### EECS 16B CSM

Bryan Ngo

Change o Basis

Vector Differential Equations

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Bryan Ngo

Computer Science Mentors

2021-09-20

Vector Differential Equations 1 Change of Basis

## Logistics

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Change of Basis

- unexcused absences in first 3 weeks → auto-dropped & NP
- excused absences: email bryanngo@berkeley.edu & cc mentors@berkeley.edu
  with subject line [Request for Absence] <course>
- Slides available at https://github.com/bdngo/16b-csm

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### Change of Basis

Vector Differential Equations

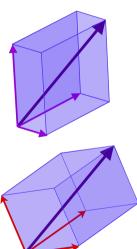
# Change of Basis

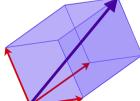
### Motivation

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### Change of Basis

- conversion from one linear coordinate system to another
- 3Blue1Brown video



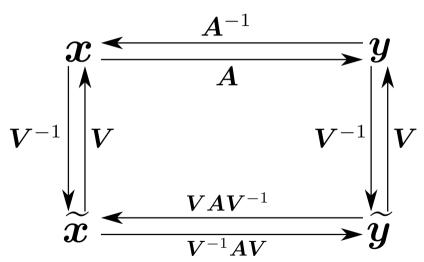


### A Visualization

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## Diagonalization

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### Change of Basis

- want the eigenvectors to be the basis for a vector space
- makes math way easier

## Diagonalization

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### Change of Basis

- want the eigenvectors to be the basis for a vector space
- makes math way easier

$$oldsymbol{V} = egin{bmatrix} oldsymbol{v}_1 & oldsymbol{v}_2 & \cdots & oldsymbol{v}_n \end{bmatrix}$$

$$AV = \begin{bmatrix} \lambda_1 v_1 & \lambda_2 v_2 & \cdots & \lambda_n v_n \end{bmatrix}$$
 (2)

$$= \begin{bmatrix} \boldsymbol{v}_1 & \boldsymbol{v}_2 & \cdots & \boldsymbol{v}_n \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_n \end{bmatrix}$$
(3)

$$= V\Lambda \implies \Lambda = V^{-1}AV \tag{4}$$

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### General Form

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Change o Basis

$$\frac{d}{dt}x(t) = Ax(t) + Bu(t)$$
(5)

- lacksquare if  $m{A}$  is diagonal, simply a bunch of exponential differential Equations
- if not, we can try to diagonalize

# Diagonalizing DEs

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$$\frac{d}{dt}x(t) = Ax(t) + Bu(t)$$
(6)

$$\frac{d}{dt}\mathbf{x}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \tag{6}$$

$$\frac{d}{dt}\mathbf{V}\mathbf{z}(t) = \mathbf{A}\mathbf{V}\mathbf{z}(t) + \mathbf{B}\mathbf{u}(t) \tag{7}$$

$$\Rightarrow \frac{d}{dt}\mathbf{z}(t) = \mathbf{V}^{-1}\mathbf{A}\mathbf{V}\mathbf{z}(t) + \mathbf{V}^{-1}\mathbf{B}\mathbf{u}(t) \tag{8}$$

$$\Rightarrow \frac{d}{dt} z(t) = V^{-1} A V z(t) + V^{-1} B u(t)$$
(8)

$$= \mathbf{\Lambda} \mathbf{z}(t) + \mathbf{V}^{-1} \mathbf{B} \mathbf{u}(t) \tag{9}$$