Bryan Ngo

Phasors

Filters

Bode Plots

# EECS 16B CSM

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UC Berkeley

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# Announcements

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- Pertinent facts
- Review session next Tuesday

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# Phasors

## **Phasors**

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### Phasors

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- Encodes information about any sinusoid: voltage, current, etc.
- If frequency is constant, then uniquely identifies

$$x(t) = A\cos(\omega t + \phi) = \frac{A}{2} \left( e^{j(\omega t + \phi)} + e^{-j(\omega t + \phi)} \right)$$
 (1)

$$= \frac{A}{2} \left( e^{j\omega t} e^{j\phi} + \overline{e^{j\omega t} e^{j\phi}} \right) \tag{2}$$

$$\iff \widetilde{X} = \frac{A}{2}e^{j\phi} \tag{3}$$

# **Properties**

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Given 
$$x_1(t) = \widetilde{X}_1 e^{j\omega t} + \overline{\widetilde{X}_1 e^{j\omega t}}, x_2(t) = \widetilde{X}_2 e^{j\omega t} + \overline{\widetilde{X}_2 e^{j\omega t}}$$
 with phasors  $\widetilde{X}_{1,2}$ ,

- Uniqueness:  $x_1(t) = x_2(t) \implies \widetilde{X}_1 = \widetilde{X}_2$
- Linearity:  $a_1x_1(t) + a_2x_2(t) \implies a_1\widetilde{X}_1 + a_2\widetilde{X}_2$  for  $a_{1,2} \in \mathbb{R}$
- Differentiation:  $x(t) \iff \widetilde{X} \implies \frac{d}{dt}x(t) = \frac{d}{dt}\left(\widetilde{X}e^{j\omega t} + \overline{\widetilde{X}e^{j\omega t}}\right) = j\omega\left(\widetilde{X}e^{j\omega t} + \overline{\widetilde{X}e^{j\omega t}}\right) \iff j\omega\widetilde{X}$

# Circuits & Phasors KVL

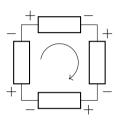
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$$\sum_{i} \widetilde{V}_{i} = 0$$

(4)

# Circuits & Phasors (cont.)

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$$\sum \widetilde{I}_{out} = \sum \widetilde{I}_{in}$$
 (5)

# Circuits & Phasors (cont.) Ohm's Law

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$$I$$
 $V$ 
 $V$ 

$$\widetilde{V} = \widetilde{I} \underbrace{Z}_{\text{impedance}}$$

(6)

# Passive Elements & Phasors

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$$V \stackrel{+ \star J}{\underset{V}{\bigvee}} L$$

$$\widetilde{V} = L \frac{d}{dt} \widetilde{I} = j\omega L \widetilde{I}$$

$$\widetilde{V} = L \frac{d}{dt} \widetilde{I}$$

$$\frac{d}{dt}\widetilde{I} = j\omega L\widetilde{I}$$

 $\widetilde{V} = \widetilde{I}R$ 

$$\frac{+ \sqrt{I}}{I} C$$

$$\widetilde{I} = C \frac{d}{dt} \widetilde{V} = j\omega C \widetilde{V} \implies \widetilde{V} = \frac{1}{i\omega C} \widetilde{I}$$

(8)

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# Filters

# Why?

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- allows us to isolate desired frequency ranges
- color organ: basically just a spectrogram
- Afrotechmods video

# Types

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lacktriangle low-pass: let in low  $\omega$ 

 $\blacksquare$  high-pass: let in high  $\omega$ 

 $\blacksquare$  band-pass: let in range of  $\omega$ 

 $\blacksquare$  band-stop: block out range of  $\omega$ 

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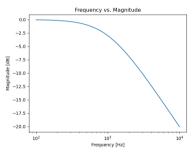
# Definition

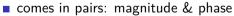
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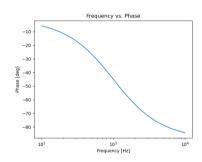
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■ above: low-pass filter



### **Features**

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### magnitude

- *x*-axis: log frequency (Hz)
- y-axis:  $|H(j\omega)|$  (dB or intensity)
- cutoff frequency:  $|H(j\omega)| = \frac{1}{\sqrt{2}} = -3 \, dB$

## phase

- *x*-axis: log frequency (Hz)
- y-axis: phase offset ( $^{\circ}$  or rad)

# Why?

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- allows us to characterize a filter very fast
- quick visual tool

## Resonance

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- whenever you see an inductor & a capacitor
- energy is oscillating back and forth
- when voltage is at resonant frequency  $\frac{1}{\sqrt{LC}}$ , inductor and capacitor act as short