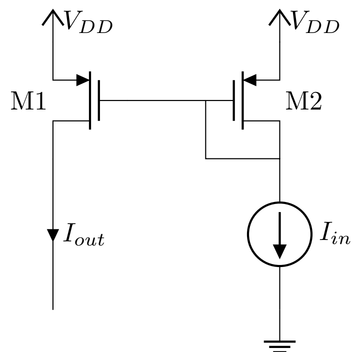
EE 105 HW 11

1

$$\lambda = 0\,\mathrm{V}^{-1} \tag{1}$$

$$I_d = \begin{cases} 0.5\mu C_{ox}\frac{W}{L}(V_{GS} - V_T)^2 & \text{NMOS} \\ 0.5\mu C_{ox}\frac{W}{L}(V_{SG} - V_T)^2 & \text{PMOS} \end{cases} \tag{2}$$

(a) The current mirror is



At M2, we know that $I_{in} = I_{SD}$, so

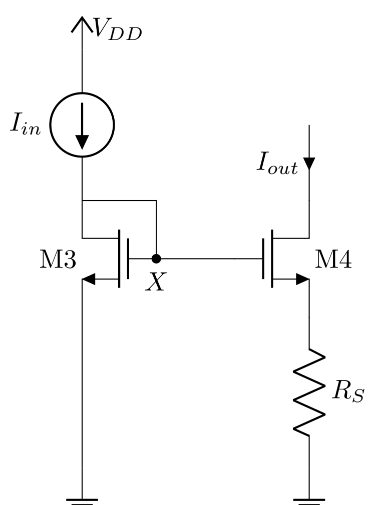
$$I_{in} = 0.5\mu C_{ox}\left(\frac{W}{L}\right)_{\mathrm{M2}}(V_{SG} - V_T)^2 \tag{3}$$

Since the drain voltages and gate voltages are shorted between the two MOSFETs, we also know that $V_{SG,\mathrm{M1}} = V_{SG,\mathrm{M2}}$. This also means that

$$I_{out} = 0.5\mu C_{ox}\left(\frac{W}{L}\right)_{\mathrm{M1}}(V_{SG} - V_T)^2 \tag{4}$$

Since the transistor aspect ratios are the same, we have that $I_{in} = I_{out}$.

(b) The weighted current mirror is



We want $I_{in} = I_{out}$ with $\left(\frac{W}{L}\right)_{\text{M4}} = 9\left(\frac{W}{L}\right)_{\text{M3}}$.

$$I_{in} = \frac{1}{2}\mu C_{ox}\left(\frac{W}{L}\right)_{\text{M3}}(V_X - V_T)^2 \tag{5}$$

$$I_{out} = \frac{1}{2}\mu C_{ox} 9\left(\frac{W}{L}\right)_{\text{M3}}(V_X - V_Y - V_T)^2 = \frac{V_Y}{R_S} \tag{6}$$

By definition, we have

$$I_{in} = I_{out} \implies (V_x - V_T)^2 = 9(V_x - V_Y - V_T)^2 \tag{7}$$

$$V_X - V_T = 3(V_X - \cancelto{I_{out}R_S}{V_Y} - V_T) \tag{8}$$

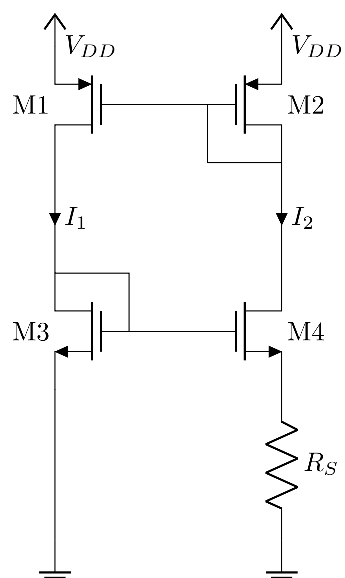
$$\implies 3I_{out}R_S = 2(V_X - V_T) \tag{9}$$

$$\implies R_S = \frac{2(V_X - V_T)}{3I_{in}} \tag{10}$$

Subtituting the equation for M3's overdrive voltage, we get that

$$R_S = \frac{2\sqrt{\frac{I_{in}}{\frac{1}{2}\mu C_{ox}\left(\frac{W}{L}\right)_{\text{M3}}}}}{3I_{in}} = \frac{2}{3}\sqrt{\frac{2}{\mu C_{ox}\left(\frac{W}{L}\right)_{\text{M3}} I_{in}}} \tag{11}$$

(c) The biased current mirror is



By the current mirror property, we have $I_1 = I_2$. Substituting $I_{in} = I_1$ into the expression for R_S, we have

$$I_1 = I_2 = \frac{4}{9}\frac{2}{\mu C_{ox}\left(\frac{W}{L}\right)_{\text{M3}} R_S^2} \tag{12}$$

(d)

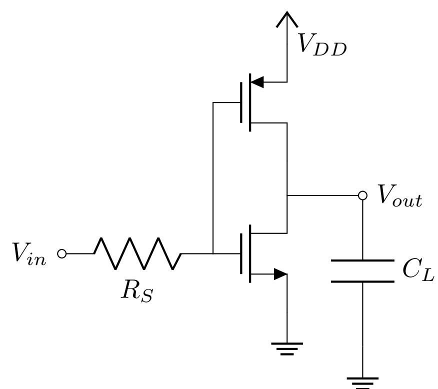
$$\left(\frac{W}{L}\right)_{\mathrm{M7}} = \left(\frac{W}{L}\right)_{\mathrm{M5}} = 4\left(\frac{W}{L}\right)_{\mathrm{M1}} \tag{13}$$
$$\left(\frac{W}{L}\right)_{\mathrm{M8}} = \left(\frac{W}{L}\right)_{\mathrm{M6}} = 2\left(\frac{W}{L}\right)_{\mathrm{M3}} \tag{14}$$
$$\tag{15}$$

Due to the current multiplcation property, we have that

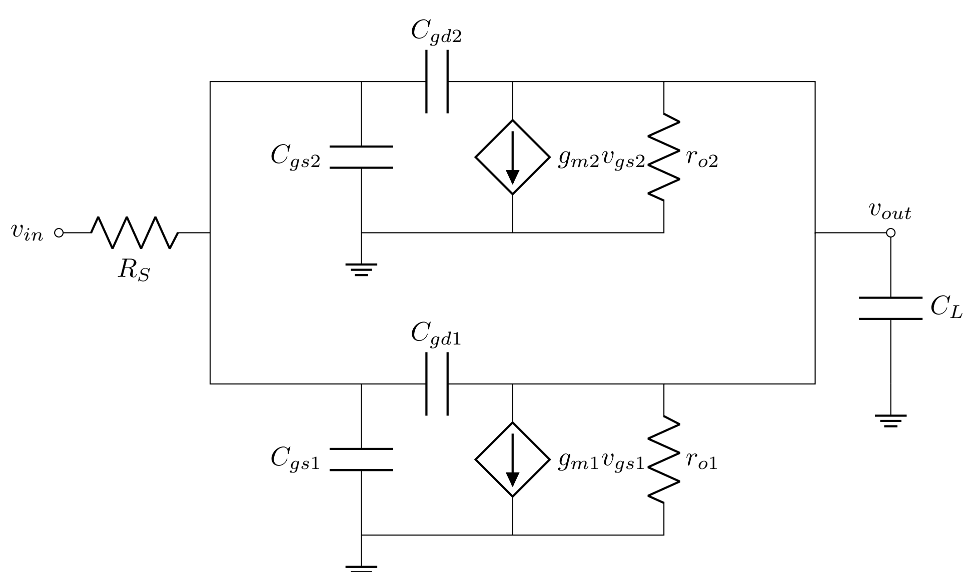
$$I_3 = 4I_1 \tag{16}$$
$$I_4 = 2I_1 \tag{17}$$
$$I_5 = 2I_1 \tag{18}$$

M8 will "win" over M7 since we can at minimum drive $2I_1$.

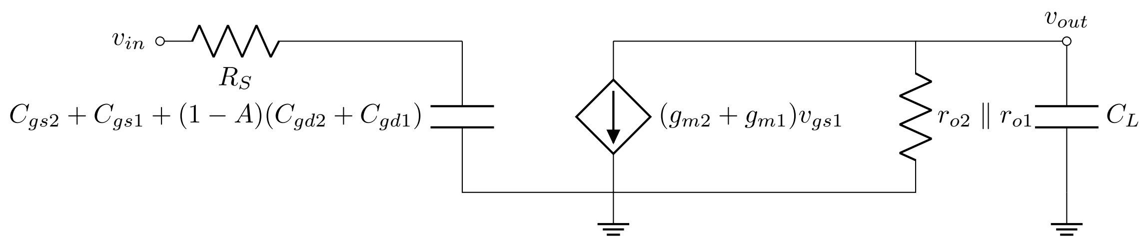
2



(a) The small-signal model for the circuit is



(b) After using the Miller approximation, we get



where $A = -(g_{m2} + g_{m1})(r_{o2} \parallel r_{o1})$. Let $C_{in} = C_{gs2} + C_{gs1} + (1 + (g_{m2} + g_{m1})(r_{o2} \parallel r_{o1}))(C_{gd2} + C_{gd1})$. The gain can be calculated as

$$v_g = \frac{1}{1 + sR_SC_{in}} v_{in} \tag{19}$$

$$(g_{m2} + g_{m1})v_g + v_{out}\left(\frac{1}{r_{o2} \parallel r_{o1}} + sC_L\right) = 0 \tag{20}$$

$$\implies v_{out} = -\frac{g_{m2} + g_{m1}}{\left(\frac{1}{r_{o2}\|r_{o1}} + sC_L\right)} v_g \tag{21}$$

$$\implies \frac{v_{out}}{v_{in}} = -\frac{g_{m2} + g_{m1}}{\left(\frac{1}{r_{o2}\|r_{o1}} + sC_L\right)} \frac{1}{1 + sR_SC_{in}} \tag{22}$$

(c) The parameters are

$$R_S = 20\,\mathrm{k\Omega} \tag{23}$$
$$V_{DD} = 1.2\,\mathrm{V} \tag{24}$$
$$V_{Tn} = |V_{Tp}| = 0.5\,\mathrm{V} \tag{25}$$
$$k_n = k_p = 1 \times 10^{-3}\,\mathrm{A\,V^{-2}} \tag{26}$$
$$\lambda = 0.2\,\mathrm{V^{-1}} \tag{27}$$
$$C_{ox} = 20\,\mathrm{fF\,\mu m^{-2}} \tag{28}$$
$$L_{ov}C_{ox} = 0.1\,\mathrm{fF\,\mu m^{-1}} \tag{29}$$
$$C_L = 200\,\mathrm{fF} \tag{30}$$
$$W = 12\,\mathrm{\mu m} \tag{31}$$
$$L = 100\,\mathrm{nm} \tag{32}$$

Finding the DC operating point,

$$I_{m2} = I_{m1} \tag{33}$$

$$\cancel{\frac{k_n}{2}}(V_{SG} - |V_{Tp}|)^2(1 + \lambda V_{SD}) = \cancel{\frac{k_p}{2}}(V_{GS} - V_{Tn})^2(1 + \lambda V_{DS}) \tag{34}$$

$$(V_{DD} - V_{in} - |V_{Tp}|)^2 \cancel{\left(1 + \lambda\left(V_{DD} - \frac{V_{DD}}{2}\right)\right)} = (V_{in} - V_{Tn})^2 \cancel{\left(1 + \lambda\frac{V_{DD}}{2}\right)} \tag{35}$$

$$V_{DD} - V_{in} - |V_{Tp}| = V_{in} - V_{Tn} \tag{36}$$

$$\implies V_{in} = \frac{V_{DD} - |V_{Tp}| + V_{Tn}}{2} = 0.6\,\mathrm{V} \tag{37}$$

Finding the transconductance parameters,

$$g_{m1} = k_n(V_{in} - V_{Tn})(1 + \lambda V_{DS}) = 0.112\,\mathrm{mS} \tag{38}$$
$$g_{m2} = k_p(V_{DD} - V_{in} - |V_{Tp}|)(1 + \lambda V_{SD}) = 0.112\,\mathrm{mS} \tag{39}$$

Finding the output resistances,

$$r_{o1} = \frac{2}{k_n(V_{in} - V_{Tn})^2\lambda} = 1\,\mathrm{M\Omega} \tag{40}$$
$$r_{o2} = \frac{2}{k_p(V_{DD} - V_{in} - |V_{Tp}|)^2\lambda} = 1\,\mathrm{M\Omega} \tag{41}$$

Finding the drain and source capacitances,

$$C_{gs1} = C_{gs2} = \frac{2}{3}WLC_{ox} + WL_{ov}C_{ox} = 17.2\,\mathrm{fF} \tag{42}$$
$$C_{gd1} = C_{gd2} = WL_{ov}C_{ox} = 1.2\,\mathrm{fF} \tag{43}$$

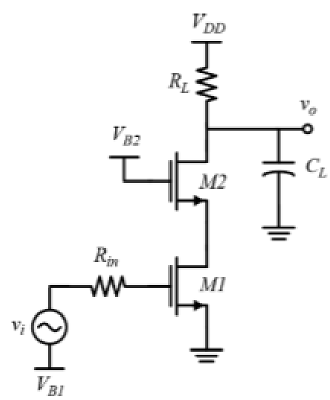
Finding the Miller capacitor,

$$C_{in} = C_{gs2} + C_{gs1} + (1 + (g_{m2} + g_{m1})(r_{o2} \parallel r_{o1}))(C_{gd2} + C_{gd1}) = 305.6\,\mathrm{fF} \tag{44}$$

Finding the poles,

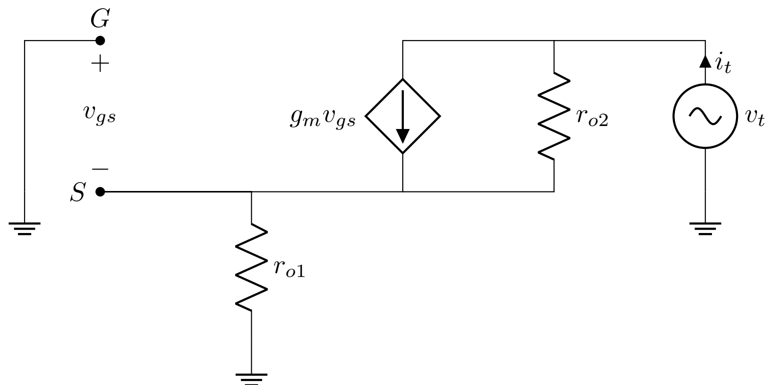
$$s_1 = \frac{1}{2\pi R_S C_{in}} = 26\,\mathrm{MHz} \tag{45}$$
$$s_2 = \frac{1}{2\pi(r_{o1} \parallel r_{o2})C_L} = 1.59\,\mathrm{MHz} \tag{46}$$

3



$$g_m = 1\,\mathrm{mS} \tag{47}$$
$$r_o = 100\,\mathrm{k\Omega} \tag{48}$$
$$C_{gd} = 3\,\mathrm{fF} \tag{49}$$
$$C_{sb} = 5\,\mathrm{fF} \tag{50}$$
$$C_{db} = 4\,\mathrm{fF} \tag{51}$$
$$R_{in} = 3\,\mathrm{k\Omega} \tag{52}$$
$$R_L = 2\,\mathrm{k\Omega} \tag{53}$$
$$C_L = 100\,\mathrm{fF} \tag{54}$$

(a) Assuming a high enough frequency such that the capacitor impedances are much larger than any other impedances in the circuit, the small-signal model of M2 is



The source voltage can be found as

$$i_t = \frac{v_t - v_s}{r_{o2}} - g_m v_s \tag{55}$$

$$\frac{v_t - v_s}{r_{o2}} - g_m v_s = \frac{v_s}{r_{o1}} \tag{56}$$

$$\implies r_o(v_t - v_s) - g_m r_o^2 v_s = r_o v_s \tag{57}$$

$$\implies v_s = \frac{r_o}{2r_o + g_m r_o^2} v_t \tag{58}$$

For M1, since both gate and source are grounded, we can ignore the transconductance, so we have that $v_s = i_t r_o$, so

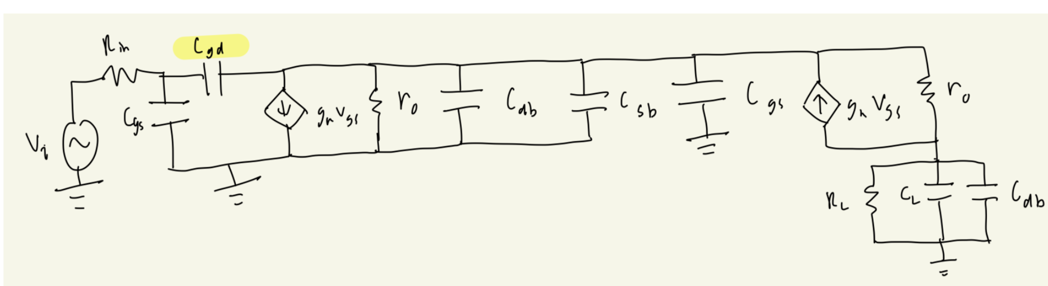
$$i_t \cancel{r_o} = \frac{\cancel{r_o}}{2r_o + g_m r_o^2} v_t \implies \frac{v_t}{i_t} = g_m r_o^2 + 2r_o = 10.2\,\mathrm{M\Omega} \tag{59}$$

(b) Assume a high enough frequency such that the capacitor impedances are much larger than any other impedances in the circuit. Since we are looking into the source of M2, we do not need to consider the effect of M1. Thus, the impedance is nothing more than the impedance of a common-gate amplifier, which is

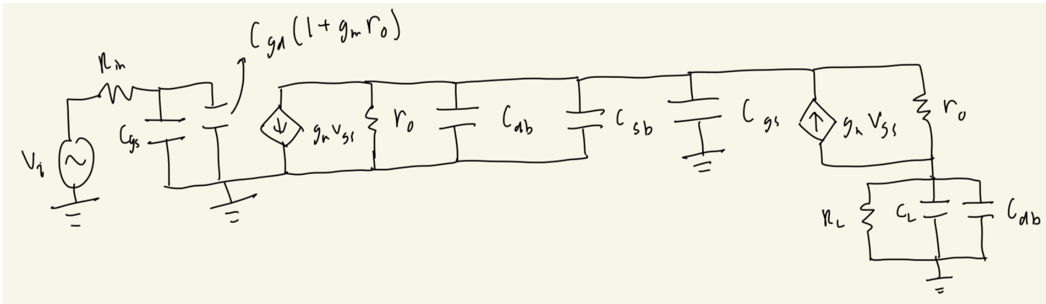
$$R_{in} = \left(g_m \frac{1 + \frac{1}{g_m r_o}}{1 + \frac{R_L}{r_0}} \right)^{-1} = 1010\,\Omega \tag{60}$$

(c) Using the $A_v = -G_m R_o$ method, we can find R_o as the result of the first part, and G_m can be found from discussion as

$$\frac{v_o}{v_i} = -g_m r_o \frac{R_{out} \parallel R_L}{r_o + R_{in}} = -1.98 \tag{61}$$

(d) 

Note that we can combine many of the capacitances into their equivalences, such as the body capacitors in the middle and the capacitors at the drain of M2.

(e) 

(f) The second-stage amplifier is a common-drain amplifier. Since the gain of a common-drain amplifier is one, the low-frequency small-signal gain is unchanged, i.e.

$$\frac{v_o}{v_i} = -g_m r_o \frac{R_{out} \parallel R_L}{r_o + R_{in}} = -1.98 \tag{62}$$