

DATA 100 HW 01

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1 Calculus

$$\sigma(x) = \frac{1}{1 + e^{-x}} \quad (1)$$

1.1

Theorem 1. *Given the function $\sigma(x)$, $\sigma(-x) = 1 - \sigma(x)$.*

Proof.

$$\sigma(-x) = \frac{1}{1 + e^x} \quad (2)$$

$$= \frac{1 + e^x - e^x}{1 + e^x} \quad (3)$$

$$= 1 - \frac{e^x}{1 + e^x} \frac{e^{-x}}{e^{-x}} \quad (4)$$

$$= 1 - \frac{1}{1 + e^{-x}} = 1 - \sigma(x) \quad (5)$$

□

1.2

Theorem 2. *Given the function $\sigma(x)$, $\frac{d}{dx}\sigma(x) = \sigma(x)(1 - \sigma(x))$.*

Proof.

$$\frac{d}{dx}\sigma(x) = \frac{d}{dx}(1 + e^{-x})^{-1} = e^{-x}(1 + e^{-x})^{-2} \quad (6)$$

$$= \frac{1}{1 + e^{-x}} \frac{e^{-x}}{1 + e^{-x}} \frac{e^x}{e^x} \quad (7)$$

$$= \sigma(x) \frac{1}{1 + e^x} = \sigma(x)(1 - \sigma(x)) \quad (8)$$

□

2 Minimization

Finding the zeroes of $f(c)$,

$$f(c) = \frac{1}{n} \sum_{i=1}^n (x_i - c)^2 \quad (9)$$

$$f'(c) = -\frac{2}{n} \sum_{i=1}^n (x_i - c) = 0 \quad (10)$$

$$\sum_{i=1}^n (x_i - c) = \sum_{i=1}^n x_i - nc = 0 \implies c = \frac{1}{n} \sum_{i=1}^n x_i \quad (11)$$

Then, by the second derivative test,

$$f''(c) = -\frac{2}{n} \sum_{i=1}^n -1 = 2 \quad (12)$$

implying a concave up function, meaning our zero is in fact a minimum.

3 Probability & Statistics

3.1

The percent of surveyed US adults who had a great deal of confidence in both scientists and religious leaders is impossible to find with information, since we only know what people trusted one at a time. To know who trusts both, we would need data for every individual and who they trusted a great deal, which is not given in the bar graph.

3.2

$$\Pr(C) = 0.01 \implies \Pr(C^c) = 0.99 \quad (13)$$

$$\Pr(T | C) = 0.8 \quad (14)$$

$$\Pr(T | C^c) = 0.096 \quad (15)$$

$$\Pr(C | T) = \frac{\Pr(C) \Pr(T | C)}{\Pr(C) \Pr(T | C) + \Pr(C^c) \Pr(T | C^c)} \quad (16)$$

$$= \frac{0.01 \cdot 0.8}{0.01 \cdot 0.8 + 0.99 \cdot 0.096} \approx 0.078 \quad (17)$$

3.3

The standard deviation is $\sigma = 6.1$ since the inflection point of the histogram is at $\pm\sigma$, and the inflection point is around 6.1.

4 Linear Algebra

4.1

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \implies \text{rank}(A) = 2 \quad (18)$$

The matrix is full rank.

4.2

$$\mathbf{B} = \begin{bmatrix} 3 & 0 \\ -4 & 0 \end{bmatrix} \implies \text{rank}(\mathbf{B}) = 1 \quad (19)$$

The matrix is not full rank, since $\mathbf{v}_1 = 0\mathbf{v}_2$.

4.3

$$\mathbf{C} = \begin{bmatrix} 0 & 5 & 10 \\ 1 & 0 & 10 \end{bmatrix} \implies \text{rank}(\mathbf{C}) = 2 \quad (20)$$

The matrix is not full rank, since $\mathbf{v}_3 = 2\mathbf{v}_1 + 10\mathbf{v}_2$.

4.4

$$\mathbf{D} = \begin{bmatrix} 0 & -2 & 2 \\ 2 & -2 & 4 \\ 3 & 5 & -2 \end{bmatrix} \implies \text{rank}(\mathbf{D}) = 2 \quad (21)$$

The matrix is not full rank, since $\mathbf{v}_3 = \mathbf{v}_1 - \mathbf{v}_2$.