EECS 16B CSM

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PCA

Linearization

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Linearization

1 PCA

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- allows us to reduce dimensionality
- preserve only most important singular components

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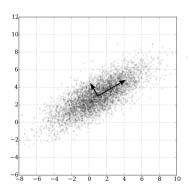
Steps

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- 1 do SVD
- $\mathbf{2}$ pick the first σ_i that you want
- $lacksquare{3}$ first v_i from $oldsymbol{V}^ op$ are the singular components



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State Space Form

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$$\frac{d}{dt}\mathbf{x}(t) = f(\mathbf{x}(t), \mathbf{u}(t)) \tag{1}$$

But what if f(x(t), u(t)) isn't linear?

Linearity

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Given a system y(t) = f(x(t)),

$$ax(t) \iff ay(t)$$
 (2)

$$x_1(t) + x_2(t) \iff y_1(t) + y_2(t)$$
 (3)

$$ax_1(t) + bx_2(t) \iff ay_1(t) + by_2(t)$$
 (4)

Taylor Approximations

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Calculus review!

$$f(x) = \sum \frac{f^{(n)}(x_0)}{n!} (x - x_0)^n = \frac{f(x_0)}{0!} + \frac{f'(x_0)}{1!} (x - x_0) + \frac{f''(x_0)}{2!} (x - x_0)^2 + \cdots$$

$$\approx f(x_0) + f'(x_0)(x - x_0) \tag{5}$$

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$$J_{x} = \begin{bmatrix} \partial_{x_{1}} f_{1} & \partial_{x_{2}} f_{1} & \cdots & \partial_{x_{n}} f_{1} \\ \partial_{x_{1}} f_{2} & \partial_{x_{2}} f_{2} & \cdots & \partial_{x_{n}} f_{2} \\ \vdots & \vdots & \ddots & \vdots \\ \partial_{x_{1}} f_{n} & \partial_{x_{2}} f_{n} & \cdots & \partial_{x_{n}} f_{n} \end{bmatrix}$$

$$J_{u} = \begin{bmatrix} \partial_{u_{1}} f_{1} & \partial_{u_{2}} f_{1} & \cdots & \partial_{u_{n}} f_{1} \\ \partial_{u_{1}} f_{2} & \partial_{u_{2}} f_{2} & \cdots & \partial_{u_{n}} f_{2} \\ \vdots & \vdots & \ddots & \vdots \\ \partial_{u_{1}} f_{n} & \partial_{u_{2}} f_{n} & \cdots & \partial_{u_{n}} f_{n} \end{bmatrix}$$

$$(7)$$

$$\frac{d}{dt}\boldsymbol{x}(t) \approx \boldsymbol{J}_{\boldsymbol{x}}(\boldsymbol{x}(t) - \boldsymbol{x}^*) + \boldsymbol{J}_{\boldsymbol{u}}(\boldsymbol{u}(t) - \boldsymbol{u}^*) \tag{9}$$