

EECS 16B CSM

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2021-03-17

Logistics

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CSM

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Stability

Feedback

- Some new guests!
- How did MT go?
- Feedback: <https://forms.gle/8g1NcqqE4m1shkVx5>

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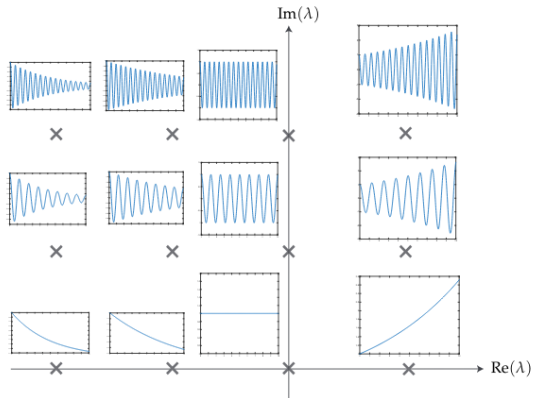
Continuous

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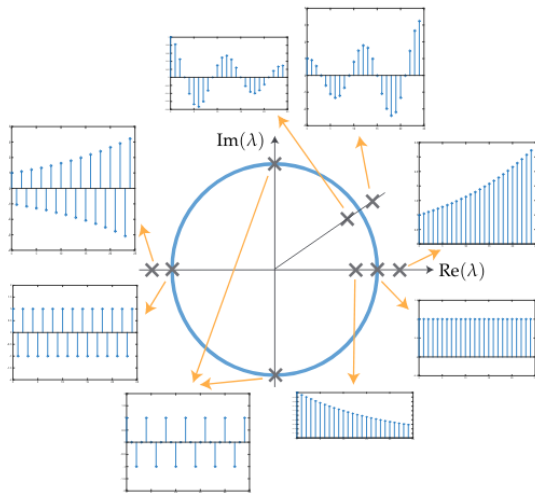
Discrete

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Controllability

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$$\mathbf{x}[t+1] = \mathbf{A}\mathbf{x}[t] + \mathbf{B}\mathbf{u}[t] \quad (1)$$

$$\mathbf{x}[1] = \mathbf{A}\mathbf{x}[0] + \mathbf{B}\mathbf{u}[0] \quad (2)$$

$$\mathbf{x}[2] = \mathbf{A}^2\mathbf{x}[0] + \mathbf{A}\mathbf{B}\mathbf{u}[0] + \mathbf{B}\mathbf{u}[1] \quad (3)$$

$$\mathbf{x}[t] = \mathbf{A}^t\mathbf{x}[0] + \sum_{i=0}^{t-1} \mathbf{A}^{t-i}\mathbf{B}\mathbf{u}[i] \quad (4)$$

$$\mathbf{x}[t] - \mathbf{A}^t\mathbf{x}[0] = \begin{bmatrix} \mathbf{B} & \mathbf{A}\mathbf{B} & \dots & \mathbf{A}^{t-1}\mathbf{B} \end{bmatrix} \begin{bmatrix} \mathbf{u}[t-1] \\ \mathbf{u}[t-2] \\ \vdots \\ \mathbf{u}[0] \end{bmatrix} \quad (5)$$

$$\Rightarrow \text{span} \left\{ \begin{bmatrix} \mathbf{B} & \mathbf{A}\mathbf{B} & \dots & \mathbf{A}^{t-1}\mathbf{B} \end{bmatrix} \right\} = \mathbb{R}^n \quad (6)$$

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Open-Loop

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$$\mathbf{x}[t + 1] = \mathbf{A}\mathbf{x}[t] + \mathbf{B}\mathbf{u}[t] \quad (7)$$

- define a certain range of use
- simpler
- no restraints apart from stability

Closed-Loop

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$$\mathbf{u}[t] = \begin{bmatrix} k_1 & k_2 & \cdots & k_n \end{bmatrix} \begin{bmatrix} x_1[t] \\ x_2[t] \\ \vdots \\ x_n[t] \end{bmatrix} \quad (8)$$

$$\mathbf{x}[t+1] = \mathbf{A}\mathbf{x}[t] + \mathbf{BK}\mathbf{x}[t] = (\mathbf{A} + \mathbf{BK})\mathbf{x}[t] \quad (9)$$

- adaptable to a wide range of use
- more complex
- self-correcting
- requires more constraints

Controller Canonical Form

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$$\mathbf{x}[t+1] = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ a_1 & a_2 & a_3 & \cdots & a_n \end{bmatrix} \mathbf{x}[t] + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} \mathbf{u}[t] \quad (10)$$

$$\det(\mathbf{A} + \mathbf{BK} - \lambda \mathbf{I}) = \lambda^n - \sum_{i=1}^n (a_i + k_i) \lambda^i \quad (11)$$