EECS 16B CSM

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Computer Science Mentors

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Updates

- Quest?
- Focus for today?

Differential Equations

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Concept check!

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$$\frac{d}{dt}x(t) = f(x, t) \tag{1}$$

- Focusing on first-order ODEs
- Relates the derivative in other terms
- 3Blue1Brown video

Exponential Differential Equation

Homogeneous

$$\frac{d}{dt}x(t) = \lambda x(t) \implies x(t) = x_0 e^{\lambda t} \tag{2}$$

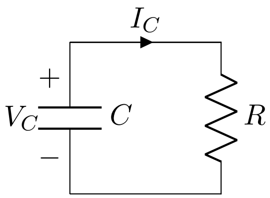
Exponential Differential Equation

Non-Homogeneous

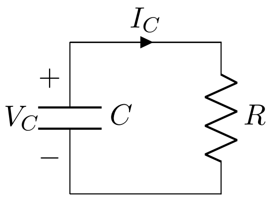
$$\frac{d}{dt}x(t) = \alpha x(t) + \beta \tag{3}$$

RC Circuits

Undamped Response



Undamped Response



$$C\frac{d}{dt}V_C = -\frac{V_C}{R} \tag{4}$$

$$\frac{d}{dt}V_C = \underbrace{-\frac{1}{RC}}_{\lambda} V_C \tag{5}$$

$$\Rightarrow V_C(t) = V_0 e^{-\frac{1}{RC}t} = V_0 e^{-\frac{1}{\tau}t} \tag{6}$$

Vector Differential Equations

General Form

$$\frac{d}{dt}\boldsymbol{x}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t) \tag{7}$$

- if $\boldsymbol{A}$ is diagonal, simply a bunch of exponential differential Equations
- if not, we can try to diagonalize

Change of Basis

Motivation

- conversion from one linear coordinate system to another
- Yet another 3Blue1Brown video

A Visualization

$$
\begin{tikzcd}
x \arrow[r, shift right, "A"'] \arrow[d, shift right, "V^{-1}"'] & y \arrow[l, shift right, "A^{-1}"'] \arrow[d, shift right, "V^{-1}"'] \\
x^* \arrow[u, shift right, "V"'] \arrow[r, shift right, "V^{-1}AV"'] & y^* \arrow[u, shift right, "V"'] \arrow[l, shift right, "VAV^{-1}"']
\end{tikzcd}
$$

Diagonalization

- want the eigenvectors to be the basis for a vector space
- makes math *way* easier

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$$\boldsymbol{V} = \begin{bmatrix} \boldsymbol{v}_1 & \boldsymbol{v}_2 & \cdots & \boldsymbol{v}_n \end{bmatrix} \tag{8}$$

$$\boldsymbol{AV} = \begin{bmatrix} \lambda_1 \boldsymbol{v}_1 & \lambda_2 \boldsymbol{v}_2 & \cdots & \lambda_n \boldsymbol{v}_n \end{bmatrix} \tag{9}$$

$$= \begin{bmatrix} \boldsymbol{v}_1 & \boldsymbol{v}_2 & \cdots & \boldsymbol{v}_n \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_n \end{bmatrix} \tag{10}$$

$$= \boldsymbol{V\Lambda} \implies \boldsymbol{\Lambda} = \boldsymbol{V}^{-1}\boldsymbol{AV} \tag{11}$$

Diagonalizing DEs

$$\frac{d}{dt}\boldsymbol{x}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t) \tag{12}$$

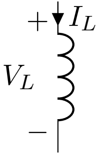
$$\frac{d}{dt}\boldsymbol{V}\boldsymbol{z}(t) = \boldsymbol{A}\boldsymbol{V}\boldsymbol{z}(t) + \boldsymbol{B}\boldsymbol{u}(t) \tag{13}$$

$$\Rightarrow \frac{d}{dt}\boldsymbol{z}(t) = \boldsymbol{V}^{-1}\boldsymbol{A}\boldsymbol{V}\boldsymbol{z}(t) + \boldsymbol{V}^{-1}\boldsymbol{B}\boldsymbol{u}(t) \tag{14}$$

$$= \boldsymbol{\Lambda}\boldsymbol{z}(t) + \boldsymbol{V}^{-1}\boldsymbol{B}\boldsymbol{u}(t) \tag{15}$$

Inductors

Basic Properties



$$V_L = L\frac{d}{dt}I_L \tag{16}$$

- like a capacitor but for magnetic fields
- resists instantaneous change in current

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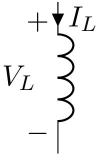

$$V_L = L\frac{d}{dt}I_L \tag{16}$$

- like a capacitor but for magnetic fields
- resists instantaneous change in current
- what happens when $\omega = 0$? $\omega = \infty$?