EECS 16B CSM

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State Space Form

$$\frac{d}{dt}\boldsymbol{x}(t) = f(\boldsymbol{x}(t), \boldsymbol{u}(t)) \tag{1}$$

But what if $f(\boldsymbol{x}(t), \boldsymbol{u}(t))$ isn't linear?

Linearity

Given a system $y(t) = f(x(t))$,

$$ax(t) \iff ay(t) \tag{2}$$

$$x_1(t) + x_2(t) \iff y_1(t) + y_2(t) \tag{3}$$

$$ax_1(t) + bx_2(t) \iff ay_1(t) + by_2(t) \tag{4}$$

Taylor Approximations

Calculus review!

$$f(x) = \sum_{n \geqslant 0} \frac{f^{(n)}(x_0)}{n!}(x - x_0)^n = \frac{f(x_0)}{0!} + \frac{f'(x_0)}{1!}(x - x_0) + \frac{f''(x_0)}{2!}(x - x_0)^2 + \cdots \tag{5}$$

$$\approx f(x_0) + f'(x_0)(x - x_0) \tag{6}$$

Linearization

$$\boldsymbol{J_x} = \begin{bmatrix} \partial_{x_1} f_1 & \partial_{x_2} f_1 & \cdots & \partial_{x_n} f_1 \\ \partial_{x_1} f_2 & \partial_{x_2} f_2 & \cdots & \partial_{x_n} f_2 \\ \vdots & \vdots & \ddots & \vdots \\ \partial_{x_1} f_n & \partial_{x_2} f_n & \cdots & \partial_{x_n} f_n \end{bmatrix} \tag{7}$$

$$\boldsymbol{J_u} = \begin{bmatrix} \partial_{u_1} f_1 & \partial_{u_2} f_1 & \cdots & \partial_{u_n} f_1 \\ \partial_{u_1} f_2 & \partial_{u_2} f_2 & \cdots & \partial_{u_n} f_2 \\ \vdots & \vdots & \ddots & \vdots \\ \partial_{u_1} f_n & \partial_{u_2} f_n & \cdots & \partial_{u_n} f_n \end{bmatrix} \tag{8}$$

$$\frac{d}{dt}\boldsymbol{x}(t) \approx \boldsymbol{J_x}(\boldsymbol{x}(t) - \boldsymbol{x}^*) + \boldsymbol{J_u}(\boldsymbol{u}(t) - \boldsymbol{u}^*) \tag{9}$$