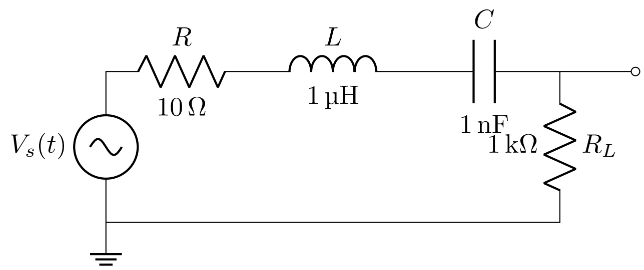
EE 105 HW 03

1

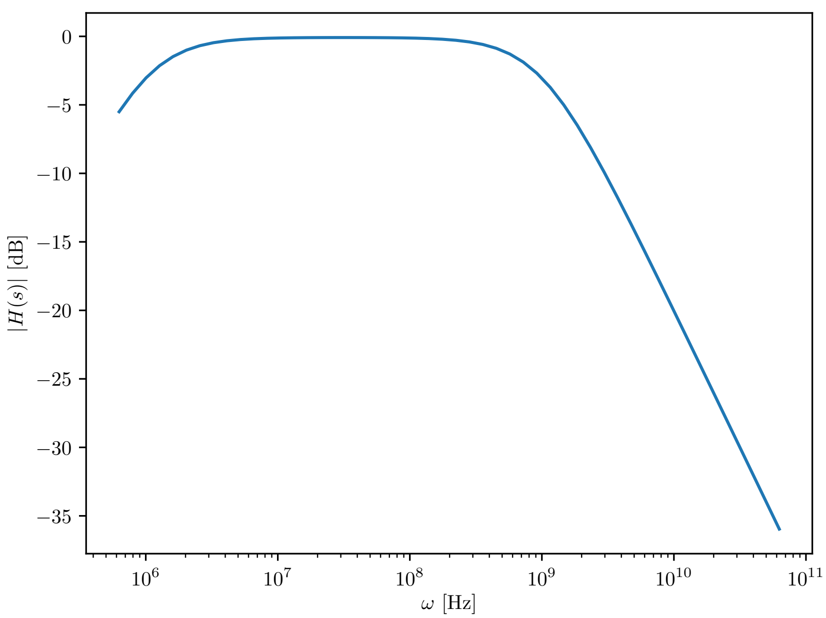


The transfer function is

$$H(s) = \frac{R_L}{R_L + R + sL + \frac{1}{sC}} = \frac{sR_LC}{1 + sC(R_L + R) + s^2LC} = \frac{s\frac{R_L}{L}}{\frac{1}{LC} + s\frac{R_L+R}{L} + s^2} = K\frac{s\frac{\omega_0}{Q'}}{\omega_0^2 + s\frac{\omega_0}{Q'} + s^2} \tag{1}$$

where $\omega_0 = \frac{1}{\sqrt{LC}}$, $Q' = \frac{1}{R_L+R}\sqrt{\frac{L}{C}}$, and $K = \frac{R_L}{R_L+R}$. At $s = j\omega_0$, we have

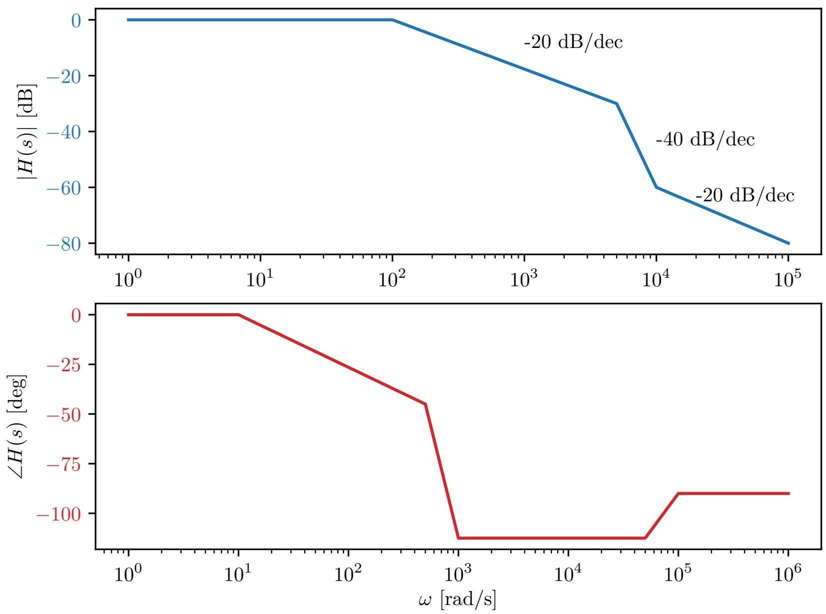
$$H(j\omega_0) = \frac{R_L}{R_L + R}\frac{\cancel{j\omega_0\frac{\omega_0}{Q'}}}{\cancel{\omega_0^2} + \cancel{j\omega_0\frac{\omega_0}{Q'}} - \cancel{\omega_0^2}} = \frac{R_L}{R_L + R} \tag{2}$$



This is a variant of a band-pass filter.

2

$$H(s) = 50\frac{s + 10000}{(s + 100)(s + 5000)} \tag{3}$$



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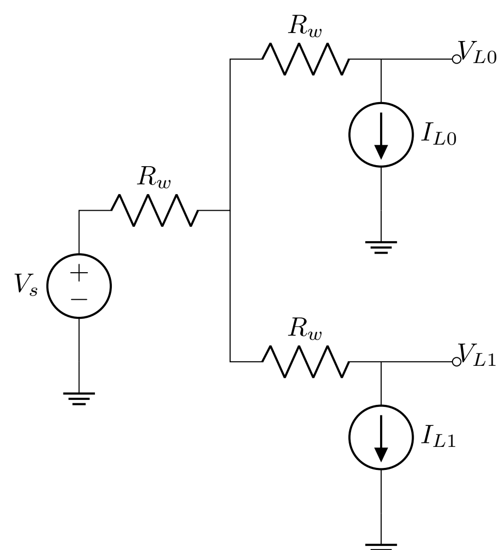
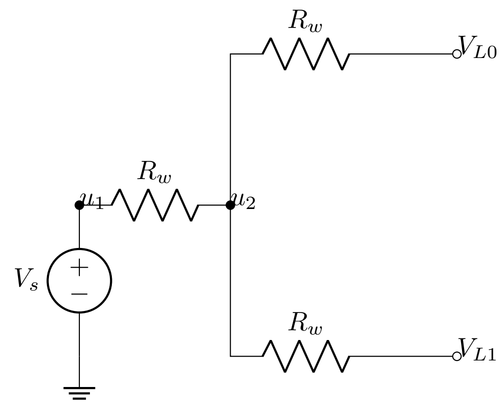
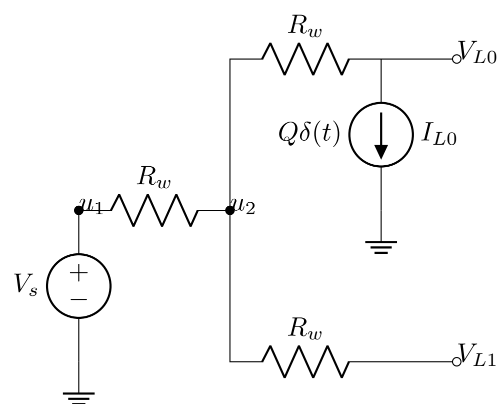


(a) With $I_{L0} = I_{L1} = 0\,\mathrm{A}$,



Since we end up with an open circuit, we have $V_{L0} = V_{L1} = V_s$.

(b) With $I_{L0} = Q\delta(t)$, $I_{L1} = 0\,\mathrm{A}$,



Performing node voltage analysis, we get the equations

$$\frac{V_s - u_2}{R_w} = \frac{u_2 - V_{L0}}{R_w} \tag{4}$$

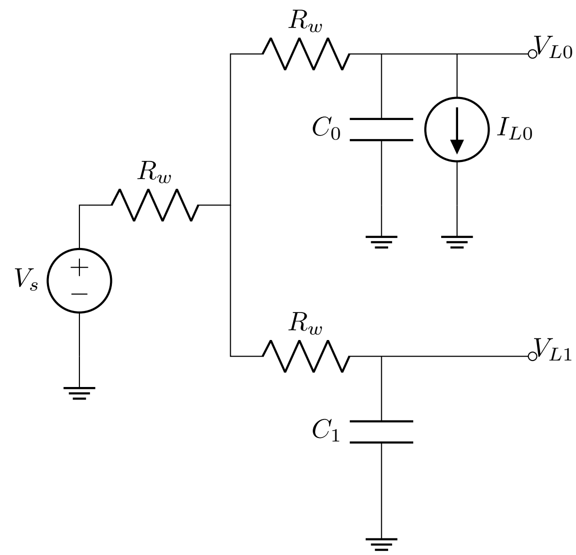
$$\frac{u_2 - V_{L0}}{R_w} = Q\delta(t) \tag{5}$$

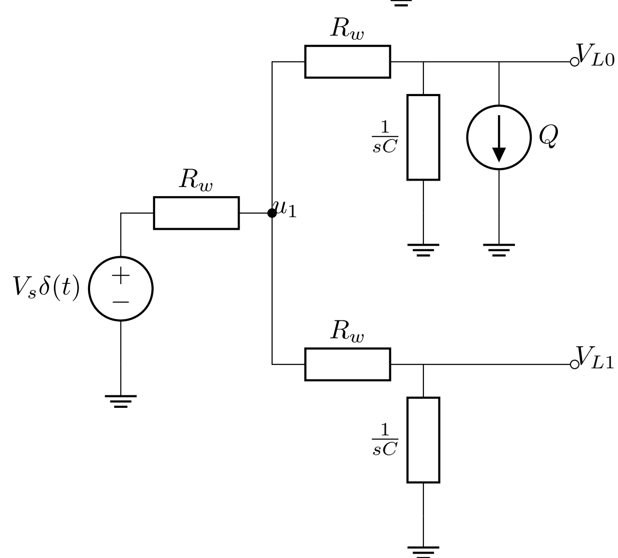
$$V_{L1} = u_2 \tag{6}$$

$$\implies V_{L1} = V_s - R_w Q\delta(t) \tag{7}$$

$$\implies V_{L0} = V_s - 2R_w Q\delta(t) \tag{8}$$

(c) The circuit is now





We can null V_s and add it in later by superposition. Performing node voltage analysis gives us the equations

$$\frac{u_1}{\cancel{R_w}} + \frac{u_1 - V_{L1}}{\cancel{R_w}} + \frac{u_1 - V_{L0}}{\cancel{R_W}} = 0 \tag{9}$$

$$\frac{u_1 - V_{L1}}{R_w} = \frac{V_{L1}}{\frac{1}{sC}} \tag{10}$$

$$\frac{u_1 - V_{L0}}{R_w} = \frac{V_{L0}}{\frac{1}{sC}} + Q \tag{11}$$

$$\implies 3u_1 - V_{L1} - V_{L0} = 0 \tag{12}$$

$$u_1 = (1 + sR_wC)V_{L1} \implies V_{L1} = \frac{u_1}{1 + sR_wC} \tag{13}$$

$$u_1 = (1 + sR_wC)V_{L0} + R_wQ \implies V_{L0} = \frac{u_1 - R_wQ}{1 + sR_wC} \tag{14}$$

$$3u_1 - \frac{u_1 - R_wQ}{1 + sR_wC} - \frac{u_1 - R_wQ}{1 + sR_wC} = 0 \tag{15}$$

$$\implies u_1 \left(3 - \frac{2}{1 + sR_wC}\right) = -\frac{R_wQ}{1 + sR_wC} \tag{16}$$

$$u_1 \left(\frac{1 + s3R_wC}{\cancel{1 + sR_wC}}\right) = -\frac{R_wQ}{\cancel{1 + sR_wC}} \tag{17}$$

$$\implies u_1 = -\frac{R_wQ}{1 + s3R_wC} \tag{18}$$

Finding V_{L1},

$$V_{L1} = \frac{-R_wQ}{(1 + s3R_wC)(1 + sR_wC)} = \frac{A}{1 + s3R_wC} + \frac{B}{1 + sR_wC} \tag{19}$$

$$\implies -R_wQ = A(1 + sR_wC) + B(1 + s3R_wC) \tag{20}$$

$$s = -\frac{1}{R_wC} \implies B = \frac{1}{2}R_wQ \tag{21}$$

$$s = -\frac{1}{3R_wC} \implies A = -\frac{3}{2}R_wQ \tag{22}$$

$$\implies V_{L1} = R_wQ\left(-\frac{3}{2}\frac{1}{1 + s3R_wC} + \frac{1}{2}\frac{1}{1 + sR_wC}\right) \tag{23}$$

$$\xRightarrow{\mathcal{L}^{-1}} V_{L1}(t) = V_s + \frac{Q}{2C}(-e^{-\frac{t}{3R_wC}} + e^{-\frac{t}{R_wC}})u(t) \tag{24}$$

Finding V_{L0},

$$V_{L0} = V_{L1} - \frac{R_wQ}{1 + sR_wC} \tag{25}$$

$$\implies V_{L0}(t) = V_s - \frac{Q}{2C}(e^{-\frac{t}{3R_wC}} + e^{-\frac{t}{R_wC}})u(t) \tag{26}$$