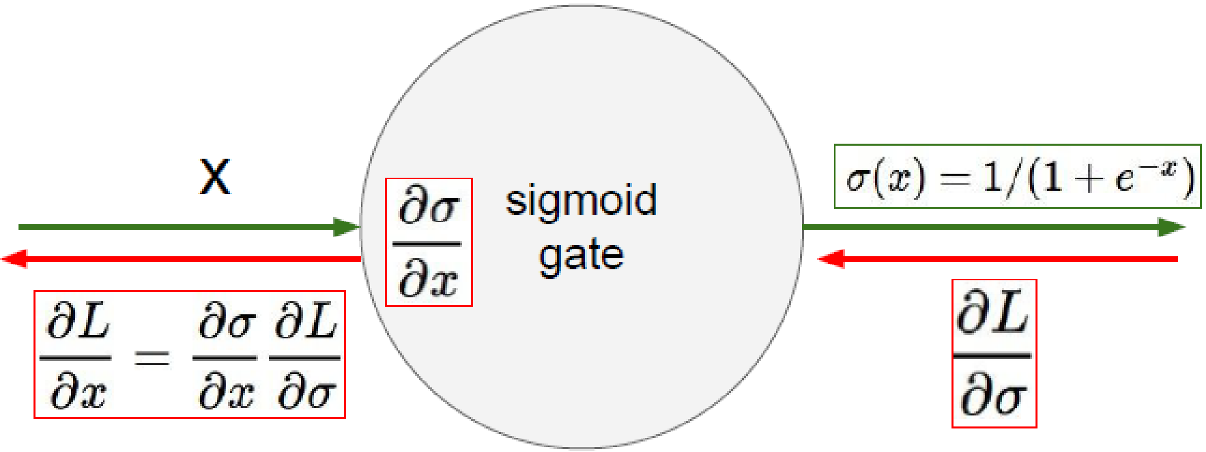
$$\sigma(x) = 1/(1+e^{-x})$$

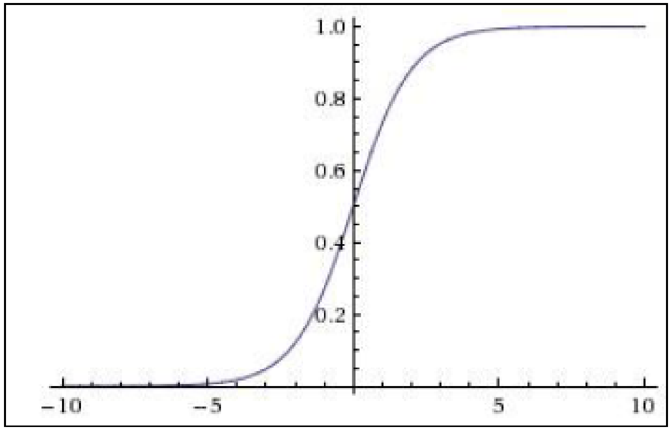


Sigmoid

- Squashes numbers to range [0,1]
- Historically popular since they have nice interpretation as a saturating "firing rate" of a neuron

1. Saturated neurons "kill" the gradients

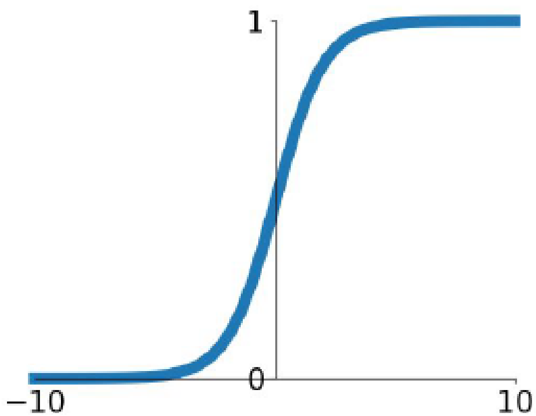




What happens when x = -10?
What happens when x = 0?
What happens when x = 10?

Activation Functions

$$\sigma(x) = 1/(1 + e^{-x})$$



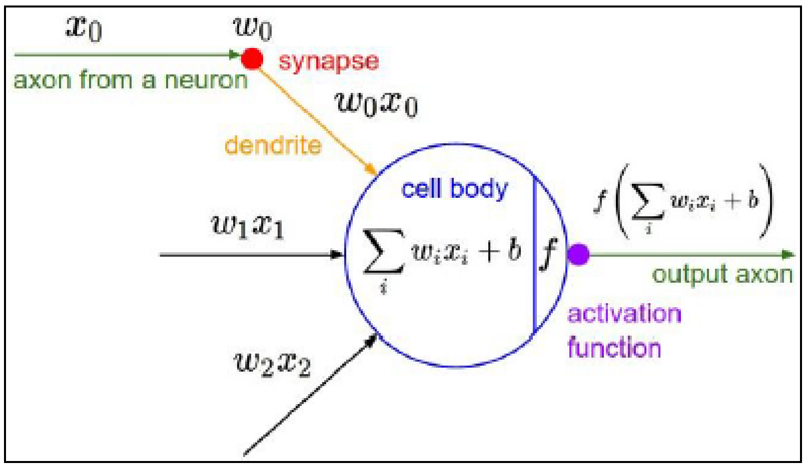
Sigmoid

- Squashes numbers to range [0,1]
- Historically popular since they have nice interpretation as a saturating “firing rate” of a neuron

3 problems:

1. Saturated neurons “kill” the gradients
2. Sigmoid outputs are not zero-centered

Consider what happens when the input to a neuron (x) is always positive:



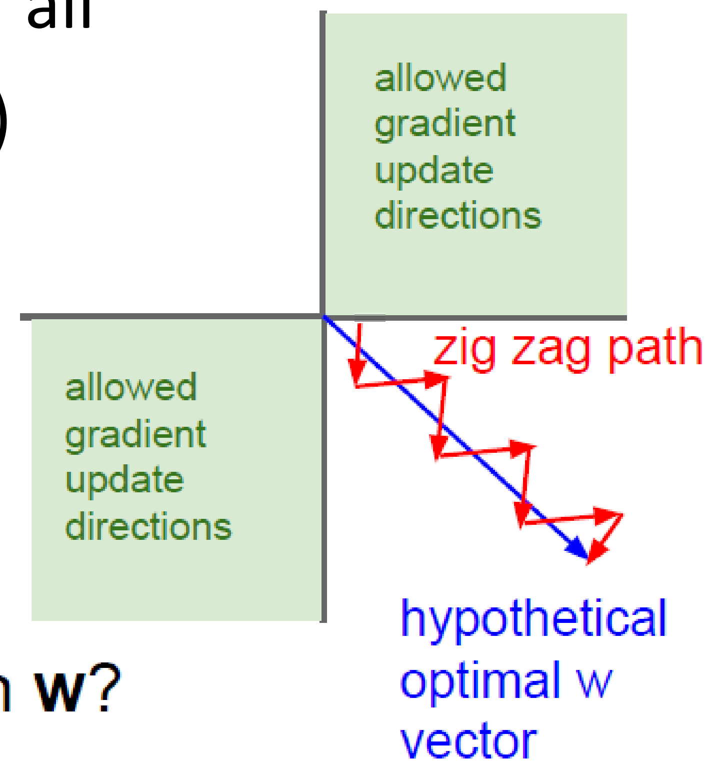
$$f\left(\sum_i w_i x_i + b\right)$$

What can we say about the gradients on **w**?

$$\frac{\partial f(\sum_i w_i x_i + b)}{\partial w} = \overbrace{\frac{1}{1+e^{-(\sum_i w_i x_i + b)}}}^{\text{positive}} \times \overbrace{(1 - \frac{1}{1+e^{-(\sum_i w_i x_i + b)}})}^{\text{positive}}$$

Since $\Delta w_i = \alpha \frac{\partial f}{\partial w} . \frac{\partial L}{\partial f}$, all Δw_i which connect to the same neuron, will be all positive or all negative (depending on the sign of $\frac{\partial L}{\partial f}$.)

$$f\left(\sum_i w_i x_i + b\right)$$



What can we say about the gradients on **w**?

Always all positive or all negative :(

(this is also why you want zero-mean data!)

For example, suppose we have:

w = (1 -1 1)

and we need to get it to:

w = (-1 1 -1)

if we can have both positive and negative gradients we could do this in one step:

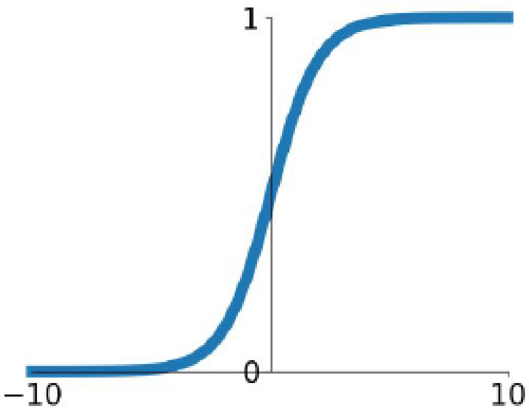
w = (1 -1 1) + (-2 2 -2) = (-1 1 -1) **===> Good. One-step update.**

However, the best we can do with only, positive or negative is two steps:

w = (1 -1 1) + (-3 -3 -3) + (1 5 1) = (-1 1 -1) **===> Bad. ZIG-ZAG Weight Update**

Activation Functions

$$\sigma(x) = 1/(1 + e^{-x})$$



Sigmoid

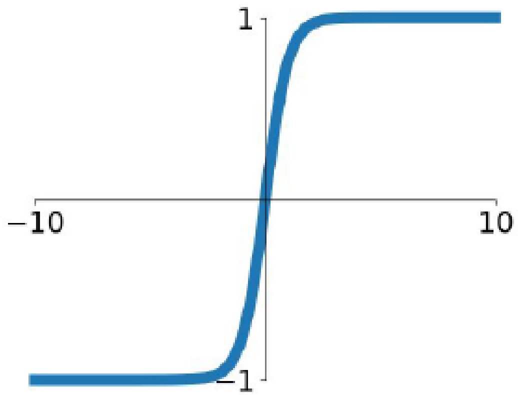
- Squashes numbers to range [0,1]
- Historically popular since they have nice interpretation as a saturating "firing rate" of a neuron

3 problems:

1. Saturated neurons "kill" the gradients
2. Sigmoid outputs are not zero-centered
3. exp() is a bit compute expensive

Activation Functions

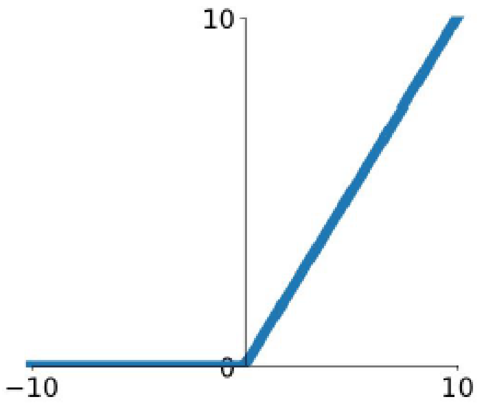
Why not tanh?



tanh(x)

- Squashes numbers to range [-1,1]
- zero centered (nice)
- still kills gradients when saturated :(

Activation Functions



ReLU
(Rectified Linear Unit)

- Computes **f(x) = max(0,x)**

- Does not saturate (in +region)
- Very computationally efficient
- Converges much faster than sigmoid/tanh in practice (e.g. 6x)
- Actually more biologically plausible than sigmoid

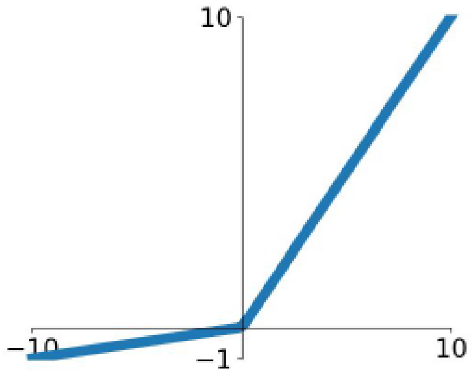
[Krizhevsky et al., 2012]

- Not zero-centered output
- An annoyance:

hint: what is the gradient when $x < 0$?

Activation Functions

[Mass et al., 2013]
[He et al., 2015]



- Does not saturate
- Computationally efficient
- Converges much faster than sigmoid/tanh in practice! (e.g. 6x)
- **will not “die”.**

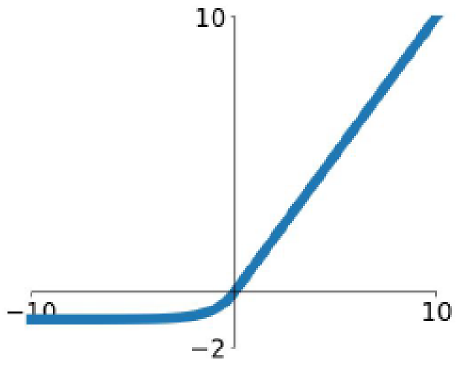
Leaky ReLU

$$f(x) = \max(0.01x, x)$$

Activation Functions

[Clevert et al., 2015]

Exponential Linear Units (ELU)



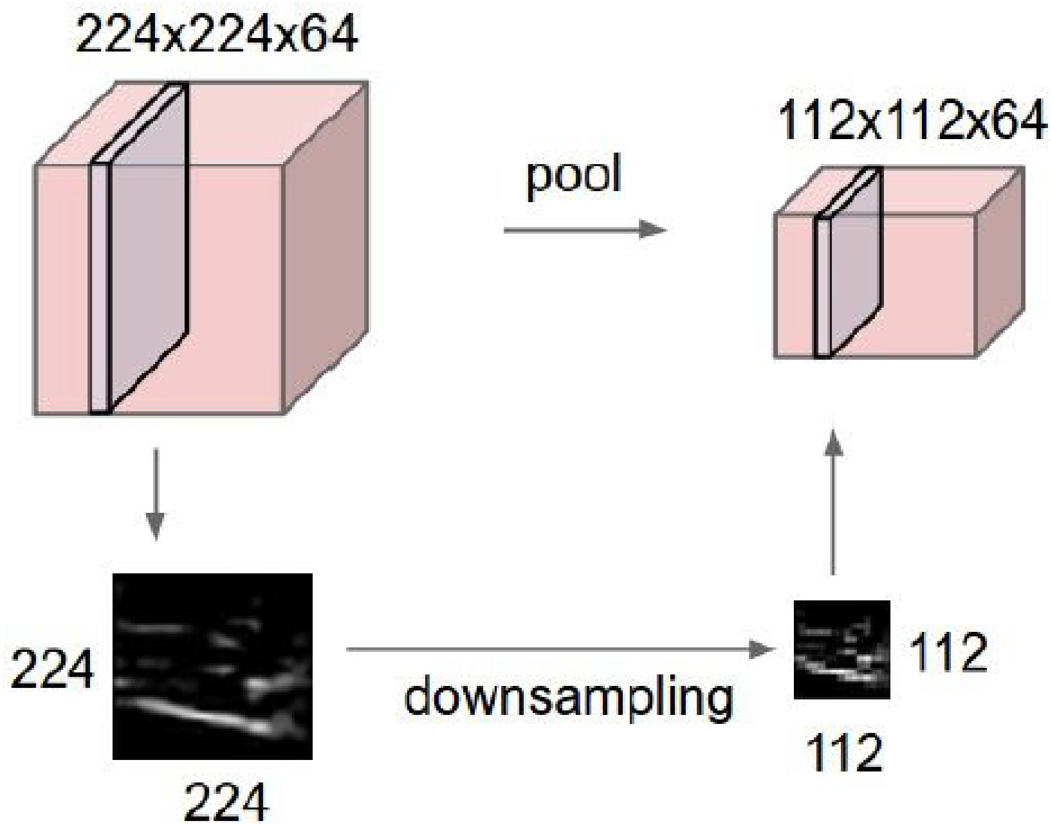
- All benefits of ReLU
- Closer to zero mean outputs
- Negative saturation regime compared with Leaky ReLU adds some robustness to noise

$$f(x) = \begin{cases} x & \text{if } x > 0 \\ \alpha\,(\exp(x) - 1) & \text{if } x \le 0 \end{cases}$$

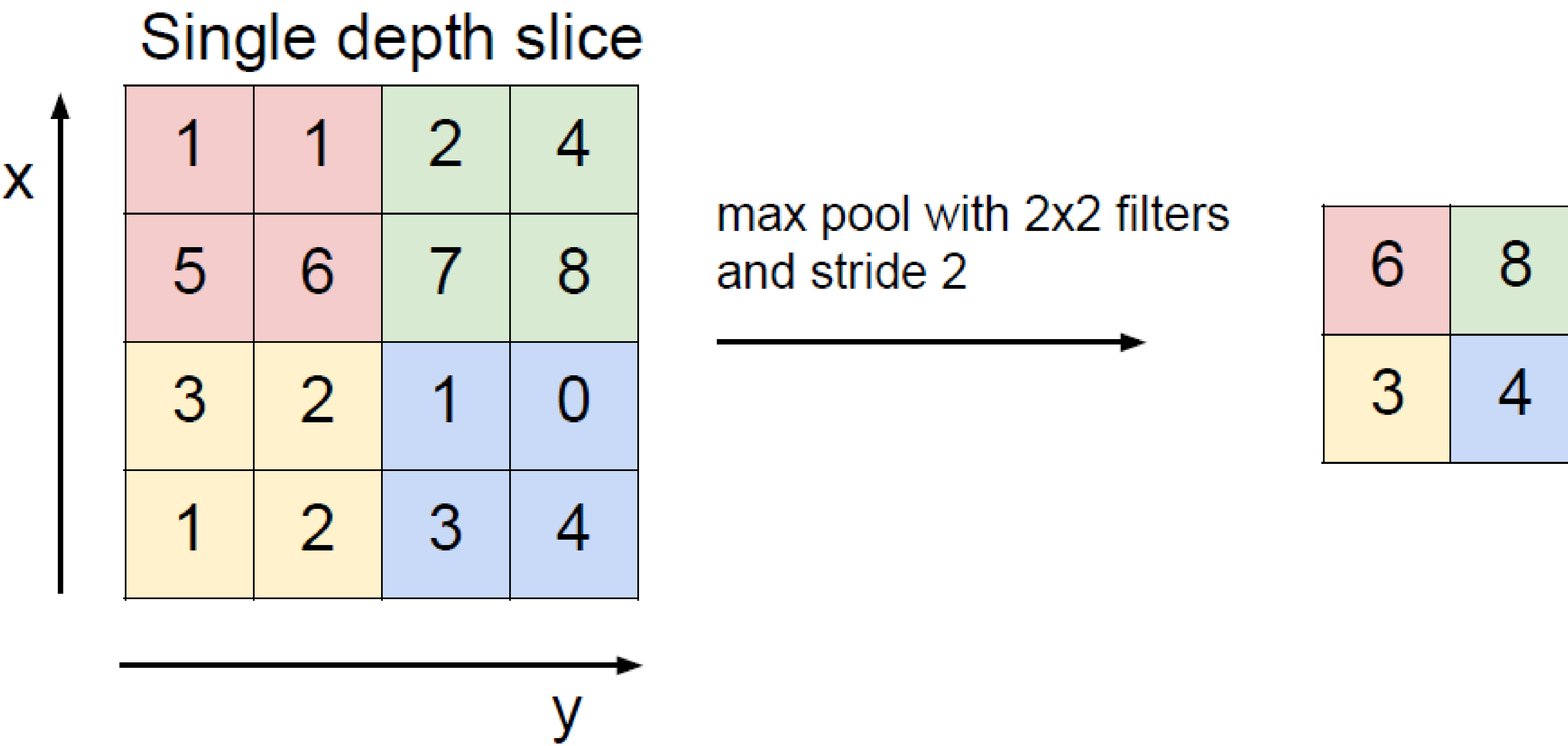
- Computation requires exp()

Pooling layer

- makes the representations smaller and more manageable
- operates over each activation map independently:



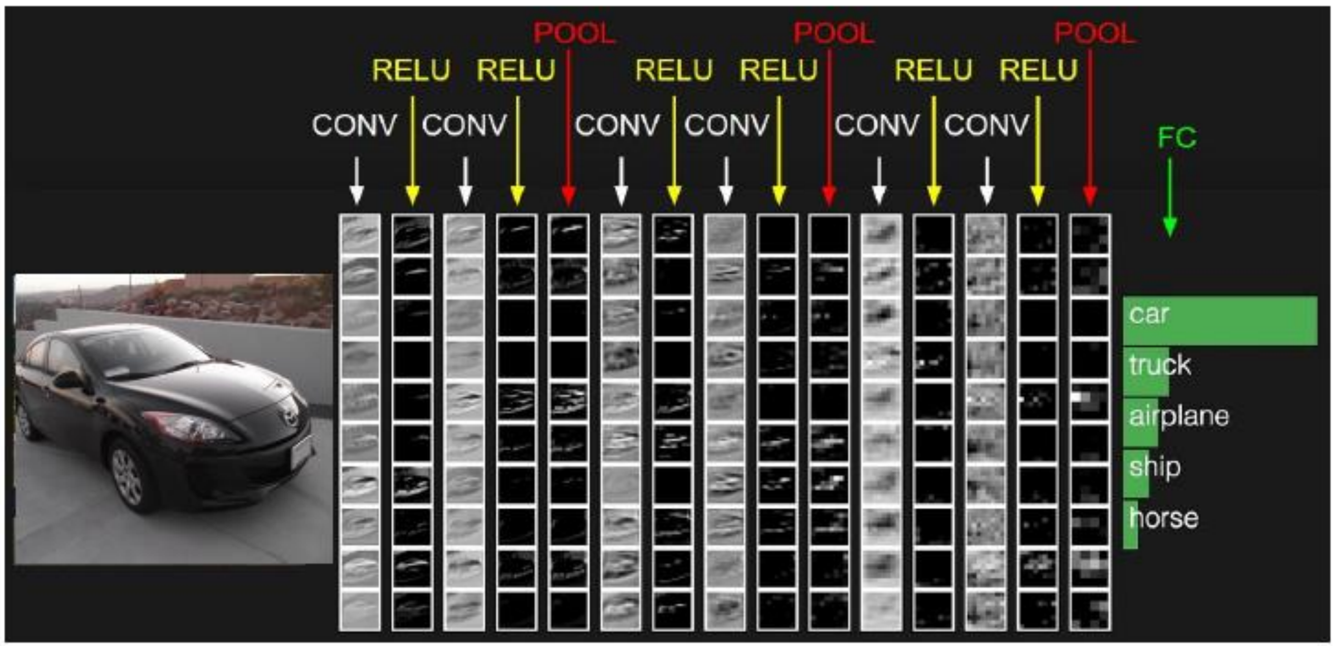
MAX POOLING


Single depth slice
x
y
1 1 2 4
5 6 7 8
3 2 1 0
1 2 3 4
max pool with 2x2 filters
and stride 2
6 8
3 4

- Accepts a volume of size $W_1 \times H_1 \times D_1$
- Requires two hyperparameters:
  - their spatial extent F,
  - the stride S,
- Produces a volume of size $W_2 \times H_2 \times D_2$ where:
  - $W_2 = (W_1 - F)/S + 1$
  - $H_2 = (H_1 - F)/S + 1$
  - $D_2 = D_1$
- Introduces zero parameters since it computes a fixed function of the input
- Note that it is not common to use zero-padding for Pooling layers

Fully Connected Layer (FC layer)

- Contains neurons that connect to the entire input volume, as in ordinary Neural Networks



- Next class
  - A simple example of Backpropagation in CNN.