Homework 1

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2.a

Theorem 1. *The system L is not time invariant.*

Proof. We can represent a time-delayed signal as a linear combination of the other 3 signals. We can represent the 3 input signals in a matrix

$$\boldsymbol{X} = \begin{bmatrix} -2 & -2 & 0 \\ 1 & 1 & 1 \\ -2 & 0 & 1 \end{bmatrix} \tag{1}$$

Then, we can find the linear combination necessary to create a time delayed signal (for example, $x_2[n-1]$) by solving the system of equations

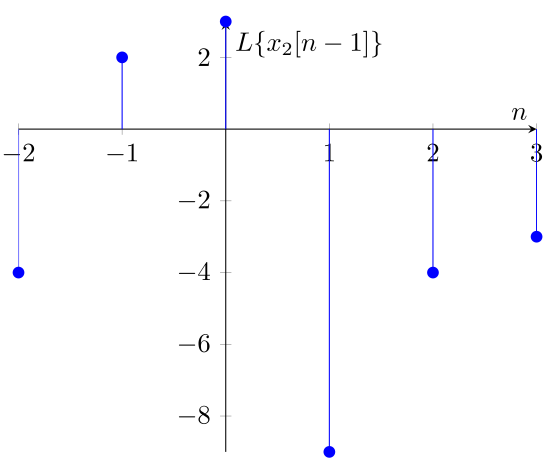
$$\boldsymbol{X}\boldsymbol{a} = \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix} \tag{2}$$

where we get $\boldsymbol{a} = \begin{bmatrix} -\frac{3}{2} & \frac{3}{2} & -2 \end{bmatrix}^\top$. Thus, we have

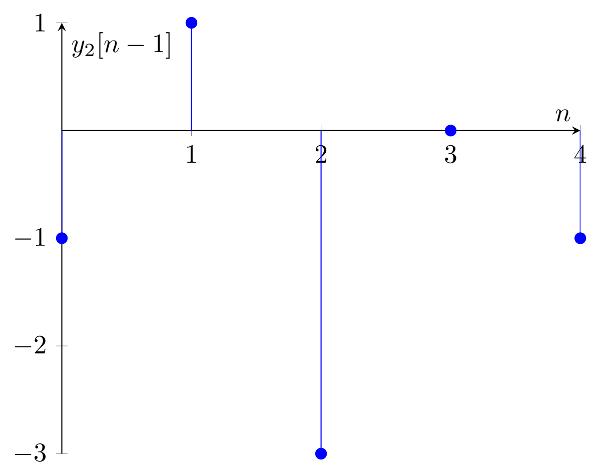
$$x_2[n-1] = -\frac{3}{2}x_1[n] + \frac{3}{2}x_2[n] - 2x_3[n] \tag{3}$$

Calculating $-\frac{3}{2}y_1[n] + \frac{3}{2}y_2[n] - 2y_3[n]$, we get the following plot:

Time invariance counterexample



Expected output signal



□

2.b

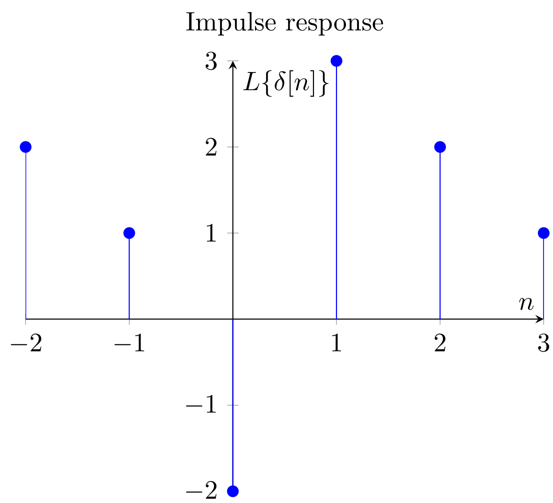
We can represent the delta function as

$$\delta[n] = \frac{1}{2}(x_1[n] - x_2[n] + 2x_3[n]) \tag{4}$$

Using the properties of linear systems, we can find the impulse response

$$h[n] = \frac{1}{2}(y_1[n] - y_2[n] + 2y_3[n]) \tag{5}$$

which gives us the following plot:



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$$y[n] - ay[n-1] = x[n] \tag{6}$$
$$y[0] = 1 \tag{7}$$

3.a

For the homogenuous solution, assume that $y_h[n] = A\lambda^n$. Then,

$$A\lambda^n - aA\lambda^{n-1} = 0 \tag{8}$$
$$1 - a\lambda^{-1} = 0 \tag{9}$$
$$\implies \lambda = a \implies y_h[n] = Aa^n \tag{10}$$

Finding the particular solution,

n	$y_p[n]$
0	1
N	$f[N] = \alpha f[0] = \alpha$
$2N$	$f[2N] = \alpha f[N] = \alpha^2$
$\vdots$	$\vdots$

(11)