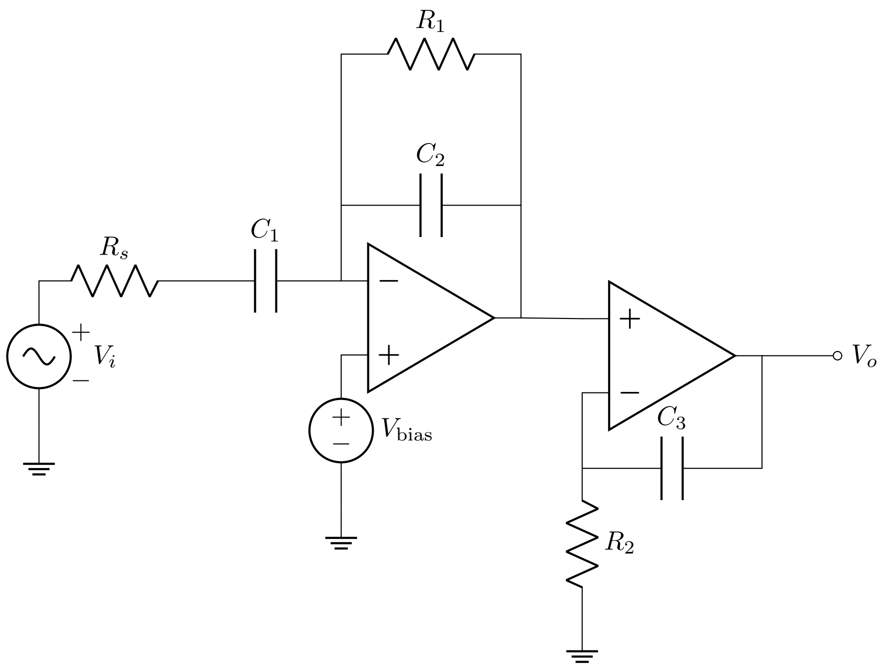
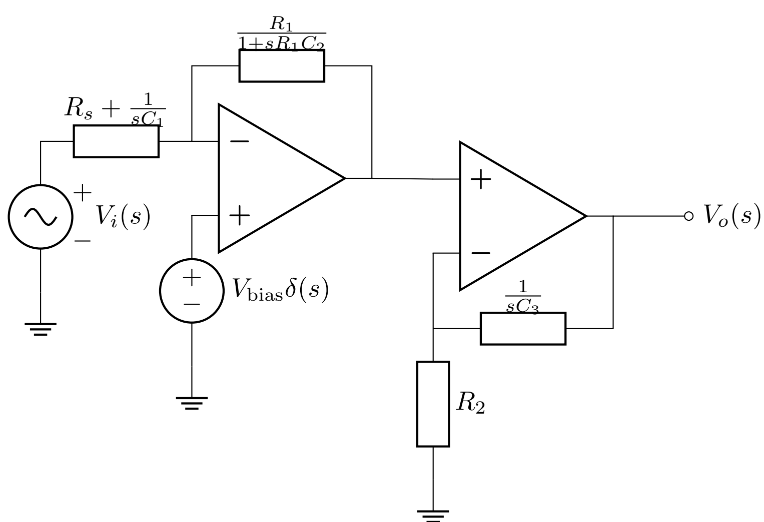
EE 105 HW 05

1



In the s-domain,



Using superposition, we can derive the output of the first op-amp as

$$
\begin{aligned}
V_x(s) &= -V_i(s)\frac{\frac{R_1}{1+sR_1C_2}}{\frac{1+sR_sC_1}{sC_1}} + V_{\text{bias}}\delta(s)\left(1+\frac{\frac{R_1}{1+sR_1C_2}}{\frac{1+sR_sC_1}{sC_1}}\right) && (1)\\
&= -V_i(s)\frac{sR_1C_1}{(1+sR_1C_2)(1+sR_sC_1)} + V_{\text{bias}}\delta(s)\left(1+\frac{sR_1C_1}{(1+sR_1C_2)(1+sR_sC_1)}\right) && (2)
\end{aligned}
$$

We can then multiply by the second op-amp's transfer function to get

$$V_o(s) = V_x\left(1 + \frac{1}{sR_2C_3}\right) \tag{3}$$

$$= \left(-V_i(s)\frac{sR_1C_1}{(1+sR_1C_2)(1+sR_sC_1)} + \cancelto{0}{V_{\text{bias}}}\delta(s)\left(1 + \frac{sR_1C_1}{(1+sR_1C_2)(1+sR_sC_1)}\right)\right)\left(1 + \frac{1}{sR_2C_3}\right) \tag{4}$$

$$\implies \frac{V_o(s)}{V_i(s)} = -\frac{sR_1C_1}{(1+sR_1C_2)(1+sR_sC_1)}\left(1 + \frac{1}{sR_2C_3}\right) \tag{5}$$

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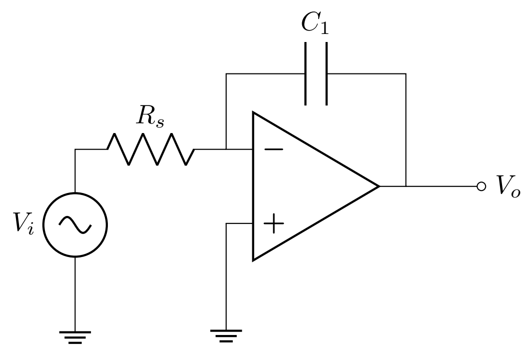
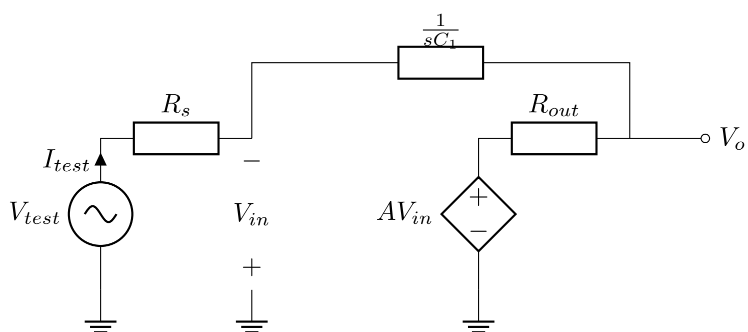


(a) Applying a test voltage at the input, the current through R_s is $I_{test} = \frac{V_{test} - \cancelto{0}{V^-}}{R_s}$, where we use the ideality of the op amp to cancel out V^-. This means that $Z_{test} = \frac{V_{test}}{I_{test}} = R_s$. Applying a test voltage at the output, since we are applying a DC source, the voltage difference at the capacitor is 0 due to the virtual short, therefore we have $Z_{out} = 0$.

(b) The circuit in the s-domain is now



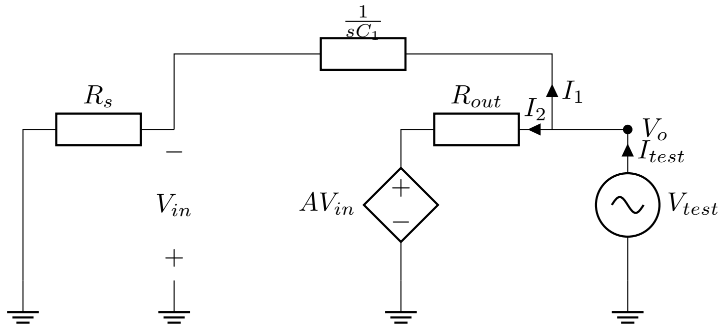
We can create the loop equation

$$V_{test} - I_{test}R_s - I_{test}\frac{1}{sC_1} - I_{test}R_{out} + A(V_{test} - I_{test}R_s) = 0 \tag{6}$$

$$V_{test}(1+A) - I_{test}\left(R_s + \frac{1}{sC_1} - R_{out} - AR_s\right) = 0 \tag{7}$$

$$Z_{in} = \frac{V_{test}}{I_{test}} = \frac{R_s + \frac{1}{sC_1} - R_{out} - AR_s}{1+A} \tag{8}$$

For the output impedance, we can apply a test voltage at V_o and null V_i to get



We can obtain the node equations

$$I_{test} = \frac{V_{test} - AV_{in}}{R_{out}} + sC_1(V_{test} + V_{in}) \tag{9}$$

$$V_{in} = V_{test} - sR_sC_1(V_{test} + V_{in}) \tag{10}$$

$$\implies V_{in} = V_{test}\frac{1 - sR_sC_1}{1 + sR_sC_1} \tag{11}$$

$$I_{test} = V_{test}\left(\frac{1}{R_{out}} - \frac{A}{R_{out}}\frac{1 - sR_sC_1}{1 + sR_sC_1} + sC_1\left(1 + \frac{1 - sR_sC_1}{1 + sR_sC_1}\right)\right) \tag{12}$$

$$\implies \frac{1}{Z_{out}} = \left(\frac{1}{R_{out}} - \frac{A}{R_{out}}\frac{1 - sR_sC_1}{1 + sR_sC_1} + sC_1\left(1 + \frac{1 - sR_sC_1}{1 + sR_sC_1}\right)\right) \tag{13}$$

When $A \to \infty$, we have $G_{out} = \infty \implies Z_{out} = 0$ and $Z_{in} = R_s$.

(c) The answer can be obtained similarly by substituting the constant gain A with the gain $\frac{A}{1+j\frac{\omega}{\omega_0}}$.

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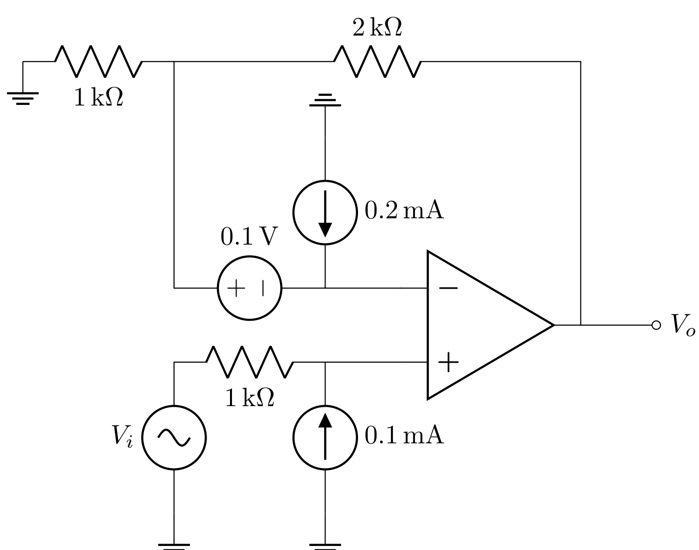


Slew rate: $2\,\mathrm{V\,\mu s^{-1}}$
Signal:

$$x(t) = \begin{cases} 1\,\mathrm{V} & 0 \leqslant t \leqslant 4\,\mathrm{\mu s} \\ 0 & \text{elsewhere} \end{cases} \tag{14}$$

We can define the node equations

$$0.2\,\mathrm{mA} = \frac{V^- + 0.1\,\mathrm{V} - V_o}{2\,\mathrm{k\Omega}} + \frac{V^- + 0.1\,\mathrm{k\Omega}}{1\,\mathrm{k\Omega}} \tag{15}$$

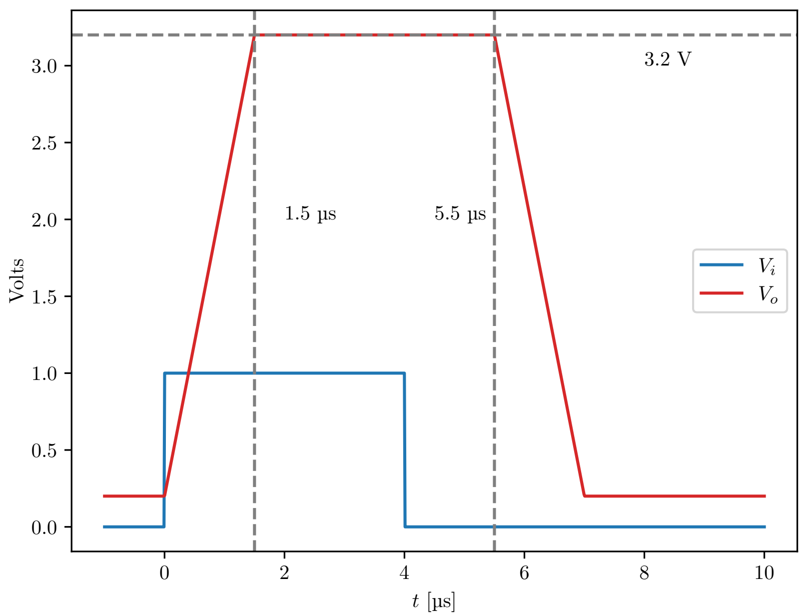
$$V_i + (0.1\,\mathrm{mA})(1\,\mathrm{k\Omega}) = V^+ \tag{16}$$

$$V^+ = V^- \tag{17}$$

$$\implies 0.2\,\mathrm{mA} = \frac{V_i + 0.1\,\mathrm{V} + 0.1\,\mathrm{V} - V_o}{2\,\mathrm{k\Omega}} + \frac{V_i + 0.1\,\mathrm{V} + 0.1\,\mathrm{V}}{1\,\mathrm{k\Omega}} \tag{18}$$

$$0.4\,\mathrm{V} = V_i + 0.2\,\mathrm{V} - V_o + 2V_i + 0.4\,\mathrm{V} \tag{19}$$

$$\implies V_o = 3V_i + 0.2\,\mathrm{V} \tag{20}$$



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$$V_{cm} = \frac{V_1 + V_2}{2} \tag{21}$$

$$V_d = V_2 - V_1 \tag{22}$$

$$V_{out} = A_{cm}V_{cm} + A_d V_d \tag{23}$$

We can solve for V_1 and V_2 in terms of V_{cm} and V_d as

$$V_1 = V_{cm} - \frac{V_d}{2} \tag{24}$$

$$V_2 = V_{cm} + \frac{V_d}{2} \tag{25}$$

Then, we can find V_{out} as

$$V_{out} = -\cancelto{10}{\frac{R_3}{R_1}} V_1 + \left(1 + \cancelto{10}{\frac{R_3}{R_1}}\right)\left(\frac{R_4}{R_2 + R_4}\right) V_2 \tag{26}$$

$$= -10V_1 + 11 \cancelto{\frac{10}{11}}{\left(1 + \frac{R_2}{R_4}\right)^{-1}} V_2 = -10V_1 + 10V_2 \tag{27}$$

$$= -10\left(\cancel{V_{cm}} - \frac{V_d}{2}\right) + 10\left(\cancel{V_{cm}} + \frac{V_d}{2}\right) = 10V_d \tag{28}$$

So $A_{cm} = 0$ and $A_d = 10$.