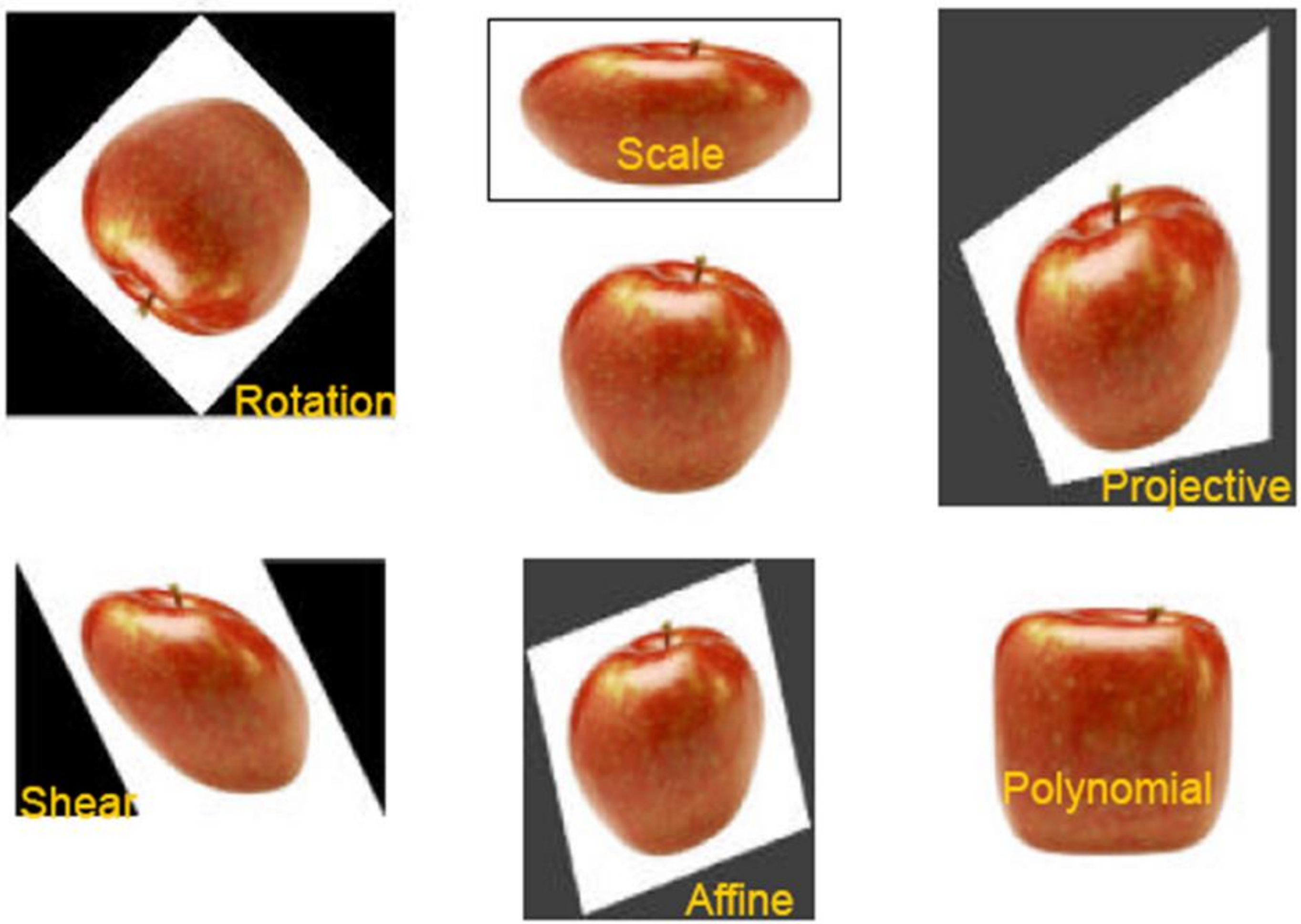
If an image is not aligned
the same way we want it to be,

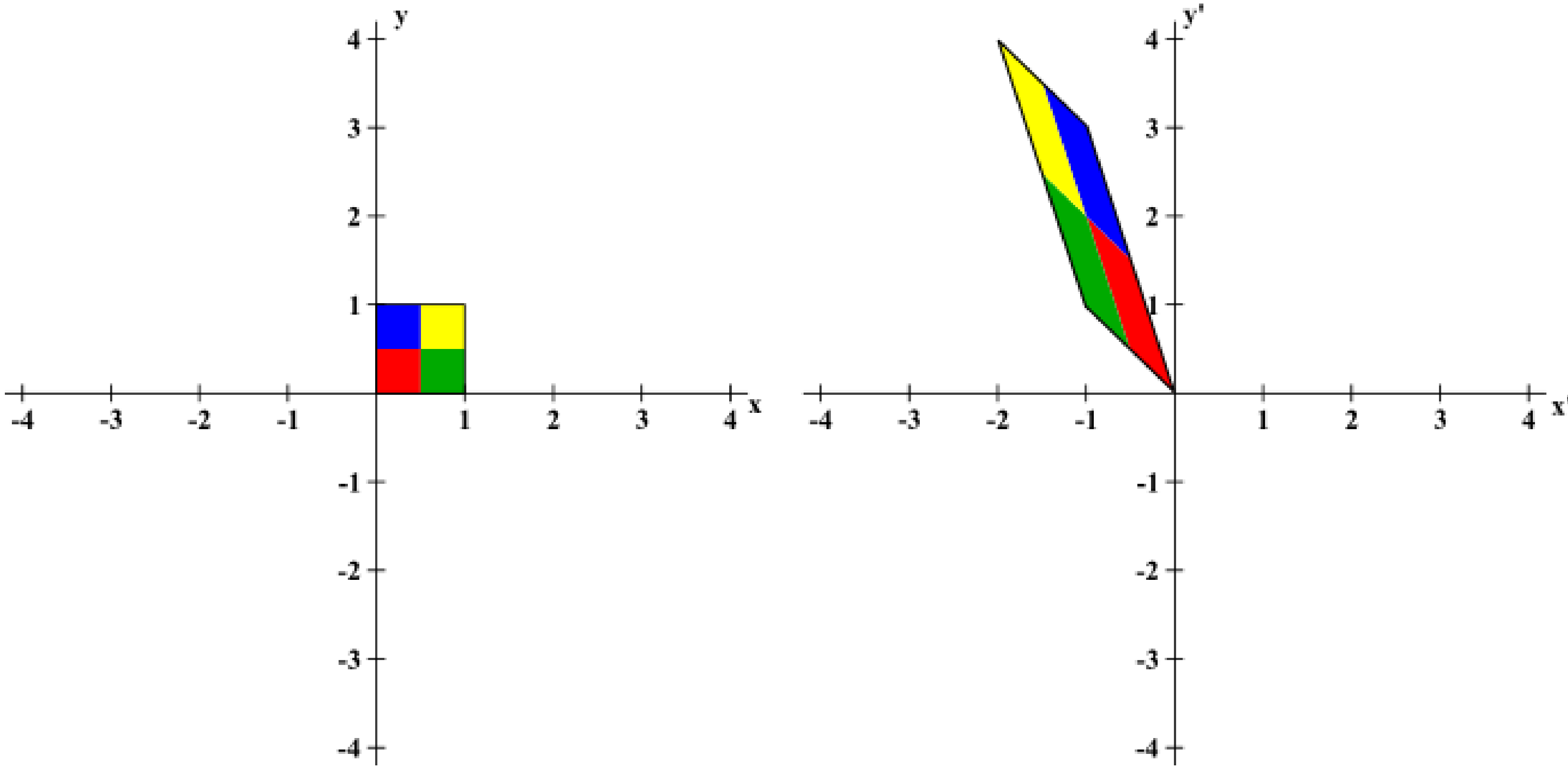
How should it be processed
to adjust its alignment

How can we obtain this look of image?



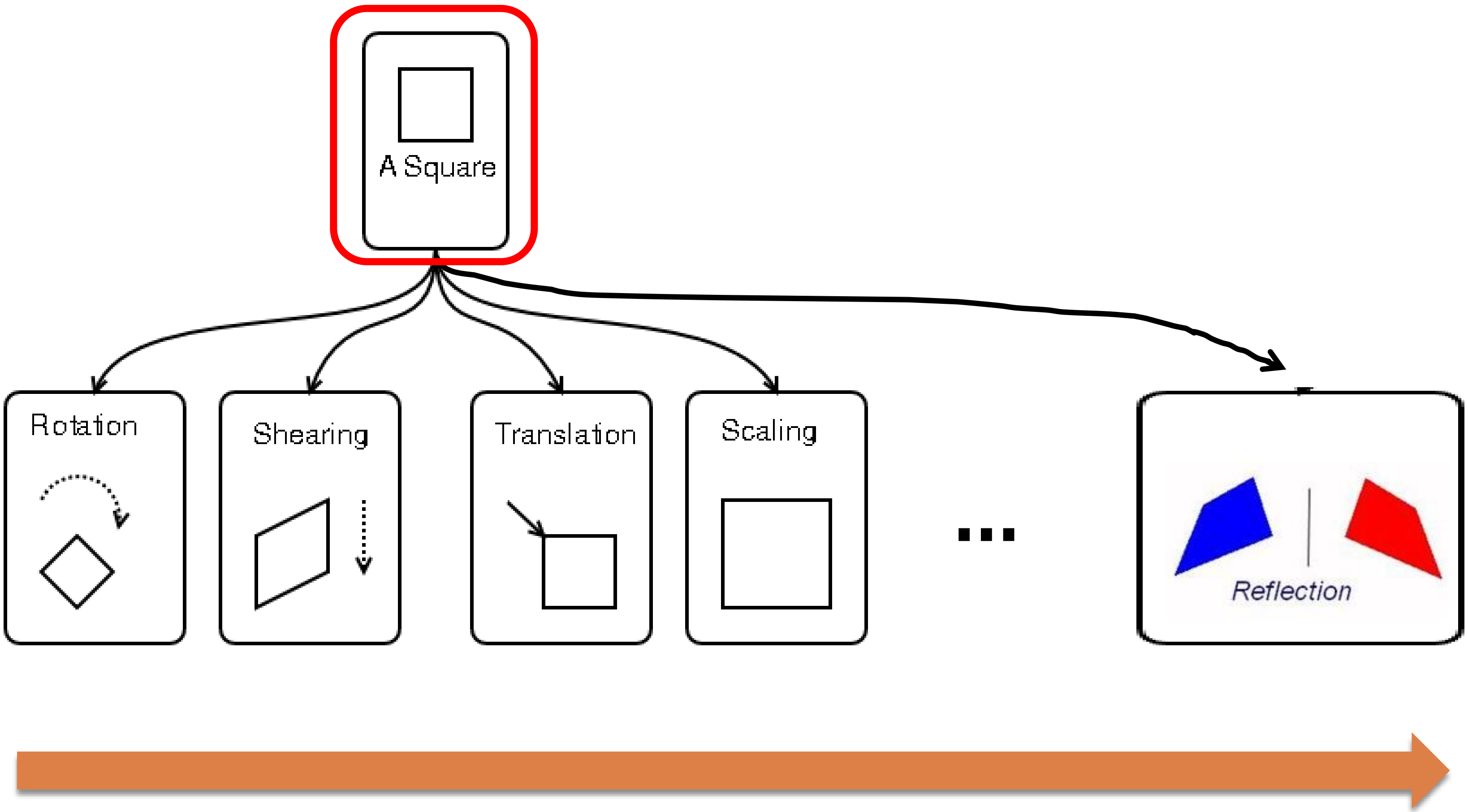
Answer is through Geometric (point) Transformation

Transforming points in 2D to a new axis

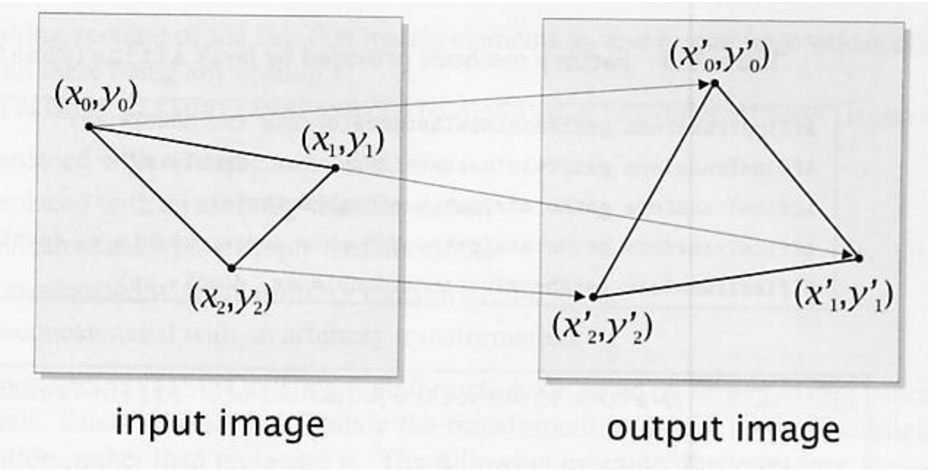


From current axis (x,y) **To new axis (x′,y′)**

What would point transform effect an image?



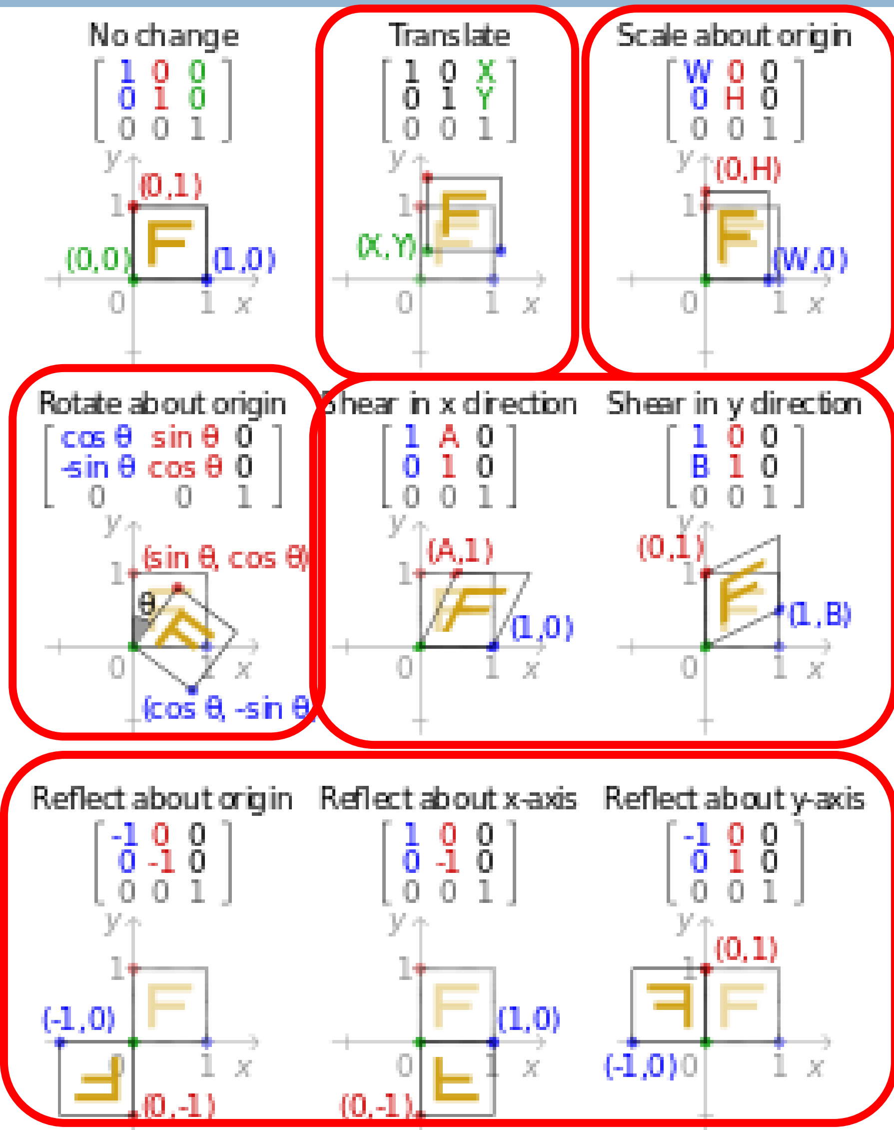
Geometric Transformation (point transform)



$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a_0 & a_1 \\ b_0 & b_1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} a_2 \\ b_2 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a_0 & a_1 & a_2 \\ b_0 & b_1 & b_2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Relation between transform action and transfer function



$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Translation

$$\begin{bmatrix} \mathbf{x}' \\ \mathbf{y}' \\ 1 \end{bmatrix} = \begin{bmatrix} \mathbf{s}_x & 0 & 0 \\ 0 & \mathbf{s}_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{y} \\ 1 \end{bmatrix}$$

Scaling

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Rotation

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & sh_x & 0 \\ sh_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

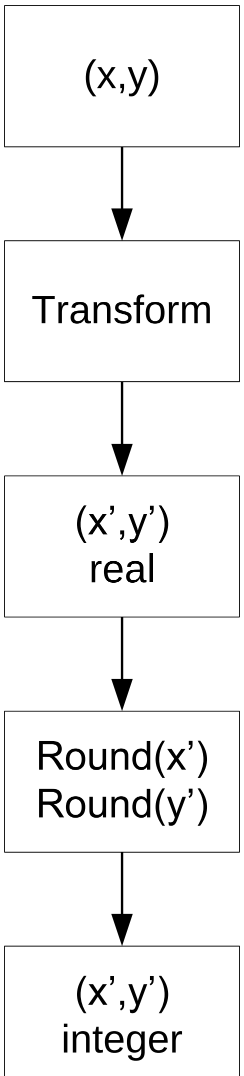
Shearing

http://en.wikipedia.org/wiki/Transformation_matrix

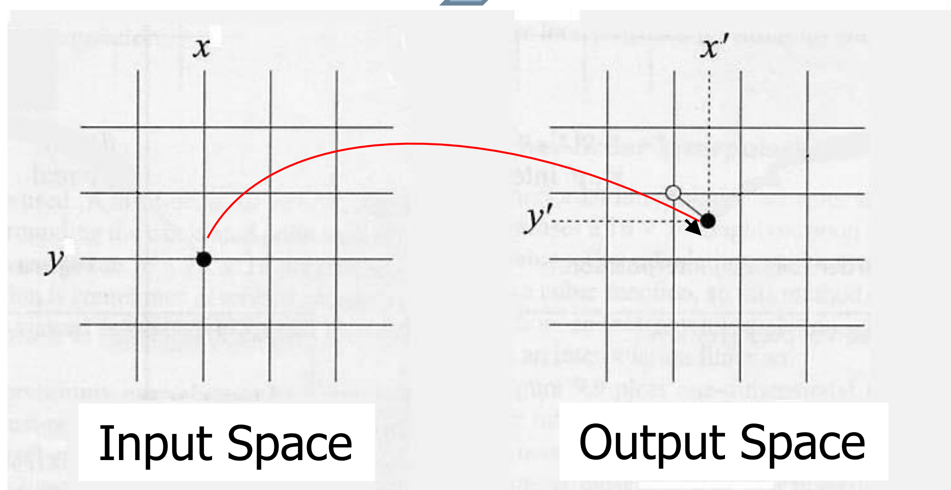
How would point transform

Apply to an image?

Forward Mapping



Integer grid (x,y) Real number grid (x',y')



$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a_0 & a_1 & a_2 \\ b_0 & b_1 & b_2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

An Image uses integer grid

$$g(round(x'), round(y'))$$

Point Transform Example

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Rotation

$$\theta = 90$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} cos(90) & -sin(90) & 0 \\ sin(90) & cos(90) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

	y=0	y=1
x=0	1	2
x=1	3	4

x	y	x'	y'
0	0		
0	1		
1	0		
1	1		

Point Transform Example

	y=0	y=1
x=0	1	2
x=1	3	4

i/p image
f(*x*,*y*)

$\theta = 90$

	y'=0	y'=1
x'=-1	2	4
x'=0	1	3

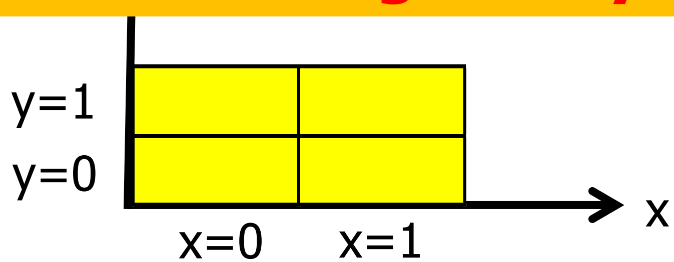
o/p image
g(*x*,*y*)

จุดหมุนที่ pixel ตำแหน่ง (0,0)

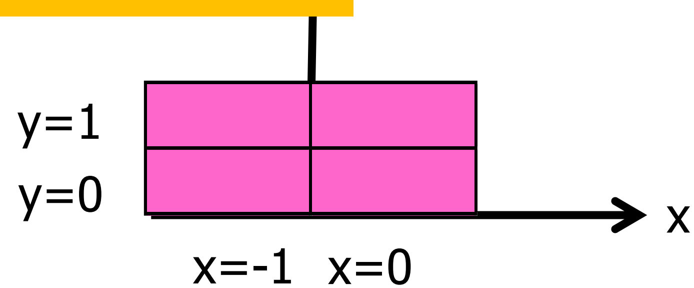
Image world

Graphic world

Can the rotation results be kept in realworld image array?



Graphic plot of
i/p image



Graphic plot of
o/p image

Shifting Back to Unsigned Integer Grid

	y'=0	y'=1
x'=-1	2	4
x'=0	1	3

o/p image
g(*x*,*y*)

Actual rotation result

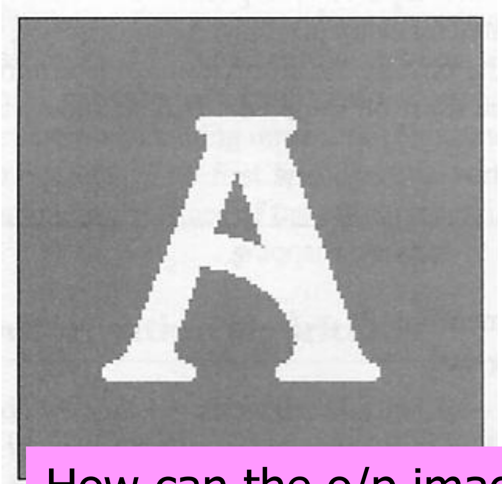
	y'=0	y'=1
x'=0	2	4
x'=1	1	3

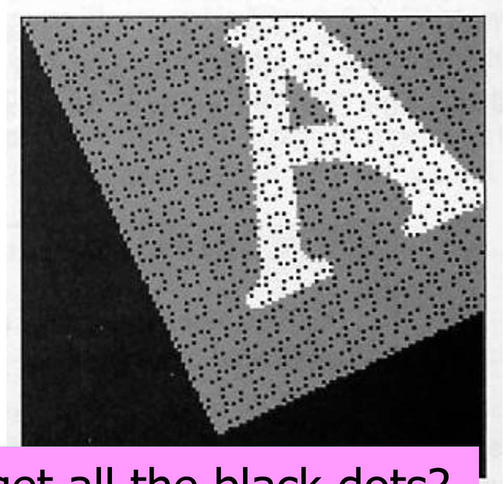
Mapping result onto unsigned integer grid

Forward Mapping Problem

Why does the o/p image can not hold all the area of the result?

i/p image
f(x,y)





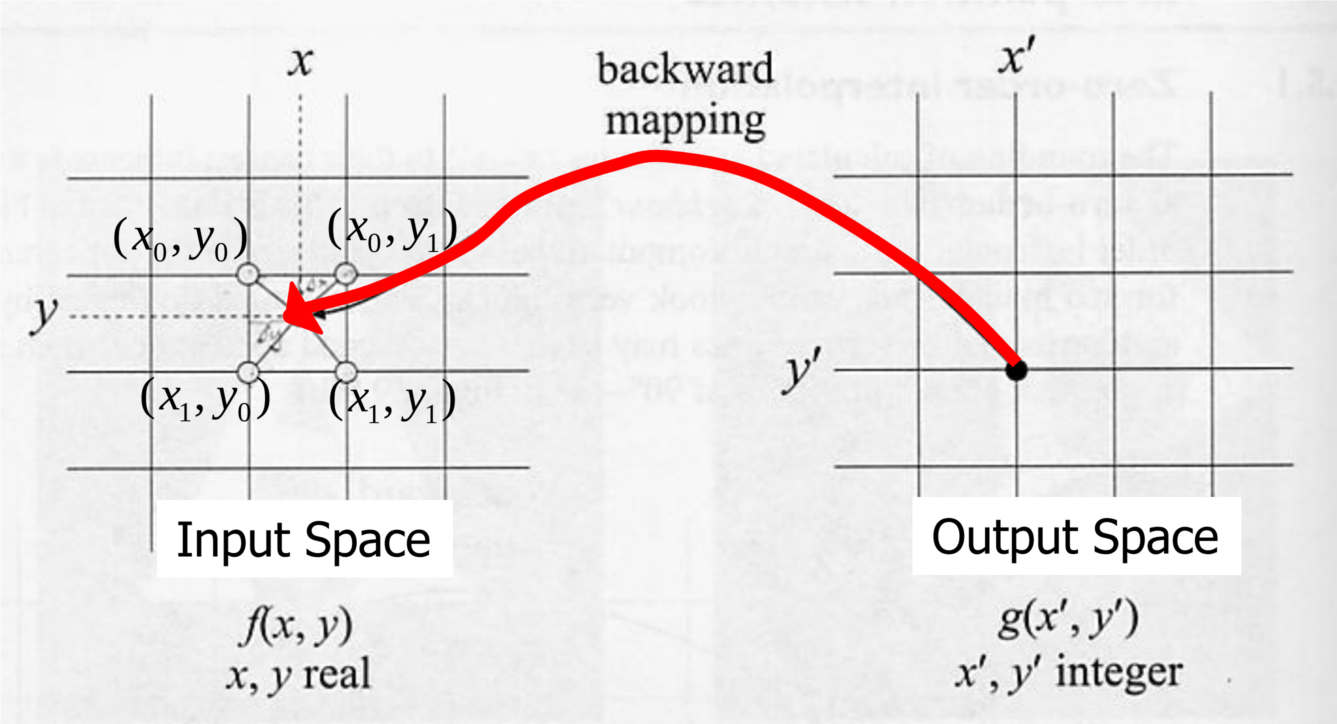
o/p image
g(x,y)

How can the o/p image get all the black dots?

$f(x, y)$ $\quad$ $g(x', y')$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos(25^\circ) & -\sin(25^\circ) \\ \sin(25^\circ) & \cos(25^\circ) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Backward Mapping



$$g(x', y') = \text{interpolation}[f(x, y)]$$

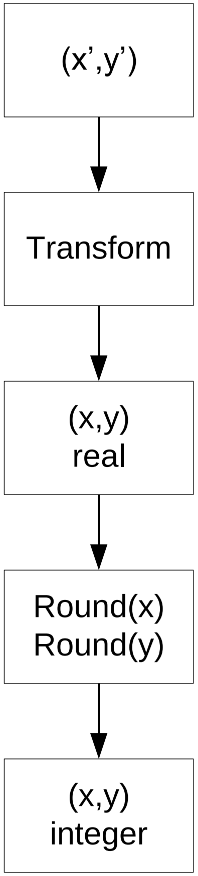
ค่าเฉลี่ยจากตำแหน่งพิกเซลเพื่อนบ้านรอบจุด (x,y)

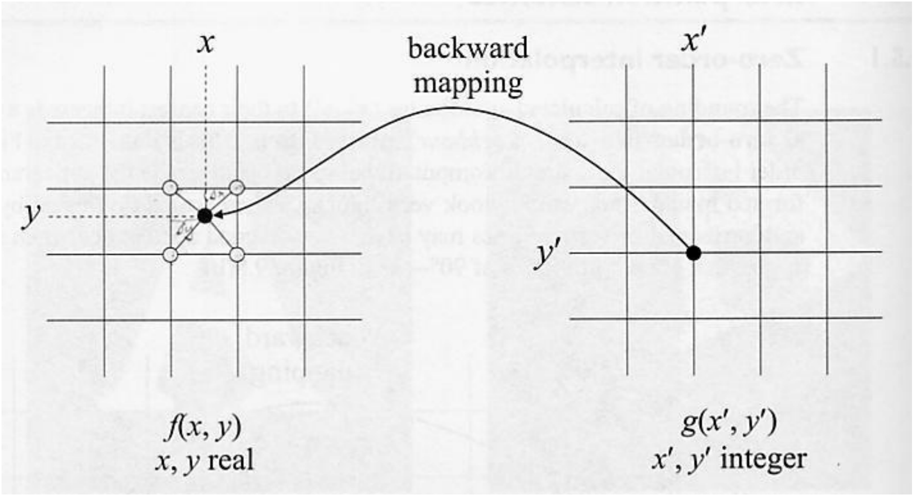
Geometric Transform Parameters (Backward Mapping)

	Backward	Forward
1) Rotation	$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix}$	$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$
2) Translation (Shift)	$x = x' - \Delta x$ $y = y' - \Delta y$	$x' = x + \Delta x$ $y' = y + \Delta y$
3) Enlarge or reduce by scaling factor (S)	$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{S_x} & 0 \\ 0 & \frac{1}{S_y} \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix}$	$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$
4) Shear (บิด)	$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{\lvert\det()\rvert} \begin{bmatrix} -1 & sh_x \\ sh_y & -1 \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix}$	$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & sh_x \\ sh_y & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

Zero-order Interpolation

(nearest neighbor)





$$g(x',y') = f[round(x), round(y)]$$

First-order interpolation

(bilinear interpolation)

(x',y')

↓

Transform

↓

(x,y)
real

↓

(x_0,y_0)

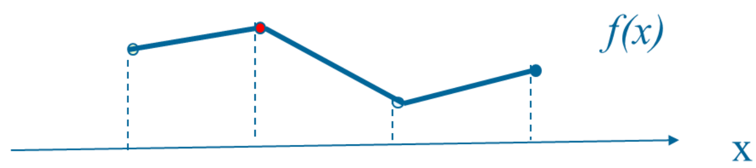
x_0=floor(x)
y_0=floor(y)

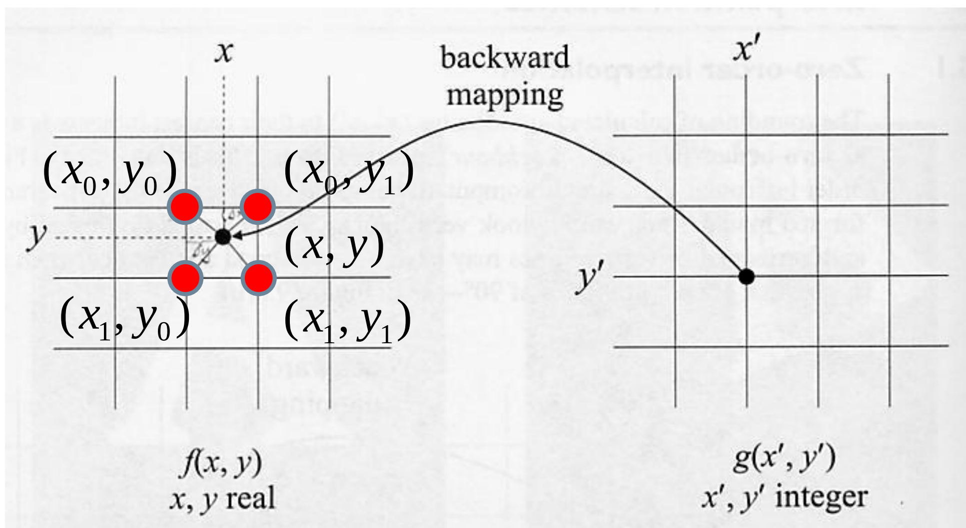
↓

First-order Interpolation

↓

g(x',y')





$$\Delta x = x - x_0$$

$$\Delta y = y - y_0$$

$$
\begin{aligned}
g(x',y') = f(x_0,y_0) &+ [f(x_1,y_0) - f(x_0,y_0)]\Delta x \\
&+ [f(x_0,y_1) - f(x_0,y_0)]\Delta y \\
&+ [f(x_1,y_1) + f(x_0,y_0) - f(x_0,y_1) - f(x_1,y_0)\Delta x\Delta y
\end{aligned}
$$

Rotation using Bilinear Interpolation

	y=0	y=1
x=0	1	2
x=1	3	4

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix}$$

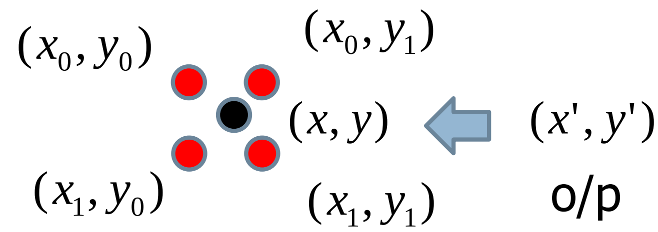
Rotate	30
cos()	0.866025
sin()	0.5

	y=-1	y=0	y=1	y=2
x=-2	**0**	**0**	**0**	**0**
x=-1	**0**	**0**	2	**0**
x=0	**0**	1	4	**0**
x=1	**0**	**0**	3	**0**
x=2	**0**	**0**	**0**	**0**

Forward mapping

x	y	f(x,y)	x'=xcos-ysin	y'=xsin+ycos	round(x')	round(y')
0	0	1	0	0	0	0
0	1	2	-0.5	0.866025	-1	1
1	0	3	0.866025	0.5	1	1
1	1	4	0.366025	1.366025	0	1
	min		-0.5	0	-1	0
	max		0.866025	1.366025	1	1

Backward Mapping
(Bilinear Interpolation)



$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos 30^\circ & \sin 30^\circ \\ -\sin 30^\circ & \cos 30^\circ \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix}$$

$(x, y)_{i/p}$

$$\begin{aligned} g(x', y') = f(x_0, y_0) &+ [f(x_1, y_0) - f(x_0, y_0)]\Delta x \\ &+ [f(x_0, y_1) - f(x_0, y_0)]\Delta y \\ &+ [f(x_1, y_1) + f(x_0, y_0) - f(x_0, y_1) - f(x_1, y_0)\Delta x \Delta y \end{aligned}$$

x'	y'	x=x'cos+y'sin	y=-x'sin+y'cos	x0	y0	f(x0,y0)	f(x0,y1)	f(x1,y0)	f(x1,y1)	dx	dy
-2	-1	-2.2321	0.1340	-3	0	0	0	0	0	0.7680	0.1340
-2	0	-1.7321	1.0000	-2	1	0	0	0	0	0.2680	0.0000
-2	1	-1.2321	1.8660	-2	1	0	0	0	0	0.7680	0.8660
-2	2	-0.7321	2.7321	-1	2	0	0	0	0	0.2680	0.7321
-1	-1	-1.3660	-0.3660	-2	-1	0	0	0	0	0.6340	0.6340
-1	0	-0.8660	0.5000	-1	0	0	0	1	2	0.1340	0.5000
-1	1	-0.3660	1.3660	-1	1	0	0	2	0	0.6340	0.3660
-1	2	0.1340	2.2321	0	2	0	0	0	0	0.1340	0.2321
0	-1	-0.5000	-0.8660	-1	-1	0	0	0	1	0.5000	0.1340
0	0	0.0000	0.0000	0	0	1	2	3	4	0.0000	0.0000
0	1	0.5000	0.8660	0	0	1	2	3	4	0.5000	0.8660
0	2	1.0000	1.7321	1	1	4	0	0	0	0.0000	0.7321
1	-1	0.3660	-1.3660	0	-2	0	0	0	0	0.3660	0.6340
1	0	0.8660	-0.5000	0	-1	0	1	0	3	0.8660	0.5000
1	1	1.3660	0.3660	1	0	3	4	0	0	0.3660	0.3660
1	2	1.8660	1.2321	1	1	4	0	0	0	0.8660	0.2321
2	-1	1.2321	-1.8660	1	-2	0	0	0	0	0.2321	0.1340
2	0	1.7321	-1.0000	1	-1	0	0	0	0	0.7321	0.0000
2	1	2.2321	-0.1340	2	-1	0	0	0	0	0.2321	0.8660
2	2	2.7321	0.7321	2	0	0	0	0	0	0.7321	0.7321

Backward Mapping
(Bilinear Interpolation)

$$g(x',y') = f(x_0,y_0) + [f(x_1,y_0) - f(x_0,y_0)]\Delta x$$
$$+ [f(x_0,y_1) - f(x_0,y_0)]\Delta y$$
$$+ [f(x_1,y_1) + f(x_0,y_0) - f(x_0,y_1) - f(x_1,y_0)\Delta x\Delta y$$

f(x0,y0)	f(x0,y1)	f(x1,y0)	f(x1,y1)	dx	dy	[f(x1,y0)-f(x0,y0)]*dx	[f(x0,y1)-f(x0,y0)]*dy	[...]*dx*dy	g(x',y')
0	0	0	0	0.7680	0.1340	0.0000	0.0000	0.0000	0.0000
0	0	0	0	0.2680	0.0000	0.0000	0.0000	0.0000	0.0000
0	0	0	0	0.7680	0.8660	0.0000	0.0000	0.0000	0.0000
0	0	0	0	0.2680	0.7321	0.0000	0.0000	0.0000	0.0000
0	0	0	0	0.6340	0.6340	0.0000	0.0000	0.0000	0.0000
0	0	1	2	0.1340	0.5000	0.1340	0.0000	0.0670	0.2010
0	0	2	0	0.6340	0.3660	1.2680	0.0000	-0.4641	0.8038
0	0	0	0	0.1340	0.2321	0.0000	0.0000	0.0000	0.0000
0	0	0	1	0.5000	0.1340	0.0000	0.0000	0.0670	0.0670
1	2	3	4	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
1	2	3	4	0.5000	0.8660	1.0000	0.8660	0.0000	2.8660
4	0	0	0	0.0000	0.7321	0.0000	-2.9282	0.0000	1.0718
0	0	0	0	0.3660	0.6340	0.0000	0.0000	0.0000	0.0000
0	1	0	3	0.8660	0.5000	0.0000	0.5000	0.8660	1.3660
3	4	0	0	0.3660	0.3660	-1.0981	0.3660	-0.1340	2.1340
4	0	0	0	0.8660	0.2321	-3.4641	-0.9282	0.8038	0.4115
0	0	0	0	0.2321	0.1340	0.0000	0.0000	0.0000	0.0000
0	0	0	0	0.7321	0.0000	0.0000	0.0000	0.0000	0.0000
0	0	0	0	0.2321	0.8660	0.0000	0.0000	0.0000	0.0000
0	0	0	0	0.7321	0.7321	0.0000	0.0000	0.0000	0.0000

Forward vs Backward Mapping
(Bilinear Interpolation)

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos 30^\circ & \sin 30^\circ \\ -\sin 30^\circ & \cos 30^\circ \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix}$$

	y=0	y=1
x=0	1	2
x=1	3	4

Forward Mapping

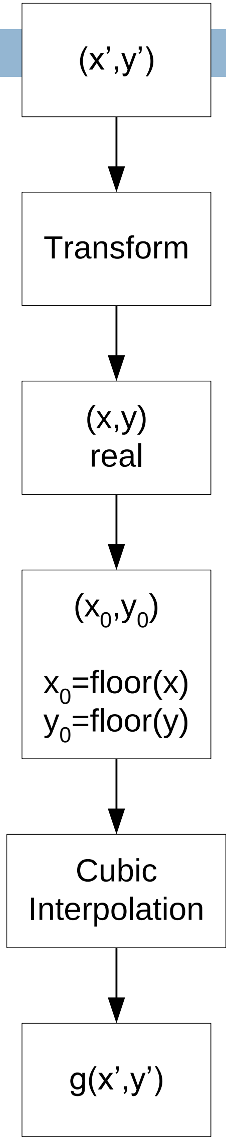
	y=-1	y=0	y=1	y=2
x=-2	0	0	0	0
x=-1	0	0	2	0
x=0	0	1	4	0
x=1	0	0	3	0
x=2	0	0	0	0

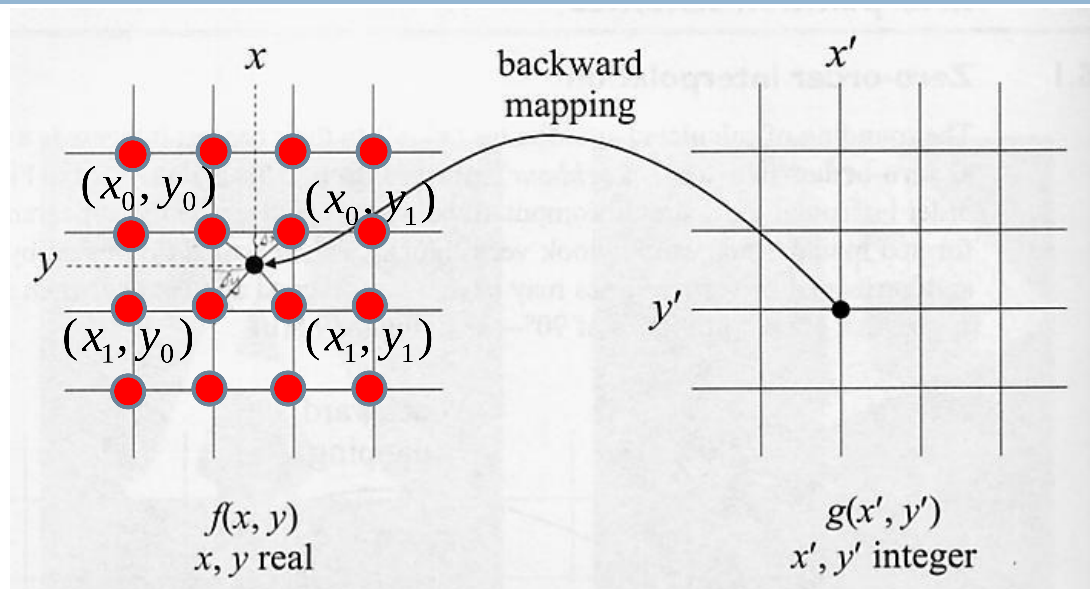
Backward Mapping

	y=-1	y=0	y=1	y=2
x=-2	0	0	0	0
x=-1	0	0.2010	0.8038	0
x=0	0.0670	1	2.8660	1.0718
x=1	0	1.3660	2.1340	0
x=2	0	0	0	0

	y=-1	y=0	y=1	y=2
x=-2	0	0	0	0
x=-1	0	0	1	0
x=0	0	1	3	1
x=1	0	1	2	0
x=2	0	0	0	0

High-order interpolation
(cubic interpolation)





$$\Delta x = x - x_0$$

$$\Delta y = y - y_0$$

$$g(x', y') = \sum_{m=-1}^{2} \sum_{n=-1}^{2} f(x_0 + m, y_0 + n) R(m - \Delta x) R(\Delta y - n)$$

$$R(k) = \frac{1}{6}[P(k+2)^3 - 4P(k+1)^3 - 4P(k-1)^3 + 6P(k)^3]$$

$$P(z) = \begin{cases} z & z > 0 \\ 0 & z \le 0 \end{cases}$$

Zero-Order Interpolation

First-Order (Bilinear) Interpolation

Cubic Interpolation

Homework

- ให้นศ.สร้างภาพขนาด 2x2 พิกเซล ในรูปของ 2D-array
- กำหนดค่าใน 2D-array ด้วยการ random ค่าระหว่าง 3-7
- ทำการหมุนภาพด้วยเทคนิค Backward mapping และใช้การกำหนดค่าระดับความเข้มแสงผลลัพธ์ด้วยเทคนิคการเฉลี่ยค่าแบบ bilinear interpolation
  - มุมที่หมุนขึ้นกับเลขท้ายของรหัสนศ. Mod 4
    - [zeta(0) zeta(1) zeta(2) zeta(3)] = [30, 60, 120, 150]
- แสดงขั้นตอนการคำนวณการหมุนภาพโดยละเอียดลงในกระดาษ A4