Empirical Regularities from Interacting Long and Short Memory Investors in an Agent Based Stock Market

Blake LeBaron *
Brandeis University

September 2000
Revised: May 2001
A revised version has appeared in *IEEE Transactions on Evolutionary Computation*, 5: 442-455 (2001).

Abstract

This paper explores some of the empirical features generated in an agent based computational stock market with market participants adapting and evolving over time. Investors view differing lengths of past information as being relevant to their investment decision making process. The interaction of these memory lengths in determining market prices creates a kind of market ecology in which it is difficult for the more stable, longer horizon agents to take over the market. What occurs is a dynamically changing market in which different types of agents arrive and depart depending on their current relative performance. This paper analyzes several key time series features of such a market. It is calibrated to the variability and growth of dividend payments in the United States. The market generates some features which are remarkably similar to those from actual data. These include magnifying the volatility from the dividend process, inducing persistence in volatility and volume, and generating fat tailed return distributions.

Keywords: Finance, Agent Based Markets, Neural Networks, Volatility, Financial Forecasting

*Graduate School of International Economics and Finance, Brandeis University, 415 South Street, Mailstop 32, Waltham, MA 02453 - 2728, blebaron@brandeis.edu, www.brandeis.edu/~blebaron. The author is also a faculty research fellow at the National Bureau of Economic Research, and an external faculty member at the Santa Fe Institute. The market model used here was built using Matlab version 5.3.

1 Introduction

Modeling the many interactions, diverse beliefs, and behaviors which are contained in a financial market is a daunting task. However, much of modern finance theory is based on careful assumptions and well crafted theories which allow simplification to analytically tractable models often involving a single representative agent. While such models appear to push ones notions of stylized representations to the limit, they are a common part of the economics toolkit for making difficult social interactions tractable. These heroic attempts at simplifying markets have generally been unsuccessful at meeting the challenge of lining up with many empirical features of real markets. Among these are relative returns, the amount and persistence of volatility and trading volume, and cross correlations between volatility, returns and volume. At the moment there is no unified theoretical model capable of replicating all these facts, although some have been replicated individually.

This paper presents the first results of calibrating agent based markets with interacting artificially intelligent agents to aggregate macroeconomic and financial data.[1] The results show that in some dimensions agent based markets show great promise in solving some of these puzzles, while in others more work remains. The model presented is designed to fit prices, trading volume, and returns for data sampled at the monthly frequency.

The model used is a relatively new agent based market. Many of its details are developed and presented in LeBaron (forthcoming 2001). Although in spirit it is similar to the Santa Fe artificial stock market it is a radically different design built to be more tractable, and closer to the world of more traditional macro finance models except for its heterogeneous agent framework.[2]

The market also stresses many of the coevolutionary features that are an interesting part of agent based modeling. In a coevolutionary setting the fitness of strategies depends critically on the current population of other strategies. In this market rules and agents are evolved, and compete with each other in their trading activities. The wealthier agents survive, along with rules that have been actively used. Agents' decision making processes differ in a very special way designed to replicate heterogeneity in the real world. They use differing amounts of past data in deciding on their optimal trading strategies. Some traders take a perspective of looking back 25 years to evaluate a trading rule, while others view only the previous 6 months

[1] Agent based approaches in which a market situation is analyzed from the bottom up using adaptive agent systems are becoming more prevalent in economics. A good summary of this research can be found at the web site maintained by Leigh Tesfatsion (http://www.econ.iastate.edu/tesfatsi/ace.htm). Information specific to finance can be found at the agent based computational finance website, www.brandeis.edu/~blebaron/acf, and in the survey paper, LeBaron (2000*a*).

[2] Results on the SFI artificial market have been presented in Arthur, Holland, LeBaron, Palmer & Tayler (1997), and LeBaron, Arthur & Palmer (1999).

as being important. This allows for interesting evolutionary races across these differing players, and helps to answer some questions about whether there is a way to judge the most rational, and to predict who should end up wealthier in the long run. This bounded memory perspective on past information has been approached in many different contexts, and is related to constant gain learning algorithms.[3]

Section 2 summarizes the model. Section 3 gives results from some computer experiments, and the final section summarizes and concludes.

2 Market Structure

The market consists of several different pieces for which some are more or less standard in economics and finance, and others are new. The market is interesting from an evolutionary standpoint for several reasons. First, as mentioned in the introduction, financial markets are coevolutionary entities where the fitness of agents depends critically on the strategies of other evolving agents. Second, this particular market setup involves evolution for both the agents and the rules. Agents are evolved over wealth, and chose from a common set of rules in an attempt to maximize their own objectives. Rules are evolved separately using only a weak notion of popularity as their fitness function. This allows some aspects of social learning and information transfer across agents.[4] It also allows for an endogenous measure of belief dispersions coming from the number of different rules or strategies in use at a given time.

The following sections describe the market in greater detail. Since this market is calibrated to actual data, time increments are meaningful. The basic time interval of the market is taken to be 1 month which corresponds to many longer range financial studies in macro economics and finance.

2.1 Securities

The market is a partial equilibrium model with two securities, a risk free asset in infinite supply paying a constant interest rate, r_f, and a risky security paying investors a random dividend each period. It is available in a fixed supply of 1 share for the population. If s_i are the share holdings of agent i, the following constraint must be met in all periods,

$$1 = \sum_{i=1}^{I} s_i, \tag{1}$$

[3]See Honkapohja & Mitra (2000), Sargent (1999), and Mullainathan (1998) for related work on bounded memory. There is also a connection to the issue of rationality of adaptive expectation forecasts as in Muth (1960).

[4]See Vriend (1998) for some comparisons of social versus individual learning.

where I is the number of agents. The log dividend follows a random walk, with an annualized growth rate of 2 percent, and a annual standard deviation of 6 percent. This corresponds roughly to actual dividend properties from the U.S.[5] The constant interest rate is set to a value of 1 percent at an annual rate (compounded monthly) which is also a common benchmark in economics for the real rate of interest.

Agents receive only three forms of income: dividends, interest payments, and capital gains from purchases and sales. These go to building wealth and current consumption. No other income streams are available.

2.2 Agents and rules

Agents are defined by intertemporal constant relative risk aversion preferences of logarithmic form,

$$u_{i,t} = E_t \sum_{s=0}^{\infty} \beta^s \log c_{i,t+s}, \tag{2}$$

subject to the intertemporal budget constraint,

$$w_{i,t} = p_t s_{i,t} + b_{i,t} + c_{i,t} = (p_t + d_t) s_{i,t-1} + (1 + r_f) b_{i,t-1}. \tag{3}$$

$s_{i,t}$ and $b_{i,t}$ are the risky and risk free asset holdings respectively, and r_f is the risk free rate of return. r_t will represent the risky asset return at time t. p_t is the price of the risky security at time t. $w_{t,i}$ and $c_{i,t}$ are the wealth and consumption of agent i. Finally, d_t is the dividend paid at time t. These heavily restricted preferences are used for tractability. It is well known that for logarithmic utility the agent's optimal consumption choice can be separated from the portfolio composition and is a constant proportion of wealth,[6]

$$c_{i,t} = (1 - \beta) w_{i,t}. \tag{4}$$

The time rate of discount, β, is set to $(0.95)^{1/12}$ which corresponds to 0.95 annual rate. This is a common time rate of discount used in macro models, and a later section will show that this provides a reasonable match for the equilibrium dividend yield in the model. It will be useful to denote the interest rate corresponding

[5] See Campbell (1999) for a summary of many of the features of aggregate financial series. Also, note that seasonalities in the aggregate dividend process are ignored.

[6] This result is well known in dynamic financial models. See Merton (1969) and Samuelson (1969) for early derivations. Altug & Labadie (1994) and Giovannini & Weil (1989) provide updated derivations in more general settings. Future versions of this model will generalize these preferences, but this brings in the added dimension of trying to determine optimal consumption given current information. Also related are the analytic policy rules for time varying returns in Barberis (2000) and Campbell & Viceira (1996). Finally, Lettau & Uhlig (1999) and Allen & Carroll (2001) present some results showing some of the difficulties in building agents with the capabilities of learning dynamic consumption plans.

to the time rate of discount as r,

$$r = \frac{1}{\beta} - 1. \tag{5}$$

A second property of logarithmic preferences is that the portfolio decision is myopic in that agents maximize the logarithm of next period's portfolio return. Agents will concentrate their learning efforts on this optimal portfolio decision. They are interested in finding a rule that will maximize the expected logarithm of the portfolio return from a dynamic strategy. The strategy recommends a fraction of savings, $\beta w_{i,t}$, to invest in the risky asset as a function of current information, z_t. The objective is to

$$\max_{\alpha_j} E_t \log[1 + \alpha_j r_{t+1} + (1 - \alpha_j) r_f], \tag{6}$$

for the set of all available rules, α_j. In general it would be impossible for agents to run this optimization each period, since the above expectation depends on the state of all other agents in the market, along with the dividend state. The portfolio decision will therefore be replaced with a simple rule which will be continually tested against other candidate rules. This continual testing forms a key part of the learning going on the market.

The trading rules, α_j, should be thought of as being separate from the actual agents. The best analogy is to that of an investment advisor or mutual fund. A population of rules is maintained, and agents select from this set as they might chose an advisor. One difference here from the world of investment advisors is that the rule is a simple function, $\alpha(z_t; \omega_j)$, where z_t is time t information, and ω_j are parameters specific to rule j. The functional form used for each rule is a feedforward neural network with a single hidden unit with restricted inputs. It is given by

$$\begin{aligned}
h_k &= g_1(\omega_{1,k} z_{t,k} + \omega_{0,k}) && (7)\\
\alpha(z_t) &= g_2(\omega_2 + \sum_{k=1}^{m} \omega_{3,k} h_k) && (8)\\
g_1(u) &= \tanh(u) && (9)\\
g_2(u) &= \frac{e^u}{1 + e^u}. && (10)
\end{aligned}$$

This framework restricts the portfolio to positive weights between zero and one, so there is no short selling or borrowing allowed. The model needs to set borrowing constraints to keep it off nonstationary bubble trajectories, and to avoid having to unwind debt positions when the agents go bankrupt. With these restrictions in place agents' wealth can go to zero, but they cannot end up with negative wealth. It is also important to note that this is a very restrictive neural network structure. Each hidden unit, h_k is connected to only one information variable. This differs from standard neural networks which let all inputs influence each hidden unit. This step was taken to enhance tractability of the learned rules, but it does limit the generality of how agents can combine input information. It also provides some intuition for why this might be a sensible and tractable way to build dynamic trading strategies. Each information input is translated into a variable between -1 and 1. This could be interpreted as a kind of buy/sell signal. The next stage of the network could then be thought of as a kind of "or" gate across these signals by summing a weighted version of them together. This creates a rule which can respond by moving the portfolio into stocks when any number of individual trading signals are active which seems like a sensible picture of actual trading behavior.

The weights are stored in a population table along with information on performance of this rule in the recent past. A simple real valued vector, ω_j, completely describes each dynamic trading strategy. All that is needed to be stored is a time series of the portfolio returns from each rule since the agent's objective only requires this as an input.

Agents chose the rule to use in the current period based on its performance in the past. They look over their own past memory length, T_i, to evaluate the performance of the rule in the future using,

$$\max_j \hat{E}(r_p) = \frac{1}{T_i} \sum_{k=1}^{T_i} \log[1 + \alpha(z_{t-k};\omega_j) r_{t-k+1} + (1 - \alpha(z_{t-k};\omega_j)) r_f]. \tag{11}$$

The only feature driving heterogeneity across agents' decisions is their memory, T_i. This can lead to relatively similar decision rules, and very unstable markets. Further heterogeneity is added by making the rule decision have a random component. Agents compare their current rule to a candidate comparison rule drawn at random from the pool of active rules. If this new rule beats the current one using the above return estimation, then it will replace the current one. If not, the agent continues to use the same rule. This might appear to give a very weak selection property for rules, but since agents get to have many chances to evaluate rules, they should move to better strategies over time as the further the explore the rule set. Further heterogeneity is generated, by having only half the agents, selected at random, update their rules each period.

2.3 Information

Agents trading strategies are based on simple information structures which are input to the neural network, and used to generate the trading strategies, $\alpha(z_t;\omega_j)$. Obviously, the choice of the information set, z_t, is important. This set will be chosen to encompass reasonable predictors that are commonly used in real markets. The information set will include, returns, past returns, the price dividend ratio, and trend following technical trading indicators. In the current version, the only types of technical rules used are exponential moving averages. The moving average is formed as

$$m_{k,t} = \rho m_{k,t-1} + (1-\rho)p_t. \tag{12}$$

Formally, the information set, z_t, will consist of 6 items.

1. $r_t = \log(\frac{p_t+d_t-p_{t-1}}{p_{t-1}})$
2. r_{t-1}
3. r_{t-2}
4. $\log(\frac{rp_t}{d_t})$
5. $\log(\frac{p_t}{m_{1,t}})$
6. $\log(\frac{p_t}{m_{2,t}})$

Several of the items are logged to make the relative units sensible. The dividend price ratio is normalized around a benchmark determined in the equilibrium presented in section 2.6. The two moving average indicators, $m_{1,t}$ and $m_{2,t}$, correspond to values of $\rho = 0.8$ and $\rho = 0.99$ respectively.

It is important to think about the timing of information as this will be important to the trading mechanisms covered in the next section. As trading begins at time t, all $t-1$ and earlier information is known. Also, the dividend at time t has been revealed and paid. This means that α_j can be written as a function of p_t and information that is known at time t,[7]

$$\alpha_j = \alpha_j(p_t; I_t). \tag{13}$$

[7]The impact of p_t on the current information vector, z_t, is taken into account as well.

All variables in I_t are known before trading begins in period t. p_t will then be determined endogenously to clear the market.

2.4 Trading

Trading is performed by finding the aggregate demand for shares, and setting it equal to the fixed aggregate supply of 1 share. Given the strategy space each agent's demand for shares, $s_{i,t}$, at time t can be written as,

$$s_{i,t}(p_t) = \frac{\alpha_i(p_t; I_t)\beta w_{i,t}}{p_t} \tag{14}$$

$$w_{i,t} = (p_t + d_t)s_{i,t-1} + (1 + r_f)b_{i,t-1}, \tag{15}$$

where $w_{t,i}$ is the total wealth of agent i, and $b_{i,t-1}$ are the bond holdings from the previous period. Summing these demands gives an aggregate demand function,

$$D(p_t) = \sum_{i=1}^{I} s_{i,t}(p_t). \tag{16}$$

Setting $D(p_t) = 1$ will find the equilibrium price, p_t. This is essentially a simple Walrasian auction in the market for the risky asset. Unfortunately, there is no analytic way to do this given the complex nonlinear demand functions. This operation will be performed numerically. Also, it is not clear that the equilibrium price at time t is unique. Given the large number of nonlinear demand functions involved it probably is not. A nonlinear search procedure will start at p_{t-1} as its initial value, and stop at the first price that sets excess demand to zero.[8]

It is important to remember the equilibrium is found by taking the current set of trading strategies as given. Once p_t is revealed then it is possible that agent i might want to change to a different trading rule. It is in this sense that the equilibrium is only temporary.[9]

2.5 Evolution

Around this structure of rules, agents, and markets is an evolutionary dynamic that controls adaptation and learning in the entire system. Evolution of agents is performed in a very simple fashion based on accumulated wealth. Every period one of the 5 least wealthiest agents is chosen at random and removed. One new agent

[8]The search uses the matlab built in function fzero. Also, the results are not sensitive to starting exactly at p_{t-1}. Some experiments have been performed starting the search at p_{t-1} plus a small amount of noise. The results were for these experiments were not different from those starting the search at p_{t-1} exactly.

[9]This is similar to the types of learning equilibrium surveyed in Grandmont (1998).

with a memory length T_i drawn randomly replaces this agent. This new agent chooses an initial trading rule randomly from the best half of the current rules judged according to its own memory length. The distribution for this draw will be different in different experimental market runs. It is initialized with the share holdings of the deleted agent, and receives bond holdings equal to the median over the population. To make evolution neutral to the overall population wealth, these new holdings are taken from other agents in the population. The burden is split evenly between the upper half of agents sorted by current bond holdings. This gives a weak redistributional effect to the evolutionary process.

Rule evolution is more complicated. Rules are evolved using a genetic algorithm.[10] This method tries to evolve the population using biologically inspired operators that take useful rules, and either modify them a little (mutation), or combine them with parts of other rules (crossover).

One of the crucial aspects of evolutionary learning is the fitness criterion which is used to select good parents from the current generation. It is not clear what makes a rule "fit" in a multiagent market. For example, it would be tempting to evolve the rules based on the historical performance on a fixed history of past data, but this would not capture the fact that agents are looking at different history lengths. To try to account for agent diversity a very weak selection criterion is used. A rule can be a parent for the next generation if at least one agent has used it over the last 10 periods. Rules that haven't been used for 10 periods are marked for replacement. This is equivalent to eliminating all mutual fund managers with no customers. This weak selection procedure also rules out more involved optimization procedures such as the many variants of hill climbing often used for neural network weight optimization. This would require a well defined objective surface which would be impossible to define for this set of heterogeneous agents. This method seems like a reasonable compromise in terms of building relatively robust rules that are not too finely tuned to the preferences of any one agent.[11]

Evolution proceeds as follows. First, the set of rules to be eliminated is identified. Then for each rule to be replaced the algorithm chooses between three methods with equal probability :

1. **Mutation:** Choose one rule from the parent set, and add a uniform random variable to one of the network weights, ω. The random increment is distributed uniform $[-0.25, 0.25]$.

[10] The genetic algorithm, Holland (1975), is a widely used technique in computational learning. Goldberg (1989) provides a good overview, and Mitchell (1996) gives a recent perspective. There are many evolutionary techniques, and this modified algorithm also contains inspiration from many of these others. Fogel (1995) provides a broad perspective to the complete set of methods.

[11] Hill climbing alone would also have to tackle the problem of over fitting in some efficient and sensible fashion. There is no easy solution to this problem. LeBaron (1998) provides one solution using an evolutionary bootstrap approach. This method is too computationally costly to be performed here.

2. **New weight:** Chose one rule from the parent set, chose one weight at random, and replace it with a new value chosen uniformly from $[-1, 1]$. This is the same distribution used at startup.

3. **Crossover:** Take two parents at random from the set of good rules. Take all weights from one parent, and replace one set of weights corresponding to one input with the weights from the other parent. This amounts to replacing the two weights that affect the input directly (linear and bias), plus the weight on the corresponding hidden unit. Visually this is equivalent to chopping off a branch of the network for parent one, and replacing it with a branch from parent two.

A new rule is initialized by evaluating its performance over the past history of prices and information. Agents will use this performance history to decide on whether this rule should be used as they do with the others.

It would be difficult to argue that there is any particular magic to this procedure for evolving rules, and it goes without saying that these mechanisms are ad hoc. However, the objective is to produce new and interesting strategies that must then survive the competition with the other rules in terms of forecasting. Experiments have shown that the results are robust to different minor modifications of these mechanisms.

2.6 Equilibrium

It is useful in multi-agent financial simulations to have a benchmark with which the results can be compared. For multi-agent simulations the homogeneous agent world is often the appropriate benchmark. It turns out that in this model for the given calibrated parameters there is a homogeneous equilibrium in which agents all hold only the risky asset. Prices, dividends, and consumption all grow at the same expected growth rate which matches the rate for dividends. In this equilibrium stock returns should be unpredictable, and trading volume should be zero. In this sense it matches a classic efficient market situation where all information is contained in prices, and agents agree on functions mapping dividends into prices. Setting current consumption equal to dividends, and assuming all agents are the same gives the function mapping dividends to prices of

$$(1 - \beta)(p_t + d_t) = c_t = d_t \tag{17}$$

$$p_t = \frac{\beta}{1 - \beta} d_t = \frac{d_t}{r} \tag{18}$$

The existence of such an equilibrium provides an important benchmark for the model. Given the complexity of agent based financial markets it is not simply enough to match empirical facts. The model should also be able to show that for some region of the parameters it can do something consistent with existing economic theory. The market can then be "taken out of the box" to perform more realistic studies of market dynamics.

2.7 Timing

The timing of the market is crucial since it is not an equilibrium model where everything happens simultaneously. A specific ordering must be prescribed to events. The following list shows how things proceed.

1. Dividends, d_t, are revealed and paid.
2. The new equilibrium price, p_t, is determined, and trades are made.
3. Rules are evolved.
4. Agents update their rule selection using the latest information.
5. Agents are evolved.

Although this appears to be a sensible ordering, it is not clear if other sequences might give different results. The fact that there needs to be an ordering is limited by usual computing tools. The best situation would be for things to be happening asynchronously, and software tools are becoming available to tackle this problem. However, this would open some very difficult problems in terms of trading and price determination.

2.8 Initialization and parameters

While this market is intended to be relatively streamlined, it still involves a fair number of parameters which may not have as much economic content as one would like. The first of these are the initialization parameters which control the agent and rule structure at startup. Rules are started with parameters, ω, drawn from a uniform $[-1, 1]$ distribution for each neural network weight. Agents begin with a memory, T_i, drawn from a specified distribution which will be set differently in various experiments. Bond holding levels are set to 0.1. The shares are divided equally among all agents. Finally, initial price and dividend series are generated using the stochastic dividend process, setting $p_t = d_t/r$.

There are several other parameters that will remain fixed in these runs, but which may be interesting to change in the future. The number of agents is set to 1000, the number of rules to 250, and the maximum history is set to 250. The period of inactivity after which a rule is deleted due to lack of use is fixed at 10 for all runs.

3 Results

The computer experiments presented here emphasize the key difference between two cases. In the first case, agents of many memory lengths, T_i, are allowed to interact in the market. This is referred to as the *all memory* case. Initial agents are drawn from a uniform $[5, 250]$ distribution, and new entrants are drawn from the same distribution. This experiment is designed to explore the dynamics of the completely heterogeneous market setup. A comparison experiment, referred to as the *long memory* case, is provided by starting the market with a set of only longer memory agents drawn from a uniform $[220, 250]$ distribution.

3.1 Run Summaries

Figure 1 presents plots of prices and volume for the final 500 periods of a 10000 period run of the all memory case. Remember that periods are being calibrated to be one month in actual calendar time, so the 500 period plot represents over 40 years of real price data. The figure shows a strong upward trend over the period which is due to the trending random walk of the underlying dividend process. It also displays a large amount of variability about this trend with some very dramatic dips, and sharp rises during the period. The corresponding volume series shows a moderate amount of trading activity with turnover rates of nearly 5 percent per month. There also appears to be some clumping to volume activity along with a connection between volume and large price moves. These features will be demonstrated in future sections.

Figure 2 shows the same features for the long memory case. This displays a price series which appears to be following a random walk, and a volume series which is nearly zero. In the homogeneous equilibrium the price series should be proportionate to the dividend series, and therefore follow a random walk as well. In the equilibrium all agents are in agreement on valuations, so trading volume should be zero. The simulation can occasionally generate some minor blips in trading as several agents may explore some out of equilibrium strategies, but they are quickly convinced to come back and join the rest of the crowd.

A dramatic comparison of the two cases is given by figure 3 which displays the continuously compounded (logged) returns including dividends in the two different cases. In the heterogenous case there is clearly greater volatility, and several very large moves. There appear to be a few more large negative than positive returns as well. The homogeneous case shows a much smaller amount of volatility, and very few large jumps in the return process. These same returns are plotted as histograms in figure 4. In both cases the histograms include a Gaussian distribution superimposed on the return distributions. In the top panel the all memory case displays strong deviations from Gaussianeity which are typical of most financial series. The lower panel displays a distribution much closer to normality, and therefore different from actual return series.

3.2 Stock returns

Monthly continuously compounded excess returns with dividends are sampled from periods 5000 to 10000, and are given by

$$\log(\frac{p_t + d_t}{p_{t-1}} - r_f) \tag{19}$$

Sampling far out into the simulation run allows the system to move beyond the initial learning phase during which time some of the worst randomly initialized strategies are removed. Table 1 presents summary statistics for these returns in the two different cases along with comparison numbers for the S&P 500 index.[12] The first four columns correspond to the series mean, standard deviation, skewness, and kurtosis. All three series show relatively similar mean monthly excess returns. The long memory case is actually the closest of the simulations to the actual data here. The most interesting value is the standard deviation. Here, the all memory case shows a clear amplification of return volatility as compared to the long memory case. There is an increase by nearly a factor of four from this benchmark. The volatility of the all memory case is much closer to the actual S&P volatility, although it does give a value slightly higher than the actual returns process. The column labeled kurtosis shows that all three series generate some excess kurtosis indicating some deviations from normality in all the cases. The columns labeled Q-ratio present quantile ratio values. These give another measure of the distribution shape. They are the ratio in the left and right tails of the distribution of the 25th to the 5th, and the 75th to the 95th quantiles respectively. For a Gaussian distribution these would be 0.41 for both tails. The table shows values close to Gaussian for the long memory case, but deviating in the tails for the all memory and actual data. The probability in the tails is slightly larger for the all memory simulation than for the actual data.

Information on return dynamics is presented in figures 5 through 7. The first, figure 5, summarizes the autocorrelation features of the monthly return series for the two simulated markets along with the S&P. All three show little evidence for strong autocorrelation with only a few slightly large values of about 0.1 coming from the all memory case. Figure 6 turns to volatility by reporting the autocorrelations of the absolute value of returns. This picture clearly shows the all memory case following the actual market data in generating large positive volatility autocorrelations. The long memory case generates no persistence to volatility. This is consistent with a picture of what appears to be a near independent returns series for this

[12] The S&P numbers are sampled from 1926-1998 and are taken from the Ibbotson data set.

important benchmark.

Finally, figure 7 tests the leverage relationship discovered by Black (1976) which documented an inverse relationship between returns today and future volatility. In other words, when the market price falls volatility tends to go up. Figure 7 displays the cross-autocorrelations between returns and volatility. The figure compares the all memory and long memory cases to the S&P. The figure shows a strong inverse relationship for the all memory case indicating both a contemporaneous and a lagged relationship from returns to volatility as in many stock return series. The results for the long memory case are dramatically different displaying a positive cross correlation. The relationship in the actual S&P data is close to zero, with a small negative correlation between lagged returns and future volatility. This is clearly different from the all horizon case in magnitude. This relationship is much more pronounced in higher frequency returns data.

3.3 Predictability

Much of modern finance has been concerned with searches for predictability of some type. Recently, the area has been filled with various standard predictability regressions. Lagged values of financial data ranging from dividend price ratios to technical indicators have shown some use in forecasting future returns.[13] However, the long term stability of these predictors has occasionally been called into question.[14] This section explores a subset of possible predictors and looks at their stability through time.

Figure 8 displays the dividend yield for both the all memory and long memory cases. This is the ratio of the dividend to the price of the risky asset. It is annualized by multiplying the dividend at each date by 12. In a stationary equilibrium this value should be constant, since there is no change in the fundamental riskiness of the equity asset. This is very nearly the case for the long memory simulations. The upper panel, corresponding to the all memory case, shows a much more realistic picture with a highly variable dividend price ratio. As we will see in the next table these wide swings are indicative of potential return predictability.

Table 2 presents results of univariate ordinary least squares regressions of current returns on several candidates of lagged information. These include the dividend yield, lagged returns, and two exponential moving average trading indicators. Each of these corresponds to information variables included in the agents' information set contributing to evolved dynamic strategies. The table presents both the t-statistic for the coefficient on the linear predictor along with the R-squared for the regression. The regressions are estimated on a 250 length time series ending at the point given in the table. Results are reported for several

[13] See Fama (1991) for a survey.

[14] See Ghysels (1998), Sullivan, Timmerman & White (1999), and LeBaron (2000*b*) for examples of changes in predictability.

different time periods. This is done both to document the unusual nature of the initial time periods, and to show how the predictability properties appear to be changing over time. The results in the table show many interesting features. Among these are an unusually large amount of predictability at time period 500. Some of the regressions generate an R-squared of nearly 75 percent which is unheard of for any financial series. This should be expected since the agents are behaving with a large amount of randomness at startup, and this probably leaves many patterns of predictability in the returns series. As learning takes over these gross market inefficiencies are dissipated. The later time period regressions reduce R-squared values to much more typical values of a few percent to near zero. For the dividend price ratio the results are fairly stable over time, but for the other values there are some changes as one moves across the simulation time periods. In some cases the predictors change sign, and also move from being insignificant to significant.

These preliminary results are suggestive of a market that is changing continuously from the perspective of these linear regressions and information variables.[15] Although markets are predictable according to normal tests of significance, the best predictors may be changing over time.[16]

3.4 Trading Volume

This agent based stock market generates trading volume series along with price series. In a less than efficient market these are just as important as returns in characterizing what is happening.[17] Figure 9 displays the autocorrelation for volume in the all memory, long memory, and New York Stock Exchange (NYSE) respectively. For the NYSE, volume is taken to be monthly shares traded divided by total shares outstanding which is known as the turnover ratio. This is normalized by a 12 month moving average. At 1 to 2 months both the NYSE and the heterogeneous market display a large amount of positive autocorrelation. The actual data displays a slightly faster decay. The long memory market shows a small amount of erratic autocorrelation. Given the small amount of volume in these series, these numbers should be viewed with some care.

Figure 10 shows the cross correlation between trading volume and return volatility measured as the absolute value of returns. This is well known to be positive for contemporaneous volume and volatility. The graph displays this property for all three series. The all memory case generates a much larger positive

[15]See Benink & Bossaerts (1996) and Timmermann (1998) for similar theoretical questions related to stationarity.

[16]Some caution should be used in interpreting these t-statistics since there is some possibility for data snooping here, and one would naturally expect the t-statistics to change purely by chance. Second, the given the excess kurtosis in the series the OLS regression estimates of significance may not be correct.

[17]See Karpov (1987) for an early survey, and Gallant, Rossi & Tauchen (1992) for a more recent display of price/volume facts.

correlation than either the long memory case, or the NYSE volume data. The differences in magnitude with the actual data could be related to the fact that trade in this market takes place over one asset yielding a very strong linkage between price and volume.

Finally, figure 11 shows the crosscorrelation between volume and returns. This has been shown to be generally positive in actual data, and it is replicated here for the NYSE series. The two market simulations generate different features with one, the all memory case, displaying a strong negative correlation, and the long memory displaying zero autocorrelation. This is one of the strongest counterfactuals produced by the market so far, and it is interesting to think about what might be different about the agent simulation in comparison to actual markets here.

3.5 Consumption

As a standard infinite horizon investment and consumption model this market generates a consumption time series as well as financial market prices. This adds another interesting dimension with which to test the results. Table 3 gives a summary of some of the results for aggregate consumption from the model. Given that all the series are nonstationary, results are given for annualized growth rates determined from the monthly consumption series aggregated to quarterly frequencies. The table shows general agreement in mean growth rates which is not surprising given the calibration done with actual data. What is interesting is the amplification in volatility in both the all memory, and long memory cases. In particular aggregate consumption for the all memory case is over 20 percent, but for the actual macro series it is only about 3 percent.[18] Since consumption is proportional to wealth in the simple log consumption case it is easy to see why the increased financial market volatility is transferred directly into consumption.

This is a very important counterfactual for the agent based approach to fitting macroeconomic facts. Even though the market is a good mechanism for magnifying fundamental volatility into stock prices, it is important to remember that part of the puzzle of financial markets is also that this volatility does not appear in other macro series. In order to match this feature it will be important to think about other aspects of the consumption decision making process. Mechanisms such as habit persistence, or some kind of lagged wealth estimation may be necessary.

[18] It should be noted that the rate of 6 percent for the long memory case is not surprising either. This is the volatility of the dividend process which is higher than the volatility of aggregate U.S. consumption. If the dividends had been lined up with consumption as in Mehra & Prescott (1985) then this would line up with the actual consumption variability.

4 Conclusions

The results in this paper show that an agent based model is capable of quantitatively replicating many features of actual financial markets. Comparisons show favorable results for returns and volatility and their persistence. The data also replicated the well known feature of excess kurtosis, or too many large moves, in the returns series. It also was able to generate pictures of volume/volatility cross correlations, along with the leverage asymmetry that matched features of real data. Given the market is forced to rely on a dividend process fitted to the U.S. aggregate, and to keep within the bounds of well defined, restrictive, intertemporal preferences these successes are quite remarkable.

In addition to these features, there were places where the market appears weak. The biggest of these is consumption. As one could easily predict given the proportionality of consumption to wealth, consumption moves around considerably given asset price volatility. This shows that while artificial markets can be viewed as a type of volatility generating engine, they cannot immediately solve one of the basic problems of macro economics and finance, the dichotomy between return and consumption variability. Other preferences and consumption rules will need to be considered to solve this puzzle. Also, it appears that returns generate too many large moves. Kurtosis levels and quantile ratio statistics reveal a distribution with too much probability mass in the tails relative to the center as compared to actual returns series. This problem is probably due to the large amount of similarity across agents. Adding further external heterogeneity will probably reduce the large moves. Finally, the volume/return relation is dramatically different in the artificial data. Markets tend to fall on rising volume while the reverse is true in the real world. Further tests on this need to be made both on the artificial markets, and on data for individual stock returns.

This market can only be viewed as an initial test of an emerging technology for finance and economics. Previous simplified analytic models have not fared well in matching financial data, and the time and technology have now arrived to turn toward agent based approaches. However, several important questions still remain. The model presented is complicated, and contains many deep parameters controlling evolution and learning for which we have only very weak notions of what their values should be. These apparent degrees of freedom could potentially be used to fit just about any feature of the data. On the other hand, these models appear to fit many features with relative ease which more traditional models do not even consider.

References

Allen, T. W. & Carroll, C. D. (2001), Individual learning about consumption, Technical Report 8234, National Bureau of Economic Research, Cambridge, MA.

Altug, S. & Labadie, P. (1994), *Dynamic Choice and Asset Markets*, Academic Press, San Diego, CA.

Arthur, W. B., Holland, J., LeBaron, B., Palmer, R. & Tayler, P. (1997), Asset pricing under endogenous expectations in an artificial stock market, *in* W. B. Arthur, S. Durlauf & D. Lane, eds, 'The Economy as an Evolving Complex System II', Addison-Wesley, Reading, MA, pp. 15–44.

Barberis, N. (2000), 'Investing for the long run when returns are predictable', *Journal of Finance* **55**, 225–264.

Benink, H. & Bossaerts, P. (1996), An exploration of neo-austrian theory applied to financial markets, Technical report, California Institute of Technology, Pasadena, CA.

Black, F. (1976), 'Studies of stock price volatility changes', *Proceedings of the American Statistical Association, Business and Economics Statistics Section* **177-181**.

Campbell, J. Y. (1999), Asset prices, consumption, and the business cycle, *in* J. B. Taylor & M. Woodford, eds, 'Handbook of Macroeconomics', North Holland, pp. 1231–1303.

Campbell, J. Y. & Viceira, L. (1996), Consumption and portfolio decisions when expected returns are time varying, Technical report, Department of Economics, Harvard University, Cambridge, MA.

Fama, E. F. (1991), 'Efficient capital markets: II', *Journal of Finance* **45**, 1575–1617.

Fogel, D. B. (1995), *Evolutionary Computation: Toward a New Philosophy of Machine Intelligence*, IEEE Press, Piscataway, NJ.

Gallant, A. R., Rossi, P. E. & Tauchen, G. (1992), 'Stock prices and volume', *The Review of Financial Studies* **5**, 199–242.

Ghysels, E. (1998), 'On stable factor structures in the pricing of risk: Do time varying betas help or hurt?', *Journal of Finance* **53**, 549–574.

Giovannini, A. & Weil, P. (1989), Risk aversion and intertemporal substitution in the capital asset pricing model, Technical Report 2824, National Bureau of Economic Research, Cambridge, MA.

Goldberg, D. E. (1989), *Genetic Algorithms in search, optimization and machine learning*, Addison Wesley, Reading, MA.

Grandmont, J.-M. (1998), 'Expectations formation and stability of large socioeconomic systems', *Econometrica* **66**, 741–782.

Holland, J. H. (1975), *Adaptation in Natural and Artificial Systems*, University of Michigan Press, Ann Arbor, MI.

Honkapohja, S. & Mitra, K. (2000), Learning with bounded memory in stochastic models, Technical report, RUESG and Department of Economics, University of Helsinki, Helsinki, Finland.

Karpov, J. M. (1987), 'The relation between price changes and trading volume: A survey', *Journal of Financial and Quantitative Analysis* **22**, 109–126.

LeBaron, B. (1998), An evolutionary bootstrap method for selecting dynamic trading strategies, *in* A. P. N. Refenes, A. N. Burgess & J. Moody, eds, 'Advances in Computational Finance', Kluwer Academic Press, pp. 141–160.

LeBaron, B. (2000*a*), 'Agent based computational finance: Suggested readings and early research', *Journal of Economic Dynamics and Control* **24**(5-7), 679–702.

LeBaron, B. (2000*b*), 'The stability of moving average technical trading rules on the Dow Jones index', *Derivatives Use, Trading, and Regulation* **5**(4), 324–338.

LeBaron, B. (forthcoming 2001), 'Evolution and time horizons in an agent based stock market', *Macroeconomic Dynamics* **5**(2).

LeBaron, B., Arthur, W. B. & Palmer, R. (1999), 'Time series properties of an artificial stock market', *Journal of Economic Dynamics and Control* **23**, 1487–1516.

Lettau, M. & Uhlig, H. (1999), 'Rules of thumb versus dynamic programming', *American Economic Review* **89**, 148–174.

Mehra, R. & Prescott, E. (1985), 'The equity premium: A puzzle', *Journal of Monetary Economics* **15**, 145–161.

Merton, R. (1969), 'Lifetime portfolio selection under uncertainty:The continuous time case', *Review of Economics and Statistics* **51**, 247–257.

Mitchell, M. (1996), *An Introduction to Genetic Algorithms*, MIT Press, Cambridge, MA.

Mullainathan, S. (1998), A memory based model of bounded rationality, Technical report, Massachusetts Institute of Technology, Cambridge, MA.

Muth, J. (1960), 'Optimal properties of exponentially weighted forecasts', *Journal of the American Statistical Association* **55**, 299–306.

Samuelson, P. (1969), 'Lifetime portfolio selection by dynamic stochastic programming', *Review of Economics and Statistics* **51**, 239–246.

Sargent, T. (1999), *The Conquest of American Inflation*, Princeton University Press, Princeton, NJ.

Sullivan, R., Timmerman, A. & White, H. (1999), 'Data-snooping, technical trading rule performance and the bootstrap', *Journal of Finance* **54**, 1647–1691.

Timmermann, A. (1998), Structural breaks, incomplete information and stock prices, Technical report, London School of Economics, London, UK.

Vriend, N. (1998), An illustration of the essential difference between individual and social learning, and its consequences for computational analysis, Technical report, Queen Mary and Westfield College, Univ. of London, UK.

Table 1: *Excess Return Summary Statistics*

	Mean	Std	Skewness	Kurtosis	Q-ratio(L)	Q-ratio(R)
All Memory	0.66	0.083	-2.80	44	0.22	0.27
Long Memory	0.58	0.018	0.10	4.9	0.40	0.40
S&P	0.58	0.056	-0.43	11.1	0.35	0.38

Mean and Std. are the annualized mean and standard deviation of the returns series inclusive of dividends. Skewness and kurtosis are estimated at the monthly horizon. Values for the S&P are the total return less the 30 day T-bill rate monthly from Jan 1926 through June 1998. Q-ratio is the ratio of the 25th to 5th quantile in the left tail, and the 75th to 95th quantile in the right tail. These values should be 0.41 for a Gaussian.

Table 2: *Forecasting Regressions: All memory*

	Period	D/P Ratio	Return lag 1	Return lag 2	MA(1)	MA(2)
500	t-ratio	-2.66	25.85	19.16	19.55	0.10
	R^2	0.028	0.728	0.596	0.606	0.000
5000	t-ratio	-3.19	-0.10	-0.91	1.03	-2.23
	R^2	0.039	0.000	0.003	0.004	0.020
7500	t-ratio	-2.58	5.23	-1.48	1.71	-1.49
	R^2	0.026	0.099	0.009	0.011	0.009
10000	t-ratio	-4.25	2.02	2.57	-1.24	-3.88
	R^2	0.068	0.016	0.026	0.006	0.057

Results for univariate predictive regressions. T-ratio refers to the t-statistic for for the OLS coefficient on the corresponding predictor. R^2 refers to the r-squared value from this linear regression. In each case the regression is estimated for the 500 periods (months) ending at Period.

Table 3: *Consumption Growth Rates*

	Mean	Std	ACF(1)
All Memory	1.89	23.3	0.34
Long Memory	1.89	6.02	-0.04
U.S.	1.77	3.26	-0.12

Annual consumption growth rates and variability. For the U.S. this comes from annual data and is measured from 1891-1995, and the values come from Campbell (1999). For the market simulations quarterly series are aggregated from the simulated monthly consumption series, and multiplied by 4, and $\sqrt{4}$, to get the annual mean and standard deviations respectively. The correlations are quarterly.

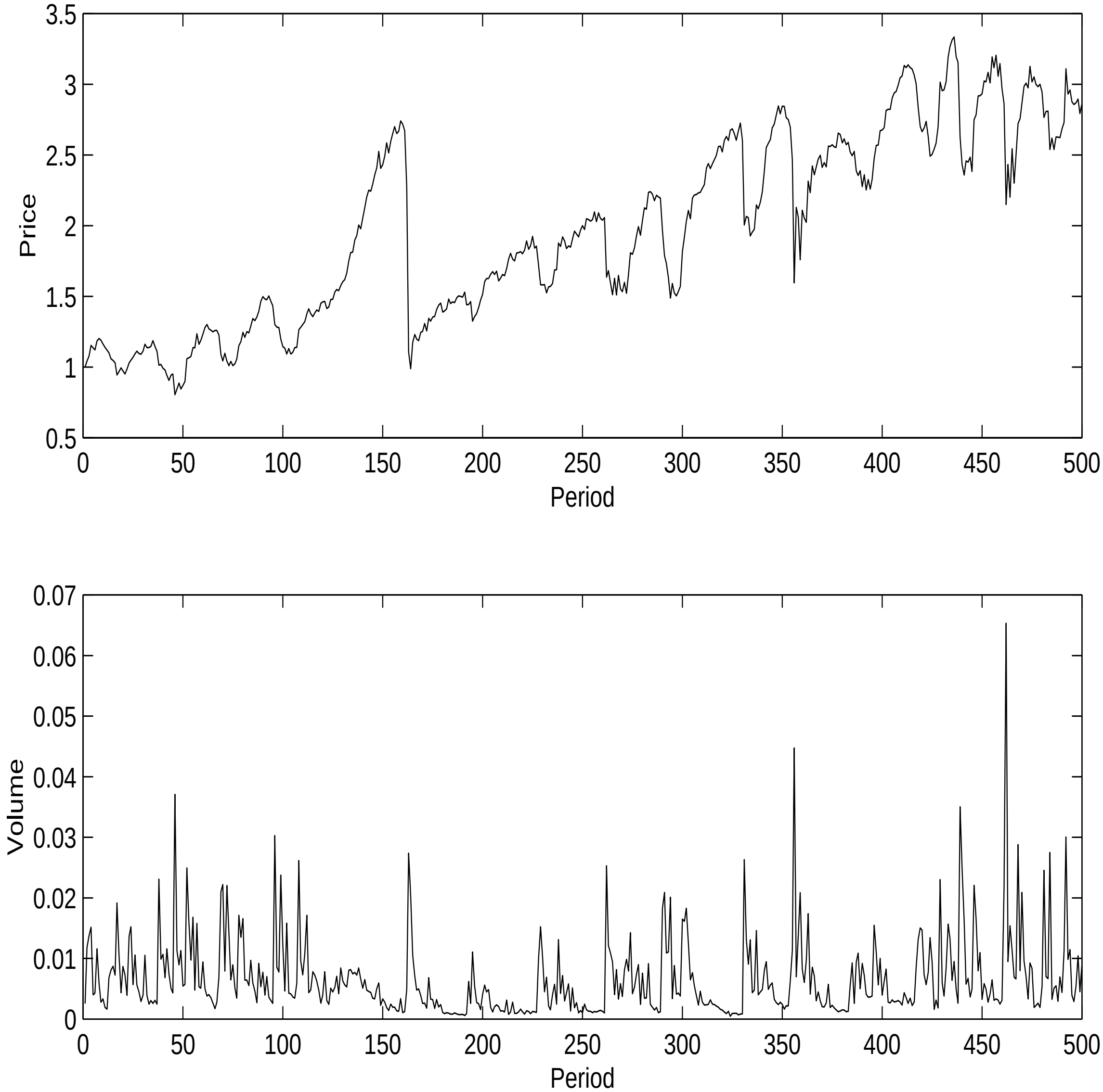


Figure 1: **Price/Trading Volume time series for all memory agents**

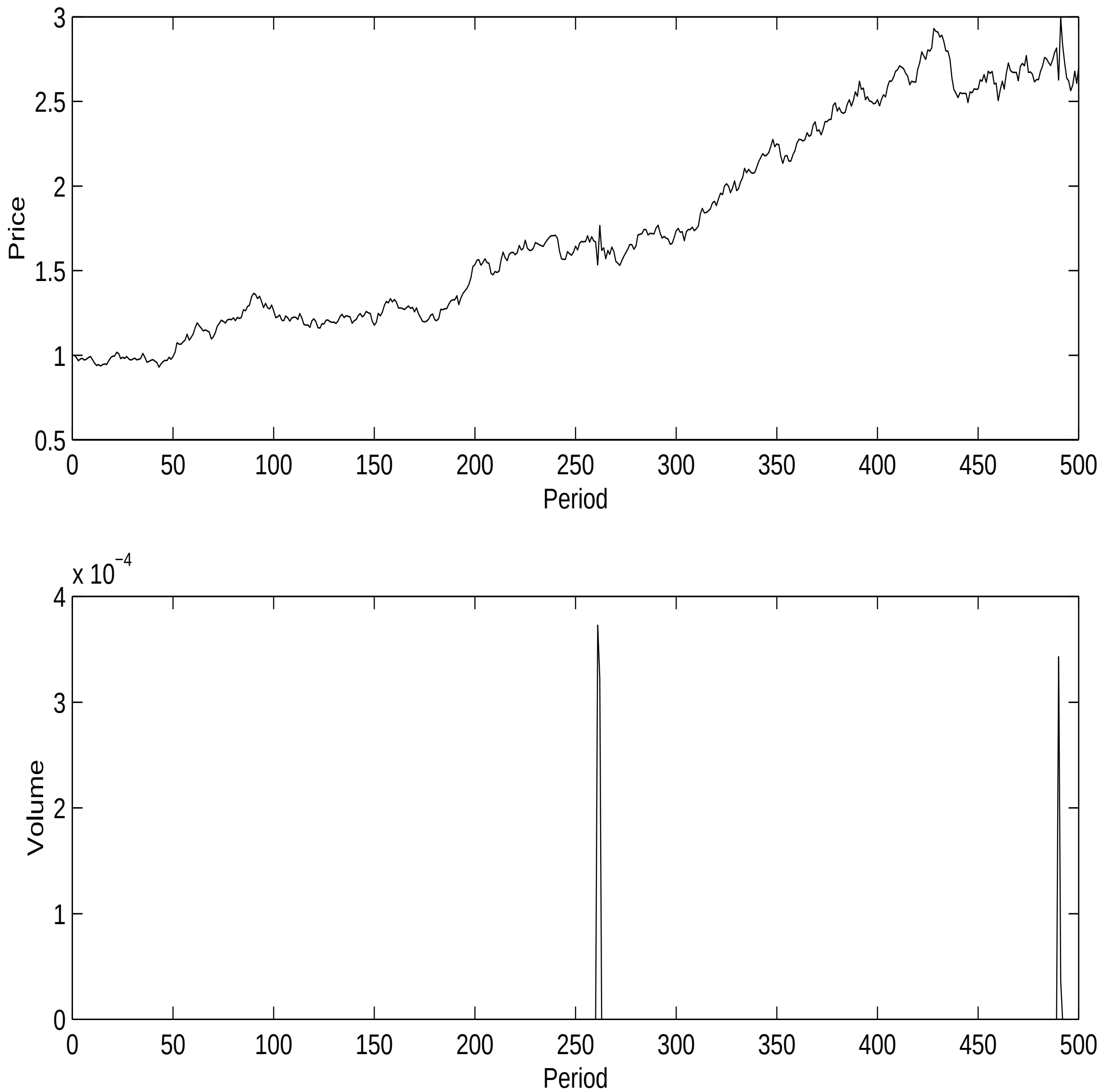


Figure 2: **Price/Trading volume time series for long memory agents**

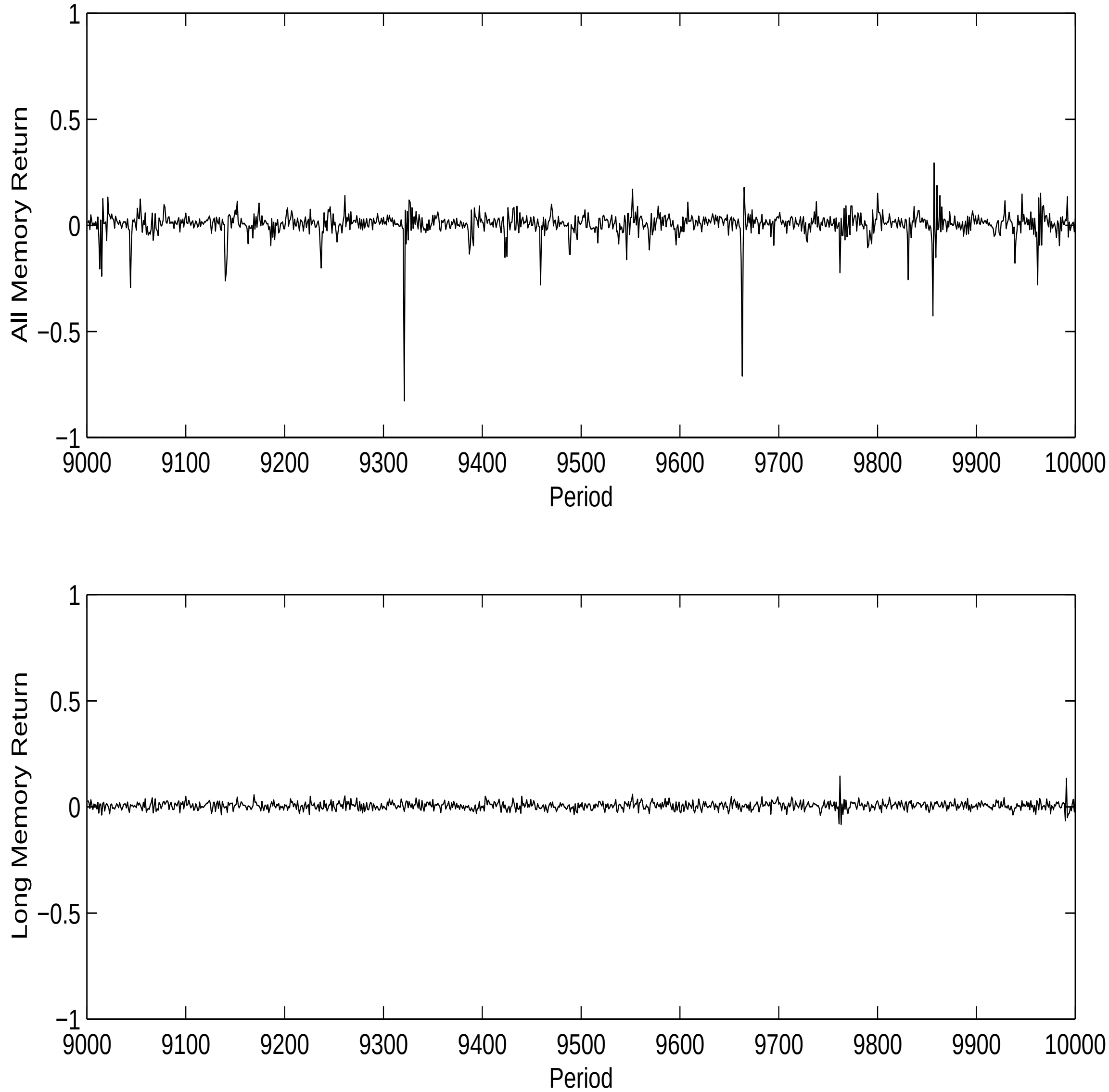


Figure 3: **Return time series**

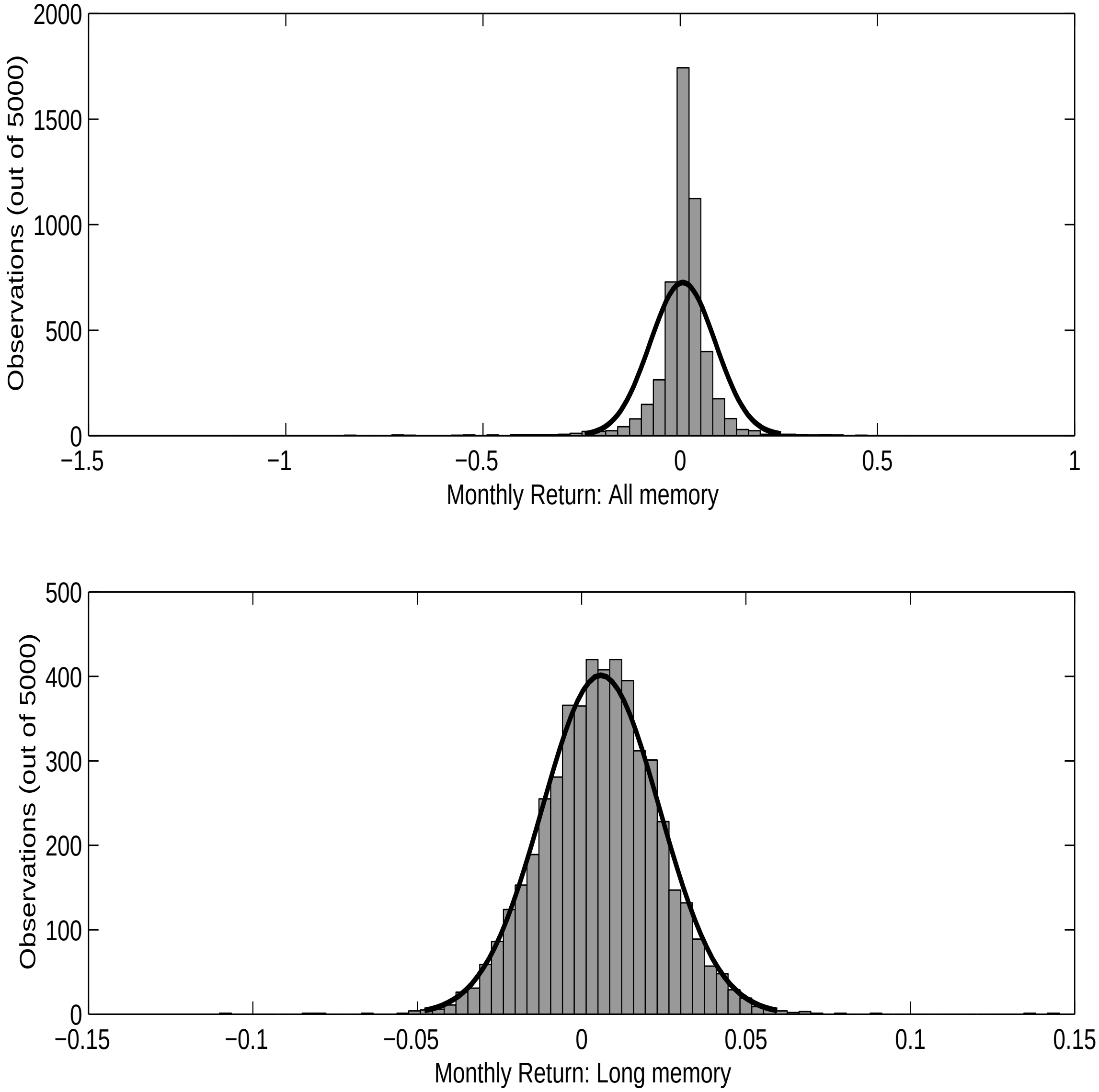


Figure 4: **Return distributions**

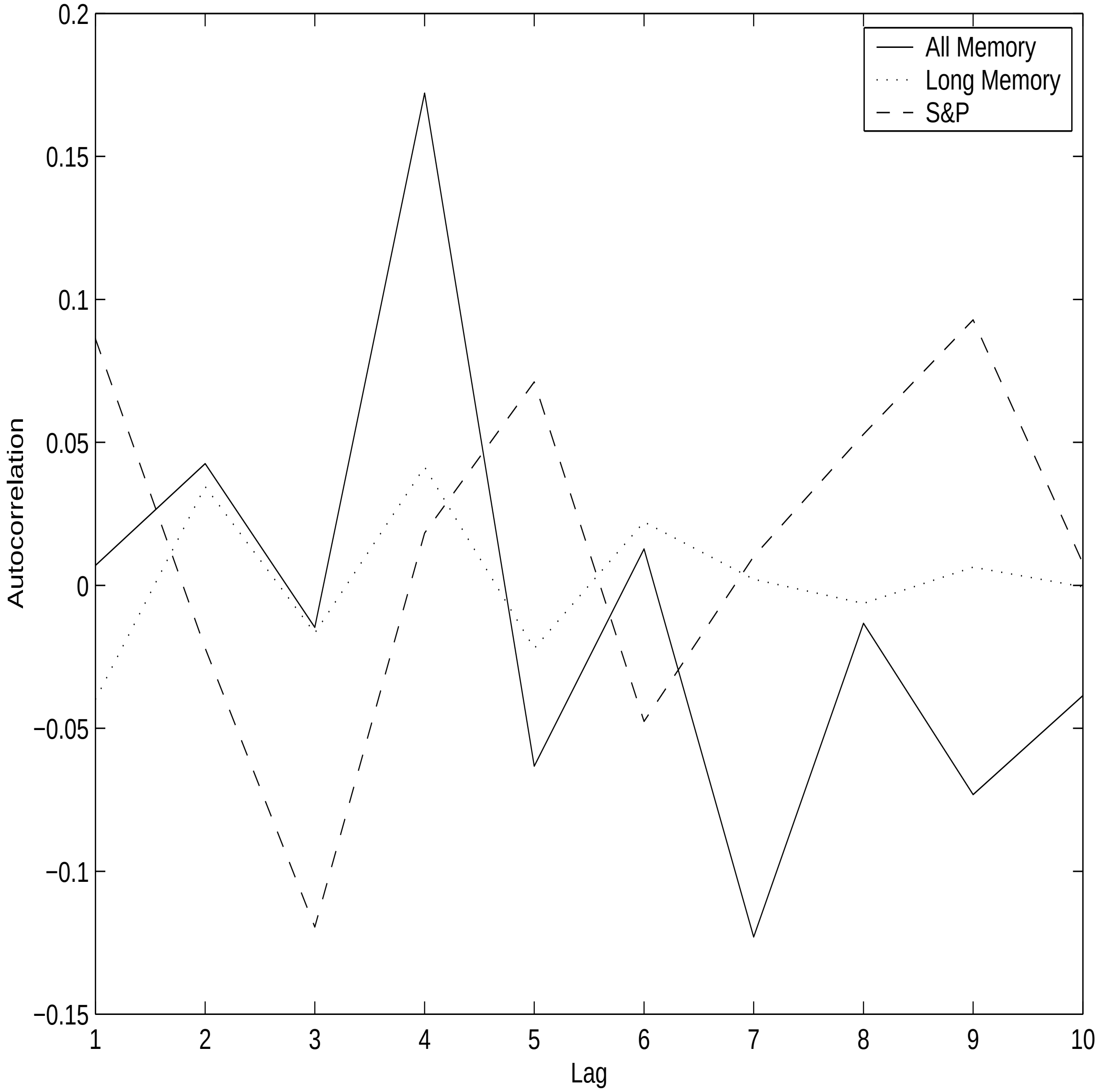


Figure 5: **Return autocorrelations**

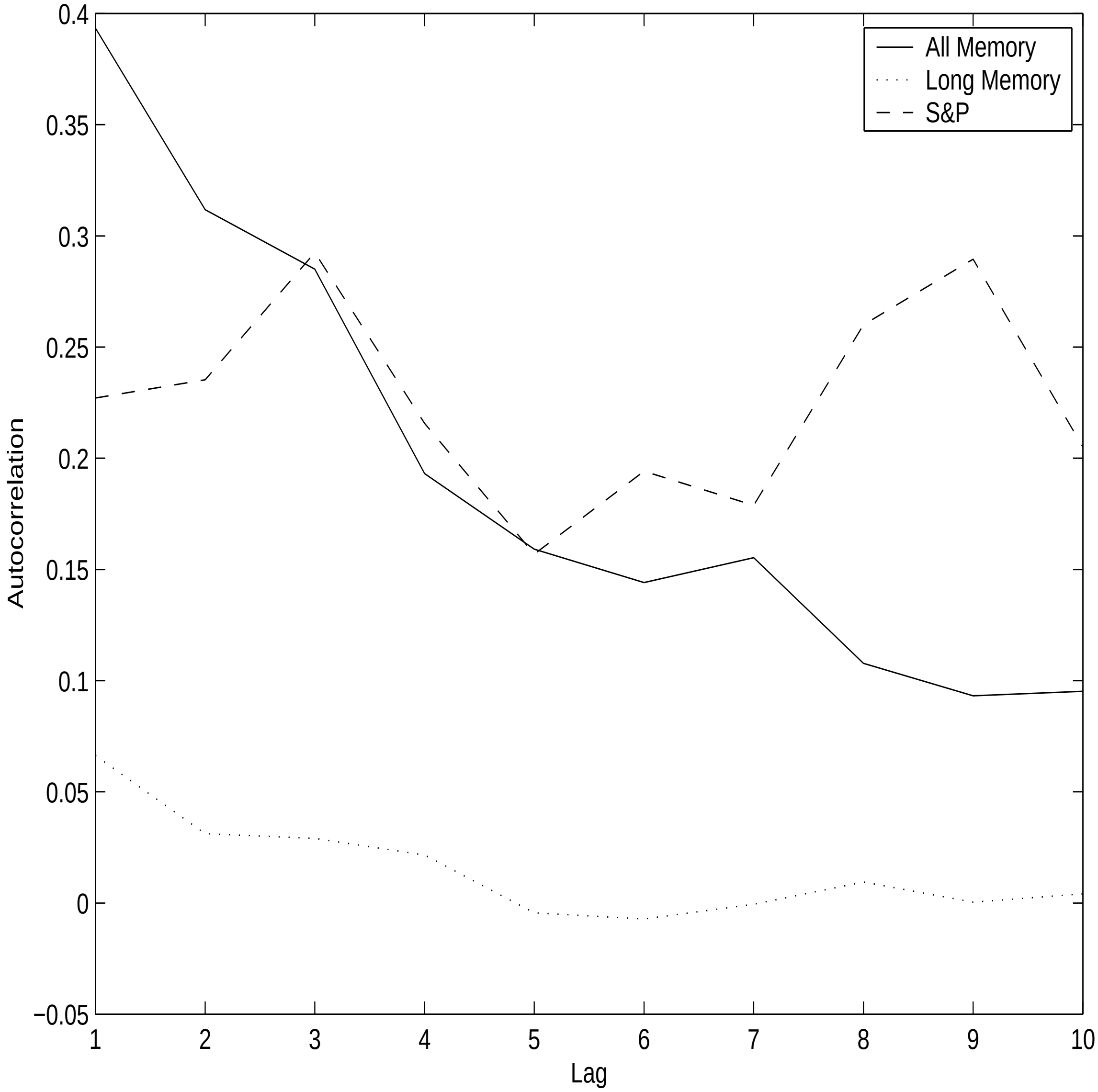


Figure 6: **Volatility (absolute return) autocorrelations**

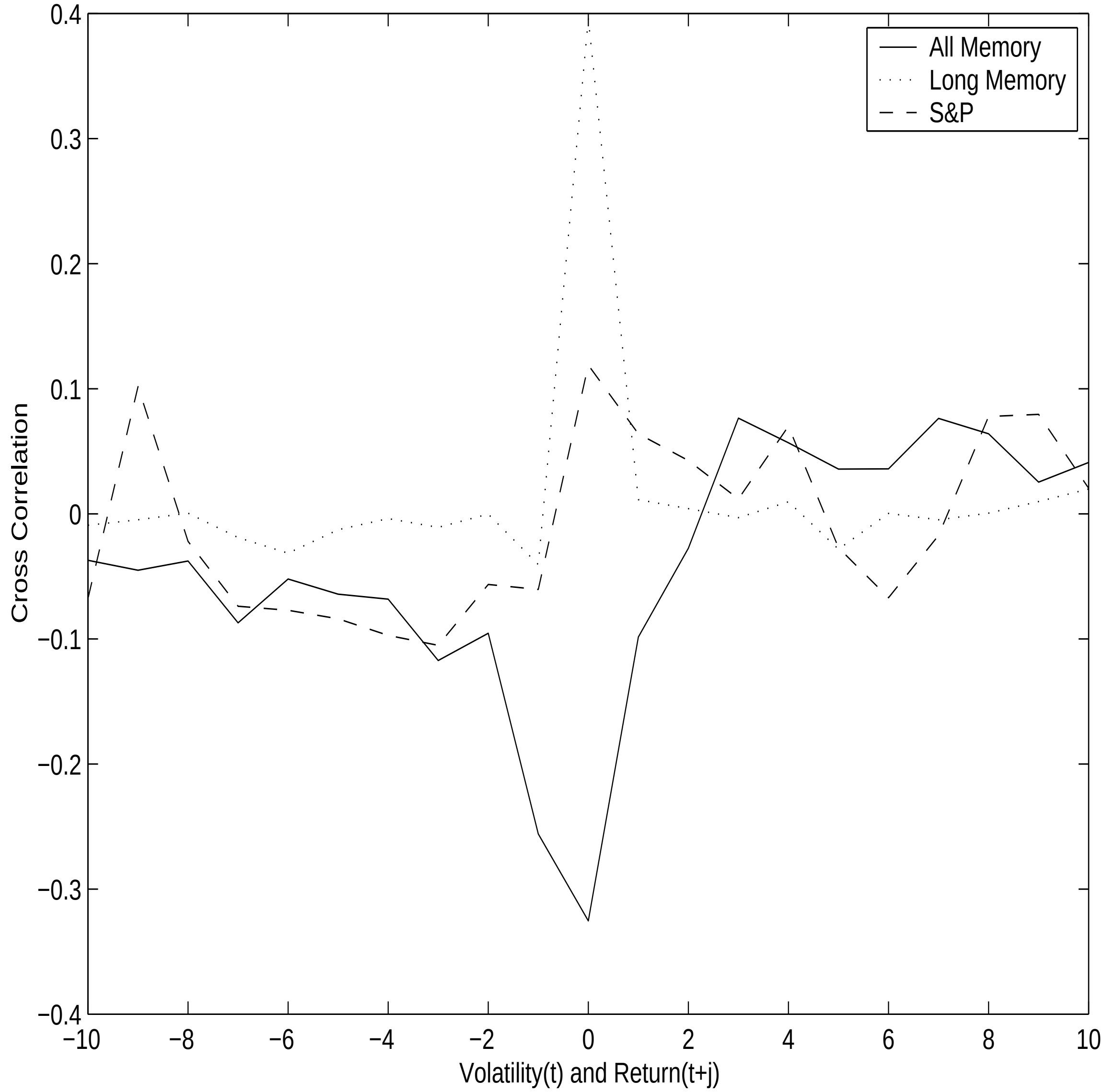
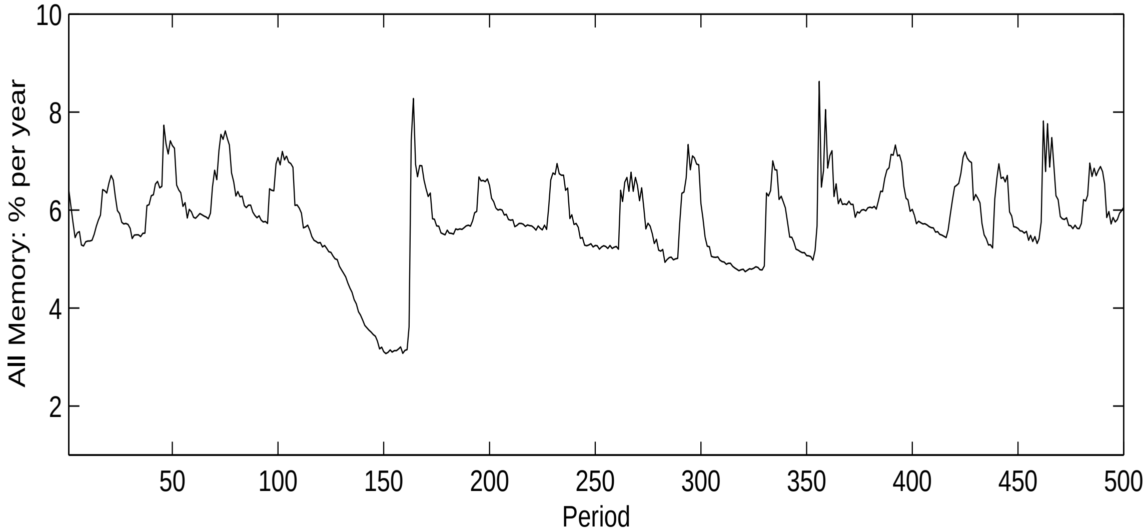


Figure 7: **Volatility and return crosscorrelations (j = 0 refers to contemporaneous correlation)**



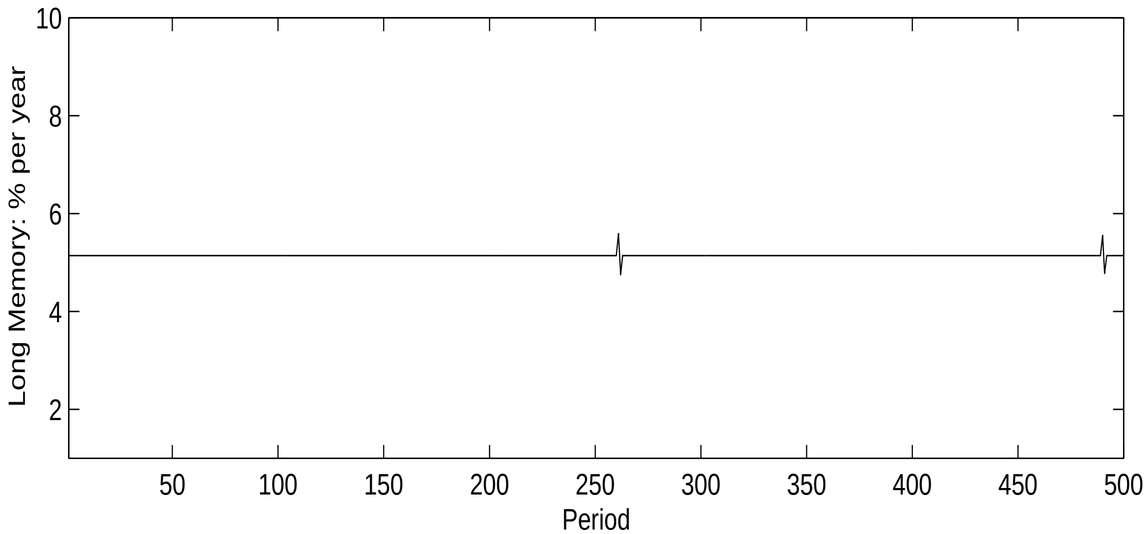


Figure 8: **Dividend yield: dividend/price ratio, annual rate**

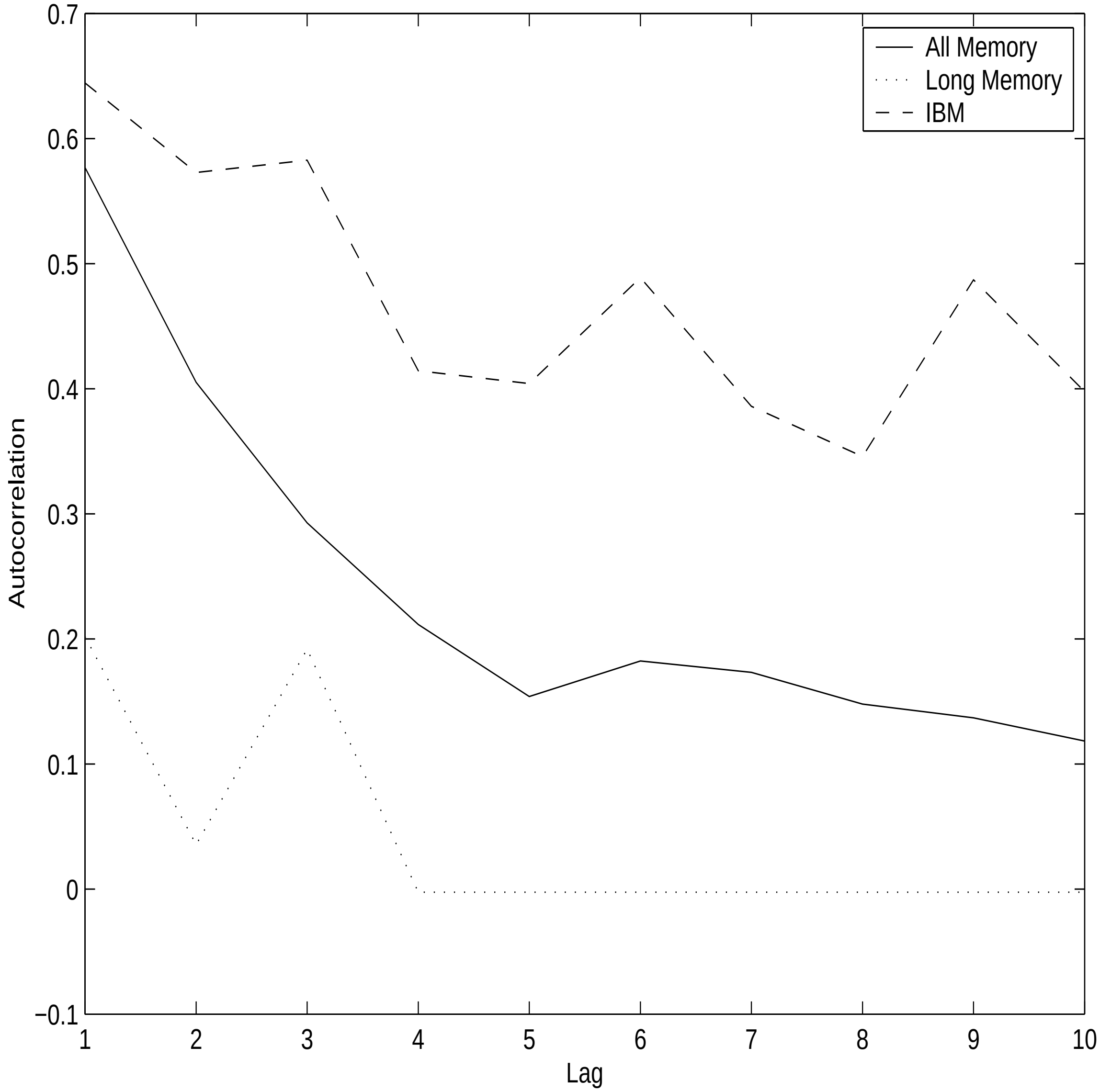


Figure 9: **Volume autocorrelation**

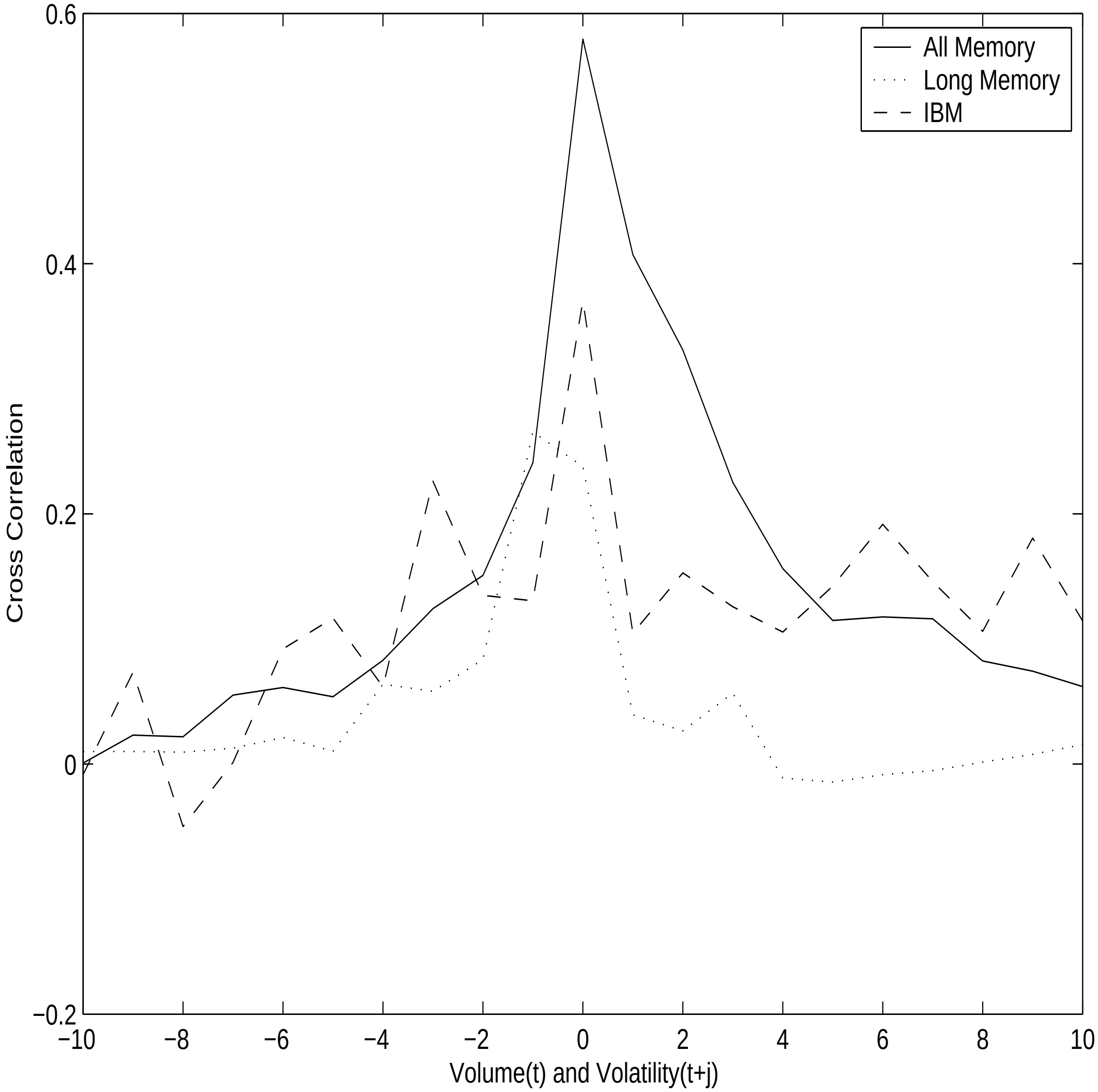


Figure 10: **Volume/Volatility cross correlation (j = 0 refers to contemporaneous correlation)**

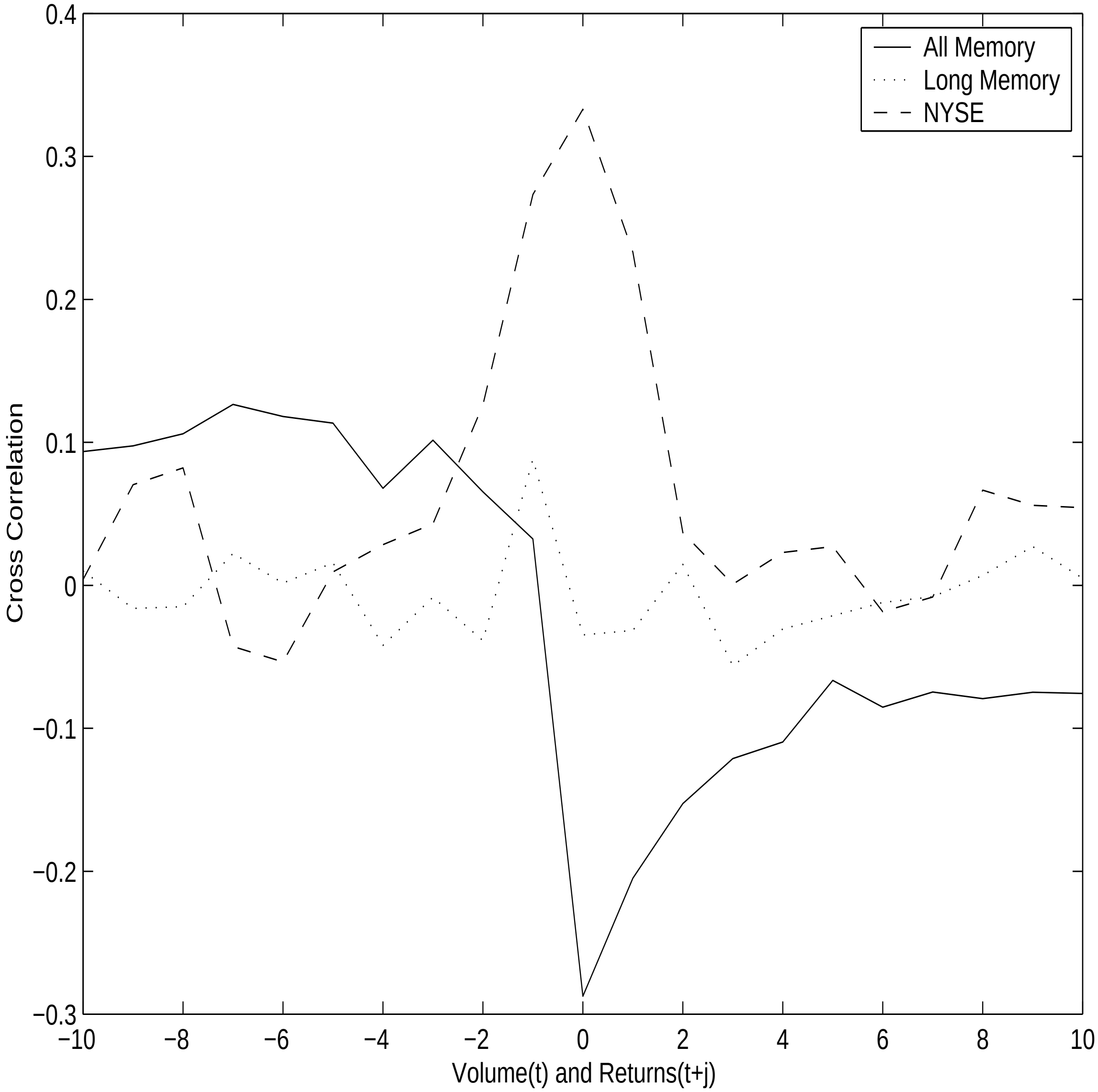


Figure 11: **Volume/Return cross correlation (j = 0 refers to contemporaneous correlation)**