EECS 16B CSM

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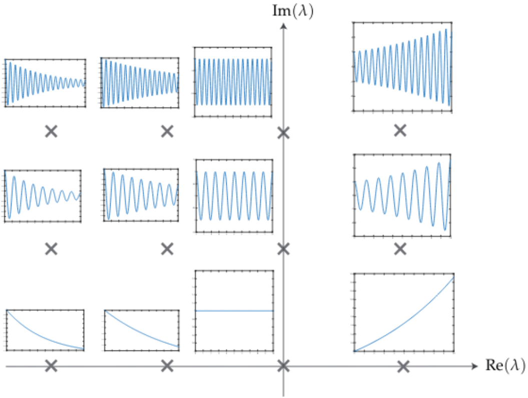
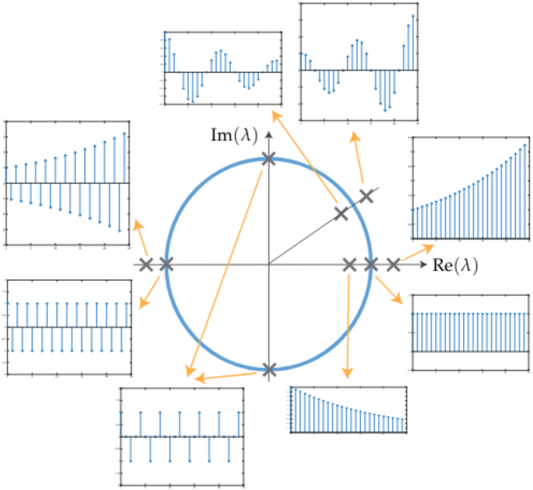
2020-10-25

Stability

Continuous



Discrete



Feedback

Open-Loop

$$\boldsymbol{x}[t+1] = \boldsymbol{A}\boldsymbol{x}[t] + \boldsymbol{B}\boldsymbol{u}[t] \tag{1}$$

- define a certain range of use
- simpler
- no restraints apart from stability

Closed-Loop

$$\boldsymbol{u}[t] = \begin{bmatrix} k_1 & k_2 & \cdots & k_n \end{bmatrix} \begin{bmatrix} x_1[t] \\ x_2[t] \\ \vdots \\ x_n[t] \end{bmatrix} \tag{2}$$

$$\boldsymbol{x}[t+1] = \boldsymbol{A}\boldsymbol{x}[t] + \boldsymbol{B}\boldsymbol{K}\boldsymbol{x}[t] = (\boldsymbol{A} + \boldsymbol{B}\boldsymbol{K})\boldsymbol{x}[t] \tag{3}$$

- adaptable to a wide range of use
- more complex
- self-correcting
- requires more constraints

Controller Canonical Form

$$\boldsymbol{x}[t+1] = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ a_1 & a_2 & a_3 & \cdots & a_n \end{bmatrix} \boldsymbol{x}[t] + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} \boldsymbol{u}[t] \tag{4}$$

$$\det(\boldsymbol{A} + \boldsymbol{BK} - \lambda \boldsymbol{I}) = \lambda^n - \sum_{i=1}^{n} (a_i + k_i) \lambda^i \tag{5}$$

Least Squares

Quick Review

$$\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b} \implies \hat{\boldsymbol{x}} \approx \left([\boldsymbol{A}]^\top \boldsymbol{A}\right)^{-1} [\boldsymbol{A}]^\top \boldsymbol{b} \tag{6}$$

- we want to minimize the error vector $\boldsymbol{e} = \boldsymbol{b} - \boldsymbol{A}\boldsymbol{x}$