EECS 16B CSM

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Computer Science Mentors

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Complex Numbers

Definition

$$z = \underbrace{a + bj}_{\text{rectangular}} = \underbrace{re^{j\theta}}_{\text{polar}} \tag{1}$$

- $a, b, r, \theta \in \mathbb{R}$
- $j^2 = -1$
- we use j in EE

Coordinate Transforms

$$r^2 = a^2 + b^2 \tag{2}$$

$$\tan(\theta) = \frac{b}{a} \tag{3}$$

$$a = r\cos(\theta) \tag{4}$$

$$b = r\sin(\theta) \tag{5}$$

Euler's Formula

$$e^{j\theta} = \cos(\theta) + j\sin(\theta) \tag{6}$$

- relevant 3Blue1Brown
- $e^{j\pi} + 1 = 0$ is a special case

Phasors

Phasors

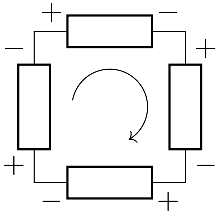
- Encodes information about any sinusoid: voltage, current, etc.
- If frequency is constant, then uniquely identifies

$$A\cos(\omega t + \phi) = \Re\{Ae^{j(\omega t + \phi)}\} = \Re\{\underbrace{Ae^{j\phi}}_{\text{phasor}} e^{j\omega t}\} \tag{7}$$

Properties

Given $x_1(t) = \Re\{A_1 e^{j\omega t}\}, x_2(t) = \Re\{A_2 e^{j\omega t}\}$ with phasors $A_{1,2}$,

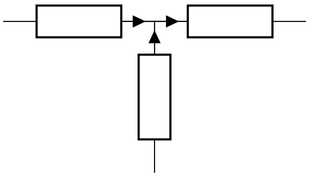
- Uniqueness: $x_1(t) = x_2(t) \implies A_1 = A_2$
- Linearity: $a_1 x_1(t) + a_2 x_2(t) \implies a_1 A_1 + a_2 A_2$ for $a_{1,2} \in \mathbb{R}$
- Differentiation: $x(t) \Leftrightarrow A \implies \frac{d}{dt} x(t) = \frac{d}{dt} \Re\{A e^{j\omega t}\} = \Re\{j\omega A e^{j\omega t}\} \Leftrightarrow j\omega A$



$$\sum_i \overline{V}_i = 0 \tag{8}$$

Circuits & Phasors (cont.)

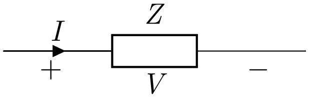
KCL



$$\sum \overline{I}_{out} = \sum \overline{I}_{in} \tag{9}$$

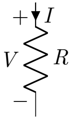
Circuits & Phasors (cont.)

Ohm's Law

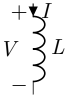


$$\overline{V} = \overline{I}\underbrace{Z}_{\text{impedance}} \tag{10}$$

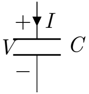
Passive Elements & Phasors



$$\overline{V} = \overline{I} R \tag{11}$$



$$\overline{V} = L \frac{d}{dt} \overline{I} = j\omega L \overline{I} \tag{12}$$



$$\overline{I} = C \frac{d}{dt} \overline{V} = j\omega C \overline{V} \implies \overline{V} = \frac{1}{j\omega C} \overline{I} \tag{13}$$

Demo

http://tinyurl.com/y5qfnqtk

Low Pass Filter

$$\overline{V}_{out} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} \overline{V}_{in} \tag{14}$$

$$\overline{V}_{out} = \frac{1}{1 + j\omega RC} \overline{V}_{in} \tag{15}$$

$$\Rightarrow H(j\omega) = \frac{\overline{V}_{out}}{\overline{V}_{in}} = \frac{1}{1 + j\omega RC} \tag{16}$$