

EECS 16B CSM

Bryan Ngo

UC Berkeley

2021-03-10

Midterm

EECS 16B
CSM

Bryan Ngo

- Good luck!
- Review vs. Worksheet

State Space Form

EECS 16B
CSM

Bryan Ngo

$$\frac{d}{dt}\mathbf{x}(t) = f(\mathbf{x}(t), \mathbf{u}(t)) \quad (1)$$

Linearization

EECS 16B
CSM

Bryan Ngo

$$\mathbf{J}_x = \begin{bmatrix} \partial_{x_1} f_1 & \partial_{x_2} f_1 & \cdots & \partial_{x_n} f_1 \\ \partial_{x_1} f_2 & \partial_{x_2} f_2 & \cdots & \partial_{x_n} f_2 \\ \vdots & \vdots & \ddots & \vdots \\ \partial_{x_1} f_n & \partial_{x_2} f_n & \cdots & \partial_{x_n} f_n \end{bmatrix} \quad (2)$$

$$\mathbf{J}_u = \begin{bmatrix} \partial_{u_1} f_1 & \partial_{u_2} f_1 & \cdots & \partial_{u_n} f_1 \\ \partial_{u_1} f_2 & \partial_{u_2} f_2 & \cdots & \partial_{u_n} f_2 \\ \vdots & \vdots & \ddots & \vdots \\ \partial_{u_1} f_n & \partial_{u_2} f_n & \cdots & \partial_{u_n} f_n \end{bmatrix} \quad (3)$$

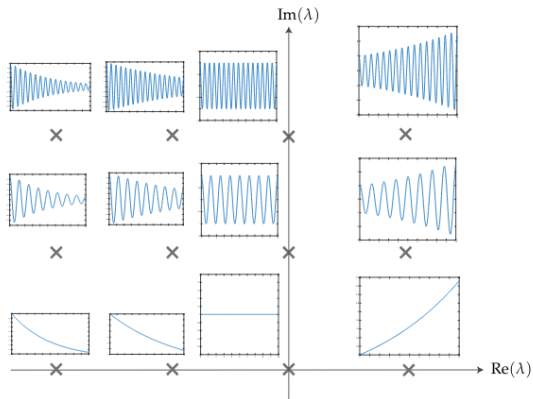
$$\frac{d}{dt} \mathbf{x}(t) \approx \mathbf{J}_x(\mathbf{x}(t) - \mathbf{x}^*) + \mathbf{J}_u(\mathbf{u}(t) - \mathbf{u}^*) \quad (4)$$

Stability

Continuous

EECS 16B
CSM

Bryan Ngo

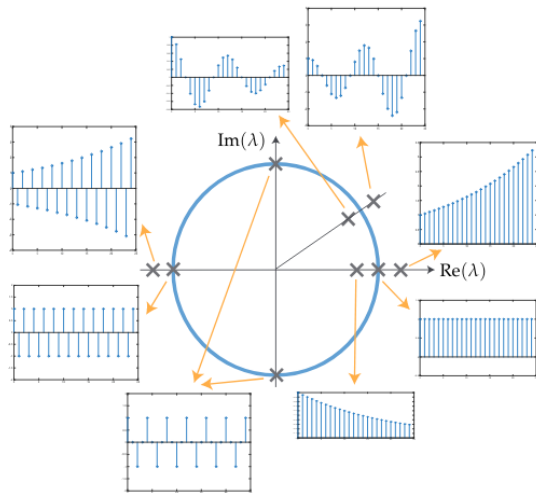


Stability

Discrete

EECS 16B
CSM

Bryan Ngo



Controllability

EECS 16B
CSM

Bryan Ngo

$$\mathbf{x}[t+1] = \mathbf{A}\mathbf{x}[t] + \mathbf{B}\mathbf{u}[t] \quad (5)$$

$$\mathbf{x}[1] = \mathbf{A}\mathbf{x}[0] + \mathbf{B}\mathbf{u}[0] \quad (6)$$

$$\mathbf{x}[2] = \mathbf{A}^2\mathbf{x}[0] + \mathbf{A}\mathbf{B}\mathbf{u}[0] + \mathbf{B}\mathbf{u}[1] \quad (7)$$

$$\mathbf{x}[t] = \mathbf{A}^t\mathbf{x}[0] + \sum_{i=0}^{t-1} \mathbf{A}^{t-i}\mathbf{B}\mathbf{u}[i] \quad (8)$$

$$\mathbf{x}[t] - \mathbf{A}^t\mathbf{x}[0] = \begin{bmatrix} \mathbf{B} & \mathbf{A}\mathbf{B} & \cdots & \mathbf{A}^{t-1}\mathbf{B} \end{bmatrix} \begin{bmatrix} \mathbf{u}[t-1] \\ \mathbf{u}[t-2] \\ \vdots \\ \mathbf{u}[0] \end{bmatrix} \quad (9)$$

$$\Rightarrow \text{span} \left\{ \begin{bmatrix} \mathbf{B} & \mathbf{A}\mathbf{B} & \cdots & \mathbf{A}^{t-1}\mathbf{B} \end{bmatrix} \right\} = \mathbb{R}^n \quad (10)$$