

EECS 16B CSM

Bryan Ngo

Computer Science Mentors

2020-09-21

- 1 Differential Equations
- 2 RC Circuits
- 3 Vector Differential Equations
- 4 Change of Basis
- 5 Inductors (Preview)

Logistics

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Differential
Equations

RC Circuits

Vector
Differential
Equations

Change of
Basis

Inductors
(Preview)

- Quest?
- Focus for today?
- messenger chat
- all slides available at <https://github.com/bdngo/16b-csm>

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Differential
Equations

RC Circuits

Vector
Differential
Equations

Change of
Basis

Inductors
(Preview)

Differential Equations

Differential Equations

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Differential
Equations

RC Circuits

Vector
Differential
Equations

Change of
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Inductors
(Preview)

Concept check!

Differential Equations

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(Preview)

Concept check!

$$\frac{d}{dt}x(t) = f(x, t) \quad (1)$$

- Focusing on first-order ODEs
- Relates the derivative in other terms
- 3Blue1Brown video

Exponential Differential Equation

Homogeneous

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$$\frac{d}{dt}x(t) = \lambda x(t) \implies x(t) = x_0 e^{\lambda t} \quad (2)$$

Exponential Differential Equation

Non-Homogeneous

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$$\frac{d}{dt}x(t) = \alpha x(t) + \beta \quad (3)$$

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RC Circuits

Undamped Response

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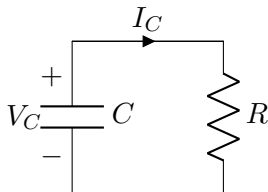
Differential
Equations

RC Circuits

Vector
Differential
Equations

Change of
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Inductors
(Preview)



Undamped Response

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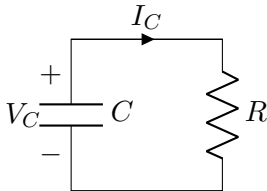
Differential
Equations

RC Circuits

Vector
Differential
Equations

Change of
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$$C \frac{d}{dt} V_C = -\frac{V_C}{R} \quad (4)$$

$$\frac{d}{dt} V_C = -\underbrace{\frac{1}{RC}}_{\lambda} V_C \quad (5)$$

$$\Rightarrow V_C(t) = V_0 e^{-\frac{1}{RC}t} = V_0 e^{-\frac{1}{\tau}t} \quad (6)$$

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Differential
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Vector
Differential
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(Preview)

Vector Differential Equations

General Form

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Differential
Equations

RC Circuits

Vector
Differential
Equations

Change of
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Inductors
(Preview)

$$\frac{d}{dt}\mathbf{x}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \quad (7)$$

- if \mathbf{A} is diagonal, simply a bunch of exponential differential Equations
- if not, we can try to diagonalize

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Differential
Equations

RC Circuits

Vector
Differential
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Change of
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Inductors
(Preview)

Change of Basis

Motivation

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Differential
Equations

RC Circuits

Vector
Differential
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Change of
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Inductors
(Preview)

- conversion from one linear coordinate system to another
- Yet another 3Blue1Brown video

A Visualization

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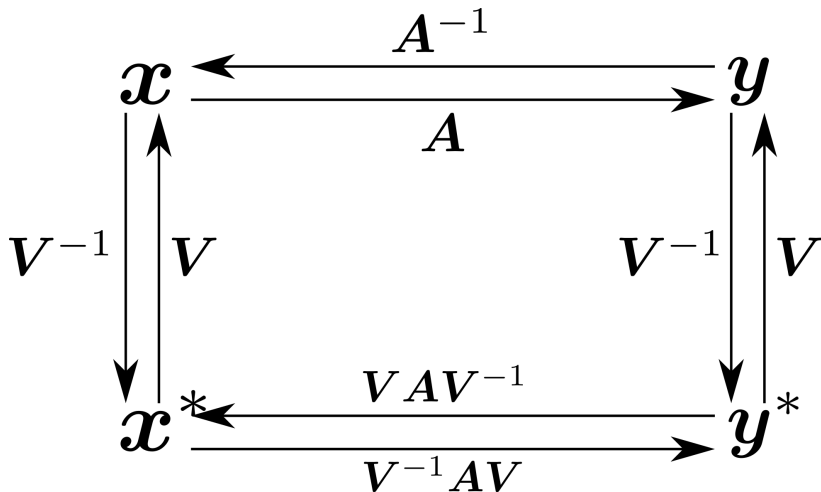
Differential
Equations

RC Circuits

Vector
Differential
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Change of
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Inductors
(Preview)



Diagonalization

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**Change of
Basis**

Inductors
(Preview)

- want the eigenvectors to be the basis for a vector space
- makes math way easier

Diagonalization

- want the eigenvectors to be the basis for a vector space
- makes math way easier

$$\mathbf{V} = [\mathbf{v}_1 \quad \mathbf{v}_2 \quad \cdots \quad \mathbf{v}_n] \quad (8)$$

$$\mathbf{A}\mathbf{V} = [\lambda_1\mathbf{v}_1 \quad \lambda_2\mathbf{v}_2 \quad \cdots \quad \lambda_n\mathbf{v}_n] \quad (9)$$

$$= [\mathbf{v}_1 \quad \mathbf{v}_2 \quad \cdots \quad \mathbf{v}_n] \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_n \end{bmatrix} \quad (10)$$

$$= \mathbf{V}\mathbf{\Lambda} \implies \mathbf{\Lambda} = \mathbf{V}^{-1}\mathbf{A}\mathbf{V} \quad (11)$$

Diagonalizing DEs

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Change of
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$$\frac{d}{dt}\mathbf{x}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \quad (12)$$

$$\frac{d}{dt}\mathbf{V}\mathbf{z}(t) = \mathbf{A}\mathbf{V}\mathbf{z}(t) + \mathbf{B}\mathbf{u}(t) \quad (13)$$

$$\Rightarrow \frac{d}{dt}\mathbf{z}(t) = \mathbf{V}^{-1}\mathbf{A}\mathbf{V}\mathbf{z}(t) + \mathbf{V}^{-1}\mathbf{B}\mathbf{u}(t) \quad (14)$$

$$= \mathbf{\Lambda}\mathbf{z}(t) + \mathbf{V}^{-1}\mathbf{B}\mathbf{u}(t) \quad (15)$$

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Inductors (Preview)

Basic Properties

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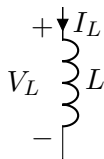
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$$V_L = L \frac{d}{dt} I_L \quad (16)$$

- like a capacitor but for magnetic fields
- resists instantaneous change in current

Basic Properties

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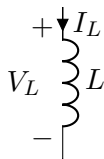
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$$V_L = L \frac{d}{dt} I_L \quad (16)$$

- like a capacitor but for magnetic fields
- resists instantaneous change in current
- what happens when $\omega = 0$? $\omega = \infty$?