

# Feature selection and dimensional reduction

# Feature selection

**How would we know which feature to be selected, combined, or removed?**

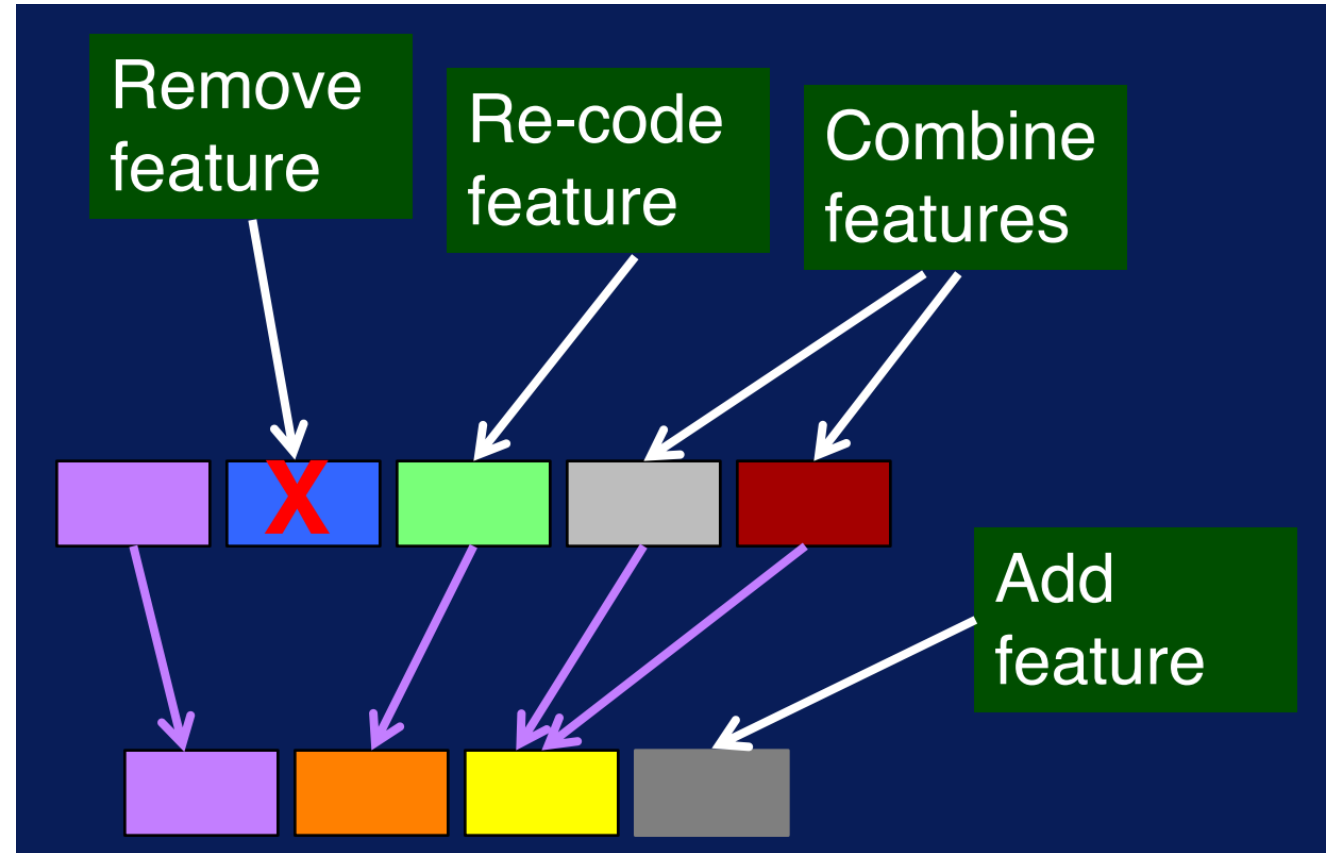
# Feature selection

## Large Dataset

- Huge amount of features are collected
- Not all are important

Ex: Removing / Filtering

**Note:** Removing carefully.



When should data be  
removed?

# Feature selection

## Removing features

- feature with  $\text{std} = 0$  or  $\text{variance} = 0$

ex. Remove 'Age' if

All students in the dataset

are in the same age / same


year

-

When should data be  
combined?

# Feature selection


Property type
Single family home
Townhouse
Condo



D1	D2
1	0
0	1
0	0

- D1: Is it a single family home?
- D2: Is it a townhouse?
- D1 = D2 = 0 indicates a condo.

Property type
Single family home
Townhouse
Condo
Coop
Multi-family
Mobile home



Property type
Single family home
Townhouse
Others

- Is it necessary to keep all categories?

When should data be transformed or replaced?



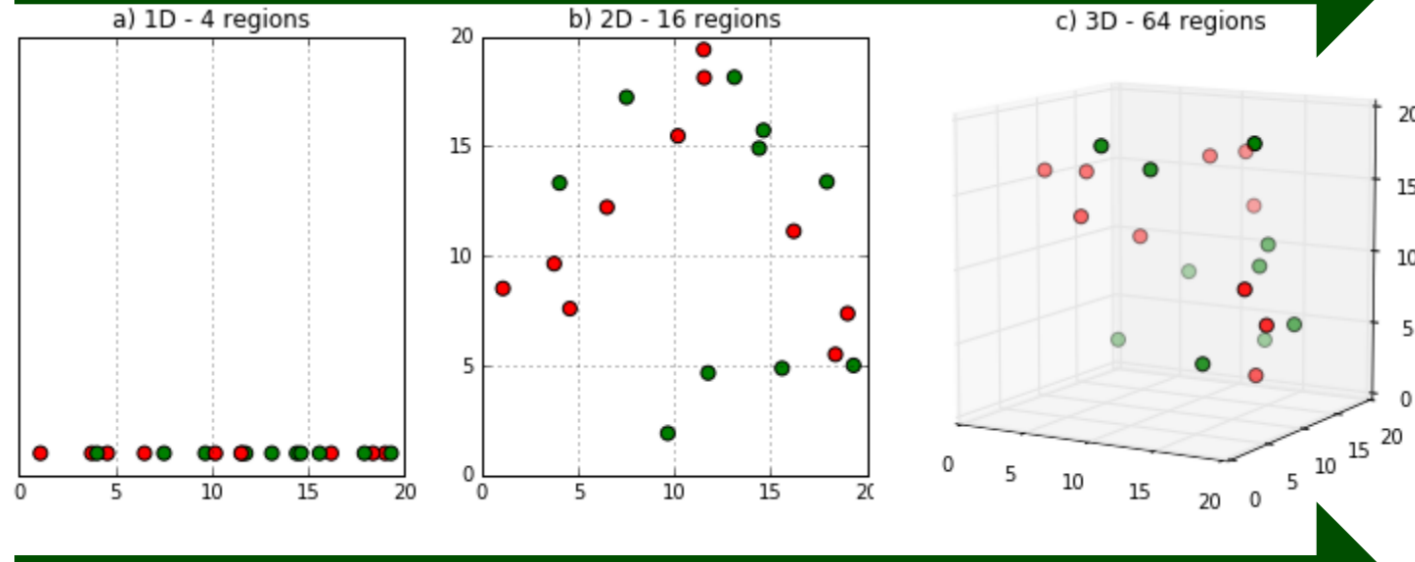
# Dimension reduction

**How would we visualize the data?**

Is the data perfect or noisy?

# Dimension of data

**Data gets increasingly sparse**

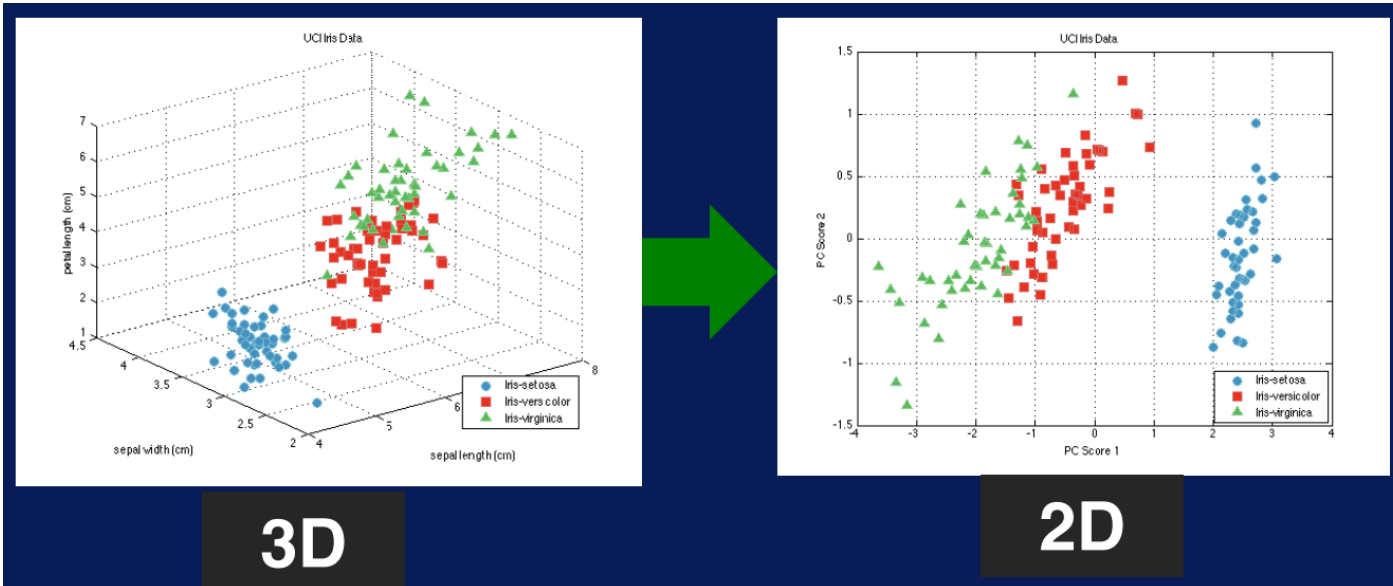


**Analysis results degrade**

- certain calculations used in analysis become much more difficult to define and calculate effectively
- distances between samples are harder to compare since all samples are far away from each other
- the difficulty of dealing with high dimensional data and as referred to as the curse of dimensionality

the number of **dimensions increases**,  
the number of **regions increases exponentially** and  
the data becomes increasingly sparse

# Dimensional reduction

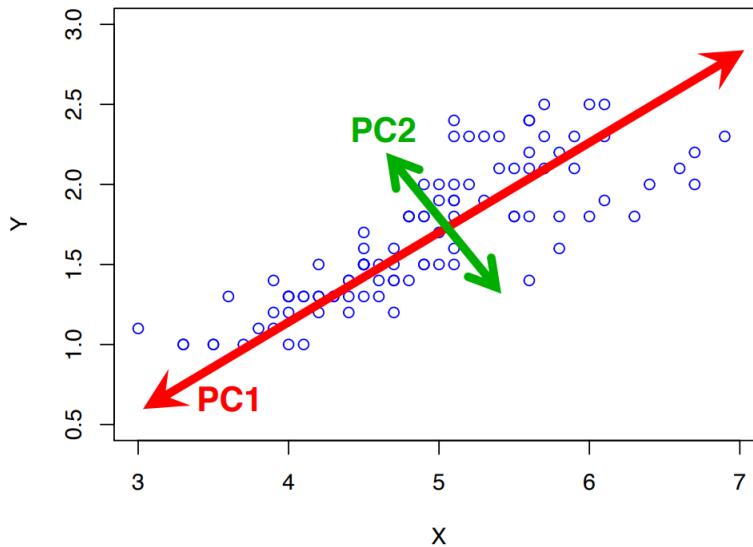


## Dimensional Reduction Techniques

- Principle Component Analysis (PCA)
- Finding new mapping on to principle (significant) space (domain)
- Eigen-based Technique

# Dimensional reduction

## Principle Component Analysis (PCA)



### - Old representation

X-Y Coordinate

feature 1 – 2 Coordinate

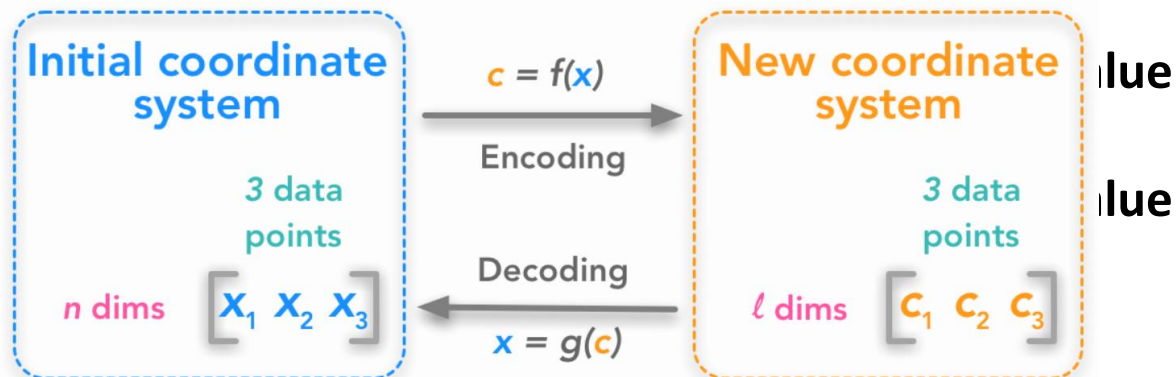
### - New PCA Coordinate

PC1 – PC2 Coordinate

PC1 = Eigen\_Vector with max

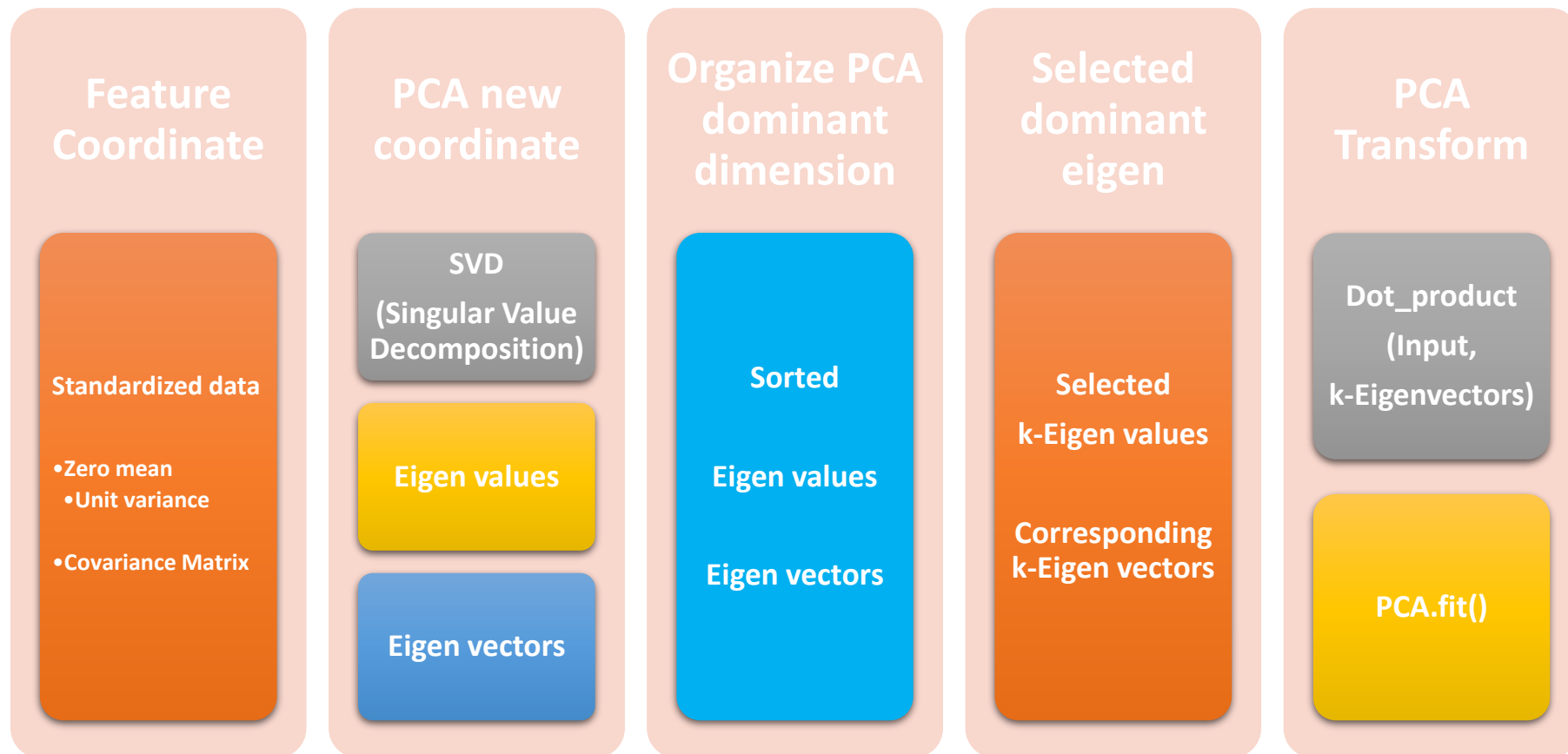
PC2 = Eigen\_Vector with 2<sup>nd</sup>

weighted sum of old coordinate



# PCA:

## Dimensional reduction



# PCA:

## #1: Standardized data

Feature  
Coordinate

Standardized data

- Zero mean
- Unit variance
- Covariance Matrix

**transformation** of the data onto

- mean subtraction

**mean = 0**

- unit scale

**mean=0 and**

**variance=1**

	a	b
1	9	39
2	15	56
3	25	93
4	14	61
5	10	50
6	18	75
7	0	32
8	16	85
9	5	42
10	19	70

# PCA:

## #1: Standardized data

Feature  
Coordinate

Standardized data

- Zero mean
- Unit variance
- Covariance Matrix

**transformation** of the data onto

- mean subtraction

**mean = 0**

- unit scale

**mean=0 and**

**variance=1**

	H	M	Hn	Mn
1	9	39		
2	15	56		
3	25	93		
4	14	61		
5	10	50		
6	18	75		
7	0	32		
8	16	85		
9	5	42		
10	19	70		
sum	131	603		
u	13.1	60.3		
std	7.279	20.29		

# PCA:

## #2: Covariance matrix

Feature  
Coordinate

Standardized data

- Zero mean
- Unit variance
- Covariance Matrix

	H	M	Hn	Mn	Hn.Hn	Mn.Mn	Hn.Mn
1	9	39					
2	15	56					
3	25	93					
4	14	61					
5	10	50					
6	18	75					
7	0	32					
8	16	85					
9	5	42					
10	19	70					
sum	131	603					
u	13.1	60.3					
std	7.279	20.29					

$$C = \begin{bmatrix} cov(HH) & cov(HM) \\ cov(MH) & cov(MM) \end{bmatrix}$$

$$cov(HH) = \frac{1}{N} \sum_{i=1}^N (H_i - \bar{H})(H_i - \bar{H})$$

$$cov(HM) = \frac{1}{N} \sum_{i=1}^N (H_i - \bar{H})(M_i - \bar{M})$$



# PCA:

## #3: Eigen-calculation

$$C = \begin{bmatrix} \text{cov}(HH) & \text{cov}(HM) \\ \text{cov}(MH) & \text{cov}(MM) \end{bmatrix}$$

Form the matrix  $A - \lambda I$ :

*eigenvalue(C), eigenvector(C)*

$$A - \lambda I = \begin{pmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{pmatrix} - \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix} = \begin{pmatrix} 1-\lambda & -3 & 3 \\ 3 & -5-\lambda & 3 \\ 6 & -6 & 4-\lambda \end{pmatrix}$$

$$A \cdot v = \lambda \cdot v$$

Calculate  $\det(A - \lambda I)$ :

$$\begin{aligned} \det(A - \lambda I) &= (1 - \lambda) \begin{vmatrix} -5 - \lambda & 3 \\ -6 & 4 - \lambda \end{vmatrix} - (-3) \begin{vmatrix} 3 & 3 \\ 6 & 4 - \lambda \end{vmatrix} + 3 \begin{vmatrix} 3 & -5 - \lambda \\ 6 & -6 \end{vmatrix} \\ &= (1 - \lambda) ((-5 - \lambda)(4 - \lambda) - (3)(-6)) + 3(3(4 - \lambda) - 3 \times 6) + 3(3 \times (-6) - (-5 - \lambda)6) \\ &= (1 - \lambda)(-20 + 5\lambda - 4\lambda + \lambda^2 + 18) + 3(12 - 3\lambda - 18) + 3(-18 + 30 + 6\lambda) \\ &= (1 - \lambda)(-2 + \lambda + \lambda^2) + 3(-6 - 3\lambda) + 3(12 + 6\lambda) \\ &= -2 + \lambda + \lambda^2 + 2\lambda - \lambda^2 - \lambda^3 - 18 - 9\lambda + 36 + 18\lambda \\ &= 16 + 12\lambda - \lambda^3. \end{aligned}$$

To find solutions to  $\det(A - \lambda I) = 0$  i.e., to solve

$$\lambda^3 - 12\lambda - 16 = 0.$$

PCA new  
coordinate

SVD  
(Singular Value  
Decomposition)

Eigen values

Eigen vectors

# PCA:

## #3: Eigen-calculation

$$C = \begin{bmatrix} \text{cov}(HH) & \text{cov}(HM) \\ \text{cov}(MH) & \text{cov}(MM) \end{bmatrix}$$

*eigenvalue(C), eigenvector(C)*

To find solutions to  $\det(A - \lambda I) = 0$  i.e., to solve

$$\lambda^3 - 12\lambda - 16 = 0.$$

eigenvalues of  $A$  are  $\lambda = 4, -2$ . ( $\lambda = -2$  is a repeated root

**Case 1:  $\lambda = 4$**

$$\left( \begin{array}{cccc|l} -3 & -3 & 3 & 0 & R1 \\ 3 & -9 & 3 & 0 & R2 \\ 6 & -6 & 0 & 0 & R3 \end{array} \right) \xrightarrow{R1 \rightarrow -1/3 \times R1} \left( \begin{array}{cccc|l} 1 & 1 & -1 & 0 & R1 \\ 3 & -9 & 3 & 0 & R2 \\ 6 & -6 & 0 & 0 & R3 \end{array} \right)$$

$$\xrightarrow{\begin{array}{l} R2 \rightarrow R2 - 3 \times R1 \\ R3 \rightarrow R3 - 6 \times R1 \end{array}} \left( \begin{array}{cccc|l} 1 & 1 & -1 & 0 & R1 \\ 0 & -12 & 6 & 0 & R2 \\ 0 & -12 & 6 & 0 & R3 \end{array} \right)$$

$$\xrightarrow{R2 \rightarrow -1/12 \times R2} \left( \begin{array}{cccc|l} 1 & 1 & -1 & 0 & R1 \\ 0 & 1 & -1/2 & 0 & R2 \\ 0 & -12 & 6 & 0 & R3 \end{array} \right)$$

$$\xrightarrow{R3 \rightarrow R3 + 12 \times R2} \left( \begin{array}{cccc|l} 1 & 1 & -1 & 0 & R1 \\ 0 & 1 & -1/2 & 0 & R2 \\ 0 & 0 & 0 & 0 & R3 \end{array} \right)$$

$$\xrightarrow{R1 \rightarrow R1 - R2} \left( \begin{array}{cccc|l} 1 & 0 & -1/2 & 0 & R1 \\ 0 & 1 & -1/2 & 0 & R2 \\ 0 & 0 & 0 & 0 & R3 \end{array} \right)$$

PCA new  
coordinate

SVD  
(Singular Value  
Decomposition)

Eigen values

Eigen vectors

# PCA:

## #3: Eigen-calculation

$$C = \begin{bmatrix} \text{cov}(HH) & \text{cov}(HM) \\ \text{cov}(MH) & \text{cov}(MM) \end{bmatrix}$$

*eigenvalue(C), eigenvector(C)*

To find solutions to  $\det(A - \lambda I) = 0$  i.e., to solve

$$\lambda^3 - 12\lambda - 16 = 0.$$

eigenvalues of  $A$  are  $\lambda = 4, -2$ . ( $\lambda = -2$  is a repeated root)

Case 1:  $\lambda = 4$

$$(A - \lambda I : 0)$$

$$\xrightarrow{R1 \rightarrow R1 - R2} \begin{pmatrix} 1 & 0 & -1/2 & 0 \\ 0 & 1 & -1/2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{matrix} \text{R1} \\ \text{R2} \\ \text{R3} \end{matrix}$$

$$x_1 - 1/2x_3 = 0$$

$$x_2 - 1/2x_3 = 0$$

$$\mathbf{x} = \begin{pmatrix} x_1 = \frac{x_3}{2} \\ x_2 = \frac{x_3}{2} \\ x_3 \end{pmatrix} = x_3 \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ 1 \end{pmatrix}$$

PCA new  
coordinate

SVD  
(Singular Value  
Decomposition)

Eigen values

Eigen vectors

# PCA:

## #3: Eigen-calculation

$$C = \begin{bmatrix} \text{cov}(HH) & \text{cov}(HM) \\ \text{cov}(MH) & \text{cov}(MM) \end{bmatrix}$$

*eigenvalue(C), eigenvector(C)*

To find solutions to  $\det(A - \lambda I) = 0$  i.e., to solve

$$\lambda^3 - 12\lambda - 16 = 0.$$

eigenvalues of  $A$  are  $\lambda = 4, -2$ . ( $\lambda = -2$  is a repeated root

Case 2:  $\lambda = -2$

$$\left( A - \lambda I : \mathbf{0} \right) \begin{array}{l} \left( \begin{array}{cccc} 3 & -3 & 3 & 0 \\ 3 & -3 & 3 & 0 \\ 6 & -6 & 6 & 0 \end{array} \right) \begin{array}{l} \text{R1} \\ \text{R2} \\ \text{R3} \end{array} \end{array} \xrightarrow{R1 \rightarrow 1/3 \times R3} \begin{array}{l} \left( \begin{array}{cccc} 1 & -1 & 1 & 0 \\ 3 & -3 & 3 & 0 \\ 6 & -6 & 6 & 0 \end{array} \right) \begin{array}{l} \text{R1} \\ \text{R2} \\ \text{R3} \end{array} \\ \xrightarrow{R2 \rightarrow R2 - 3 \times R1, R3 \rightarrow R3 - 6 \times R1} \left( \begin{array}{cccc} 1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right) \begin{array}{l} \text{R1} \\ \text{R2} \\ \text{R3} \end{array} \end{array}$$

$$x_1 + x_2 - x_3 = 0,$$

$$\mathbf{x} = \begin{pmatrix} x_1 = x_3 - x_2 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_3 - x_2 \\ x_2 \\ x_3 \end{pmatrix} = x_3 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + x_2 \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \quad \text{for any } x_2, x_3 \in \mathbb{R} \setminus \{0\}$$

PCA new  
coordinate

SVD  
(Singular Value  
Decomposition)

Eigen values

Eigen vectors

# PCA:

## #3: Eigen-decomposition

$$C = \begin{bmatrix} \text{cov}(HH) & \text{cov}(HM) \\ \text{cov}(MH) & \text{cov}(MM) \end{bmatrix}$$

*eigenvalue(C), eigenvector(C)*

Sorted eigen\_values

[377.32003303, 5.66607808]

Corresponding eigen\_vectors of sorted eigen\_values

$\begin{bmatrix} -0.32008244 & -0.94738969 \\ -0.94738969 & 0.32008244 \end{bmatrix}$

Organize  
PCA  
dominant  
dimension

Sorted

Eigen values

Eigen vectors

```
eig_vals, eig_vecs = np.linalg.eig(C)
print('Eigenvectors \n%s' % eig_vecs)
print('\nEigenvalues \n%s' % eig_vals)
```

Eigenvectors  
[[-0.94738969 -0.32008244]  
[ 0.32008244 -0.94738969]]

Eigenvalues  
[ 5.66607808 377.32003303]

Eigenvalues and eigenvectors of C  
is learned (fitted) from data (H,M)  
cannot be controlled  
sometimes resulted in unsolvable eigenvalue

# PCA:

## #3: Eigen-calculation

Through SVD (Singular Value Decomposition)

Organize  
PCA  
dominant  
dimension

Sorted

Eigen values

Eigen vectors

$$C = \begin{bmatrix} \text{cov}(HH) & \text{cov}(HM) \\ \text{cov}(MH) & \text{cov}(MM) \end{bmatrix}$$

$V$ : *eigenvalue*( $C^T C$ ), *eigenvector*( $C^T C$ )

$U$ : *eigenvalue*( $CC^T$ ), *eigenvector*( $CC^T$ )

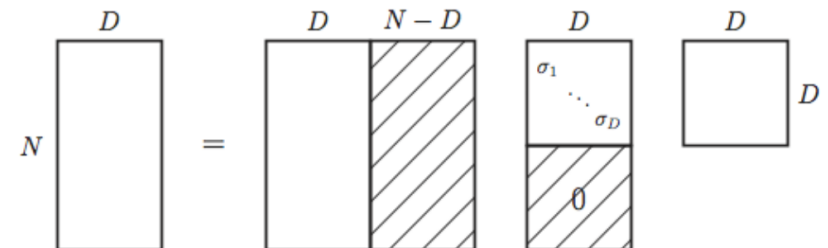
*eigenvalue* = diagonal (D)

*eigenvector* = column vector (U)

$$A = U D V^T$$

Left singular vectors (U), Singular values (D), Right singular vectors (V)

$$m \begin{bmatrix} n \\ \end{bmatrix} = m \begin{bmatrix} m \\ \end{bmatrix} m \begin{bmatrix} n \\ \end{bmatrix} n \begin{bmatrix} n \\ \end{bmatrix}$$



# PCA:

## #3: Eigen-calculation

Through SVD (Singular Value Decomposition)

Organize  
PCA  
dominant  
dimension

Sorted

Eigen values

Eigen vectors

$$C = \begin{bmatrix} cov(HH) & cov(HM) \\ cov(MH) & cov(MM) \end{bmatrix}$$

*eigenvalue*( $C^T C$ ), *eigenvector*( $C^T C$ )

*eigenvalue* = diagonal (D)

*eigenvector* = column vector (U)

```
u,s,v = np.linalg.svd(C)
print('\nEigenvalues \n%s\n' %s)
print('Eigenvectors_u \n%s\n' %u)
print('Eigenvectors_v \n%s\n' %v)
```

Eigenvalues  
[377.32003303 5.66607808]

Eigenvectors\_u  
[[-0.32008244 -0.94738969]  
 [-0.94738969 0.32008244]]

Eigenvectors\_v  
[[-0.32008244 -0.94738969]  
 [-0.94738969 0.32008244]]

# PCA:

## #4: Selecting and transforming to principle component

Through SVD (Singular Value Decomposition)

**K = 1**

**PCA( i ) = X (i) • [Eigen vector (pca i)]**

**Selected  
dominant  
eigen**

Selected  
k-Eigen values

Corresponding  
k-Eigen vectors

**PCA  
Transform**

Dot\_product  
(Input,  
k-Eigenvectors)

PCA.fit()

```
u,s,v = np.linalg.svd(C)
print('\nEigenvalues \n%s\n' %s)
print('Eigenvectors_u \n%s\n' %u)
print('Eigenvectors_v \n%s\n' %v)
```

Eigenvalues  
[377.32003303 5.66607808]

Eigenvectors\_u  
[[-0.32008244 -0.94738969]  
[-0.94738969 0.32008244]]

Eigenvectors\_v  
[[-0.32008244 -0.94738969]  
[-0.94738969 0.32008244]]

**10x1**

**10x2**

**2x1**

	H	M
1	9	39
2	15	56
3	25	93
4	14	61
5	10	50
6	18	75
7	0	32
8	16	85
9	5	42
10	19	70

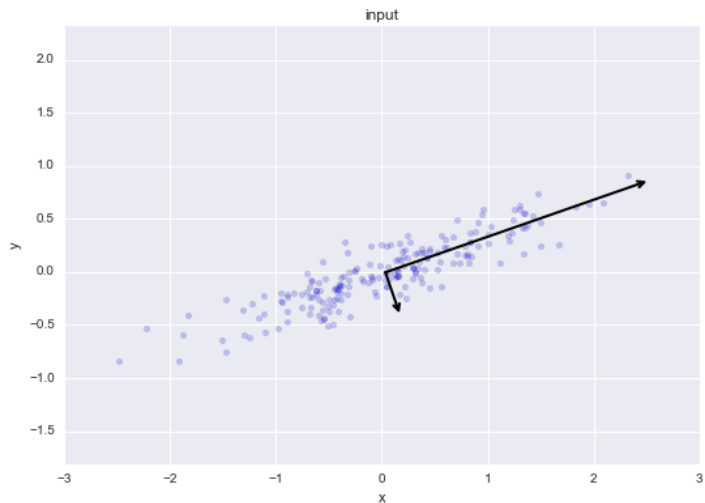
pca1  
[[-0.32008244]  
[-0.94738969]]



# PCA:

## #4: Selecting and transforming to principle component

Through SVD (Singular Value Decomposition)



**X-Y coordinate**



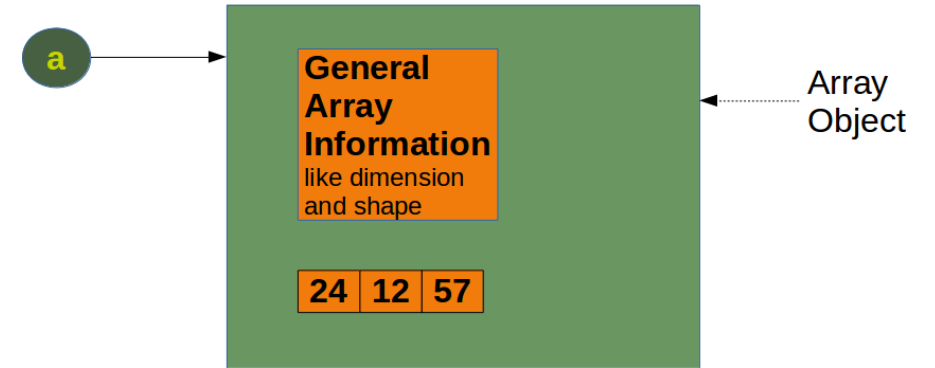
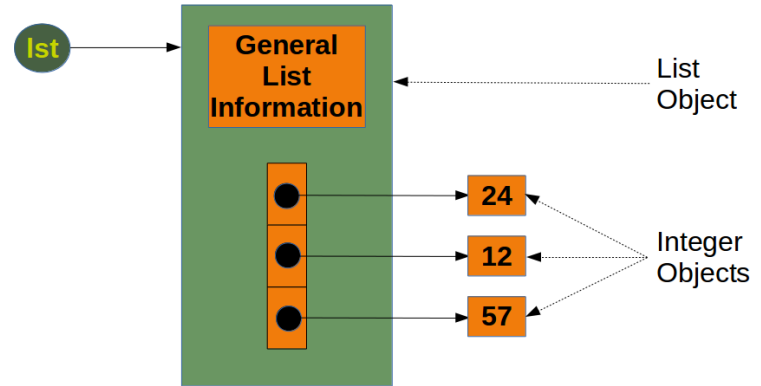
**Transformed on to pca1**

# Organizing Data

**How would we arrange data for the model?**

**Is the data in correct format?**

# Numpy array vs list



## • Numpy Array

- Numpy data structures perform better in:
- Size - less space
- Performance faster speed
- Functionality
  - optimize for math functions

## Lists

Every new element  
-> additional 8 bytes  
for the reference to new object.  
-> The integer object itself  
consumes 28 bytes.

# Data Structure

## ❑ How can we convert from Pandas dataframe to numpy array

```
import pandas as pd
import numpy as np
```

```
df = pd.DataFrame({'a':[21, 23, 32, 52],
                   'b':[72, 78, 74, 54],
                   'c':[67.1, 69.5, 56.6, 76.2]})
df
```

	a	b	c
0	21	72	67.1
1	23	78	69.5
2	32	74	56.6
3	52	54	76.2

df.shape

(4, 3)

df.dtypes

age           int64  
height       int64  
weight       int64  
dtype: object

# Data Structure

## ❑ From Pandas dataframe to numpy array

	a	b	c
0	21	72	67.1
1	23	78	69.5
2	32	74	56.6
3	52	54	76.2

**df.values( ) vs df.to\_numpy( ) ????**

# Data Structure

## ❑ From Pandas dataframe to numpy array

	a	b	c
0	21	72	67.1
1	23	78	69.5
2	32	74	56.6
3	52	54	76.2

**df.values( ) vs df.to\_numpy( ) ????**

```
x=df.to_numpy()  
x
```

```
array([[21. , 72. , 67.1],  
       [23. , 78. , 69.5],  
       [32. , 74. , 56.6],  
       [52. , 54. , 76.2]])
```

```
df.values
```

```
array([[21. , 72. , 67.1],  
       [23. , 78. , 69.5],  
       [32. , 74. , 56.6],  
       [52. , 54. , 76.2]])
```

# Data Structure

## ❑ From Pandas dataframe to numpy array

	a	b	c
0	21	72	67.1
1	23	78	69.5
2	32	74	56.6
3	52	54	76.2

**`df.transpose( )` vs `df.values.transpose( )` ????**

# Data Structure

## ❑ From Pandas dataframe to numpy array

	a	b	c
0	21	72	67.1
1	23	78	69.5
2	32	74	56.6
3	52	54	76.2

**df.transpose( ) vs df.values.transpose( ) ????**

```
df.transpose()
```

	0	1	2	3
a	21.0	23.0	32.0	52.0
b	72.0	78.0	74.0	54.0
c	67.1	69.5	56.6	76.2

```
df.values.transpose()
```

```
array([[21. , 23. , 32. , 52. ],  
       [72. , 78. , 74. , 54. ],  
       [67.1, 69.5, 56.6, 76.2]])
```



# Data Structure

## ❑ From Pandas dataframe to numpy array

	a	b	c
0	21	72	67.1
1	23	78	69.5
2	32	74	56.6
3	52	54	76.2

`df.values.transpose( )` vs `df.values.reshape ( )` ????

# Data Structure

## ❑ From Pandas dataframe to numpy array

	a	b	c
0	21	72	67.1
1	23	78	69.5
2	32	74	56.6
3	52	54	76.2

**df.values.transpose( ) vs df.values.reshape ( ) ????**

```
df.values.transpose()
```

```
array([[21. , 23. , 32. , 52. ],  
       [72. , 78. , 74. , 54. ],  
       [67.1, 69.5, 56.6, 76.2]])
```

```
df.values.reshape(3,4)
```

```
array([[21. , 72. , 67.1, 23. ],  
       [78. , 69.5, 32. , 74. ],  
       [56.6, 52. , 54. , 76.2]])
```

# Data Structure

## ❑ From Pandas dataframe to numpy array

	a	b	c
0	21	72	67.1
1	23	78	69.5
2	32	74	56.6
3	52	54	76.2

### df.values.transpose( ) vs df.values.reshape ( ) ????

```
df.values.transpose()
```

```
array([[21. , 23. , 32. , 52. ],  
       [72. , 78. , 74. , 54. ],  
       [67.1, 69.5, 56.6, 76.2]])
```

```
df.values.reshape(3,4,order='C')
```

```
array([[21. , 72. , 67.1, 23. ],  
       [78. , 69.5, 32. , 74. ],  
       [56.6, 52. , 54. , 76.2]])
```

```
df.values.reshape(3,4,order='F')
```

```
array([[21. , 52. , 74. , 69.5],  
       [23. , 72. , 54. , 56.6],  
       [32. , 78. , 67.1, 76.2]])
```

# Data Structure

❑ Reshape 2D to 3D ????

`df.values.reshape(1,3,4)` vs `df.values.reshape(3,4,1)`

	a	b	c
0	21	72	67.1
1	23	78	69.5
2	32	74	56.6
3	52	54	76.2

# Data Structure

## ❑ Reshape 2D to 3D ????

	a	b	c
0	21	72	67.1
1	23	78	69.5
2	32	74	56.6
3	52	54	76.2

**df.values.reshape(1,3,4) vs df.values.reshape(3,4,1)**

```
df.values.reshape(3,4,1)
```

```
array([[[21. ],  
        [72. ],  
        [67.1],  
        [23. ]],  
       [[78. ],  
        [69.5],  
        [32. ],  
        [74. ]],  
       [[56.6],  
        [52. ],  
        [54. ],  
        [76.2]]])
```

```
df.values.reshape(1,3,4)
```

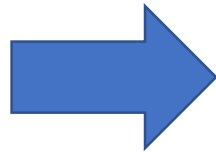
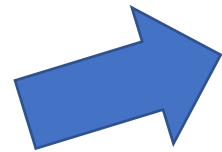
```
array([[[21. , 72. , 67.1, 23. ],  
        [78. , 69.5, 32. , 74. ],  
        [56.6, 52. , 54. , 76.2]]])
```

# Data Structure

## ❑ Time-series split

	a	b	c
0	21	72	67.1
1	23	78	69.5
2	32	74	56.6
3	52	54	76.2

Time step = 2  
Time stride = 1



	a	b	c
0	21	72	67.1
1	23	78	69.5
1	23	78	69.5
2	32	74	56.6
2	32	74	56.6
3	52	54	76.2

**.shape = ???**

# Activity: Data Preparation

## ☐ Time-series split

No	uts	Time Diff	x	y	z	Type A	Type B
1	2019-01-17 09:14:17+07:00		2.729	-3.035	8.684	1	1
2	2019-01-17 09:14:38+07:00		2.586	-2.633	500	1	1
3	2019-01-17 09:15:02+07:00		1.67	-2.117	8.512	0	0
4	2019-01-17 09:15:26+07:00			-9.788	2.519	0	1
5	2019-01-17 09:15:48+07:00		0.241		3.03	1	0
6	2019-01-17 09:16:08+07:00		1.78		3.114	1	0
7	2019-01-17 09:16:59+07:00		1.823		4.414	0	0
8	2019-01-17 09:17:23+07:00		0.103	-8.909	5.472	0	0
9	2019-01-17 09:17:44+07:00		2.046	-2.218	8.572	0	1
10	2019-01-17 09:18:05+07:00		2.28	-2.421	8.761	0	1

1. Clean / Combine Data

2. Calculate Time diff  
What would be a problem?

3. What would be the shape for  
Time-series split with  
Time step = 3 / Time stride = 2?  
(#sample, #time\_step, #Feature)