## EECS 16B CSM

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## Logistics

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- Spring break???
- CSM feedback form results

## Controllability

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$$egin{aligned} oldsymbol{x}[t+1] &= oldsymbol{A} oldsymbol{x}[t] + oldsymbol{B} oldsymbol{u}[t] & (1) \ oldsymbol{x}[1] &= oldsymbol{A} oldsymbol{x}[0] + oldsymbol{B} oldsymbol{u}[0] & (2) \ oldsymbol{x}[2] &= oldsymbol{A}^2 oldsymbol{x}[0] + oldsymbol{A} oldsymbol{B} oldsymbol{u}[0] + oldsymbol{B} oldsymbol{u}[1] & (3) \ oldsymbol{x}[t] &= oldsymbol{A}^t oldsymbol{x}[0] + \sum_{i=0}^{t-1} oldsymbol{A}^{t-i} oldsymbol{B} oldsymbol{u}[i] & (4) \end{aligned}$$

$$oldsymbol{x}[t] - oldsymbol{A}^t oldsymbol{x}[0] = egin{bmatrix} oldsymbol{B} & oldsymbol{A} oldsymbol{B} & oldsymbol{A} oldsymbol{B} & oldsymbol{A}^{t-1} oldsymbol{B} \end{bmatrix} egin{bmatrix} oldsymbol{u}[t-1] \ oldsymbol{u}[t-2] \ dots \ oldsymbol{u}[0] \end{bmatrix}$$
 (5)

$$\Rightarrow \operatorname{span}\left\{\left[\boldsymbol{B} \quad \boldsymbol{A}\boldsymbol{B} \quad \cdots \quad \boldsymbol{A}^{t-1}\boldsymbol{B}\right]\right\} = \mathbb{R}^{n} \tag{6}$$

## Gram-Schmidt

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$$oldsymbol{p}_i = oldsymbol{v}_i - \sum_{j \neq i} ([oldsymbol{v}_i]^{\!\top} oldsymbol{w}_j) oldsymbol{w}_j$$
 (7)

$$\boldsymbol{w}_i = \frac{\boldsymbol{p}_i}{\|\boldsymbol{p}_i\|} \tag{8}$$

- lacktriangle Turn basis vector set V into orthonormal basis W
- Steps
  - 1 Subtract all  $\operatorname{proj}_{\boldsymbol{v}_i}(\boldsymbol{v}_n)$  for  $i \neq n$
  - 2 Normalize result
  - 3 repeat for all vectors
- systematically removing the parallel component of every other vector
- cool GIF of GS

