Mention and explain the components of First Order Logic

**First-order logic** is a collection of [formal systems](https://en.wikipedia.org/wiki/Formal_system) used in [mathematics](https://en.wikipedia.org/wiki/Mathematics), [philosophy](https://en.wikipedia.org/wiki/Philosophy), [linguistics](https://en.wikipedia.org/wiki/Linguistics), and [computer science](https://en.wikipedia.org/wiki/Computer_science). It is also known as **first-order predicate calculus**, the**lower predicate calculus**, **quantification theory**, and [**predicate logic**](https://en.wikipedia.org/wiki/Predicate_logic). First-order logic uses [quantified variables](https://en.wikipedia.org/wiki/Quantification_(logic)) over non-logical objects. It allows the use of sentences that contain variables, so that rather than propositions such as *Socrates is a man* one can have expressions in the form *X is a man* where X is a variable.[[1]](https://en.wikipedia.org/wiki/First-order_logic#cite_note-1) This distinguishes it from[propositional logic](https://en.wikipedia.org/wiki/Propositional_logic), which does not use quantifiers.

A theory about a topic is usually a first-order logic together with a specified [domain of discourse](https://en.wikipedia.org/wiki/Domain_of_discourse) over which the quantified variables range, finitely many functions from that domain to itself, finitely many predicates defined on that domain, and a set of axioms believed to hold for those things. Sometimes "theory" is understood in a more formal sense, which is just a set of sentences in first-order logic.

The adjective "first-order" distinguishes first-order logic from [higher-order logic](https://en.wikipedia.org/wiki/Higher-order_logic) in which there are predicates having predicates or functions as arguments, or in which one or both of predicate quantifiers or function quantifiers are permitted.[[2]](https://en.wikipedia.org/wiki/First-order_logic#cite_note-2) In first-order theories, predicates are often associated with sets. In interpreted higher-order theories, predicates may be interpreted as sets of sets.

There are many [deductive systems](https://en.wikipedia.org/wiki/Deductive_system) for first-order logic which are both [sound](https://en.wikipedia.org/wiki/Soundness#Logical_systems) (all provable statements are true in all models) and [complete](https://en.wikipedia.org/wiki/Completeness_(logic)) (all statements which are true in all models are provable). Although the [logical consequence](https://en.wikipedia.org/wiki/Logical_consequence) relation is only [semidecidable](https://en.wikipedia.org/wiki/Semidecidability), much progress has been made in [automated theorem proving](https://en.wikipedia.org/wiki/Automated_theorem_proving) in first-order logic. First-order logic also satisfies several [metalogical](https://en.wikipedia.org/wiki/Metalogic) theorems that make it amenable to analysis in [proof theory](https://en.wikipedia.org/wiki/Proof_theory), such as the [Löwenheim–Skolem theorem](https://en.wikipedia.org/wiki/L%C3%B6wenheim%E2%80%93Skolem_theorem) and the [compactness theorem](https://en.wikipedia.org/wiki/Compactness_theorem).

First-order logic is the standard for the formalization of mathematics into [axioms](https://en.wikipedia.org/wiki/Axiomatic_system) and is studied in the [foundations of mathematics](https://en.wikipedia.org/wiki/Foundations_of_mathematics). [Peano arithmetic](https://en.wikipedia.org/wiki/Peano_arithmetic) and [Zermelo–Fraenkel set theory](https://en.wikipedia.org/wiki/Zermelo%E2%80%93Fraenkel_set_theory) are axiomatizations of [number theory](https://en.wikipedia.org/wiki/Number_theory) and [set theory](https://en.wikipedia.org/wiki/Set_theory), respectively, into first-order logic. No first-order theory, however, has the strength to uniquely describe a structure with an infinite domain, such as the [natural numbers](https://en.wikipedia.org/wiki/Natural_number) or the [real line](https://en.wikipedia.org/wiki/Real_line). Axioms systems that do fully describe these two structures (that is, [categorical](https://en.wikipedia.org/wiki/Categorical_theory) axiom systems) can be obtained in stronger logics such as [second-order logic](https://en.wikipedia.org/wiki/Second-order_logic).

For a history of first-order logic and how it came to dominate formal logic, see José Ferreirós (2001).

Syntax

There are two key parts of first-order logic. The [syntax](https://en.wikipedia.org/wiki/Syntax) determines which collections of symbols are legal expressions in first-order logic, while the [semantics](https://en.wikipedia.org/wiki/Semantics) determine the meanings behind these expressions.

**Alphabet**[

Unlike natural languages, such as English, the language of first-order logic is completely formal, so that it can be mechanically determined whether a given expression is legal. There are two key types of legal expressions: **terms**, which intuitively represent objects, and **formulas**, which intuitively express predicates that can be true or false. The terms and formulas of first-order logic are strings of **symbols** which together form the **alphabet** of the language. As with all [formal languages](https://en.wikipedia.org/wiki/Formal_language), the nature of the symbols themselves is outside the scope of formal logic; they are often regarded simply as letters and punctuation symbols.

It is common to divide the symbols of the alphabet into **logical symbols**, which always have the same meaning, and **non-logical symbols**, whose meaning varies by interpretation. For example, the logical symbol {\displaystyle \land } always represents "and"; it is never interpreted as "or". On the other hand, a non-logical predicate symbol such as Phil(*x*) could be interpreted to mean "*x* is a philosopher", "*x* is a man named Philip", or any other unary predicate, depending on the interpretation at hand.

**Logical symbols**[[edit](https://en.wikipedia.org/w/index.php?title=First-order_logic&action=edit&section=4)]

There are several logical symbols in the alphabet, which vary by author but usually include:

* The quantifier symbols [∀](https://en.wikipedia.org/wiki/Universal_quantification) and [∃](https://en.wikipedia.org/wiki/Existential_quantification)
* The [logical connectives](https://en.wikipedia.org/wiki/Logical_connective): ∧ for [conjunction](https://en.wikipedia.org/wiki/Logical_conjunction), ∨ for [disjunction](https://en.wikipedia.org/wiki/Disjunction), → for [implication](https://en.wikipedia.org/wiki/Material_conditional), ↔ for [biconditional](https://en.wikipedia.org/wiki/Logical_biconditional), ¬ for negation. Occasionally other logical connective symbols are included. Some authors use C*pq*, instead of →, and E*pq*, instead of ↔, especially in contexts where → is used for other purposes. Moreover, the horseshoe ⊃ may replace →; the triple-bar ≡ may replace ↔; a tilde (~), N*p*, or F*pq*, may replace ¬; *||*, or A*pq* may replace ∨; and &, K*pq*, or the middle dot, ⋅, may replace [∧](https://en.wikipedia.org/wiki/%E2%88%A7), especially if these symbols are not available for technical reasons. (*Note*: the aforementioned symbols C*pq*, E*pq*, N*p*, A*pq*, and K*pq* are used in [Polish notation](https://en.wikipedia.org/wiki/Polish_notation).)
* Parentheses, brackets, and other punctuation symbols. The choice of such symbols varies depending on context.
* An infinite set of **variables**, often denoted by lowercase letters at the end of the alphabet *x*, *y*, *z*, ... . Subscripts are often used to distinguish variables: *x*0, *x*1, *x*2, ... .
* An **equality symbol** (sometimes, **identity symbol**) =; see [the section on equality below](https://en.wikipedia.org/wiki/First-order_logic#Equality_and_its_axioms).

It should be noted that not all of these symbols are required – only one of the quantifiers, negation and conjunction, variables, brackets and equality suffice. There are numerous minor variations that may define additional logical symbols:

* Sometimes the truth constants T, V*pq*, or ⊤, for "true" and F, O*pq*, or ⊥, for "false" are included. Without any such logical operators of valence 0, these two constants can only be expressed using quantifiers.
* Sometimes additional logical connectives are included, such as the [Sheffer stroke](https://en.wikipedia.org/wiki/Sheffer_stroke), D*pq* (NAND), and [exclusive or](https://en.wikipedia.org/wiki/Exclusive_or), J*pq*.

**Non-logical symbols**[[edit](https://en.wikipedia.org/w/index.php?title=First-order_logic&action=edit&section=5)]

The [non-logical symbols](https://en.wikipedia.org/wiki/Non-logical_symbols) represent predicates (relations), functions and constants on the domain of discourse. It used to be standard practice to use a fixed, infinite set of non-logical symbols for all purposes. A more recent practice is to use different non-logical symbols according to the application one has in mind. Therefore, it has become necessary to name the set of all non-logical symbols used in a particular application. This choice is made via a [**signature**](https://en.wikipedia.org/wiki/Signature_(mathematical_logic)).[[4]](https://en.wikipedia.org/wiki/First-order_logic#cite_note-4)

The traditional approach is to have only one, infinite, set of non-logical symbols (one signature) for all applications. Consequently, under the traditional approach there is only one language of first-order logic.[[5]](https://en.wikipedia.org/wiki/First-order_logic#cite_note-5) This approach is still common, especially in philosophically oriented books.

1. For every integer *n* ≥ 0 there is a collection of [*n*-**ary**](https://en.wikipedia.org/wiki/Arity), or *n*-**place**, **predicate symbols**. Because they represent [relations](https://en.wikipedia.org/wiki/Relation_(mathematics)) between *n* elements, they are also called **relation symbols**. For each arity *n* we have an infinite supply of them:

*Pn*0, *Pn*1, *Pn*2, *Pn*3, ...

1. For every integer *n* ≥ 0 there are infinitely many *n*-ary **function symbols**:

*f n*0, *f n*1, *f n*2, *f n*3, ...

In contemporary mathematical logic, the signature varies by application. Typical signatures in mathematics are {1, ×} or just {×} for [groups](https://en.wikipedia.org/wiki/Group_(mathematics)), or {0, 1, +, ×, <} for [ordered fields](https://en.wikipedia.org/wiki/Ordered_field). There are no restrictions on the number of non-logical symbols. The signature can be [empty](https://en.wikipedia.org/wiki/Empty_set), finite, or infinite, even [uncountable](https://en.wikipedia.org/wiki/Uncountable). Uncountable signatures occur for example in modern proofs of the [Löwenheim-Skolem theorem](https://en.wikipedia.org/wiki/L%C3%B6wenheim-Skolem_theorem).

In this approach, every non-logical symbol is of one of the following types.

1. A **predicate symbol** (or **relation symbol**) with some **valence** (or **arity**, number of arguments) greater than or equal to 0. These are often denoted by uppercase letters*P*, *Q*, *R*,... .
   * Relations of valence 0 can be identified with [propositional variables](https://en.wikipedia.org/wiki/Propositional_variable). For example, *P*, which can stand for any statement.
   * For example, *P*(*x*) is a predicate variable of valence 1. One possible interpretation is "*x* is a man".
   * *Q*(*x*,*y*) is a predicate variable of valence 2. Possible interpretations include "*x* is greater than *y*" and "*x* is the father of *y*".
2. A **function symbol**, with some valence greater than or equal to 0. These are often denoted by lowercase letters *f*, *g*, *h*,... .
   * Examples: *f*(*x*) may be interpreted as for "the father of *x*". In [arithmetic](https://en.wikipedia.org/wiki/Arithmetic), it may stand for "-x". In [set theory](https://en.wikipedia.org/wiki/Set_theory), it may stand for "the [power set](https://en.wikipedia.org/wiki/Power_set) of x". In arithmetic, *g*(*x*,*y*) may stand for "*x*+*y*". In set theory, it may stand for "the union of *x* and *y*".
   * Function symbols of valence 0 are called **constant symbols**, and are often denoted by lowercase letters at the beginning of the alphabet *a*, *b*, *c*,... . The symbol *a*may stand for Socrates. In arithmetic, it may stand for 0. In set theory, such a constant may stand for the empty set.

The traditional approach can be recovered in the modern approach by simply specifying the "custom" signature to consist of the traditional sequences of non-logical symbols.

**Formation rules**[[edit](https://en.wikipedia.org/w/index.php?title=First-order_logic&action=edit&section=6)]

The [formation rules](https://en.wikipedia.org/wiki/Formation_rule) define the terms and formulas of first order logic. When terms and formulas are represented as strings of symbols, these rules can be used to write a [formal grammar](https://en.wikipedia.org/wiki/Formal_grammar) for terms and formulas. These rules are generally [context-free](https://en.wikipedia.org/wiki/Context-free_grammar) (each production has a single symbol on the left side), except that the set of symbols may be allowed to be infinite and there may be many start symbols, for example the variables in the case of [terms](https://en.wikipedia.org/wiki/First-order_logic#Terms).

**Terms**

The set of [**terms**](https://en.wikipedia.org/wiki/Term_(mathematics)) is [inductively defined](https://en.wikipedia.org/wiki/Inductive_definition) by the following rules:

1. **Variables.** Any variable is a term.
2. **Functions.** Any expression *f*(*t*1,...,*tn*) of *n* arguments (where each argument *ti* is a term and *f* is a function symbol of valence *n*) is a term. In particular, symbols denoting individual constants are 0-ary function symbols, and are thus terms.

Only expressions which can be obtained by finitely many applications of rules 1 and 2 are terms. For example, no expression involving a predicate symbol is a term.

**Formulas**

The set of [**formulas**](https://en.wikipedia.org/wiki/Formula_(mathematical_logic)) (also called well-formed formulas [[6]](https://en.wikipedia.org/wiki/First-order_logic#cite_note-6) or **wff**s) is inductively defined by the following rules:

1. **Predicate symbols.** If *P* is an *n*-ary predicate symbol and *t1*, ..., *tn* are terms then *P*(*t*1,...,*t*n) is a formula.
2. **Equality.** If the equality symbol is considered part of logic, and *t1* and *t*2 are terms, then *t*1 = *t*2 is a formula.
3. **Negation.** If φ is a formula, then {\displaystyle \lnot }φ is a formula.
4. **Binary connectives.** If φ and ψ are formulas, then (φ {\displaystyle \rightarrow } ψ) is a formula. Similar rules apply to other binary logical connectives.
5. **Quantifiers.**

Only expressions which can be obtained by finitely many applications of rules 1–5 are formulas. The formulas obtained from the first two rules are said to be [**atomic formulas**](https://en.wikipedia.org/wiki/Atomic_formula).