

EECS 16B CSM

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Computer Science Mentors

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1 Complex Numbers

2 Phasors

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Numbers

Phasors

Complex Numbers

Definition

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Phasors

$$z = \underbrace{a + bj}_{\text{rectangular}} = \underbrace{re^{j\theta}}_{\text{polar}} \quad (1)$$

- $a, b, r, \theta \in \mathbb{R}$
- $j^2 = -1$
- we use j in EE

Coordinate Transforms

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Phasors

$$r^2 = a^2 + b^2 \quad (2)$$

$$\tan(\theta) = \frac{b}{a} \quad (3)$$

$$a = \Re\{z\} = r \cos(\theta) \quad (4)$$

$$b = \Im\{z\} = r \sin(\theta) \quad (5)$$

Euler's Formula

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$$e^{j\theta} = \cos(\theta) + j \sin(\theta) \quad (6)$$

- relevant 3Blue1Brown
- $e^{j\pi} + 1 = 0$ is a special case

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Phasors

- Encodes information about any sinusoid: voltage, current, etc.
- If frequency is constant, then uniquely identifies

$$A \cos(\omega t + \phi) = \Re\{A e^{j(\omega t + \phi)}\} = \Re\{\underbrace{A e^{j\phi}}_{\text{phasor}} e^{j\omega t}\} \quad (7)$$

Properties

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Phasors

Given $x_1(t) = \Re\{A_1 e^{j\omega t}\}$, $x_2(t) = \Re\{A_2 e^{j\omega t}\}$ with phasors $A_{1,2}$,

- Uniqueness: $x_1(t) = x_2(t) \implies A_1 = A_2$
- Linearity: $a_1 x_1(t) + a_2 x_2(t) \implies a_1 A_1 + a_2 A_2$ for $a_{1,2} \in \mathbb{R}$
- Differentiation: $x(t) \Leftrightarrow A \implies \frac{d}{dt} x(t) = \frac{d}{dt} \Re\{A e^{j\omega t}\} = \Re\{j\omega A e^{j\omega t}\} \Leftrightarrow j\omega A$

Circuits & Phasors

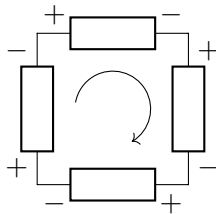
KVL

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Phasors



$$\sum_i \bar{V}_i = 0 \quad (8)$$

Circuits & Phasors (cont.)

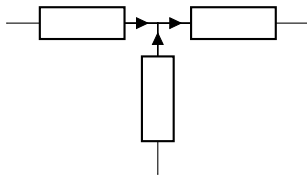
KCL

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Phasors



$$\sum \bar{I}_{out} = \sum \bar{I}_{in} \quad (9)$$

Circuits & Phasors (cont.)

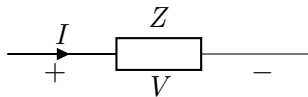
Ohm's Law

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Phasors



$$\bar{V} = \bar{I} \underbrace{Z}_{\text{impedance}}$$

(10)

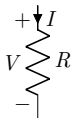
Passive Elements & Phasors

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Phasors



$$\bar{V} = \bar{I}R \quad (11)$$



$$\bar{V} = L \frac{d}{dt} \bar{I} = j\omega L \bar{I} \quad (12)$$



$$\bar{I} = C \frac{d}{dt} \bar{V} = j\omega C \bar{V} \implies \bar{V} = \frac{1}{j\omega C} \bar{I} \quad (13)$$

Demo

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Phasors

`http://tinyurl.com/y5qfnqtk`

Low Pass Filter

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$$\overline{V}_{out} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} \overline{V}_{in} \quad (14)$$

$$\overline{V}_{out} = \frac{1}{1 + j\omega RC} \overline{V}_{in} \quad (15)$$

$$\Rightarrow H(j\omega) = \frac{\overline{V}_{out}}{\overline{V}_{in}} = \frac{1}{1 + j\omega RC} \quad (16)$$