EE 123 HW 02

Bryan Ngo

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$$a^n u[n] \iff \frac{1}{1 - ae^{-j\omega}}, \ |a| < 1 \tag{1}$$

3.a

$$x[n] = -b^n u[-n-1] = \begin{cases} -b^n & n \leqslant -1 \\ 0 & n \geqslant 0 \end{cases} \tag{2}$$

Using the definition of the DTFT,

$$X(e^{j\omega}) = \sum_{k \in \mathbb{Z}} -b^k u[-k-1] e^{-j\omega k} \tag{3}$$

$$= -\sum_{k \leqslant -1} b^k e^{-j\omega k} \tag{4}$$

Letting $k' = -k$,

$$X(e^{j\omega}) = \sum_{k' \geqslant 1} -b^{-k'} e^{j\omega k'} \tag{5}$$

$$= \sum_{k \geqslant 0} -(b^{-1} e^{j\omega})^k - 1 \tag{6}$$

$$= 1 - \frac{1}{1 - be^{j\omega}} \tag{7}$$

$$= \frac{\cancel{1} - be^{j\omega} - \cancel{1}}{1 - be^{j\omega}} \cdot \frac{-be^{-j\omega}}{-be^{-j\omega}} \tag{8}$$

$$= \frac{1}{1 - be^{-j\omega}} \tag{9}$$

where $|b^{-1}| < 1 \implies |b| > 1$.

3.b

$$Y(e^{j\omega}) = 2e^{-j\omega} \frac{1}{1 - (-2)e^{-j\omega}} \tag{10}$$

$$\xRightarrow{\mathcal{F}^{-1}} y[n] = 2(-(-2)^{n-1} u[-(n-1)-1]) \tag{11}$$

$$= (-2)^n u[-n] \tag{12}$$

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$$H(z) = \frac{1 - z^{-1}}{1 - 0.25z^{-2}} = \frac{1 - z^{-1}}{(1 - 0.5z^{-1})(1 + 0.5z^{-1})} \tag{13}$$

4.a

Using the Z-transform multiplication property,

$$Y(z) = H(z)X(z) = \frac{1 - z^{-1}}{(1 - 0.5z^{-1})(1 + 0.5z^{-1})(1 - z^{-1})} = \frac{1}{(1 - 0.5z^{-1})(1 + 0.5z^{-1})} \tag{14}$$

Then, using partial fraction decomposition,

$$Y(z) = \frac{1}{(1 - 0.5z^{-1})(1 + 0.5z^{-1})} = \frac{A}{1 - 0.5z^{-1}} + \frac{B}{1 + 0.5z^{-1}} \tag{15}$$

$$= 1 = A(1 + 0.5z^{-1}) + B(1 - 0.5z^{-1}) \tag{16}$$

$$\tag{17}$$

Letting $z = -0.5$, we get $B = \frac{1}{2}$. Letting $z = 0.5$, we get $A = \frac{1}{2}$. Then,

$$Y(z) = \frac{1}{2} \frac{1}{1 - 0.5z^{-1}} + \frac{1}{2} \frac{1}{1 + 0.5z^{-1}} \tag{18}$$

$$\xrightarrow{\mathcal{Z}^{-1}} y[n] = \frac{1}{2} \left(\frac{1}{2}\right)^n u[n] + \frac{1}{2} \left(-\frac{1}{2}\right)^n u[n] \tag{19}$$

4.b

$$y[n] = \delta[n] - \delta[n - 1] \xrightarrow{\mathcal{Z}} Y(z) = 1 - z^{-1} = H(z)X(z) \tag{20}$$

meaning that $X(z) = 1 - 0.25z^{-2} = (1 - 0.5z^{-1})(1 + 0.5z^{-1})$. By the convolution property of the Z-transform,

$$x[n] = \left(\left(\frac{1}{2}\right)^n u[n]\right) * \left(\left(-\frac{1}{2}\right)^n u[n]\right) \tag{21}$$

4.c

$$y[n] = \cos(0.5\pi n) \xrightarrow{\mathcal{Z}} Y(z) = 1 - z^{-1} = H(z)X(z) \tag{22}$$

meaning that $X(z) = 1 - 0.25z^{-2} = (1 - 0.5z^{-1})(1 + 0.5z^{-1})$.