EECS 16B CSM

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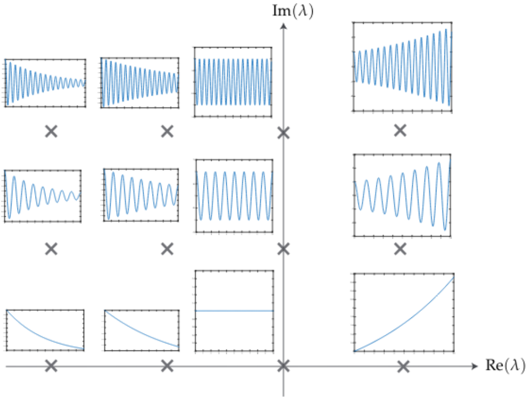
Logistics

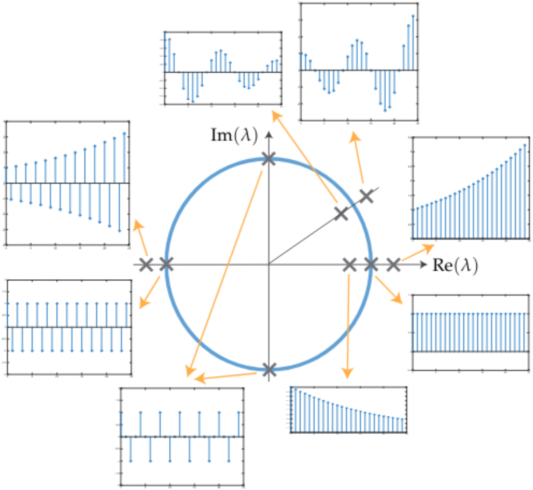
- Some new guests!
- How did MT go?
- Feedback: `https://forms.gle/8g1NcqqE4m1shkVx5`

Stability

Continuous



Discrete



Feedback

Open-Loop

$$\boldsymbol{x}[t+1] = \boldsymbol{A}\boldsymbol{x}[t] + \boldsymbol{B}\boldsymbol{u}[t] \tag{1}$$

- define a certain range of use
- simpler
- no restraints apart from stability

Controllability

$$\boldsymbol{x}[t+1] = \boldsymbol{A}\boldsymbol{x}[t] + \boldsymbol{B}\boldsymbol{u}[t] \tag{2}$$

$$\boldsymbol{x}[1] = \boldsymbol{A}\boldsymbol{x}[0] + \boldsymbol{B}\boldsymbol{u}[0] \tag{3}$$

$$\boldsymbol{x}[2] = \boldsymbol{A}^2\boldsymbol{x}[0] + \boldsymbol{A}\boldsymbol{B}\boldsymbol{u}[0] + \boldsymbol{B}\boldsymbol{u}[1] \tag{4}$$

$$\boldsymbol{x}[t] = \boldsymbol{A}^t\boldsymbol{x}[0] + \sum_{i=0}^{t-1} \boldsymbol{A}^{t-i}\boldsymbol{B}\boldsymbol{u}[i] \tag{5}$$

$$\boldsymbol{x}[t] - \boldsymbol{A}^t\boldsymbol{x}[0] = \begin{bmatrix} \boldsymbol{B} & \boldsymbol{A}\boldsymbol{B} & \cdots & \boldsymbol{A}^{t-1}\boldsymbol{B} \end{bmatrix} \begin{bmatrix} \boldsymbol{u}[t-1] \\ \boldsymbol{u}[t-2] \\ \vdots \\ \boldsymbol{u}[0] \end{bmatrix} \tag{6}$$

$$\Rightarrow \operatorname{span}\left\{\begin{bmatrix} \boldsymbol{B} & \boldsymbol{A}\boldsymbol{B} & \cdots & \boldsymbol{A}^{t-1}\boldsymbol{B} \end{bmatrix}\right\} = \mathbb{R}^n \tag{7}$$

Closed-Loop

$$\boldsymbol{u}[t] = \begin{bmatrix} k_1 & k_2 & \cdots & k_n \end{bmatrix} \begin{bmatrix} x_1[t] \\ x_2[t] \\ \vdots \\ x_n[t] \end{bmatrix} \tag{8}$$

$$\boldsymbol{x}[t+1] = \boldsymbol{A}\boldsymbol{x}[t] + \boldsymbol{B}\boldsymbol{K}\boldsymbol{x}[t] = (\boldsymbol{A} + \boldsymbol{B}\boldsymbol{K})\boldsymbol{x}[t] \tag{9}$$

- adaptable to a wide range of use
- more complex
- self-correcting
- requires more constraints

Controller Canonical Form

$$\boldsymbol{x}[t+1] = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ a_1 & a_2 & a_3 & \cdots & a_n \end{bmatrix} \boldsymbol{x}[t] + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} \boldsymbol{u}[t] \tag{10}$$

$$\det(\boldsymbol{A} + \boldsymbol{B}\boldsymbol{K} - \lambda \boldsymbol{I}) = \lambda^n - \sum_{i=1}^{n} (a_i + k_i)\lambda^i \tag{11}$$