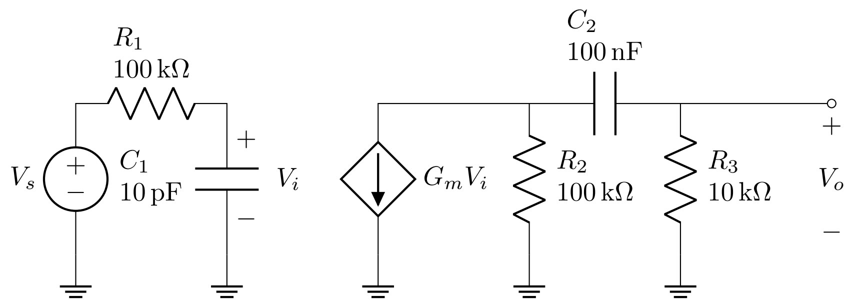
EE 105 HW 04

1



By inspection, we can see that $T_i(s)$ is a simple RC low-pass filter, so we have

$$T_i(s) = \frac{1}{1 + sR_1C_1} \tag{1}$$

Performing node voltage analysis on the second circuit in the s-domain (assuming the current through C_2 is going left to right), we have

$$G_m V_i + \frac{u_1}{R_2} + \frac{u_1 - V_o}{\frac{1}{sC_2}} = 0 \tag{2}$$

$$\frac{u_1 - V_o}{\frac{1}{sC_2}} = \frac{V_o}{R_3} \tag{3}$$

$$\implies u_1 = V_o\left(1 + \frac{1}{sR_3C_2}\right) \tag{4}$$

Substituting this expression back into Equation 2,

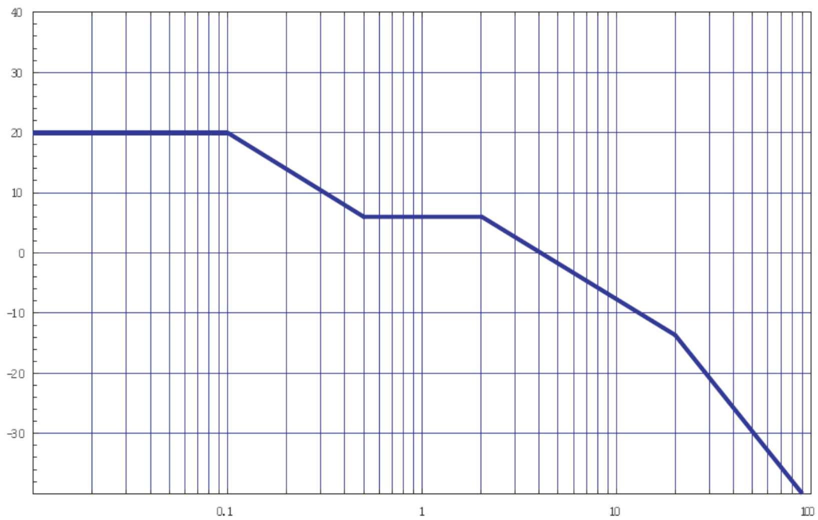
$$G_m V_i + \frac{V_o}{R_2}\left(1 + \frac{1}{sR_3C_2}\right) + sC_2\left(\cancel{V_o} + \frac{V_o}{sR_3C_2} - \cancel{V_o}\right) = 0 \tag{5}$$

$$G_m V_i + V_o\left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{sR_2R_3C_2}\right) = 0 \tag{6}$$

$$\implies T_o(s) = \frac{V_o}{V_i} = -\frac{G_m}{\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{sR_2R_3C_2}} \tag{7}$$

$$\implies T(s) = T_i(s)T_o(s) = -\frac{G_m}{(1 + sR_1C_1)\left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{sR_2R_3C_2}\right)} \tag{8}$$

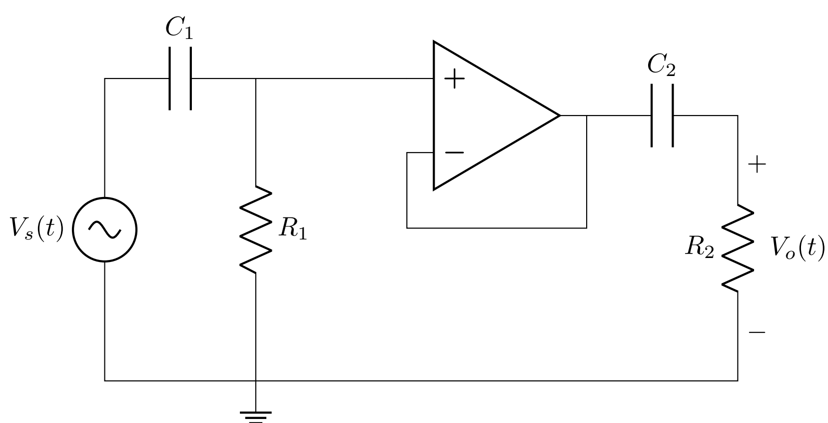
2



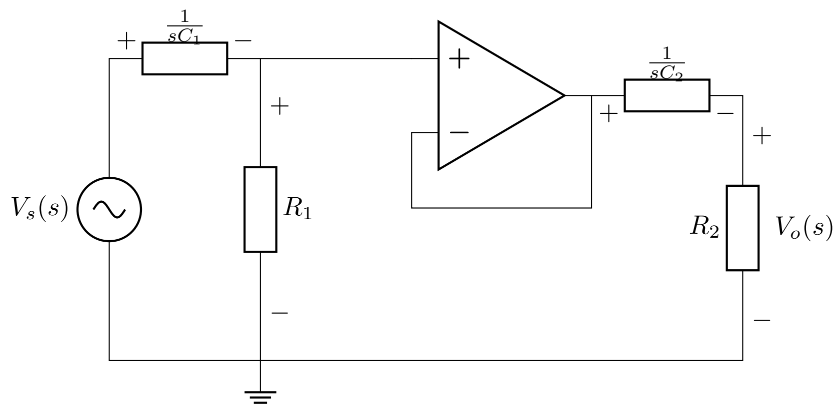
By inspection, we have poles at $\omega = 0.1, 2, 20\,\mathrm{rad\,s^{-1}}$. We also have zeroes at $\omega = 0.5\,\mathrm{rad\,s^{-1}}$. We also have a DC gain of $20\,\mathrm{dB}$. The transfer function is

$$H(s) = 80\frac{s+0.5}{(s+0.1)(s+2)(s+20)} \tag{9}$$

3



In the s-domain,



Using the voltage divider formula, we have

$$V^+ = \frac{sR_1C_1}{1+sR_1C_1} V_s(s) \tag{10}$$

$$V_o(s) = \frac{sR_2C_2}{1+sR_2C_2} V^- \tag{11}$$

By the golden op-amp rules, $V^+ = V^-$, so

$$H(s) = \frac{V_o(s)}{V_s(s)} = \frac{sR_2C_2}{1+sR_2C_2} \frac{sR_1C_1}{1+sR_1C_1} \tag{12}$$

4

$$x(t) = \begin{cases} t+1 & 0 \leqslant t \leqslant 1 \\ 2-t & 1 < t \leqslant 2 \\ 0 & \text{elsewhere} \end{cases} \tag{13}$$

$$h(t) = \delta(t+2) + 2\delta(t+1) \tag{14}$$

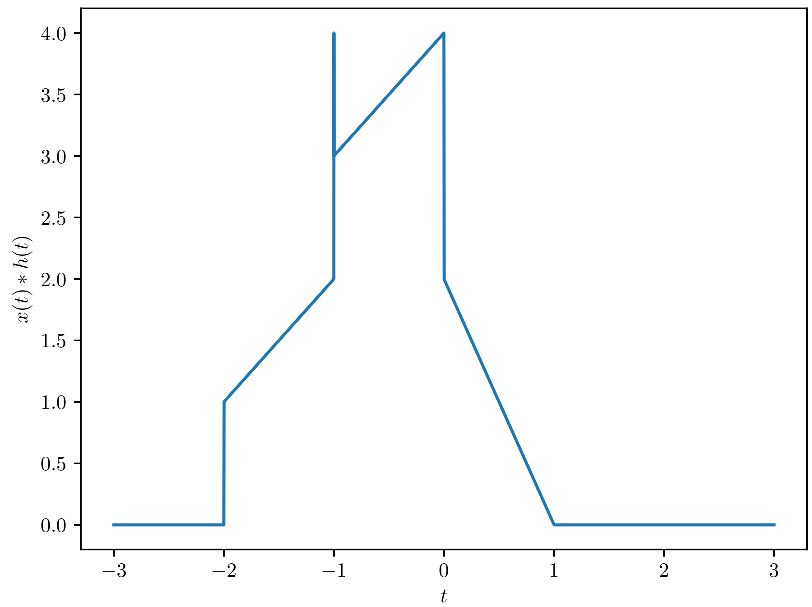
The convolution is

$$x(t) * h(t) = \int_{\mathbb{R}} x(\tau)h(t-\tau)\ \mathrm{d}\tau = \int_{\mathbb{R}} x(\tau)\delta(t+2-\tau)\ \mathrm{d}\tau + 2\int_{\mathbb{R}} x(\tau)\delta(t+1-\tau)\ \mathrm{d}\tau \tag{15}$$

By the sifting property, we have

$$x(t) * h(t) = x(t+2) + 2x(t+1) = \begin{cases} t+3 & -2 \leqslant t \leqslant -1 \\ -t & -1 < t \leqslant 0 \\ 0 & \text{elsewhere} \end{cases} + \begin{cases} 2t+4 & -1 \leqslant t \leqslant 0 \\ 2-2t & 0 < t \leqslant 1 \\ 0 & \text{elsewhere} \end{cases} \tag{16}$$

$$= \begin{cases} t+3 & -2 \leqslant t < -1 \\ 4 & t = -1 \\ t+4 & -1 \leqslant t \leqslant 0 \\ 2-2t & 0 < t \leqslant 1 \\ 0 & \text{elsewhere} \end{cases} \tag{17}$$



5

$$x(t) = u(t-3) - u(t-5) \tag{18}$$
$$h(t) = e^{-3t}u(t) \tag{19}$$

(a)

$$\begin{aligned}
y(t) &= \int_{\mathbb{R}} (u(\tau-3) - u(\tau-5))e^{-3(t-\tau)}u(t-\tau)\ \mathrm{d}\tau & (20)\\
&= e^{-3t}\int_{\mathbb{R}} e^{3\tau}u(\tau-3)u(t-\tau)\ \mathrm{d}\tau - e^{-3t}\int_{\mathbb{R}} e^{3\tau}u(\tau-5)u(t-\tau)\ \mathrm{d}\tau & (21)\\
&= e^{-3t}\left(\int_3^t e^{3\tau}\ \mathrm{d}\tau - \int_5^t e^{3\tau}\ \mathrm{d}\tau\right) & (22)\\
&= \frac{1}{3}e^{-3t}\left((e^{3t} - e^{9})u(t-3) - (e^{3t} - e^{15})u(t-5)\right) & (23)\\
&= \frac{1}{3}\left(\left(1 - e^{-3(t-3)}\right)u(t-3) - \left(1 - e^{-3(t-5)}\right)u(t-5)\right) & (24)
\end{aligned}$$

(b)

$$\begin{aligned}
y(t) &= \int_{\mathbb{R}} (\delta(\tau-3) - \delta(\tau-5))e^{-3(t-\tau)}u(t-\tau)\ \mathrm{d}\tau & (25)\\
&= e^{-3t}\int_{\mathbb{R}} e^{3\tau}\delta(\tau-3)u(t-\tau)\ \mathrm{d}\tau - e^{-3t}\int_{\mathbb{R}} e^{3\tau}\delta(\tau-5)u(t-\tau)\ \mathrm{d}\tau & (26)\\
&= e^{-3t}(e^{9}u(t-3) - e^{15}u(t-5)) & (27)\\
&= e^{-3(t-3)}u(t-3) - e^{-3(t-5)}u(t-5) & (28)
\end{aligned}$$

(c) We have that $y(t) = \int_3^t g(\tau)\ \mathrm{d}\tau$ for $3 \leqslant t \leqslant 5$.

(d) (a) In the s-domain,

$$Y(s) = X(s)H(s) = \left(\frac{e^{-3s}}{s} - \frac{e^{-5s}}{s}\right)\left(\frac{1}{s+3}\right) \tag{29}$$

$$= \frac{e^{-3s}}{s(s+3)} - \frac{e^{-5s}}{s(s+3)} \tag{30}$$

$$\overset{\mathcal{L}^{-1}}{\Longrightarrow} y(t) = \frac{1}{3}(1 - e^{-3(t-3)})u(t-3) - \frac{1}{3}(1 - e^{-3(t-5)})u(t-5) \tag{31}$$

(b) In the s-domain,

$$G(s) = (sX(s) - \cancelto{0}{x(0^-)})H(s) = sX(s)H(s) \tag{32}$$

$$= \frac{e^{-3s}}{s+3} - \frac{e^{-5s}}{s+3} \tag{33}$$

$$\overset{\mathcal{L}^{-1}}{\Longrightarrow} g(t) = e^{-3t}u(t-3) - e^{-3t}u(t-5) \tag{34}$$