EECS 16B CSM

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Announcements

- Pertinent facts
- Review session next Tuesday

Phasors

Phasors

- Encodes information about any sinusoid: voltage, current, etc.
- If frequency is constant, then uniquely identifies

$$x(t) = A\cos(\omega t + \phi) = \frac{A}{2}\left(e^{j(\omega t+\phi)} + e^{-j(\omega t+\phi)}\right) \tag{1}$$

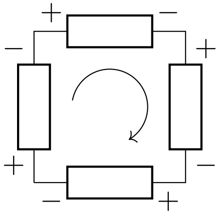
$$= \frac{A}{2}\left(e^{j\omega t}e^{j\phi} + \overline{e^{j\omega t}e^{j\phi}}\right) \tag{2}$$

$$\iff \widetilde{X} = \frac{A}{2}e^{j\phi} \tag{3}$$

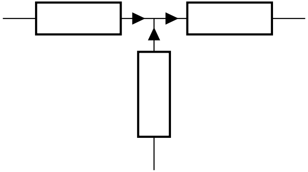
Properties

Given $x_1(t) = \widetilde{X}_1 e^{j\omega t} + \overline{\widetilde{X}_1 e^{j\omega t}}$, $x_2(t) = \widetilde{X}_2 e^{j\omega t} + \overline{\widetilde{X}_2 e^{j\omega t}}$ with phasors $\widetilde{X}_{1,2}$,

- Uniqueness: $x_1(t) = x_2(t) \implies \widetilde{X}_1 = \widetilde{X}_2$
- Linearity: $a_1 x_1(t) + a_2 x_2(t) \implies a_1 \widetilde{X}_1 + a_2 \widetilde{X}_2$ for $a_{1,2} \in \mathbb{R}$
- Differentiation: $x(t) \iff \widetilde{X} \implies \frac{d}{dt} x(t) = \frac{d}{dt}\left(\widetilde{X} e^{j\omega t} + \overline{\widetilde{X} e^{j\omega t}}\right) = j\omega\left(\widetilde{X} e^{j\omega t} + \overline{\widetilde{X} e^{j\omega t}}\right) \iff j\omega \widetilde{X}$



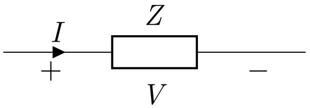
$$\sum_i \widetilde{V}_i = 0 \tag{4}$$



$$\sum \widetilde{I}_{out} = \sum \widetilde{I}_{in} \tag{5}$$

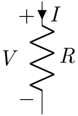
Circuits & Phasors (cont.)

Ohm's Law

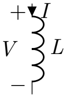


$$\tilde{V} = \tilde{I} \underbrace{Z}_{\text{impedance}} \tag{6}$$

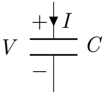
Passive Elements & Phasors



$$\widetilde{V} = \widetilde{I} R \tag{7}$$



$$\widetilde{V} = L \frac{d}{dt} \widetilde{I} = j\omega L \widetilde{I} \tag{8}$$



$$\widetilde{I} = C \frac{d}{dt} \widetilde{V} = j\omega C \widetilde{V} \implies \widetilde{V} = \frac{1}{j\omega C} \widetilde{I} \tag{9}$$

Filters

Why?

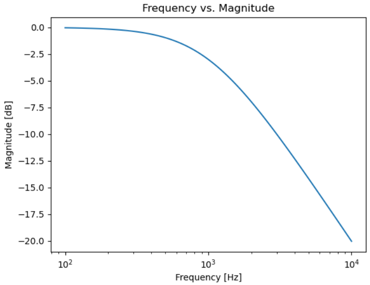
- allows us to isolate desired frequency ranges
- color organ: basically just a spectrogram
- Afrotechmods video

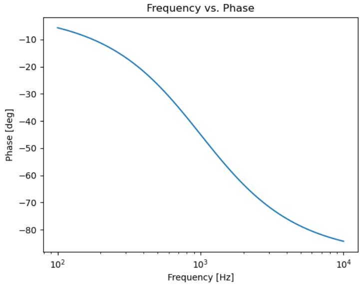
Types

- low-pass: let in low ω
- high-pass: let in high ω
- band-pass: let in range of ω
- band-stop: block out range of ω

Bode Plots

Definition





- comes in pairs: magnitude & phase
- above: low-pass filter

Features

- magnitude
  - x-axis: log frequency (Hz)
  - y-axis: $|H(j\omega)|$ (dB or intensity)
  - **cutoff frequency**: $|H(j\omega)| = \frac{1}{\sqrt{2}} = -3\,\mathrm{dB}$
- phase
  - x-axis: log frequency (Hz)
  - y-axis: phase offset ($^\circ$ or rad)

Why?

- allows us to characterize a filter very fast
- quick visual tool

Resonance

- whenever you see an inductor & a capacitor
- energy is oscillating back and forth
- when voltage is at resonant frequency $\frac{1}{\sqrt{LC}}$, inductor and capacitor act as short