

EE 123 HW 04

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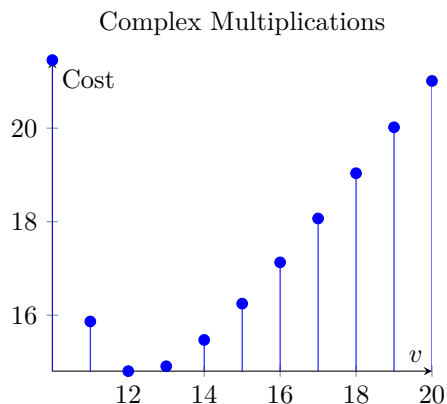
3.a

Each circular convolution in the overlap-save method will result in a length $2^v - P + 1$ signal. Then, the cost of the FFT and IFFT is $\frac{2^v}{2} \log_2(2^v) = v2^{v-1}$. Then, there is a 2^v -pointwise multiplication. The total number of multiplications is $2^v(v+1)$. Thus, the FFT for each sample will require

$$\frac{2^v(v+1)}{2^v - P + 1}. \quad (1)$$

complex multiplications per output sample.

3.b



with a minimum cost of $v = 12$. The direct evaluation would cost 500 complex multiplications per output sample, since that is the length of a given sample.

3.c

$$\lim_{v \rightarrow \infty} \frac{2^v(v+1)}{2^v - P + 1} = \lim_{v \rightarrow \infty} \frac{v+1}{1 - \left(\frac{P-1}{2^v}\right)} = v \quad (2)$$

Thus, for $P = 500$, the direct method will be more efficient for $v > 500$.

4 Fun with FFT

- a. For $0 \leq k < N$, $H[k] = X_r[k] + W_{2N}^k X_i[k]$.
- b. For $0 \leq k < N$, $H[k + N] = X_r[k] - W_{2N}^k X_i[k]$.
- c. $X[k] = \frac{1}{2}(H[k] + H[k + N]) + \frac{j}{2}W_{2N}^{-k}(H[k] - H[k + N])$, which takes 3 multiplications and 3 additions.

5 Hadamard Transform

5.a

$$H_3 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \end{bmatrix} \quad (3)$$

The order that represents increasing frequency content is the sequency ordering.

5.b

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6.a

$$X[3k] = \sum_{n=0}^{N-1} x[n] W_N^{3kn} \quad (4)$$

$$= \sum_{n=0}^{\frac{N}{3}-1} x[n] W_N^{3kn} + \sum_{n=\frac{N}{3}}^{\frac{2N}{3}-1} x[n] W_N^{3kn} + \sum_{n=\frac{2N}{3}}^{N-1} x[n] W_N^{3kn} \quad (5)$$

$$= \sum_{n=0}^{\frac{N}{3}-1} x[n] W_N^{3kn} + \sum_{n=0}^{\frac{N}{3}-1} x \left[n + \frac{N}{3} \right] W_N^{3kn} + \sum_{n=0}^{\frac{N}{3}-1} x \left[n + \frac{2N}{3} \right] W_N^{3kn} \quad (6)$$

$$= \sum_{n=0}^{\frac{N}{3}-1} \left(x[n] + x \left[n + \frac{N}{3} \right] + x \left[n + \frac{2N}{3} \right] \right) W_N^{3kn} \quad (7)$$

$$= \sum_{n=0}^{\frac{N}{3}-1} \underbrace{\left(x[n] + x \left[n + \frac{N}{3} \right] + x \left[n + \frac{2N}{3} \right] \right)}_{x_1[n]} W_N^{kn} \quad (8)$$

6.b

$$X[3k+1] = \sum_{n=0}^{N-1} x[n] W_N^{n(3k+1)} \quad (9)$$

$$= \sum_{n=0}^{\frac{N}{3}-1} x[n] W_N^{n(3k+1)} + \sum_{n=\frac{N}{3}}^{\frac{2N}{3}-1} x[n] W_N^{n(3k+1)} + \sum_{n=\frac{2N}{3}}^{N-1} x[n] W_N^{n(3k+1)} \quad (10)$$

$$= \sum_{n=0}^{\frac{N}{3}-1} x[n] W_N^{n(3k+1)} + \sum_{n=0}^{\frac{N}{3}-1} x\left[n + \frac{N}{3}\right] W_N^{\frac{N}{3}} W_N^{n(3k+1)} + \sum_{n=0}^{\frac{N}{3}-1} x\left[n + \frac{2N}{3}\right] W_N^{\frac{2N}{3}} W_N^{n(3k+1)} \quad (11)$$

$$= \sum_{n=0}^{\frac{N}{3}-1} \left(x[n] + x\left[n + \frac{N}{3}\right] W_N^{\frac{N}{3}} + x\left[n + \frac{2N}{3}\right] W_N^{\frac{2N}{3}} \right) W_N^{n(3k+1)} \quad (12)$$

$$= \sum_{n=0}^{\frac{N}{3}-1} \underbrace{\left(x[n] + x\left[n + \frac{N}{3}\right] W_N^{\frac{N}{3}} + x\left[n + \frac{2N}{3}\right] W_N^{\frac{2N}{3}} \right) W_N^n W_N^{\frac{kn}{3}}}_{x_2[n]} \quad (13)$$

$$X[3k+2] = \sum_{n=0}^{N-1} x[n] W_N^{n(3k+2)} \quad (14)$$

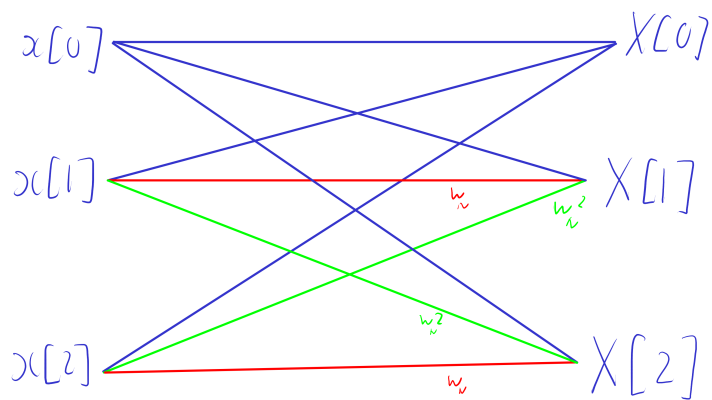
$$= \sum_{n=0}^{\frac{N}{3}-1} x[n] W_N^{n(3k+2)} + \sum_{n=\frac{N}{3}}^{\frac{2N}{3}-1} x[n] W_N^{n(3k+2)} + \sum_{n=\frac{2N}{3}}^{N-1} x[n] W_N^{n(3k+2)} \quad (15)$$

$$= \sum_{n=0}^{\frac{N}{3}-1} x[n] W_N^{n(3k+2)} + \sum_{n=0}^{\frac{N}{3}-1} x\left[n + \frac{N}{3}\right] W_N^{\frac{2N}{3}} W_N^{n(3k+2)} + \sum_{n=0}^{\frac{N}{3}-1} x\left[n + \frac{2N}{3}\right] W_N^{\frac{4N}{3}} W_N^{n(3k+2)} \quad (16)$$

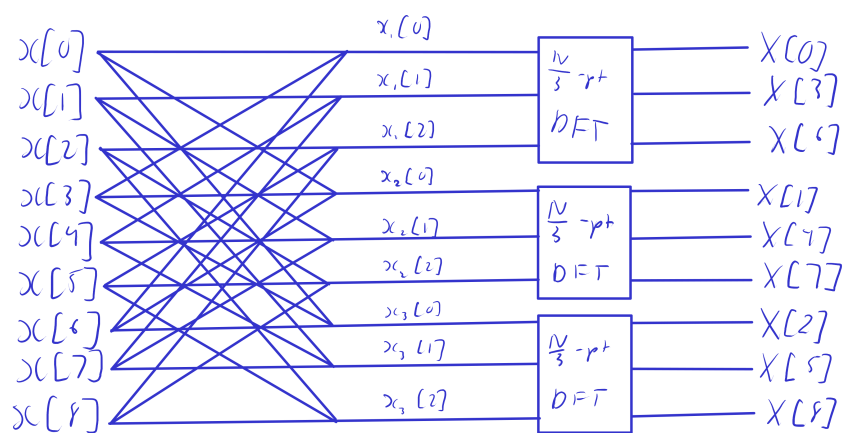
$$= \sum_{n=0}^{\frac{N}{3}-1} \left(x[n] + x\left[n + \frac{N}{3}\right] W_N^{\frac{2N}{3}} + x\left[n + \frac{2N}{3}\right] W_N^{\frac{4N}{3}} \right) W_N^{n(3k+2)} \quad (17)$$

$$= \sum_{n=0}^{\frac{N}{3}-1} \underbrace{\left(x[n] + x\left[n + \frac{N}{3}\right] W_N^{\frac{2N}{3}} + x\left[n + \frac{2N}{3}\right] W_N^{\frac{4N}{3}} \right) W_N^{2n} W_N^{\frac{kn}{3}}}_{x_3[n]} \quad (18)$$

6.c



6.d



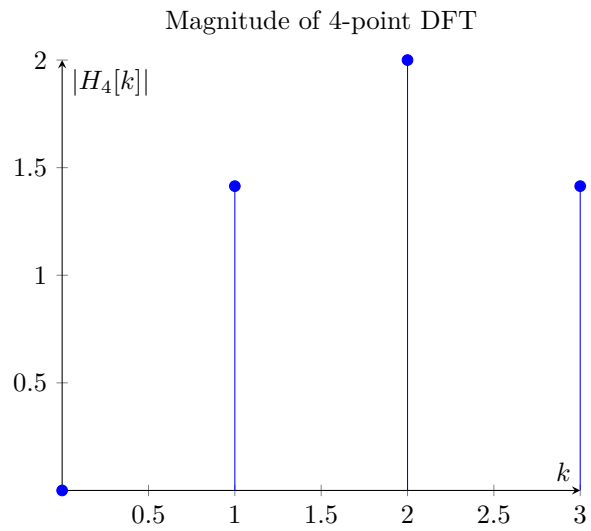
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- a. $|X[k]| \leq N$ for $k = 0$.
- b. We want $x[n]$ to be a constant under the DFT, so we can cancel out the complex exponential terms to obtain $x[n] = e^{j\theta} W_N^{-kn}$ for all $\theta \in \mathbb{R}$, and $k, n \in \mathbb{Z}$.

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8.a

$$H_4[k] = \sum_{n=0}^3 h[n] W_4^{kn} = 1 - W_4^k = 1 - e^{j\frac{2\pi}{N}k} = \{0, 1 + j, 2, 1 - j\} \quad (19)$$



The DFT is not even, not odd, is conjugate symmetric. The DFT is a high-pass filter since it lets in $\omega = \pi$, which is the highest frequency.

8.b