EECS 16B CSM

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Midterm

- Good luck!
- Review vs. Worksheet

State Space Form

$$\frac{d}{dt}\boldsymbol{x}(t) = f(\boldsymbol{x}(t), \boldsymbol{u}(t)) \tag{1}$$

Linearization

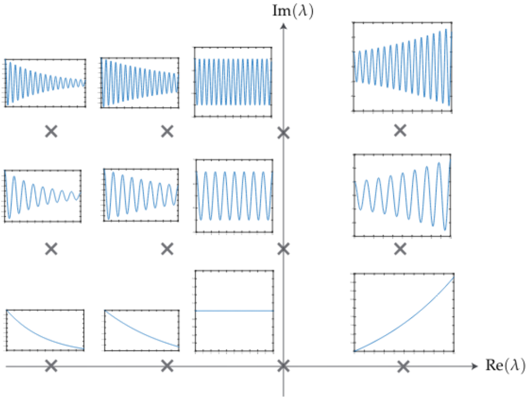
$$\boldsymbol{J_x} = \begin{bmatrix} \partial_{x_1} f_1 & \partial_{x_2} f_1 & \cdots & \partial_{x_n} f_1 \\ \partial_{x_1} f_2 & \partial_{x_2} f_2 & \cdots & \partial_{x_n} f_2 \\ \vdots & \vdots & \ddots & \vdots \\ \partial_{x_1} f_n & \partial_{x_2} f_n & \cdots & \partial_{x_n} f_n \end{bmatrix} \tag{2}$$

$$\boldsymbol{J_u} = \begin{bmatrix} \partial_{u_1} f_1 & \partial_{u_2} f_1 & \cdots & \partial_{u_n} f_1 \\ \partial_{u_1} f_2 & \partial_{u_2} f_2 & \cdots & \partial_{u_n} f_2 \\ \vdots & \vdots & \ddots & \vdots \\ \partial_{u_1} f_n & \partial_{u_2} f_n & \cdots & \partial_{u_n} f_n \end{bmatrix} \tag{3}$$

$$\frac{d}{dt}\boldsymbol{x}(t) \approx \boldsymbol{J_x}(\boldsymbol{x}(t) - \boldsymbol{x}^*) + \boldsymbol{J_u}(\boldsymbol{u}(t) - \boldsymbol{u}^*) \tag{4}$$

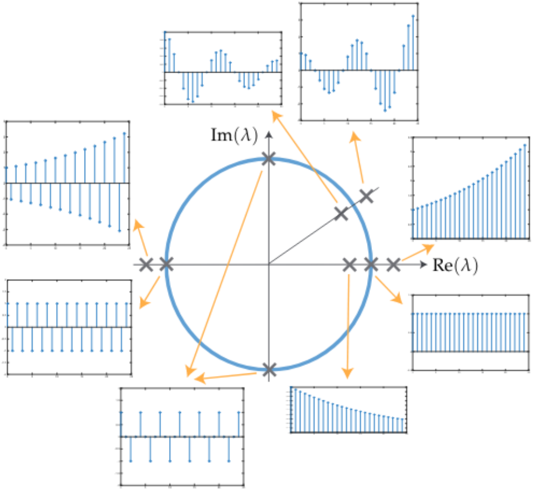
Stability

Continuous



Stability

Discrete



Controllability

$$\boldsymbol{x}[t+1] = \boldsymbol{A}\boldsymbol{x}[t] + \boldsymbol{B}\boldsymbol{u}[t] \tag{5}$$

$$\boldsymbol{x}[1] = \boldsymbol{A}\boldsymbol{x}[0] + \boldsymbol{B}\boldsymbol{u}[0] \tag{6}$$

$$\boldsymbol{x}[2] = \boldsymbol{A}^2\boldsymbol{x}[0] + \boldsymbol{A}\boldsymbol{B}\boldsymbol{u}[0] + \boldsymbol{B}\boldsymbol{u}[1] \tag{7}$$

$$\boldsymbol{x}[t] = \boldsymbol{A}^t\boldsymbol{x}[0] + \sum_{i=0}^{t-1} \boldsymbol{A}^{t-i}\boldsymbol{B}\boldsymbol{u}[i] \tag{8}$$

$$\boldsymbol{x}[t] - \boldsymbol{A}^t\boldsymbol{x}[0] = \begin{bmatrix} \boldsymbol{B} & \boldsymbol{A}\boldsymbol{B} & \cdots & \boldsymbol{A}^{t-1}\boldsymbol{B} \end{bmatrix} \begin{bmatrix} \boldsymbol{u}[t-1] \\ \boldsymbol{u}[t-2] \\ \vdots \\ \boldsymbol{u}[0] \end{bmatrix} \tag{9}$$

$$\Rightarrow \operatorname{span}\left\{\begin{bmatrix} \boldsymbol{B} & \boldsymbol{A}\boldsymbol{B} & \cdots & \boldsymbol{A}^{t-1}\boldsymbol{B} \end{bmatrix}\right\} = \mathbb{R}^n \tag{10}$$