EECS 16B CSM

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2020-10-19

Midterm

- Good luck!
- Review vs. Worksheet

State Space Form

$$\frac{d}{dt}\mathbf{x}(t) = f(\mathbf{x}(t), \mathbf{u}(t)) \tag{1}$$

Linearization

$$J_{x} = \begin{bmatrix} \partial_{x_{1}} f_{1} & \partial_{x_{2}} f_{1} & \cdots & \partial_{x_{n}} f_{1} \\ \partial_{x_{1}} f_{2} & \partial_{x_{2}} f_{2} & \cdots & \partial_{x_{n}} f_{2} \\ \vdots & \vdots & \ddots & \vdots \\ \partial_{x_{1}} f_{n} & \partial_{x_{2}} f_{n} & \cdots & \partial_{x_{n}} f_{n} \end{bmatrix}$$

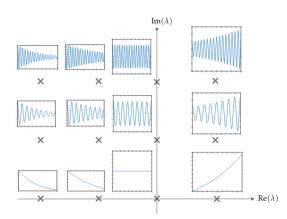
$$J_{u} = \begin{bmatrix} \partial_{u_{1}} f_{1} & \partial_{u_{2}} f_{1} & \cdots & \partial_{u_{n}} f_{1} \\ \partial_{u_{1}} f_{2} & \partial_{u_{2}} f_{2} & \cdots & \partial_{u_{n}} f_{2} \\ \vdots & \vdots & \ddots & \vdots \\ \partial_{u_{1}} f_{n} & \partial_{u_{2}} f_{n} & \cdots & \partial_{u_{n}} f_{n} \end{bmatrix}$$

$$(2)$$

$$\frac{d}{dt}\boldsymbol{x}(t) \approx \boldsymbol{J}_{\boldsymbol{x}}(\boldsymbol{x}(t) - \boldsymbol{x}^*) + \boldsymbol{J}_{\boldsymbol{u}}(\boldsymbol{u}(t) - \boldsymbol{u}^*) \tag{4}$$

Stability

Continuous

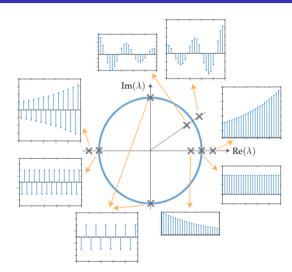


Stability

Discrete

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Controllability

$$x[t+1] = Ax[t] + Bu[t]$$

$$x[1] = Ax[0] + Bu[0]$$
(5)

$$\mathbf{a}[\mathbf{a}] = \mathbf{A}^{2} \mathbf{a}[\mathbf{a}] + \mathbf{A} \mathbf{B} \mathbf{a}[\mathbf{a}] + \mathbf{B} \mathbf{a}[\mathbf{a}]$$
 (7

$$x[2] = A^2x[0] + ABu[0] + Bu[1]$$
 (7)

$$\boldsymbol{x}[t] = \boldsymbol{A}^{t}\boldsymbol{x}[0] + \sum_{i=0}^{t-1} \boldsymbol{A}^{t-i}\boldsymbol{B}\boldsymbol{u}[i]$$
 (8)

$$oldsymbol{x}[t] - oldsymbol{A}^t oldsymbol{x}[0] = egin{bmatrix} oldsymbol{B} & oldsymbol{A} oldsymbol{B} & oldsymbol{A} oldsymbol{B} & oldsymbol{A}^{t-1} oldsymbol{B} \end{bmatrix} egin{bmatrix} oldsymbol{u}[t-1] \ oldsymbol{u}[t-2] \ dots \ oldsymbol{u}[0] \end{bmatrix} \end{cases}$$
 (9)

$$\Rightarrow \operatorname{span}\left\{\left[\boldsymbol{B} \quad \boldsymbol{A}\boldsymbol{B} \quad \cdots \quad \boldsymbol{A}^{t-1}\boldsymbol{B}\right]\right\} = \mathbb{R}^{n} \tag{10}$$