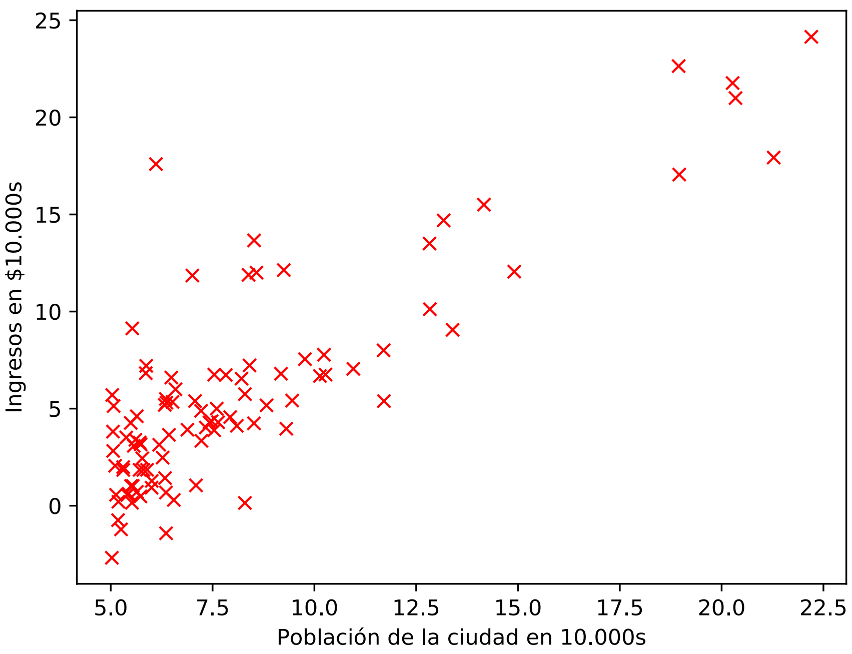
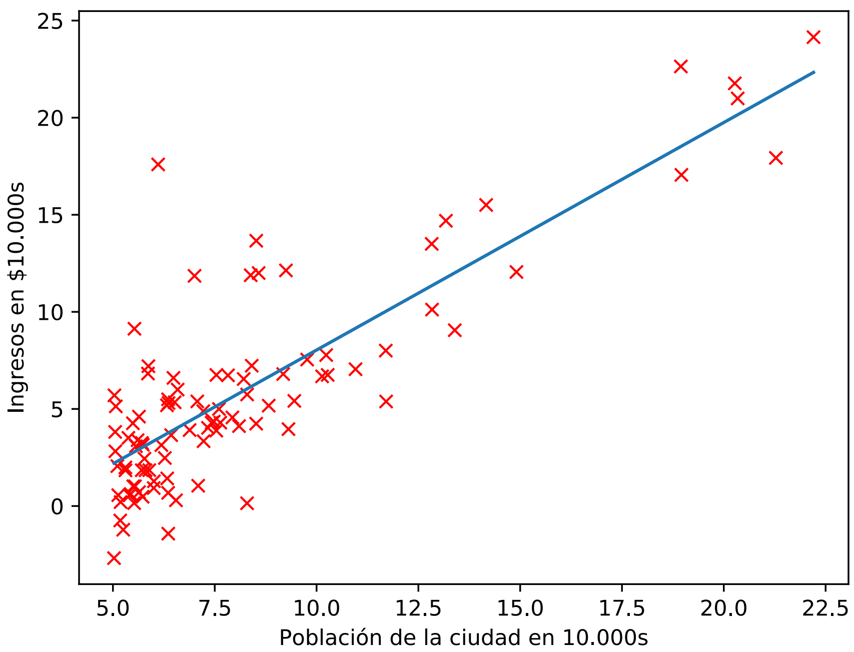
Práctica 0: vectorización

solución vectorizada

```python
def integra_mc_vec(fun, a, b, num_puntos=100):
    """Calcula la integral de f entre a y b por el método de
    Monte Carlo sin usar bucles
    """
    eje_x = np.linspace(a, b, num_puntos)
    eje_y = fun(eje_x)
    max_f = max(eje_y)
    min_f = 0
    random_x = np.random.uniform(a, b, num_puntos)
    random_y = np.random.uniform(min_f, max_f, num_puntos)
    f_random_x = fun(random_x)

    plt.figure()
    plt.plot(random_x, random_y, 'x', c='red')
    plt.plot(eje_x, eje_y, '-')
    plt.savefig('mc.pdf')
    plt.close()

    debajo = sum(random_y < f_random_x)
    area_total = abs(b - a) * abs(max_f - min_f)
    return area_total * (debajo / num_puntos)
```

Práctica 1:
regresión lineal


25
20
15
10
5
0
5.0
7.5
10.0
12.5
15.0
17.5
20.0
22.5
Población de la ciudad en 10.000s
Ingresos en $10.000s


25
20
15
10
5
0
5.0
7.5
10.0
12.5
15.0
17.5
20.0
22.5
Población de la ciudad en 10.000s
Ingresos en $10.000s

```python
import numpy as np
from pandas.io.parsers import read_csv

def carga_csv(file_name):
    """carga el fichero csv especificado y lo
       devuelve en un array de numpy
    """
    valores = read_csv(file_name, header=None).to_numpy()
    # suponemos que siempre trabajaremos con float
    return valores.astype(float)
```

```
array([[ 6.1101 , 17.592  ],
       [ 5.5277 ,  9.1302 ],
       [ 8.5186 , 13.662  ],
       [ 7.0032 , 11.854  ],
       ...
       [ 5.3054 ,  1.9869 ],
       [ 8.2934 ,  0.14454],
       [13.394  ,  9.0551 ],
       [ 5.4369 ,  0.61705]])
```

Práctica 1: regresión lineal

solución de la primera parte

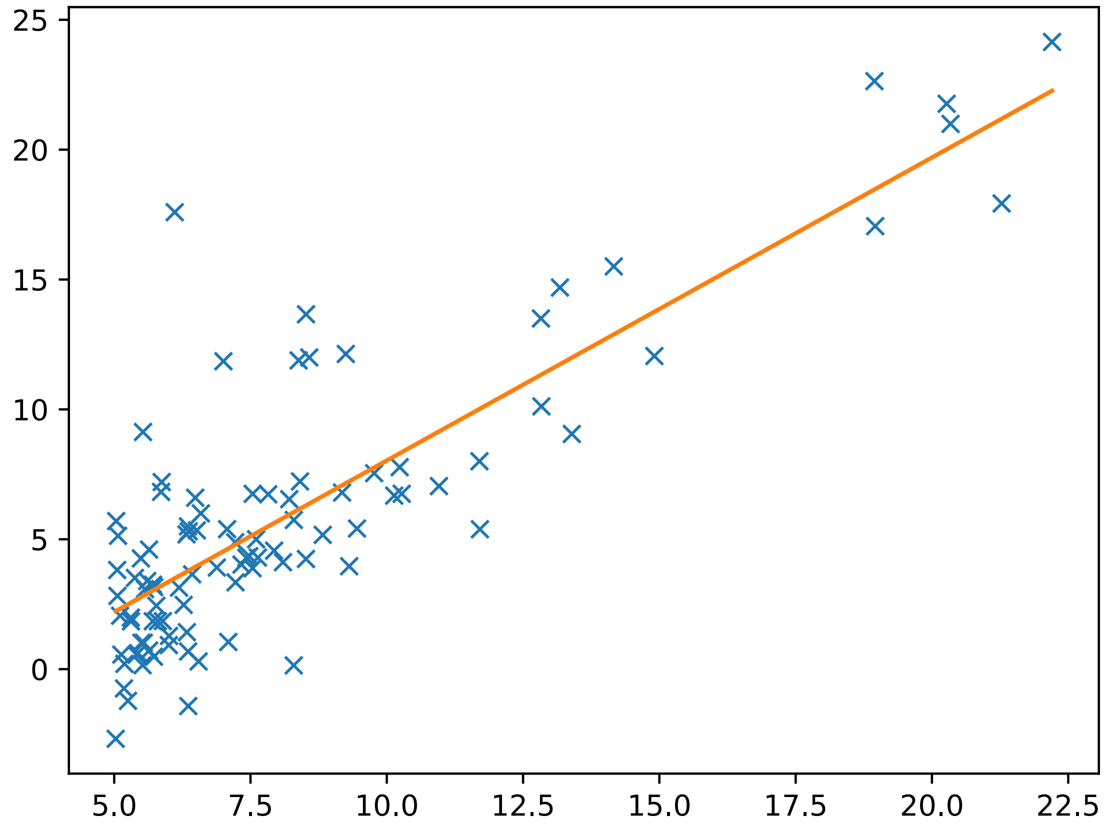
$$
\begin{aligned}
&\text{repeat until convergence } \{ \\
&\quad \theta_0 := \theta_0 - \alpha \frac{1}{m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right) \\
&\quad \theta_1 := \theta_1 - \alpha \frac{1}{m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right) \cdot x^{(i)} \\
&\}
\end{aligned}
$$

update θ_0 and θ_1 simultaneously

```
array([[ 6.1101 , 17.592  ],
       [ 5.5277 ,  9.1302 ],
       [ 8.5186 , 13.662  ],
       [ 7.0032 , 11.854  ],
       ...
       [ 5.3054 ,  1.9869 ],
       [ 8.2934 ,  0.14454],
       [13.394  ,  9.0551 ],
       [ 5.4369 ,  0.61705]])
```

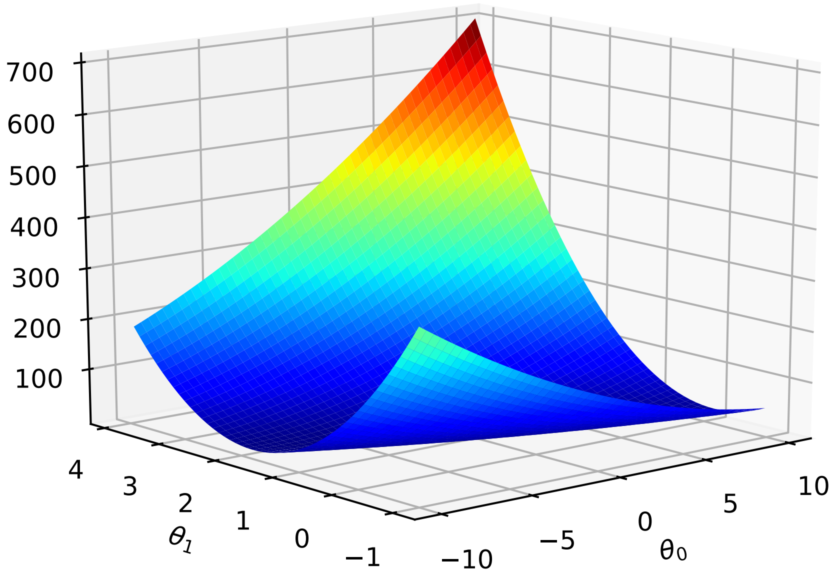
```
datos = carga_csv('ex1data1.csv')
X = datos[:, 0]
Y = datos[:, 1]
m = len(X)
alpha = 0.01
theta_0 = theta_1 = 0
for _ in range(1500):
    sum_0 = sum_1 = 0
    for i in range(m):
        sum_0 +=  (theta_0 + theta_1 * X[i]) - Y[i]
        sum_1 += ((theta_0 + theta_1 * X[i]) - Y[i]) * X[i]
    theta_0 = theta_0 - (alpha / m) * sum_0
    theta_1 = theta_1 - (alpha / m) * sum_1
```

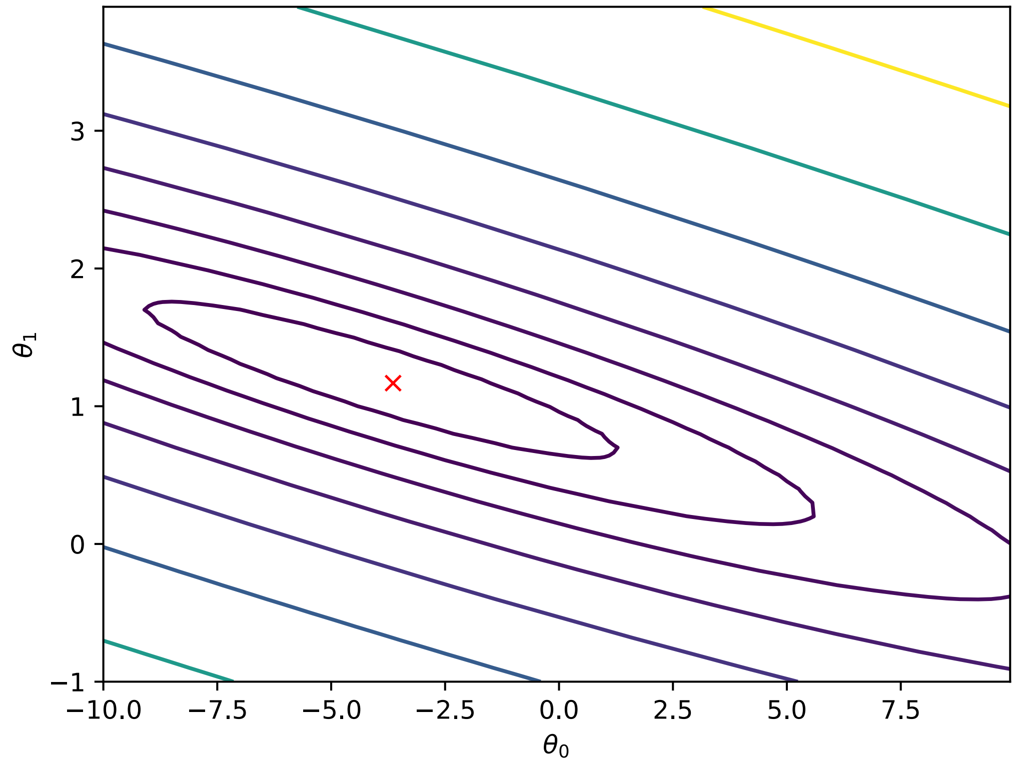
```python
plt.plot(X, Y, "x")
min_x = min(X)
max_x = max(X)
min_y = theta_0 + theta_1 * min_x
max_y = theta_0 + theta_1 * max_x
plt.plot([min_x, max_x], [min_y, max_y])
plt.savefig("resultado.pdf")
```



Práctica 1: regresión lineal

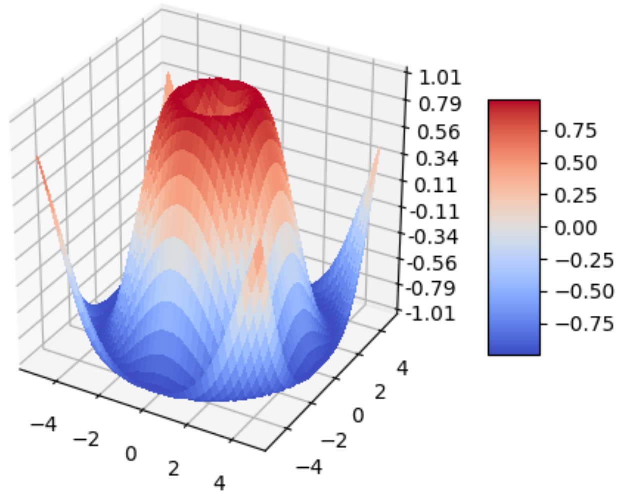
gráficas en 3D


700
600
500
400
300
200
100
4
3
2
1
0
−1
θ_1
−10
−5
0
5
10
θ_0


3
2
1
0
−1
θ_1
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
θ_0

```
import numpy as np

In [3]: x = np.array([1,2,3])

In [4]: y = np.array([4,5,6])

In [5]: xx, yy = np.meshgrid(x,y)

In [6]: xx
Out[6]:
array([[1, 2, 3],
       [1, 2, 3],
       [1, 2, 3]])

In [7]: yy
Out[7]:
array([[4, 4, 4],
       [5, 5, 5],
       [6, 6, 6]])
```

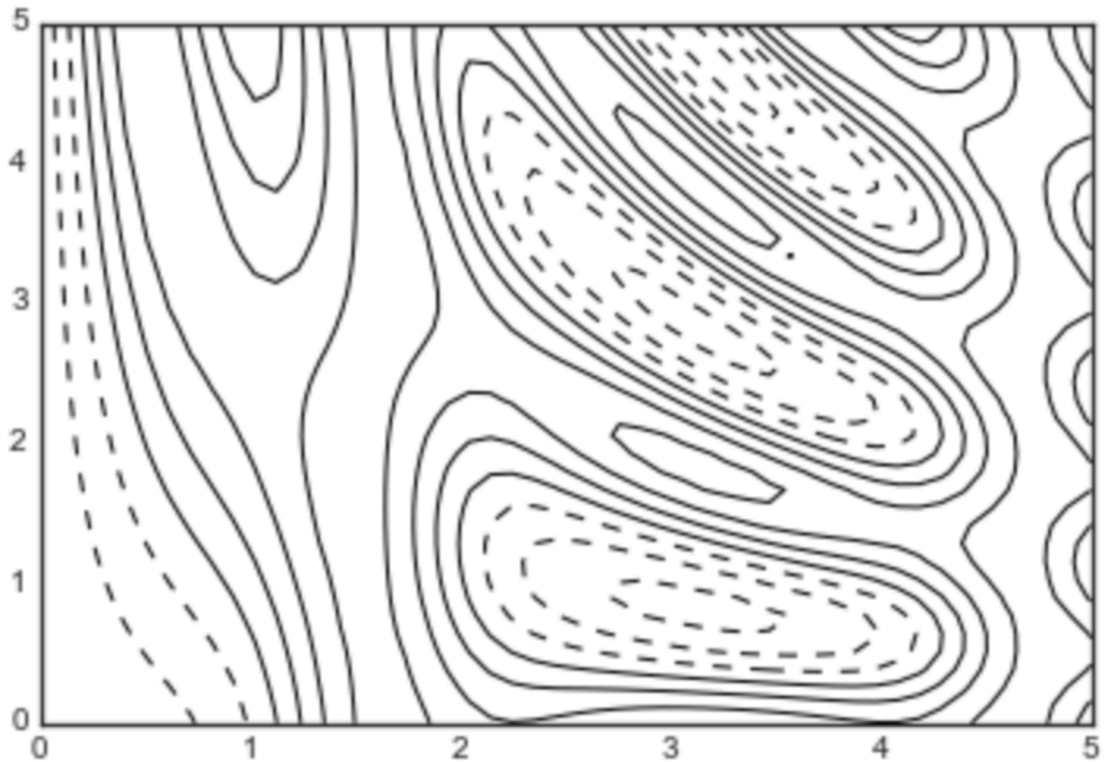
```python
from mpl_toolkits.mplot3d import Axes3D
import matplotlib.pyplot as plt
from matplotlib import cm
from matplotlib.ticker import LinearLocator, FormatStrFormatter
import numpy as np


fig = plt.figure()
ax = fig.gca(projection='3d')    #  ax = Axes3D(fig)

# Make data.
X = np.arange(-5, 5, 0.25)
Y = np.arange(-5, 5, 0.25)
X, Y = np.meshgrid(X, Y)
R = np.sqrt(X**2 + Y**2)
Z = np.sin(R)

# Plot the surface.
surf = ax.plot_surface(X, Y, Z, cmap=cm.coolwarm,
                       linewidth=0, antialiased=False)

# Customize the z axis.
ax.set_zlim(-1.01, 1.01)
ax.zaxis.set_major_locator(LinearLocator(10))
ax.zaxis.set_major_formatter(FormatStrFormatter('%.02f'))

# Add a color bar which maps values to colors.
fig.colorbar(surf, shrink=0.5, aspect=5)

plt.show()
```



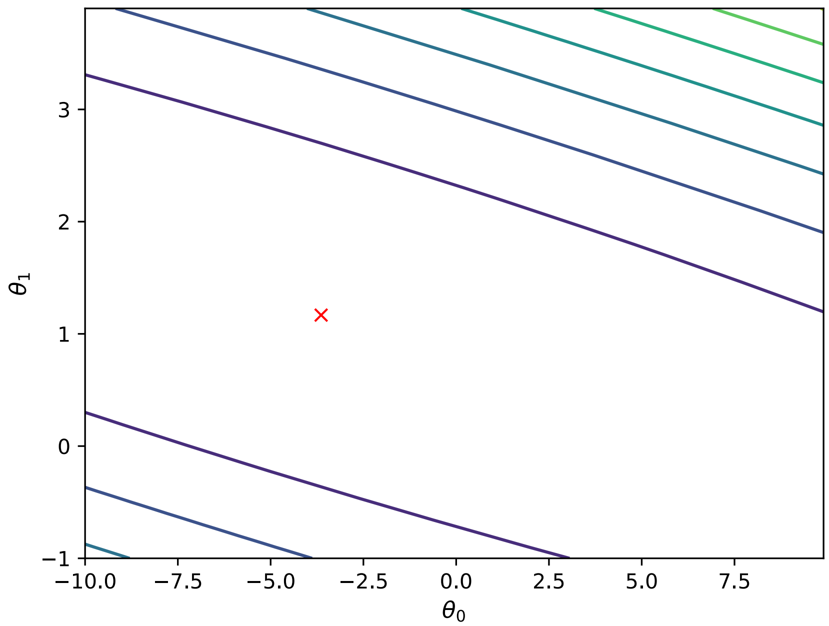
```python
def f(x, y):
    return np.sin(x) ** 10 + np.cos(10 + y * x) * np.cos(x)
```

```python
x = np.linspace(0, 5, 50)
y = np.linspace(0, 5, 40)

X, Y = np.meshgrid(x, y)
Z = f(X, Y)
```

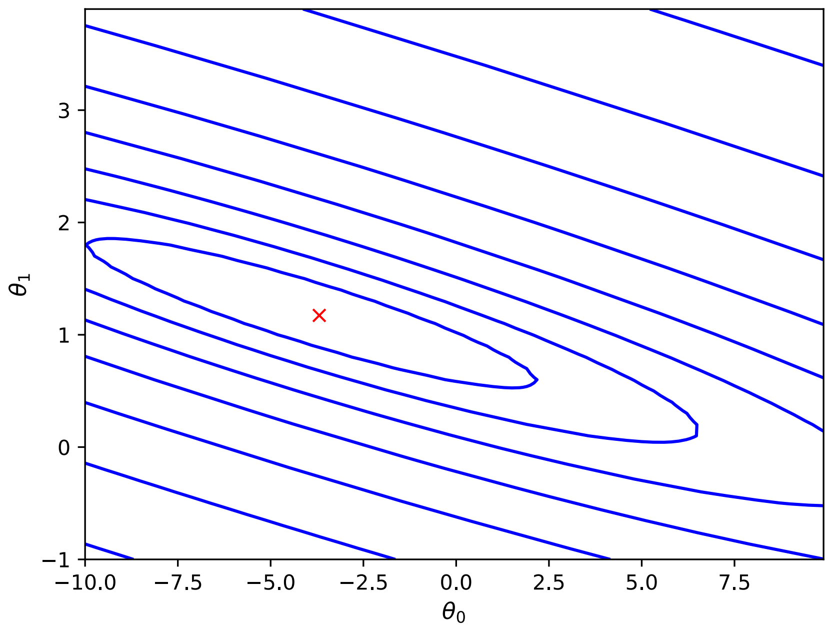
```python
plt.contour(X, Y, Z, colors='black');
```



```
plt.contour(Theta0, Theta1, Coste)
```




```
plt.contour(Theta0, Theta1, Coste,
            np.logspace(-2, 3, 20), colors='blue')
```



```python
def make_data(t0_range, t1_range, X, Y):
    """Genera las matrices X,Y,Z para generar un plot en 3D
    """

    step = 0.1
    Theta0 = np.arange(t0_range[0], t0_range[1], step)
    Theta1 = np.arange(t1_range[0], t1_range[1], step)

    Theta0, Theta1 = np.meshgrid(Theta0, Theta1)
    # Theta0 y Theta1 tienen las misma dimensiones, de forma que
    # cogiendo un elemento de cada uno se generan las coordenadas x,y
    # de todos los puntos de la rejilla

    Coste = np.empty_like(Theta0)
    for ix, iy in np.ndindex(Theta0.shape):
        Coste[ix, iy] = coste(X, Y, [Theta0[ix, iy], Theta1[ix, iy]])

    return [Theta0, Theta1, Coste]
```

Práctica 1: regresión lineal

solución vectorizada

Gradient descent algorithm

repeat until convergence {

$$\theta_0 := \theta_0 - \alpha \frac{1}{m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right)$$

$$\theta_1 := \theta_1 - \alpha \frac{1}{m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right) \cdot x^{(i)}$$

}

update θ_0 and θ_1 simultaneously

Repeat {

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta_0, \ldots, \theta_n)$$

}

(simultaneously update for every $j = 0, \ldots, n$)

$$\theta_j := \theta_j - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) x_j^{(i)}$$

$$h_\theta(x) = \theta_0 + \theta_1 x \qquad h_\theta(x) = \theta^T x$$

```python
datos = carga_csv('ex1data1.csv')

X = datos[:, :-1]
np.shape(X)         # (97, 1)
Y = datos[:, -1]
np.shape(Y)         # (97,)

m = np.shape(X)[0]
n = np.shape(X)[1]

# añadimos una columna de 1's a la X
X = np.hstack([np.ones([m, 1]), X])

alpha = 0.01
Thetas, costes = descenso_gradiente(X, Y, alpha)
```

$$J(\theta) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right)^2$$

$$X = \begin{bmatrix} — & (x^{(1)})^T & — \\ — & (x^{(2)})^T & — \\ & \vdots & \\ — & (x^{(m)})^T & — \end{bmatrix} \quad \vec{y} = \begin{bmatrix} y^{(1)} \\ y^{(2)} \\ \vdots \\ y^{(m)} \end{bmatrix}$$

$$X\theta = \begin{bmatrix} (x^{(1)})^T\theta \\ (x^{(2)})^T\theta \\ \vdots \\ (x^{(m)})^T\theta \end{bmatrix} = \begin{bmatrix} \theta^T(x^{(1)}) \\ \theta^T(x^{(2)}) \\ \vdots \\ \theta^T(x^{(m)}) \end{bmatrix}$$

$$J(\theta) = \frac{1}{2m}(X\theta - \vec{y})^T(X\theta - \vec{y})$$

$$J(\theta) = \frac{1}{2m}(X\theta - \vec{y})^T(X\theta - \vec{y})$$

```
def coste(X, Y, Theta):
    H = np.dot(X, Theta)
    Aux = (H - Y) ** 2
    return Aux.sum() / (2 * len(X))
```

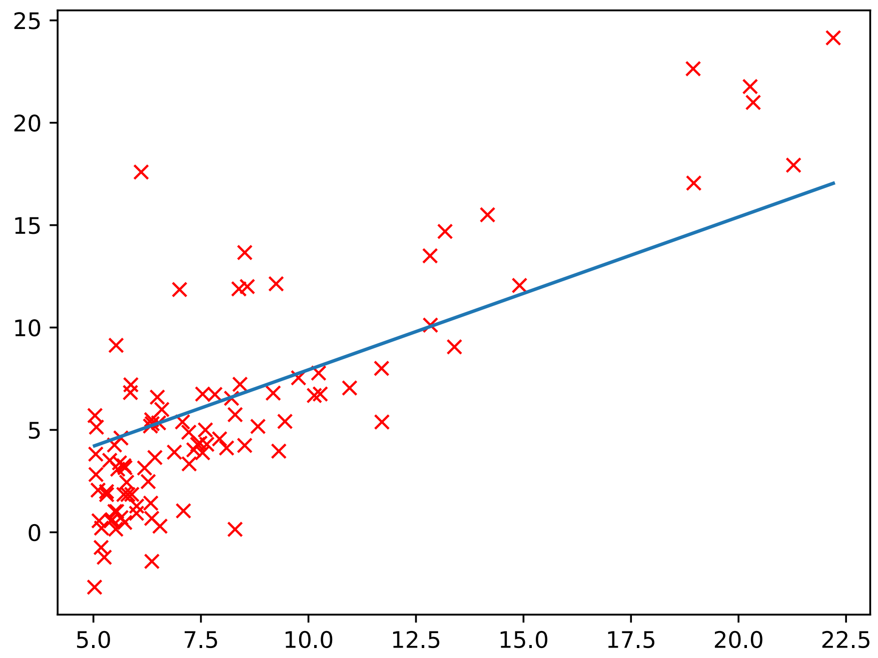
$$\frac{\partial J(\theta)}{\partial \theta_j} = \frac{1}{m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})x_j^{(i)}$$

$$\begin{aligned}
\begin{bmatrix} \frac{\partial J(\theta)}{\partial \theta_0} \\ \frac{\partial J(\theta)}{\partial \theta_1} \\ \vdots \\ \frac{\partial J(\theta)}{\partial \theta_n} \end{bmatrix}
&= \frac{1}{m} \begin{bmatrix} \sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})x_0^{(i)} \\ \sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})x_1^{(i)} \\ \vdots \\ \sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})x_n^{(i)} \end{bmatrix} \\
&= \frac{1}{m}\sum_{i=1}^{m}\left((h_\theta(x^{(i)}) - y^{(i)})x^{(i)}\right) \\
&= \frac{1}{m}X^T(h_\theta(x) - y)
\end{aligned}$$

$$h_\theta(x) - y = \begin{bmatrix} h_\theta(x^{(1)}) - y^{(1)} \\ h_\theta(x^{(2)}) - y^{(2)} \\ \vdots \\ h_\theta(x^{(m)}) - y^{(m)} \end{bmatrix}$$

$$\frac{\delta J(\theta)}{\delta \theta_j} = \ \frac{1}{m}X^T(h_\theta(x) - y)$$

```
def gradiente(X, Y, Theta, alpha):
    NuevaTheta = Theta
    m = np.shape(X)[0]
    n = np.shape(X)[1]
    H = np.dot(X, Theta)
    Aux = (H - Y)
    for i in range(n):
        Aux_i = Aux * X[:, i]
        NuevaTheta -= (alpha / m) * Aux_i.sum()
    return NuevaTheta
```



```python
def gradiente(X, Y, Theta, alpha):
    NuevaTheta = Theta
    m = np.shape(X)[0]
    n = np.shape(X)[1]
    H = np.dot(X, Theta)
    Aux = (H - Y)
    for i in range(n):
        Aux_i = Aux * X[:, i]
        NuevaTheta[i] -= (alpha / m) * Aux_i.sum()
    return NuevaTheta
```

```
"""
X: matriz bidimensional de numpy de dimensiones (m, n)
Y: matriz unidimensional de numpy de dimensiones (m,)
Theta: matriz unidimensional de numpy de dimensiones (n,)
"""
```