EECS 16B CSM

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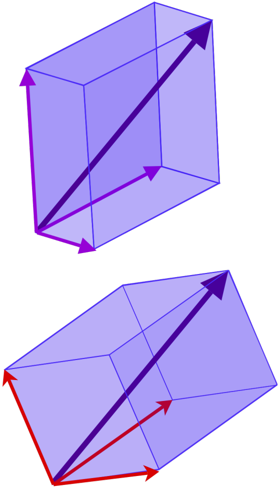
Logistics

- fill out temperature check
- pertinent facts

Change of Basis

Motivation

- conversion from one linear coordinate system to another
- 3Blue1Brown video



A Visualization

$$
\begin{tikzcd}[column sep=huge, row sep=huge]
\boldsymbol{x} \arrow[r, shift right, "\boldsymbol{A}"'] \arrow[d, shift right, "\boldsymbol{V}^{-1}"'] & \boldsymbol{y} \arrow[l, shift right, "\boldsymbol{A}^{-1}"'] \arrow[d, shift right, "\boldsymbol{V}^{-1}"'] \\
\widetilde{\boldsymbol{x}} \arrow[r, shift right, "\boldsymbol{V}^{-1}\boldsymbol{A}\boldsymbol{V}"'] \arrow[u, shift right, "\boldsymbol{V}"'] & \widetilde{\boldsymbol{y}} \arrow[l, shift right, "\boldsymbol{V}\boldsymbol{A}\boldsymbol{V}^{-1}"'] \arrow[u, shift right, "\boldsymbol{V}"']
\end{tikzcd}
$$

Diagonalization

- want the eigenvectors to be the basis for a vector space
- makes math *way* easier

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$$\boldsymbol{V} = \begin{bmatrix} \boldsymbol{v}_1 & \boldsymbol{v}_2 & \cdots & \boldsymbol{v}_n \end{bmatrix} \tag{1}$$

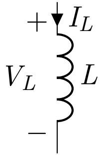
$$\boldsymbol{A}\,\boldsymbol{V} = \begin{bmatrix} \lambda_1 \boldsymbol{v}_1 & \lambda_2 \boldsymbol{v}_2 & \cdots & \lambda_n \boldsymbol{v}_n \end{bmatrix} \tag{2}$$

$$= \begin{bmatrix} \boldsymbol{v}_1 & \boldsymbol{v}_2 & \cdots & \boldsymbol{v}_n \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_n \end{bmatrix} \tag{3}$$

$$= \boldsymbol{V}\boldsymbol{\Lambda} \implies \boldsymbol{\Lambda} = \boldsymbol{V}^{-1}\boldsymbol{A}\,\boldsymbol{V} \tag{4}$$

Inductors

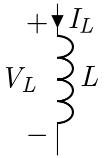
Basic Properties



$$V_L = L \frac{d}{dt} I_L \tag{5}$$

- like a capacitor but for magnetic fields
- resists instantaneous change in current

Basic Properties



$$V_L = L\frac{d}{dt}I_L \tag{5}$$

- like a capacitor but for magnetic fields
- resists instantaneous change in current
- what happens when $\omega = 0$? $\omega = \infty$?

Complex Numbers

Definition

$$z = \underbrace{a + bj}_{\text{rectangular}} = \underbrace{re^{j\theta}}_{\text{polar}} \tag{6}$$

- $a, b, r, \theta \in \mathbb{R}$
- $j^2 = -1$
- we use j in EE

Coordinate Transforms

$$r^2 = a^2 + b^2 \tag{7}$$

$$\tan(\theta) = \frac{b}{a} \tag{8}$$

$$a = \Re\{z\} = r\cos(\theta) \tag{9}$$

$$b = \Im\{z\} = r\sin(\theta) \tag{10}$$

Euler's Formula

$$e^{j\theta} = \cos(\theta) + j\sin(\theta) \tag{11}$$

- relevant 3Blue1Brown
- $e^{j\pi} + 1 = 0$ is a special case