EECS 16B CSM

Bryan Ngo

Complex Numbers

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Computer Science Mentors

2021-02-17



Logistics

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Comple: Number

- keep up the attendance!
- public Piazza: https://piazza.com/berkeley/spring2021/csm16b
- Phasors or discretization?

hasors

1 Complex Numbers

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Complex Numbers

Definition

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 $\mathsf{Phasors}$

$$z = \underbrace{a + bj}_{\text{rectangular}} = \underbrace{re^{j\theta}}_{\text{polar}} \tag{1}$$

- $a, b, r, \theta \in \mathbb{R}$
- $j^2 = -1$
- lacktriangle we use j in EE

Coordinate Transforms

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$$r^2 = a^2 + b^2 (2)$$

$$an(\theta) = \frac{b}{a} \tag{3}$$

$$a = \Re\{z\} = r\cos(\theta) \tag{4}$$

$$b = \Im\{z\} = r\sin(\theta) \tag{5}$$

Euler's Formula

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$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$
 (6)

- relevant 3Blue1Brown
- $e^{j\pi} + 1 = 0$ is a special case

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Complex Numbers

Phasors

Phasors

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Complex Numbers

- Encodes information about any sinusoid: voltage, current, etc.
- If frequency is constant, then uniquely identifies

$$x(t) = A\cos(\omega t + \phi) = \frac{A}{2} \left(e^{j(\omega t + \phi)} + e^{-j(\omega t + \phi)} \right)$$
 (7)

$$= \frac{A}{2} \left(e^{j\omega t} e^{j\phi} + \overline{e^{j\omega t} e^{j\phi}} \right) \tag{8}$$

$$\iff \widetilde{X} = \frac{A}{2}e^{j\phi} \tag{9}$$

Properties

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Given
$$x_1(t) = \widetilde{X}_1 e^{j\omega t} + \overline{\widetilde{X}_1 e^{j\omega t}}, x_2(t) = \widetilde{X}_2 e^{j\omega t} + \overline{\widetilde{X}_2 e^{j\omega t}}$$
 with phasors $\widetilde{X}_{1,2}$,

- Uniqueness: $x_1(t) = x_2(t) \implies \widetilde{X}_1 = \widetilde{X}_2$
- Linearity: $a_1x_1(t) + a_2x_2(t) \implies a_1\widetilde{X}_1 + a_2\widetilde{X}_2$ for $a_{1,2} \in \mathbb{R}$
- Differentiation: $x(t) \iff \widetilde{X} \implies \frac{d}{dt}x(t) = \frac{d}{dt}\left(\widetilde{X}e^{j\omega t} + \widetilde{X}e^{j\omega t}\right) =$ $j\omega\left(\widetilde{X}e^{j\omega t} + \overline{\widetilde{X}e^{j\omega t}}\right) \iff j\omega\widetilde{X}$

Circuits & Phasors KVL

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Comple:

Phasors

$$\sum_{i} \widetilde{V}_{i} = 0$$

(10)

Circuits & Phasors (cont.)

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$$\sum \widetilde{I}_{out} = \sum \widetilde{I}_{in} \tag{11}$$

Circuits & Phasors (cont.) Ohm's Law

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$$\widetilde{V} = \widetilde{I} \underbrace{Z}_{\text{impedance}}$$
(12)

Passive Elements & Phasors

CSM

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Phasors

 $\widetilde{I} = C \frac{d}{dt} \widetilde{V} = j\omega C \widetilde{V} \implies \widetilde{V} = \frac{1}{j\omega C} \widetilde{I}$

 $\widetilde{V} = \widetilde{I}R$

 $\widetilde{V} = L \frac{d}{dt} \widetilde{I} = j\omega L \widetilde{I}$

(13)

(14)

Demo

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Phasors

http://tinyurl.com/y5qfnqtk

Low Pass Filter

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$$\widetilde{V}_{out} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{i\omega C}} \widetilde{V}_{in} \tag{16}$$

$$\widetilde{V}_{out} = \frac{1}{1 + i\omega RC} \widetilde{V}_{in} \tag{17}$$

$$\widetilde{V}_{out} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} \widetilde{V}_{in} \tag{16}$$

$$\widetilde{V}_{out} = \frac{1}{1 + j\omega RC} \widetilde{V}_{in} \tag{17}$$

$$\Rightarrow H(j\omega) = \frac{\widetilde{V}_{out}}{\widetilde{V}_{in}} = \frac{1}{1 + j\omega RC} \tag{18}$$