

EECS 16B CSM

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Computer Science Mentors

2021-09-27

Logistics

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Complex
Numbers

Phasors

- keep up the attendance!
- public Piazza: <https://piazza.com/berkeley/fall2021/csm16b>
- VDEs or Phasors?



1 Complex Numbers

2 Phasors

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Complex Numbers

Definition

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Phasors

$$z = \underbrace{a + bj}_{\text{rectangular}} = \underbrace{re^{j\theta}}_{\text{polar}} \quad (1)$$

- $a, b, r, \theta \in \mathbb{R}$
- $j^2 = -1$
- we use j in EE

Coordinate Transforms

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Phasors

$$r^2 = a^2 + b^2 \quad (2)$$

$$\tan(\theta) = \frac{b}{a} \quad (3)$$

$$a = \Re\{z\} = r \cos(\theta) \quad (4)$$

$$b = \Im\{z\} = r \sin(\theta) \quad (5)$$

Euler's Formula

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$$e^{j\theta} = \cos(\theta) + j \sin(\theta) \quad (6)$$

- relevant 3Blue1Brown
- $e^{j\pi} + 1 = 0$ is a special case

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Phasors

- Encodes information about any sinusoid: voltage, current, etc.
- If frequency is constant, then uniquely identifies

$$x(t) = A \cos(\omega t + \phi) = \frac{A}{2} \left(e^{j(\omega t + \phi)} + e^{-j(\omega t + \phi)} \right) \quad (7)$$

$$= \frac{A}{2} \left(e^{j\omega t} e^{j\phi} + \overline{e^{j\omega t} e^{j\phi}} \right) \quad (8)$$

$$\iff \tilde{X} = \frac{A}{2} e^{j\phi} \quad (9)$$

Properties

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Phasors

Given $x_1(t) = \tilde{X}_1 e^{j\omega t} + \overline{\tilde{X}_1 e^{j\omega t}}$, $x_2(t) = \tilde{X}_2 e^{j\omega t} + \overline{\tilde{X}_2 e^{j\omega t}}$ with phasors $\tilde{X}_{1,2}$,

- Uniqueness: $x_1(t) = x_2(t) \implies \tilde{X}_1 = \tilde{X}_2$
- Linearity: $a_1 x_1(t) + a_2 x_2(t) \implies a_1 \tilde{X}_1 + a_2 \tilde{X}_2$ for $a_{1,2} \in \mathbb{R}$
- Differentiation: $x(t) \iff \tilde{X} \implies \frac{d}{dt} x(t) = \frac{d}{dt} \left(\tilde{X} e^{j\omega t} + \overline{\tilde{X} e^{j\omega t}} \right) = j\omega \left(\tilde{X} e^{j\omega t} + \overline{\tilde{X} e^{j\omega t}} \right) \iff j\omega \tilde{X}$

Circuits & Phasors

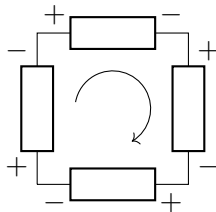
KVL

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Complex
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Phasors



$$\sum_i \tilde{V}_i = 0 \quad (10)$$

Circuits & Phasors (cont.)

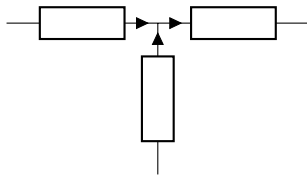
KCL

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Phasors



$$\sum \tilde{I}_{out} = \sum \tilde{I}_{in} \quad (11)$$

Circuits & Phasors (cont.)

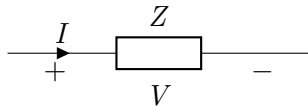
Ohm's Law

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Complex
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Phasors



$$\tilde{V} = \tilde{I} \underbrace{Z}_{\text{impedance}}$$

(12)

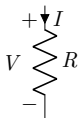
Passive Elements & Phasors

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Numbers

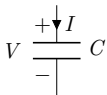
Phasors



$$\tilde{V} = \tilde{I}R \quad (13)$$



$$\tilde{V} = L \frac{d}{dt} \tilde{I} = j\omega L \tilde{I} \quad (14)$$



$$\tilde{I} = C \frac{d}{dt} \tilde{V} = j\omega C \tilde{V} \implies \tilde{V} = \frac{1}{j\omega C} \tilde{I} \quad (15)$$

Demo

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Phasors

<http://tinyurl.com/y5qfnqtk>



Low Pass Filter

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Phasors

$$\tilde{V}_{out} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} \tilde{V}_{in} \quad (16)$$

$$\tilde{V}_{out} = \frac{1}{1 + j\omega RC} \tilde{V}_{in} \quad (17)$$

$$\Rightarrow H(j\omega) = \frac{\tilde{V}_{out}}{\tilde{V}_{in}} = \frac{1}{1 + j\omega RC} \quad (18)$$