EECS 16B CSM

Bryan Ngo

Computer Science Mentors

2021-02-17

Logistics

- keep up the attendance!
- public Piazza: `https://piazza.com/berkeley/spring2021/csm16b`
- Phasors or discretization?

Complex Numbers

Definition

$$z = \underbrace{a + bj}_{\text{rectangular}} = \underbrace{re^{j\theta}}_{\text{polar}} \tag{1}$$

- $a, b, r, \theta \in \mathbb{R}$
- $j^2 = -1$
- we use j in EE

Coordinate Transforms

$$r^2 = a^2 + b^2 \tag{2}$$

$$\tan(\theta) = \frac{b}{a} \tag{3}$$

$$a = \Re\{z\} = r\cos(\theta) \tag{4}$$

$$b = \Im\{z\} = r\sin(\theta) \tag{5}$$

Euler's Formula

$$e^{j\theta} = \cos(\theta) + j\sin(\theta) \tag{6}$$

- relevant 3Blue1Brown
- $e^{j\pi} + 1 = 0$ is a special case

Phasors

Phasors

- Encodes information about any sinusoid: voltage, current, etc.
- If frequency is constant, then uniquely identifies

$$x(t) = A\cos(\omega t + \phi) = \frac{A}{2}\left(e^{j(\omega t+\phi)} + e^{-j(\omega t+\phi)}\right) \tag{7}$$

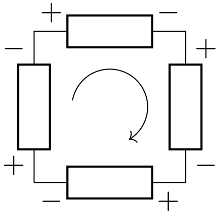
$$= \frac{A}{2}\left(e^{j\omega t}e^{j\phi} + \overline{e^{j\omega t}e^{j\phi}}\right) \tag{8}$$

$$\iff \widetilde{X} = \frac{A}{2}e^{j\phi} \tag{9}$$

Properties

Given $x_1(t) = \widetilde{X}_1 e^{j\omega t} + \overline{\widetilde{X}_1 e^{j\omega t}}$, $x_2(t) = \widetilde{X}_2 e^{j\omega t} + \overline{\widetilde{X}_2 e^{j\omega t}}$ with phasors $\widetilde{X}_{1,2}$,

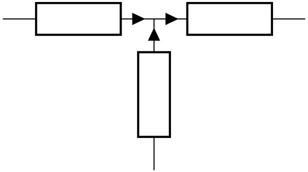
- Uniqueness: $x_1(t) = x_2(t) \implies \widetilde{X}_1 = \widetilde{X}_2$
- Linearity: $a_1 x_1(t) + a_2 x_2(t) \implies a_1 \widetilde{X}_1 + a_2 \widetilde{X}_2$ for $a_{1,2} \in \mathbb{R}$
- Differentiation: $x(t) \iff \widetilde{X} \implies \frac{d}{dt} x(t) = \frac{d}{dt}\left(\widetilde{X} e^{j\omega t} + \overline{\widetilde{X} e^{j\omega t}}\right) = j\omega\left(\widetilde{X} e^{j\omega t} + \overline{\widetilde{X} e^{j\omega t}}\right) \iff j\omega\widetilde{X}$



$$\sum_i \widetilde{V}_i = 0 \tag{10}$$

Circuits & Phasors (cont.)

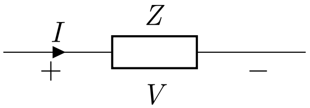
KCL



$$\sum \widetilde{I}_{out} = \sum \widetilde{I}_{in} \tag{11}$$

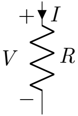
Circuits & Phasors (cont.)

Ohm's Law

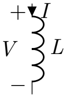


$$\widetilde{V} = \widetilde{I} \underbrace{Z}_{\text{impedance}} \tag{12}$$

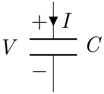
Passive Elements & Phasors



$$\widetilde{V} = \widetilde{I} R \tag{13}$$



$$\widetilde{V} = L \frac{d}{dt} \widetilde{I} = j\omega L \widetilde{I} \tag{14}$$



$$\widetilde{I} = C \frac{d}{dt} \widetilde{V} = j\omega C \widetilde{V} \implies \widetilde{V} = \frac{1}{j\omega C} \widetilde{I} \tag{15}$$

Demo

http://tinyurl.com/y5qfnqtk

Low Pass Filter

$$\widetilde{V}_{out} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} \widetilde{V}_{in} \tag{16}$$

$$\widetilde{V}_{out} = \frac{1}{1 + j\omega RC} \widetilde{V}_{in} \tag{17}$$

$$\Rightarrow H(j\omega) = \frac{\widetilde{V}_{out}}{\widetilde{V}_{in}} = \frac{1}{1 + j\omega RC} \tag{18}$$