EECS 16B CSM

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2020-11-02

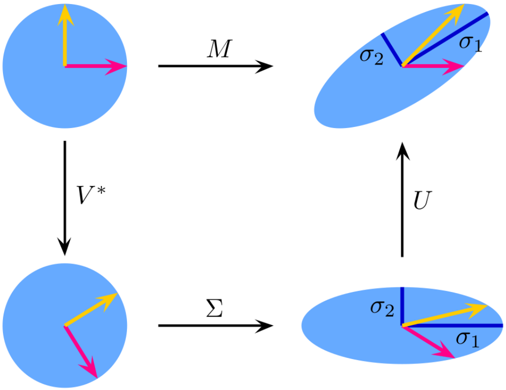
SVD

Definition

$$\boldsymbol{A} = \sum_{i=1}^{r} \sigma_i \boldsymbol{u}_i \left[\boldsymbol{v}_i\right]^\top = \boldsymbol{U}\boldsymbol{\Sigma}\left[\boldsymbol{V}\right]^\top = \begin{bmatrix} \boldsymbol{u}_1 & \boldsymbol{u}_2 & \cdots & \boldsymbol{u}_r \end{bmatrix} \begin{bmatrix} \sigma_1 & 0 & \cdots & 0 \\ 0 & \sigma_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_n \end{bmatrix} \begin{bmatrix} \left[\boldsymbol{v}_1\right]^\top \\ \left[\boldsymbol{v}_2\right]^\top \\ \vdots \\ \left[\boldsymbol{v}_n\right]^\top \end{bmatrix} \tag{1}$$

- $\boldsymbol{U}$: orthonormal matrix
- $\boldsymbol{\Sigma}$: diagonal matrix
- $\left[\boldsymbol{V}\right]^\top$: orthonormal matrix

Visualization



$$M = U \cdot \Sigma \cdot V^*$$

Gram-Schmidt Process

$$\boldsymbol{p}_i = \boldsymbol{v}_i - \sum_{j=1}^{i-1} ([\boldsymbol{v}_i]^\top \boldsymbol{w}_j)\boldsymbol{w}_j \tag{2}$$

$$\boldsymbol{w}_i = \frac{\boldsymbol{p}_i}{\|\boldsymbol{p}_i\|} \tag{3}$$

- turns set of basis vectors to set of *orthogonal* basis vectors
- systematically removing parallel components of our vector