Bryan Ng

Differentia Equations

RC Circuit

Vector
Differential
Equations

Change of

Inductors (Preview)

EECS 16B CSM

Bryan Ngo

Computer Science Mentors

2020-09-21

RC Circuit

Differentia Equations

Change of Basis

- 1 Differential Equations
- 2 RC Circuits
- 3 Vector Differential Equations
- 4 Change of Basis
- 5 Inductors (Preview)

Logistics

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Differentia Equations

RC Circuit

Vector Differentia Equations

Change o

- Quest?
- Focus for today?
- messenger chat
- all slides available at https://github.com/bdngo/16b-csm

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Differential Equations

BC Circuit

Vector Differentia

Change of

Inductors (Preview)

Differential Equations

Differential Equations

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Differential Equations

RC Circui

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Differential Equations

Change o Basis

Inductors (Preview) Concept check!

Differential Equations

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Differential Equations

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Differentia
Equations

Change of Basis

Inductors (Preview) Concept check!

$$\frac{d}{dt}x(t) = f(x,t) \tag{1}$$

- Focusing on first-order ODEs
- Relates the derivative in other terms
- 3Blue1Brown video

Exponential Differential Equation

Homogeneous

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Differential Equations

RC Circui

Vector Differentia

Differentia Equations

Basis

$$\frac{d}{dt}x(t) = \lambda x(t) \implies x(t) = x_0 e^{\lambda t}$$
 (2)

Exponential Differential Equation

Non-Homogeneous

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Differential Equations

$$\frac{d}{dt}x(t) = \alpha x(t) + \beta \tag{3}$$

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Differentia Equations

RC Circuits

Vector Differential Equations

Change of

Inductors (Preview)

RC Circuits

Undamped Response

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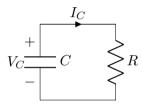
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RC Circuits

Vector
Differential

Change o



Undamped Response

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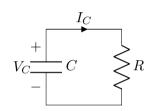
Differentia Equations

RC Circuits

Vector Differentia

Change of

Inductors (Preview)



$$C\frac{d}{dt}V_C = -\frac{V_C}{R} \tag{4}$$

$$\frac{d}{dt}V_C = -\frac{1}{RC}V_C$$

$$\Rightarrow V_C(t) = V_0 e^{-\frac{1}{RC}t} = V_0 e^{-\frac{1}{\tau}t} \tag{6}$$



(5)

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Differentia Equations

RC Circuit

Vector Differential Equations

Basis

Vector Differential Equations

General Form

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Differentia Equations

RC Circuits

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Vector Differential Equations

Change of Basis

$$\frac{d}{dt}x(t) = Ax(t) + Bu(t)$$
(7)

- lacksquare if $m{A}$ is diagonal, simply a bunch of exponential differential Equations
- if not, we can try to diagonalize

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Differentia Equations

RC Circuit

Vector
Differentia
Equations

Change of Basis

Inductors (Preview)

Change of Basis

Motivation

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Differentia Equations

RC Circuit

Vector Differentia Equations

Change of Basis

- conversion from one linear coordinate system to another
- Yet another 3Blue1Brown video

A Visualization

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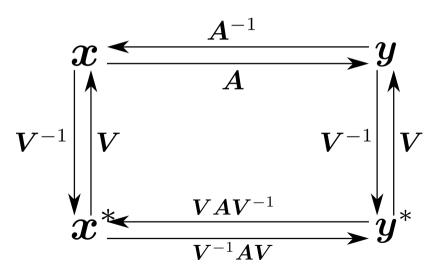
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Differentia Equations

RC Circuit

Differentia Equations

Change of Basis



Diagonalization

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Differentia Equations

RC Circuit

Vector Differentia

Change of Basis

- want the eigenvectors to be the basis for a vector space
- makes math way easier

Diagonalization

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Differentia Equations

RC Circuit

Vector
Differentia
Equations

Change of Basis

Inductors (Preview) want the eigenvectors to be the basis for a vector space

makes math way easier

$$V = \begin{bmatrix} v_1 & v_2 & \cdots & v_n \end{bmatrix} \tag{8}$$

$$AV = \begin{bmatrix} \lambda_1 v_1 & \lambda_2 v_2 & \cdots & \lambda_n v_n \end{bmatrix}$$
 (9)

$$=egin{bmatrix} oldsymbol{v}_1 & oldsymbol{v}_2 & \cdots & oldsymbol{v}_n \end{bmatrix} egin{bmatrix} \lambda_1 & 0 & \cdots & 0 \ 0 & \lambda_2 & \cdots & 0 \ dots & dots & \ddots & dots \ 0 & 0 & \cdots & \lambda_n \end{bmatrix}$$

$$= V\Lambda \implies \Lambda = V^{-1}AV \tag{11}$$

(10)

Diagonalizing DEs

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Change of Basis

$$\frac{d}{dt}x(t) = Ax(t) + Bu(t)$$
(12)

$$\frac{d}{dt}\mathbf{x}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \tag{12}$$

$$\frac{d}{dt}\mathbf{V}\mathbf{z}(t) = \mathbf{A}\mathbf{V}\mathbf{z}(t) + \mathbf{B}\mathbf{u}(t) \tag{13}$$

$$\Rightarrow \frac{d}{dt}\mathbf{z}(t) = \mathbf{V}^{-1}\mathbf{A}\mathbf{V}\mathbf{z}(t) + \mathbf{V}^{-1}\mathbf{B}\mathbf{u}(t) \tag{14}$$

$$\Rightarrow \frac{d}{dt} z(t) = V^{-1} A V z(t) + V^{-1} B u(t)$$
(14)

$$= \mathbf{\Lambda} \boldsymbol{z}(t) + \boldsymbol{V}^{-1} \boldsymbol{B} \boldsymbol{u}(t) \tag{15}$$

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Inductors (Preview)

Basic Properties

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Change of

Inductors

Inductors (Preview)

$$V_L \begin{cases} + \bigvee_{L} I_L \\ V_L \\ - \bigvee_{L} I_L \end{cases}$$

$$V_L = L \frac{d}{dt} I_L$$

(16)

- like a capacitor but for magnetic fields
- resists instantaneous change in current

Basic Properties

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Inductors (Preview)

$$V_L \begin{cases} + \sqrt[4]{I_L} \\ V_L \\ - \sqrt{1 - I_L} \end{cases}$$

$$V_L = L \frac{d}{dt} I_L$$

(16)

- like a capacitor but for magnetic fields
- resists instantaneous change in current
- what happens when $\omega = 0$? $\omega = \infty$?