#### EECS 16B CSM

Bryan Ngc

Complex Numbers

## EECS 16B CSM

Bryan Ngo

Computer Science Mentors

2020-09-27



#### EECS 16B CSM

Bryan Ngo

Complex Numbers

hasors

## Complex Numbers

## Definition

EECS 16B CSM

Bryan Ngo

Complex Numbers

 $\mathsf{Phasors}$ 

$$z = \underbrace{a + bj}_{\text{rectangular}} = \underbrace{re^{j\theta}}_{\text{polar}} \tag{1}$$

- $a, b, r, \theta \in \mathbb{R}$
- $j^2 = -1$
- lacktriangle we use j in EE

## Coordinate Transforms

EECS 16B CSM

Bryan Ng

Complex Numbers

Phasor:

$$r^2 = a^2 + b^2 (2)$$

$$\operatorname{an}(\theta) = \frac{b}{a} \tag{3}$$

$$a = r\cos(\theta) \tag{4}$$

$$b = r\sin(\theta) \tag{5}$$

## Euler's Formula

EECS 16B CSM

Bryan Ngo

Complex Numbers

$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$
 (6)

- relevant 3Blue1Brown
- $e^{j\pi} + 1 = 0$  is a special case

#### EECS 16B CSM

Bryan Ng

Complex Numbers

Phasors

### Phasors

#### EECS 16B CSM

Bryan Ngo

Complex Numbers

- Encodes information about any sinusoid: voltage, current, etc.
- If frequency is constant, then uniquely identifies

$$A\cos(\omega t + \phi) = \Re\{Ae^{j(\omega t + \phi)}\} = \Re\{\underbrace{Ae^{j\phi}}_{\mathsf{phasor}}e^{j\omega t}\}$$
 (7)

## **Properties**

#### EECS 16B CSM

Bryan Ng

Complex Numbers

Given 
$$x_1(t)=\Re\{A_1e^{j\omega t}\}, x_2(t)=\Re\{A_2e^{j\omega t}\}$$
 with phasors  $A_{1,2}$ ,

- Uniqueness:  $x_1(t) = x_2(t) \implies A_1 = A_2$
- Linearity:  $a_1x_1(t) + a_2x_2(t) \implies a_1A_1 + a_2A_2$  for  $a_{1,2} \in \mathbb{R}$
- $\qquad \text{ Differentiation: } x(t) \Leftrightarrow A \implies \frac{d}{dt}x(t) = \frac{d}{dt}\Re\{Ae^{j\omega t}\} = \Re\{j\omega Ae^{j\omega t}\} \Leftrightarrow j\omega Ae^{j\omega t}$

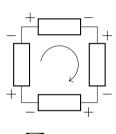
# Circuits & Phasors KVL

EECS 16B CSM

Bryan Ng

Comple:

Phasors



$$\sum_{i} \overline{V}_{i} = 0$$

(8)

## Circuits & Phasors (cont.)

EECS 16B CSM

Bryan Ng

Comple:

$$\sum \overline{I}_{out} = \sum \overline{I}_{in} \tag{9}$$

## Circuits & Phasors (cont.) Ohm's Law

EECS 16B CSM

Bryan Ngo

Complex Number

$$\overline{V} = \overline{I} \underbrace{Z}_{\text{impedance}}$$
(10)

## EECS 16B CSM $\overline{V} = \overline{I}R$ (11)**Phasors** $\overline{V} = L \frac{d}{dt} \overline{I} = j\omega L \overline{I}$ (12)

Passive Elements & Phasors

$$V = \frac{1}{V - | C} C$$

$$\overline{I} = C \frac{d}{dt} \overline{V} = j\omega C \overline{V} \implies \overline{V} = \frac{1}{j\omega C} \overline{I}$$
(13)

## Demo

EECS 16B CSM

Bryan N

Comple:

Phasors

http://tinyurl.com/y5qfnqtk

### Low Pass Filter

EECS 16B CSM

$$\overline{V}_{out} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} \overline{V}_{in}$$

$$\overline{V}_{out} = \frac{1}{1 + j\omega RC} \overline{V}_{in}$$
(14)

$$\overline{V}_{out} = \frac{1}{1 + i\omega RC} \overline{V}_{in} \tag{15}$$

$$\Rightarrow H(j\omega) = \frac{\overline{V}_{out}}{\overline{V}_{in}} = \frac{1}{1 + i\omega RC}$$
 (16)