The Impact of Imitation on Long-memory in an Order Driven Market

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Abstract

Recent research has documented that learning and evolution are capable of generating many well known features in financial times series. We extend the results of LeBaron & Yamamoto (2007) to explore the impact of varying amounts of imitation and agent learning in a simple order driven market. We show that in our framework, imitation is critical to the generation of long memory persistence in many financial time series. This shows that imitation across trader behavior is probably crucial for understanding the dynamics of prices and trading volume.

JEL Classification: G12, G14, D83

Keywords: Learning, Evolution, Market microstructure, Long memory

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1 Introduction

Agent-based financial markets provide a useful testbed for understanding the implications of changing levels of market heterogeneity and its relationship with important observed time series such as trading volume, volatility, and order flow information. This paper performs controlled computer experiments on a population of trading agents while observing the time series behavior of these simulated markets. Our experiments are based on a stylized order driven market first used in Chiarella & Iori (2002). In this market agents place their orders on an electronic order book which is observable to all agents. Recently, LeBaron & Yamamoto (2007) added a form of evolutionary learning to the model and examined various properties of the market's time series. We conjectured that many time series features were related to the levels of imitation in the market. This paper performs experiments which directly support this conjecture.

The order driven framework that we use is an important benchmark for modern financial markets. Its clearing mechanism is completely algorithmic, and the market contains no dealers, or designated market makers. In this sense, it is similar in spirit to electronic crossing markets (ECN's), and several public markets, such as the London Stock Exchange, which have substantial electronic components. This very simple market style eliminates many difficult institutional questions which would appear in a setup with different types of traders playing many different market making functions.

We populate this market with a set of agents following relatively simple trading strategies. There is a long, but sporadic, tradition of using low intelligence, or random, order flow combined with an automated market clearing mechanism. Cohen, Maier, Schwartz & Whitcomb (1983) and Domowitz & Wang (1994) are early examples stressing the statistical properties of incoming order flow data layered with a simple algorithmic trade clearing mechanism. More recently, the availability of higher quality trading data has influenced papers such as Farmer, Patelli & Zovko (2005) and Ladley & Schenk-Hoppe (2007) that demonstrate many features can be generated from random order flow alone. Our trading agents follow in this tradition by using relatively simple, but not random, trading strategies. In this way they are probably closer to the tradition of Zero-Intelligence (ZI) traders as in Gode & Sunder (1993), and not more traditional fully rational strategies as implemented in Goettler, Parlour & Rajan (2005). Our work complements that of the early ZI literature by showing that some structure on trading strategies is necessary to generate many empirical microstructure features. However, the strategies can still be relatively simple.

Time series from financial markets have given us many interesting features which remain difficult to explain. Some of these features pertain to very long horizons, such as the dynamics of dividend/price ratios, but others are measured at very high frequencies using trade by trade data that are now recorded in most electronic markets. In this paper we explore the possible causes for some of these high frequency features. We show that many of them might be related to the level of imitation going on across traders in a market. We follow most closely the empirical results from Bouchaud, Gefen, Potters & Wyart (2004) and Lillo & Farmer (2004) who document a large amount of persistence in various high frequency time series. These include trading volume, price volatility, and signed order flow. They also document little or no correlation in stock returns themselves even at very high frequencies.

In this paper we explore the importance of imitative behavior in generating these phenomena.[1] As agents imitate they reduce population heterogeneity which impacts the dynamics of prices in the financial market. This may correspond to recent conjectures about quantitative trading strategies and market volatility.[2] Imitation is not the only explanation that has been proposed for generating these features. Lillo, Mike & Farmer (2005) explore a model in which traders split larger orders into smaller ones to minimize price impact. We confirm that the order splitting approach generates some of the same features, but it also differs from our model in some important areas.

Section 2 outlines the basic model structure. Section 3 presents the results form the computer experiments, and section 4 concludes and summarizes.

2 Market structure

This section describes our market which is based on the order driven setup used in Chiarella & Iori (2002). It is designed to replicate trading in a simple electronic clearing mechanism which mimics modern electronic markets with an open order book. This paper follows LeBaron & Yamamoto (2007) by introducing adaptive learning into the market using a genetic algorithm.

The market explicitly models the details of trading behavior in an order book. Agents submit orders to buy (bids), and orders to sell (offers), based on forecasts of the current price of the asset. An observed price, p_t, is recorded when a transaction actually takes place, or if no transaction takes place it will be an average of the best bid, b_t, and best ask, a_t.

The market consists of a heterogeneous group of agents, indexed by i, who use past prices to make

[1] See LeBaron (2001*b*) for an example of how endogenously changing heterogeneity in investor strategies can impact observed market behavior.

[2] See Khandani & Lo (2007) for information on how this situation might occur with quantitative hedge funds.

predictions of the future. For all runs the number of agents is fixed at 1000. Define the return at time t as,

$$r_t = \frac{p_t - p_{t-1}}{p_{t-1}}. \tag{1}$$

Traders try to estimate a local momentum indicator as an average of the past L_i returns given by,

$$\bar{r}_{L_i} = \frac{1}{L_i} \sum_{j=1}^{L_i} r_{t-j}. \tag{2}$$

This will be used as the chartist component in the forecast since it only involves past prices. Also, assume that there is a well defined fundamental value for this security given by p^f. Traders make weighted forecasts about the future return of the asset combining both a fundamental and a momentum (technical) based forecast. This is formalized as,

$$\hat{r}^i_{t,t+\tau} = g^i_1(\frac{p^f - p_t}{p_t}) + g^i_2 \bar{r}_{L_i} + n_i \epsilon_t, \tag{3}$$

where g^i_1, g^i_2, and n_i weight the fundamental, chartist, and noise induced components of agent i's return forecast respectively. The initial distributions of these parameters are independent over agents, normally distributed, and given by $g^i_1 \sim |N(0, \sigma_1)|$, $g^i_2 \sim N(0, \sigma_2)$, $n_i \sim |N(0, \sigma_n)|$, and $\epsilon_t \sim N(0, 1)$. Table 1 presents the parameter values used in our simulations.

Agents build expectations about future prices at time $t + \tau$ according to

$$\hat{p}^i_{t+1} = p_t e^{\hat{r}^i_{t,t+\tau}}. \tag{4}$$

Agents intend to buy or sell around this forecast price according to a constant spread variable, k^i, and

$$b^i_t = \hat{p}^i_{t,t+\tau}(1 - k^i) \tag{5}$$

$$a^i_t = \hat{p}^i_{t,t+\tau}(1 + k^i), \tag{6}$$

where k^i is randomly assigned across agents and is drawn uniformly from $(0, k_{max})$.[3]

All orders to buy and sell are restricted to one unit of the asset at the appropriate bid or ask. Agents

[3]The use of k here represents a crude form of heterogeneous risk aversion in our population. Both buyers and sellers demand some premium beyond their risk neutral price. In this sense we are assuming that the actual risk of placing the order dominates any impact on the agents' portfolio. As a practical matter this also keeps traders from churning their portfolios at the same buy and sell prices. We model k as symmetric since the we are not modeling any specific differences between buying and selling behavior. Also, there is some empirical evidence on the symmetry of order placement in Mike & Farmer (2008).

decide to buy or sell based on whether they predict a price increase or decrease. Buyers compare their current bid price to the current best ask. If the bid is higher than the current best ask, then the trade takes place and the limit ask is removed from the book. If the bid is less than the best ask, then it is added to the book on the bid side. A similar mechanism takes place for sellers with the asks by comparing these to the bid side of the market. All orders have a limited time on the book, and are removed if they remain unexecuted after ω time periods go by. Trading takes place sequentially and each round runs through all agents in a randomly determined order.

A *click* is the smallest time step in the market, and is the unit of time in which a single agent makes an order and executes it, or leaves it in the book. Also, after each click agents update their forecasts according to equation 3. Both the trend and fundamental equations are updated at each tick. The trend returns in equation 2 are indexed by ticks, and are updated to reflect the mean return over the last L_i ticks. After 5 trading rounds go by agents look at the performance of their current forecasting models, and evaluate them according to forecast accuracy over the last 5 rounds,

$$f_i = \frac{1}{\sum_{5 \text{ rounds}} (p_t - E_i(p_t))^2}, \tag{7}$$

this defines the "fitness" of agent i. The forecast error is evaluated at every tick, so this sum represents performance measured over 5000 tick increments, corresponding to actions by all 1000 agents over the 5 rounds.

In our previous papers, this fitness value is used to update the entire population of traders. In this paper we are concentrating this evolutionary process on a limited fraction of the population given by γ. We will refer to this as the degree of imitation. Every 5 rounds this fixed γ fraction of the population will update their rules using a form of genetic algorithm (GA). Probabilities are assigned to each rule using,

$$p_i = \frac{f_i}{\sum_i f_i}. \tag{8}$$

For each of the γ fraction of agents the algorithm selects a parameter to modify out of the set, g_1^i, g_2^i, L_i, and n_i. This is then replaced with one drawn from the entire set of agents according to probability p_i. This parameter is then mutated with probability $p_m = 0.08$.[4] Mutation replaces the parameter with a new value

[4] We have performed robustness checks on our mutation rate. For values of $p_m = 0, 0.08, 0.12, 0.16$ our results are unchanged. As the mutation rate gets very high, the selective impact of the GA is driven to zero, and the results will disappear for relatively large values of p_m. We have performed further robustness checks on other parameters used in our simulations. Specifically, we have varied σ_1, the standard deviation of the fundamental forecasts from 0.5 to 1.5 from our original value of 1. We have also varied σ_2 the chartist forecasting component from 1 to 2 from the original value of 1.5, and the noise standard deviation, σ_n,

drawn from the original distribution used when the parameters were initialized.

This mechanism will be driving imitative behavior in our market. The GA selects for parameters from rules which have done well in the past using the probabilities p_i. Strategies built from these parameters will be emphasized in the population going forward. Evolutionary learning of this type does make some requirements about what agents know about other's strategies. It is a form of social learning where traders are able to infer exactly what their successful counterparts are doing.[5] Agent-based financial models have taken many different approaches as to how much strategy information could be shared outside the price system. Early markets such as Arthur, Holland, LeBaron, Palmer & Tayler (1997) assumed zero information sharing, but later markets such as Chen & Yeh (2001) and LeBaron (2001*a*) use intermediate mechanisms where some information sharing is allowed. Obviously, the correct model for information sharing is not identifiable, but it is clear that some imitation must take place in financial markets. In our experiments, that actual amount of imitation will be limited, since a fixed fraction will not adjust their strategies at all.

3 Model experiments

Our main goal is to understand the sensitivity of our previous results, LeBaron & Yamamoto (2007), to the degree of imitation. In a model with the degree of imitation set to $\gamma = 1$ we found that our stylized trading model generated evidence for long memory in generated time series. Specifically, trading volume, volatility, and buyer/seller initiations all generate long memory. In this paper we will explore this evidence as γ is varied from 0.2 to 1.

An important issue in a microstructure model which is driven by ticks that define agents' behavior is how to convert this into wall clock time. We do this by sampling prices every 50 ticks, and summing the number of shares traded over this interval to measure trading volume. Returns are measured as the percentage change in prices over the 50 period interval, and volatility is measured as the standard deviation in the price over the interval. We define each period as buyer or seller dominated (-1 or 1 respectively) depending on whether there are more buyer or seller initiated trades in the market.

The feature of the series that we are interested in is known as long memory. This is a form of extreme persistence in a time series. Autocorrelations in such series decay slowly as one moves to longer lags.[6]

from 0.3 to 0.7 from our original value of 0.5. Finally, we have adjusted k_{max}, the spread parameter length from 0.3 to 0.7. In all these cases we find that our results do no change significantly.

[5]See Vriend (2000) for a general discussion.

[6]It is beyond the scope of this paper to discuss long memory processes. Interested readers should consult some recent surveys which include Baillie (1996), Robinson (2003), and Doukhan, Oppenheim & M. S. Taqqu (2003). See also Parke (1999) for examples, intuition and discussion. Recent work in finance shows that volatility is a likely candidate for long memory even at

Recently, high frequency financial data have revealed evidence for long memory in several series. Specifically, trading volume, price volatility and signed order flow all appear to be reasonable candidates for long memory. We will examine these series in our simulated markets. We test for long memory using the modified R/S statistic developed in Lo (1991).[7] We will report both averages of this statistic over 10 runs, and the fraction of runs which reject the no long-memory null hypothesis at the 95 percent level of significance.

Figure 1 presents the average R/S statistic for trading volume over the 10 runs, for various values of γ. The dotted line indicates significance at the 95 percent level. The Lo version of the R/S statistic also depends on a parameter q which adjusts for possible short term dependence in the series. We estimate the test statistic for $q = 4, 6, 8, 10$. The figure clearly shows that for low levels of imitation we see very little evidence for long memory. These results are repeated in figure 2 which reports the fraction of rejections at the 95 percent level of the short memory null hypothesis. This shows a dramatic shift as γ is increased. The fraction of rejections is below 0.20 for all values of q and $\gamma < 0.30$. However, they increase quickly, and are almost 100 percent once γ goes beyond 50 percent.

Figures 3 and 4 repeat the previous results for the time series of price volatility measured as the standard deviation of prices in each 50 tick block. Once again the average of the modified R/S statistic increases dramatically as γ increases for all values of q. The fraction of rejections, plotted in figure 4, also increase with γ. The results show a very dramatic change with less than 20 percent rejections for $\gamma < 0.40$, and a shift to nearly 100 percent rejections for $\gamma > 0.70$. This parallels our results on trading volume where the presence of long memory appeared quite suddenly as imitative behavior γ is increased.

Figures 5 and 6 report the results for the signs of order flow in each period. We again see increases as γ increases. The average R/S statistic increases more gradually than in the other cases, but the fraction of rejections again moves very quickly, and is nearly 100 percent once $\gamma > 0.60$. This again indicates that the simulation cross section is moving together in terms of evidence for long memory in the time series.

Figure 7 reports the mean R/S statistic for the returns series. For all values of q these are well below the 5 percent critical value. None of the 10 runs rejected the short memory null hypothesis which is consistent with our previous results, and the empirical features of the actual returns. Models with full imitative behavior in the population, $\gamma = 1$, still do not produce evidence for long memory. Figure 8, looks at the return first order

longer horizons than intraday. For examples see Ding, Granger & Engle (1993), Baillie, Bollerslev & Mikkelsen (1996), and Andersen, Bollerslev, Diebold & Labys (2003).

[7] The Lo test is one of many long memory tests, and is based on earlier R/S analysis. The test has been criticized in Teverosvsky, Taqqu & Willinger (1999) and Willinger, Taqqu & Teverosvsky (1999). Their monte-carlo experiments show that the test can accept the null of no long range dependence as the band-width parameter is increased. We are concerned about this, but in most of our runs this low power problem is not an issue since we are rejecting the null hypothesis. We are also exploring the use of some other long range diagnostics such as Giraitis, Kokosza, Leypus & Teyssiere (2003*b*) and Giraitis, Kokosza, Leypus & Teyssiere (2003*a*).

autocorrelations across the different experiments. These show that returns are not positively correlated at all. In fact, they are slightly negatively correlated across the entire range of imitation levels, γ. This negative correlation is probably due to short range market liquidity issues, and movements from the best bid to the best ask. As buyers move up the order book, and the price rises, there will be eventual selling pressure to bring it back down. In a market with finite numbers of agents we should see some of these short range liquidity effects. These should be magnified as imitation levels are increased, since for these cases traders behave in a more similar fashion.

Figures 9 and 10 explore some of the connections between our results and the underlying dynamics of the forecast parameters. Figure 9 displays the mean and one standard deviation band on the parameter g_1, the fundamental forecast from equation 3, for the runs with 100 percent imitative behavior, $\gamma = 1$. It is clear that this value does not settle down over time, and goes through some wide swings. Also, the dispersion of this parameter varies considerably, and appears to widen after large changes. Figure 10 presents the results for a run with only 20 percent imitative behavior. The results are dramatically different. Here, the mean and dispersion on g_1 are nearly constant. These results are suggestive that many of the time series features are connected to the population dynamics in the underlying forecast parameters.

So far, we have shown that some form of imitative behavior of stock traders is key for generating long memory features. However, imitation may not be the only explanation. Lillo et al. (2005) demonstrate that the signs of order flows are autocorrelated when orders are broken up into small pieces. Motivated by their results, we will briefly examine if order splitting behavior would be another possible source for the long memory of signed order flows as well as volume and volatility. We analyze a simple automated system of order splitting with an iceberg type limit order.

The market is similar to our previous experiments with no evolution. We have 100 agents who are each given 50 units of orders to split. They determine their forecast and execution prices as in the previous experiments. This determines whether they are buyers or sellers, and their bid or ask price. For a buyer, if the bid price is larger than the best ask, the agent will execute 1 trade, and drops the block of orders by 1. The agent will continue executing buy orders until the best ask price on the book exceeds the agent's bid price. This is known as "walking up the book". At this point the agent will submit a limit order for the next unit. The remaining part of the order is entered as an iceberg order at this bid price. It does not show up on the book directly, but when that order is removed, a unit of the iceberg order is moved onto the book. If the buyer's initial bid price is less than the best ask, then 1 unit is entered on the book, and the remaining 49 are entered as iceberg orders. A similar mechanism takes place for sell orders. We assume that other agents cannot enter the market while one agent is executing orders. Once the iceberg order is in place the next agent enters the market.

We did 10 simulations, and the averages of Lo's R/S statistics over 10 runs are given in table 2. The results in table 2 show that we find long-memory of volatility as well as order signs. However, we do not see any evidence for long-memory of volume. Signed order flows are persistent in our system mainly because agents are assumed to keep submitting their orders of the same sign without allowing others to enter the market. Volatility also shows long-memory. Since agents continue to submit orders of the same sign, they have a bigger impact on reducing depth on one side of the order book. As a result, the book will become sparser, tending to have persistently larger price changes. However, once agents submit large size iceberg orders without executing any of them, the book suddenly becomes thicker so that the price changes become smaller and these smaller changes will persist. Our most interesting result is that we do not see volume persistence. This differs from both the data, and our results with imitation. Given our results on the time series on the forecast parameter g_1, it seems likely that volume persistence requires large swings in agent heterogeneity. Since iceberg orders alone maintain constant agent heterogeneity, they are not able to generate volume persistence.[8]

4 Conclusions

We have shown that a stylized agent-based order driven market model is capable of generating many interesting stylized facts in high frequency financial series. Specifically, the long memory persistence in trading volume, volatility, and order signs. Our model generates this strong persistence without generating large autocorrelations in return time series. All of these features are consistent with a growing body of literature which has documented these facts in several electronic trading systems.

We have also compared the impact of imitative behavior to another simpler mechanism based on order splitting. We find some interesting similarities and differences in our results. Most importantly, imitation was necessary to generate long-memory persistence in trading volume. We also believe there may be deeper connections between these simple trading mechanisms, and some older mechanisms for generating long memory in time series that were developed in Granger (1980) and Robinson (1978).

Our earlier conjectures were that the results were connected to the evolutionary learning process involved

[8]We have also experimented with smaller order blocks, specifically 10 units. These blocks were not able to generate long-memory in any of our time series. Obviously, there are many ways to implement order splitting, and we are continuing to explore the differences between these mechanisms, and our imitative framework.

in our simulated markets, and the extent to which it led to imitation in trader behavior. In this paper we have run a sequence of controlled experiments, carefully varying the amount of imitative behavior in the population. We have shown that long memory features depend critically on the degree of imitation in trader behavior. Our evidence for long memory varies dramatically as we change these levels. It is interesting that for low levels of imitation our long memory evidence is close to zero. Also, the appearance of long memory appears to be quite abrupt as the imitation level is increased.

This paper is part of our larger agenda of using an agent-based framework to explore the underlying causes for the levels of persistence we observe in high frequency financial time series. We think the link between imitation and population heterogeneity is interesting, and may help to understand the more general problem of what drives market prices during calm and turbulent periods. We also plan to use market structures similar to this one to explore the dynamics of other institutional details involved in trading financial securities.

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Table 1: *Parameter Values*

Parameter	Value
Std. of fundamental component: σ_1	1
Std. of chartist component: σ_2	1.5
Std. of noise trader component: σ_n	0.5
Order life: ω	200
Spread max: k_{max}	0.5
Fundamental price: p^f	1000
Max chartist time horizon: L_{max}	100
Tick size: Δ	0.1

Simulation parameter values. ω and L_{max} are measured in ticks which correspond to a single agent action period.

Table 2: *Order Splitting Tests*

	Volume	Volatility	Signs of Market Orders
$q = 4$	0.07	5.15**	1.72*
$q = 6$	0.06	4.49**	1.71*
$q = 8$	0.05	4.04**	1.70*
$q = 10$	0.05	3.70**	1.68*

Averages of R/S test statistic over 10 simulations. ** indicates that we were able to reject the null hypothesis of short-range dependence at the 95 percent confidence level in all 10 runs. * indicates that we reject the short-range dependence null hypothesis in at least 6 of the 10 simulation runs. No *'s indicates that we were not able to reject the short-range dependence null in any of the 10 runs.

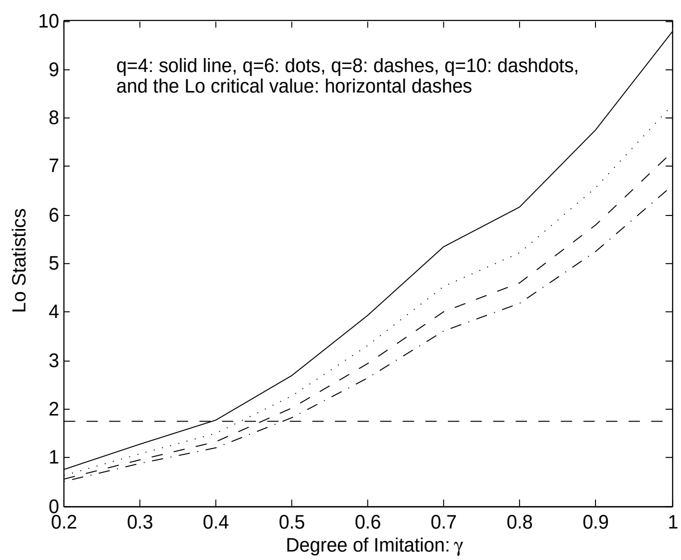

Figure 1: *Average Lo R/S statistics: Trading volume*

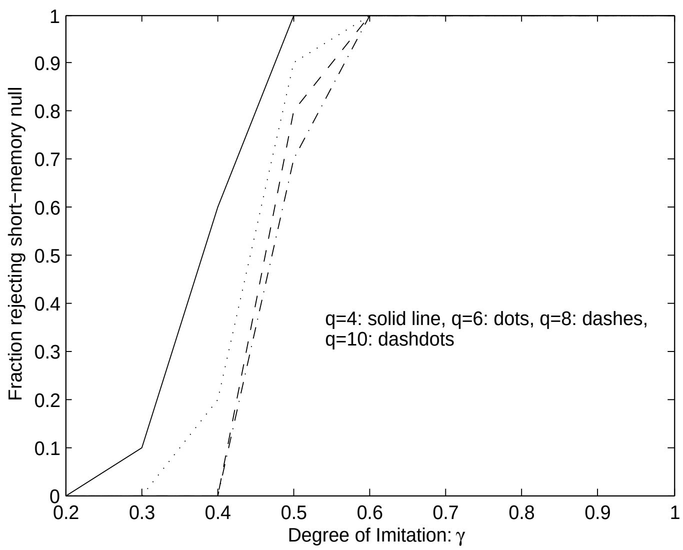

Figure 2: *Fraction of Rejections: Trading volume*
Fraction of the 10 runs rejecting the short-memory null hypothesis at the 95 percent confidence interval.

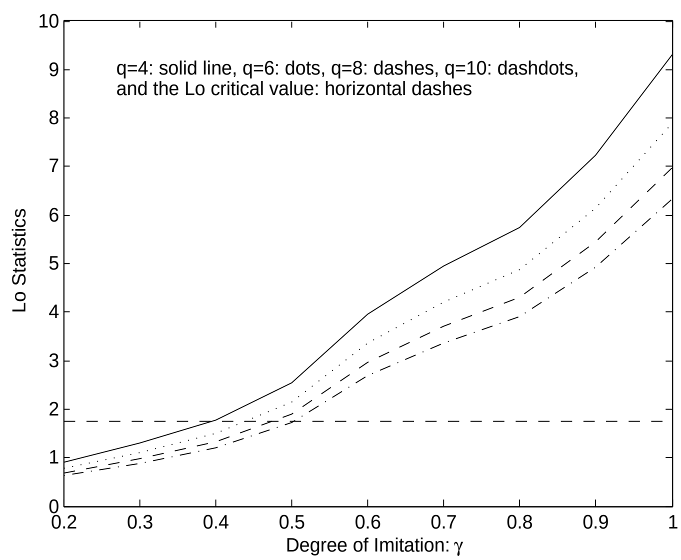

Figure 3: *Average Lo R/S statistics: Price volatility*

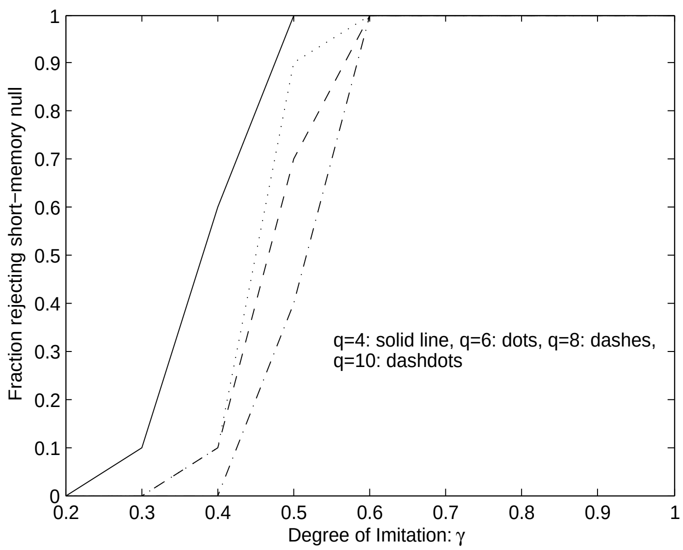

Figure 4: *Fraction of Rejections: Price volatility*
Fraction of the 10 runs rejecting the short-memory null hypothesis at the 95 percent confidence interval.

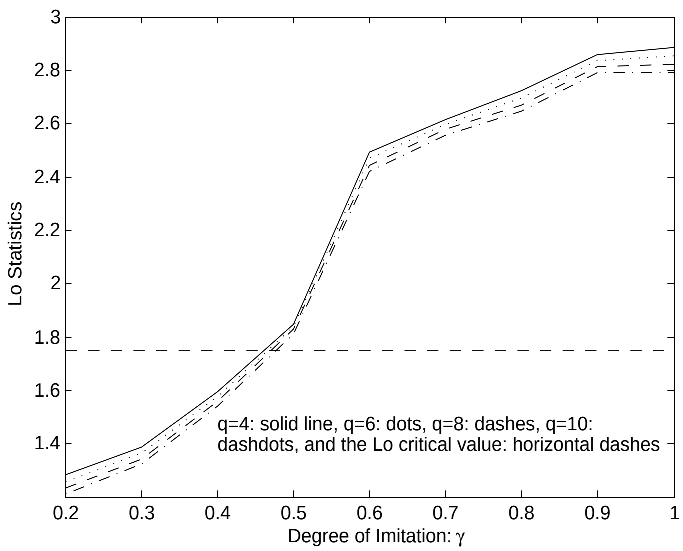

Figure 5: *Average Lo R/S statistics: Order Signs*

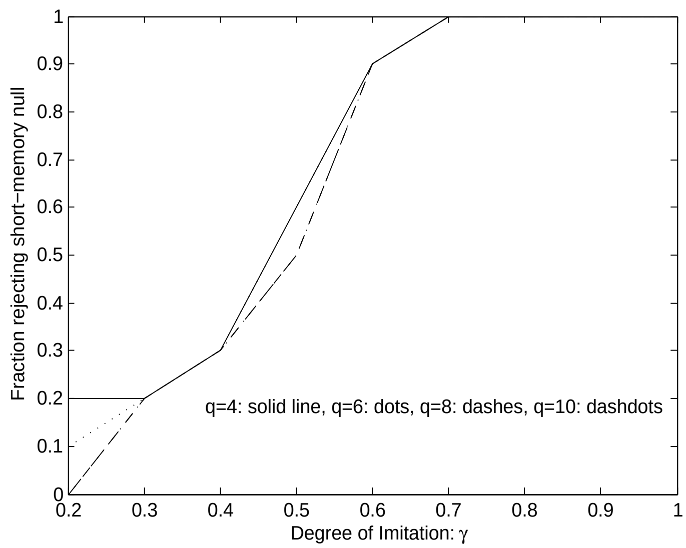

Figure 6: *Fraction of Rejections: Order Signs*
Fraction of the 10 runs rejecting the short-memory null hypothesis at the 95 percent confidence interval.

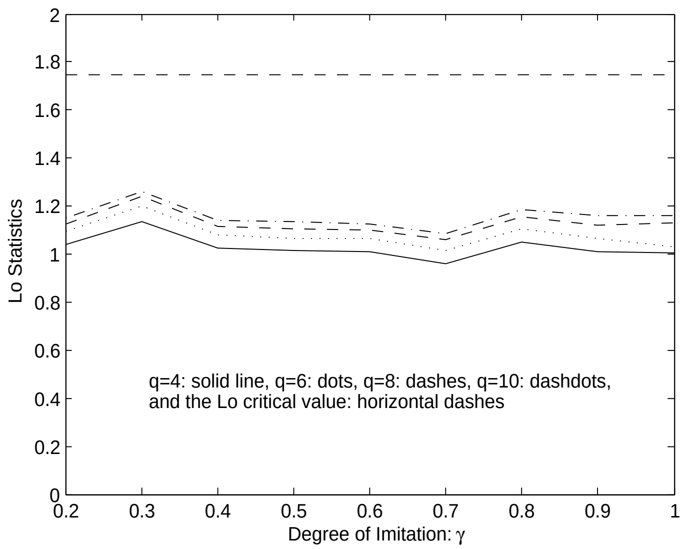

Figure 7: *Average Lo R/S statistics: Returns*

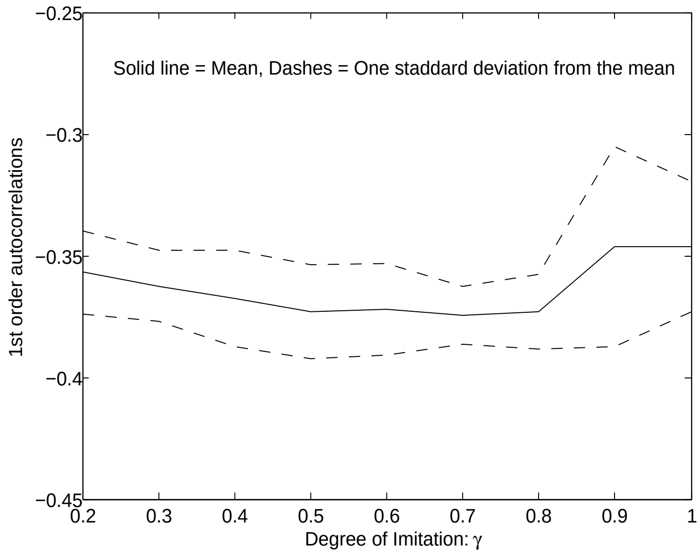

Figure 8: *First order autocorrelation: Returns*

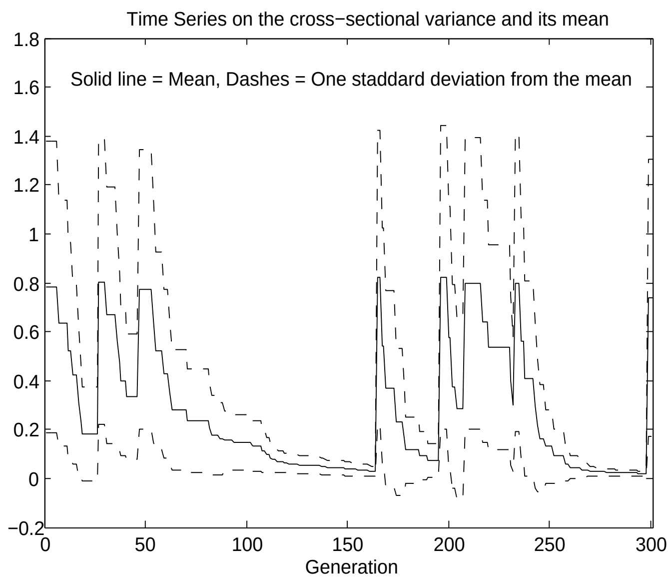

Figure 9: *Mean and Std. of* g_1 $(\gamma = 1)$

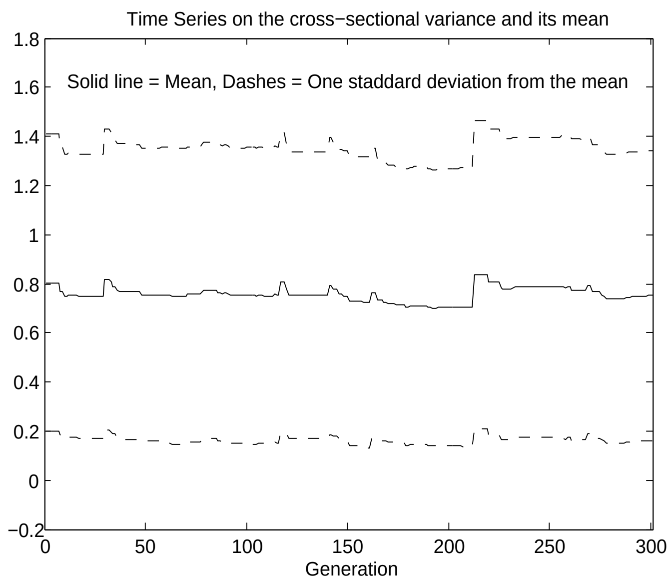

Figure 10: *Mean and Std. of* g_1 $(\gamma = 0.20)$