EECS 16B CSM

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2021-03-17

Logistics

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Stability

- Some new guests!
- How did MT go?
- Feedback: https://forms.gle/8g1NcqqE4m1shkVx5

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Feedback

Stability

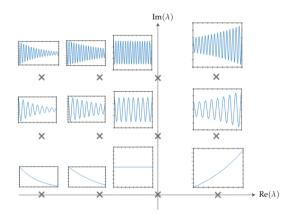
Continuous

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Feedbac



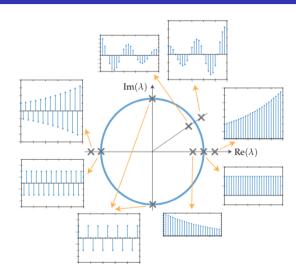
Discrete

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Feedback

Feedback

Open-Loop

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Feedback

$$x[t+1] = Ax[t] + Bu[t]$$
(1)

- define a certain range of use
- simpler
- no restraints apart from stability

Controllability

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Feedback

$$x[t+1] = Ax[t] + Bu[t]$$
(2)

$$x[1] = Ax[0] + Bu[0] \tag{3}$$

$$x[2] = A^2x[0] + ABu[0] + Bu[1]$$
 (4)

$$\boldsymbol{x}[t] = \boldsymbol{A}^{t}\boldsymbol{x}[0] + \sum_{i=0}^{t-1} \boldsymbol{A}^{t-i}\boldsymbol{B}\boldsymbol{u}[i]$$
 (5)

$$oldsymbol{x}[t] - oldsymbol{A}^t oldsymbol{x}[0] = egin{bmatrix} oldsymbol{B} & oldsymbol{A} oldsymbol{B} & oldsymbol{A} oldsymbol{B} & oldsymbol{A}^{t-1} oldsymbol{B} \end{bmatrix} egin{bmatrix} oldsymbol{u}[t-1] \ oldsymbol{u}[t-2] \ dots \ oldsymbol{u}[0] \end{bmatrix} \end{cases}$$
 (6)

$$\Rightarrow \operatorname{span}\left\{ \begin{bmatrix} \boldsymbol{B} & \boldsymbol{A}\boldsymbol{B} & \cdots & \boldsymbol{A}^{t-1}\boldsymbol{B} \end{bmatrix} \right\} = \mathbb{R}^{n}$$
 (7)

Closed-Loop

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Feedback
$$m{u}[t] = egin{bmatrix} k_1 & k_2 & \cdots & k_n \end{bmatrix} egin{bmatrix} x_1[t] \\ x_2[t] \\ \vdots \\ x_n[t] \end{bmatrix}$$

$$x[t+1] = Ax[t] + BKx[t] = (A+BK)x[t]$$
(9)

- adaptable to a wide range of use
- more complex
- self-correcting
- requires more constraints



Controller Canonical Form

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$$\boldsymbol{x}[t+1] = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ a_1 & a_2 & a_3 & \cdots & a_n \end{bmatrix} \boldsymbol{x}[t] + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} \boldsymbol{u}[t]$$
(10)

$$\det(\mathbf{A} + \mathbf{B}\mathbf{K} - \lambda \mathbf{I}) = \lambda^n - \sum_{i=1}^n (a_i + k_i)\lambda^i$$
(11)