EECS 16B CSM

Bryan Ngo

Computer Science Mentors

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Who am I?

- 2nd year majoring in EECS
- took EECS 16B Spring 2020
- Pertinent fact: ate a chicken bake for lunch

Who are you?

- Name
- Pronouns
- Year/Major
- Pertinent fact

Logistics

- unexcused absences in first 3 weeks $\rightarrow$ **auto-dropped & NP**
- excused absences: email bryanngo@berkeley.edu & cc mentors@berkeley.edu with subject line `[Request for Absence] <course>`
- Slides available at `https://github.com/bdngo/16b-csm`

Expectations

Me to You

- Be skeptical
- Constant feedback
- Become passionate about 16B

Expectations

You to Me

Vector Differential Equations

General Form

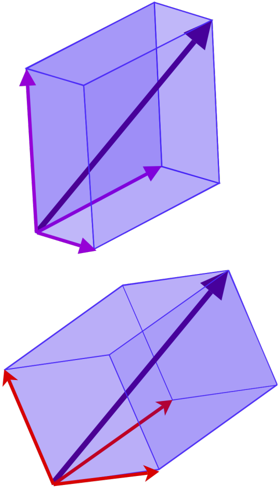
$$\frac{d}{dt}\boldsymbol{x}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t) \tag{1}$$

- if $\boldsymbol{A}$ is diagonal, simply a bunch of exponential differential Equations
- if not, we can try to diagonalize

Change of Basis

Motivation

- conversion from one linear coordinate system to another
- 3Blue1Brown video



A Visualization

$$
\begin{tikzcd}[column sep=huge, row sep=huge]
\boldsymbol{x} \arrow[r, shift right, "\boldsymbol{A}"'] \arrow[d, shift right, "\boldsymbol{V}^{-1}"'] & \boldsymbol{y} \arrow[l, shift right, "\boldsymbol{A}^{-1}"'] \arrow[d, shift right, "\boldsymbol{V}^{-1}"'] \\
\widetilde{\boldsymbol{x}} \arrow[r, shift right, "\boldsymbol{V}^{-1}\boldsymbol{A}\boldsymbol{V}"'] \arrow[u, shift right, "\boldsymbol{V}"'] & \widetilde{\boldsymbol{y}} \arrow[l, shift right, "\boldsymbol{V}\boldsymbol{A}\boldsymbol{V}^{-1}"'] \arrow[u, shift right, "\boldsymbol{V}"']
\end{tikzcd}
$$

Diagonalization

- want the eigenvectors to be the basis for a vector space
- makes math *way* easier

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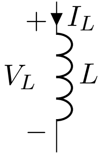
$$
\begin{aligned}
\boldsymbol{V} &= \begin{bmatrix} \boldsymbol{v}_1 & \boldsymbol{v}_2 & \cdots & \boldsymbol{v}_n \end{bmatrix} && (2) \\
\boldsymbol{AV} &= \begin{bmatrix} \lambda_1 \boldsymbol{v}_1 & \lambda_2 \boldsymbol{v}_2 & \cdots & \lambda_n \boldsymbol{v}_n \end{bmatrix} && (3) \\
&= \begin{bmatrix} \boldsymbol{v}_1 & \boldsymbol{v}_2 & \cdots & \boldsymbol{v}_n \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_n \end{bmatrix} && (4) \\
&= \boldsymbol{V\Lambda} \implies \boldsymbol{\Lambda} = \boldsymbol{V}^{-1}\boldsymbol{AV} && (5)
\end{aligned}
$$

Diagonalizing DEs

$$
\begin{aligned}
\frac{d}{dt}\boldsymbol{x}(t) &= \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t) && (6)\\
\frac{d}{dt}\boldsymbol{V}\boldsymbol{z}(t) &= \boldsymbol{A}\boldsymbol{V}\boldsymbol{z}(t) + \boldsymbol{B}\boldsymbol{u}(t) && (7)\\
\Rightarrow \frac{d}{dt}\boldsymbol{z}(t) &= \boldsymbol{V}^{-1}\boldsymbol{A}\boldsymbol{V}\boldsymbol{z}(t) + \boldsymbol{V}^{-1}\boldsymbol{B}\boldsymbol{u}(t) && (8)\\
&= \boldsymbol{\Lambda}\boldsymbol{z}(t) + \boldsymbol{V}^{-1}\boldsymbol{B}\boldsymbol{u}(t) && (9)
\end{aligned}
$$

Inductors (Preview)

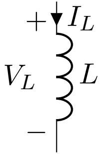
Basic Properties



$$V_L = L\frac{d}{dt}I_L \tag{10}$$

- like a capacitor but for magnetic fields
- resists instantaneous change in current

Basic Properties



$$V_L = L\frac{d}{dt}I_L \tag{10}$$

- like a capacitor but for magnetic fields
- resists instantaneous change in current
- what happens when $\omega = 0$? $\omega = \infty$?