EECS 16B CSM

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2021-10-25

Logistics

- CSM feedback form hopefully filled out

Controllability

$$\boldsymbol{x}[t+1] = \boldsymbol{A}\boldsymbol{x}[t] + \boldsymbol{B}\boldsymbol{u}[t] \tag{1}$$

$$\boldsymbol{x}[1] = \boldsymbol{A}\boldsymbol{x}[0] + \boldsymbol{B}\boldsymbol{u}[0] \tag{2}$$

$$\boldsymbol{x}[2] = \boldsymbol{A}^2\boldsymbol{x}[0] + \boldsymbol{A}\boldsymbol{B}\boldsymbol{u}[0] + \boldsymbol{B}\boldsymbol{u}[1] \tag{3}$$

$$\boldsymbol{x}[t] = \boldsymbol{A}^t\boldsymbol{x}[0] + \sum_{i=0}^{t-1} \boldsymbol{A}^{t-i}\boldsymbol{B}\boldsymbol{u}[i] \tag{4}$$

$$\boldsymbol{x}[t] - \boldsymbol{A}^t\boldsymbol{x}[0] = \begin{bmatrix} \boldsymbol{B} & \boldsymbol{A}\boldsymbol{B} & \cdots & \boldsymbol{A}^{t-1}\boldsymbol{B} \end{bmatrix} \begin{bmatrix} \boldsymbol{u}[t-1] \\ \boldsymbol{u}[t-2] \\ \vdots \\ \boldsymbol{u}[0] \end{bmatrix} \tag{5}$$

$$\Rightarrow \text{span}\left\{ \begin{bmatrix} \boldsymbol{B} & \boldsymbol{A}\boldsymbol{B} & \cdots & \boldsymbol{A}^{t-1}\boldsymbol{B} \end{bmatrix} \right\} = \mathbb{R}^n \tag{6}$$

Gram-Schmidt

$$\boldsymbol{p}_i = \boldsymbol{v}_i - \sum_{j \neq i} ([\boldsymbol{v}_i]^\top \boldsymbol{w}_j)\boldsymbol{w}_j \tag{7}$$

$$\boldsymbol{w}_i = \frac{\boldsymbol{p}_i}{\|\boldsymbol{p}_i\|} \tag{8}$$

- Turn basis vector set V into orthonormal basis W
- Steps
  1. Subtract all $\text{proj}_{\boldsymbol{v}_i}(\boldsymbol{v}_n)$ for $i \neq n$
  2. Normalize result
  3. repeat for all vectors
- systematically removing the parallel component of every other vector
- cool GIF of GS