EECS 16B HW04

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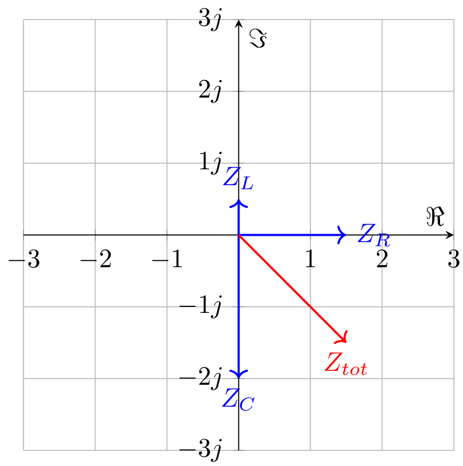
1 Phasors

1.a

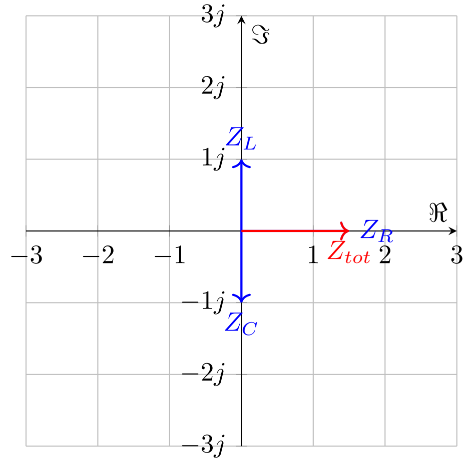
$$Z_R = R = 1.5\,\Omega \tag{1}$$
$$Z_C = \frac{1}{j\omega C} = -\frac{j}{\omega}\,\Omega \tag{2}$$
$$Z_L = j\omega L = j\omega\,\Omega \tag{3}$$

1.b



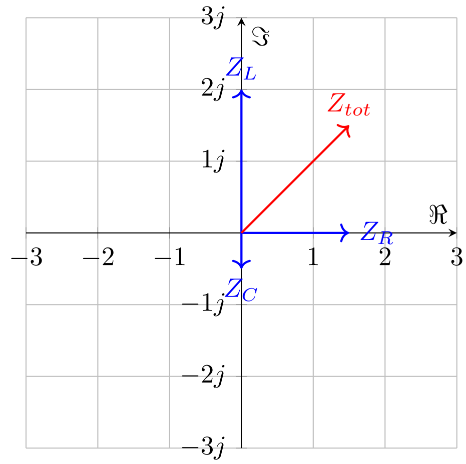
$$|Z_{tot}| = 1.5\sqrt{2}\,\Omega \tag{4}$$
$$\theta = -\frac{\pi}{4}\,\text{rad} \tag{5}$$

1.c



$$|Z_{tot}| = 1.5\,\Omega \tag{6}$$
$$\theta = 0\,\text{rad} \tag{7}$$

1.d

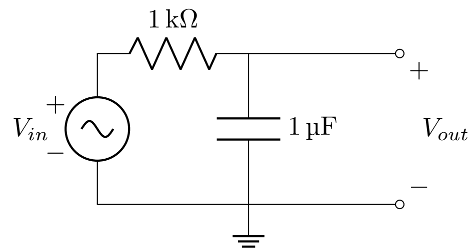


$$|Z_{tot}| = 1.5\sqrt{2}\,\Omega \tag{8}$$
$$\theta = -\frac{\pi}{4}\,\text{rad} \tag{9}$$

1.e

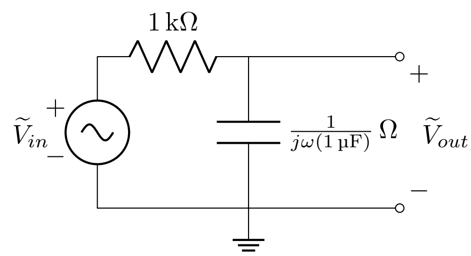
$\omega_n = 1\,\text{rad}\,\text{s}^{-1}$

2 Low-pass Filter



2.a

Phasorizing, the circuit is now transformed into



2.b

$$H(j\omega) = \frac{\widetilde{V}_{out}}{\widetilde{V}_{in}} = \underbrace{\frac{\frac{1}{j\omega(1\,\mu\mathrm{F})}}{1\,\mathrm{k\Omega} + \frac{1}{j\omega(1\,\mu\mathrm{F})}}}_{\text{voltage divider}} = \frac{1}{1 + (1\,\mathrm{k\Omega})j\omega(1\,\mu\mathrm{F})} = \frac{1}{1 + j\omega(1\,\mathrm{ms})} \tag{10}$$

2.c

$$H(\mathrm{j} \times 10^6\,\mathrm{rad\,s^{-1}}) = \frac{1}{1 + (\mathrm{j} \times 10^6\,\mathrm{rad\,s^{-1}})(1\,\mathrm{ms})} = \frac{1}{1 + \mathrm{j} \times 10^3} \approx (0.001 + \mathrm{j}) \times 10^{-3} \tag{11}$$

$$\Rightarrow |H(\mathrm{j} \times 10^6\,\mathrm{rad\,s^{-1}})| \approx 10^{-3}\,\Omega \tag{12}$$

2.d

$$H(\mathrm{j\,rad\,s^{-1}}) = \frac{1}{1 + (\mathrm{j\,rad\,s^{-1}})(1\,\mathrm{ms})} = \frac{1}{1 + \mathrm{j} \times 10^{-3}} \approx (1 + 0.001\mathrm{j}) \times 10^{-3} \tag{13}$$

$$\angle H(\mathrm{j\,rad\,s^{-1}}) \approx -10^{-3}\,\mathrm{rad} \tag{14}$$

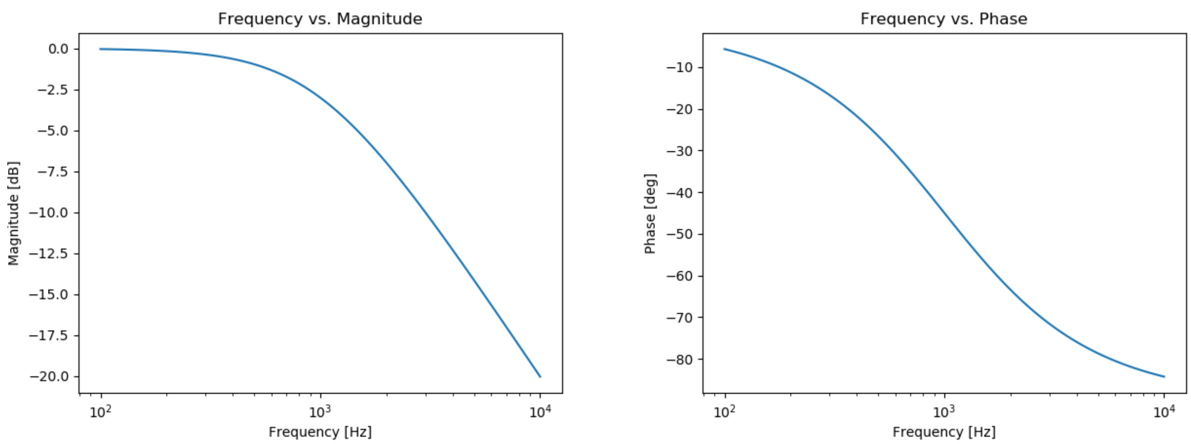
2.e

Since the input is a sine wave, our input $V_{in}(t) = \Re\{e^{j\frac{\pi}{2}} e^{1000jt}\}$. Then,

$$\widetilde{V}_{out}\Big|_{\omega=1000} = \frac{1}{1+j}\widetilde{V}_{in} \tag{15}$$

$$\begin{aligned}
V_{out}(t) &= \Re\left\{\widetilde{V}_{out} e^{1000jt}\right\} &(16)\\
&= \Re\left\{\frac{1}{1+j}\widetilde{V}_{in} e^{1000jt}\right\} &(17)\\
&= \Re\left\{\frac{1}{1+j} e^{j\frac{\pi}{2}} e^{1000jt}\right\} &(18)\\
&= \Re\left\{\frac{1}{|1+j|} e^{-j\frac{\pi}{4}} e^{j\frac{\pi}{2}} e^{1000jt}\right\} &(19)\\
&= \frac{1}{\sqrt{2}}\cos\left(1000t + \frac{\pi}{4}\right) &(20)
\end{aligned}$$

2.f



3 Color Organ Filter Design

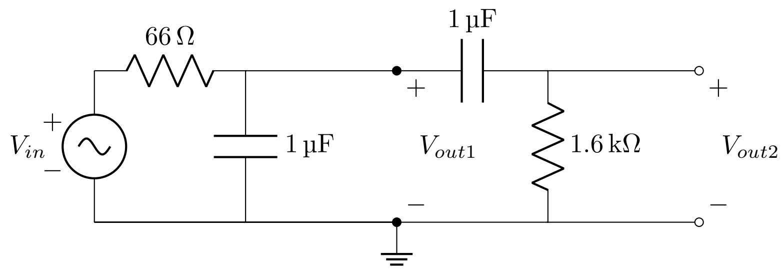
3.a

For the low-pass filter, the cutoff frequency is

$$f_{cut} = \frac{1}{2\pi RC} \Rightarrow R = \frac{1}{2\pi f_{cut} C} = 66\,\Omega \tag{21}$$

For the high-pass filter,

$$f_{cut} = \frac{1}{2\pi RC} \Rightarrow R = \frac{1}{2\pi f_{cut} C} = 1.6\,\mathrm{k}\Omega \tag{22}$$



3.b

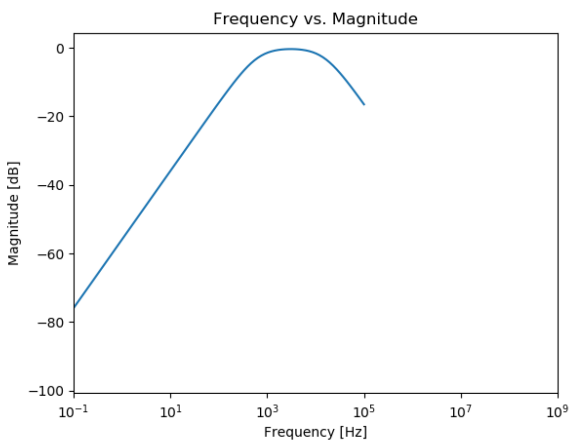
Since there is no buffer, we cannot simply take the product. Using equivalent impedance, the voltage V_{out1} is

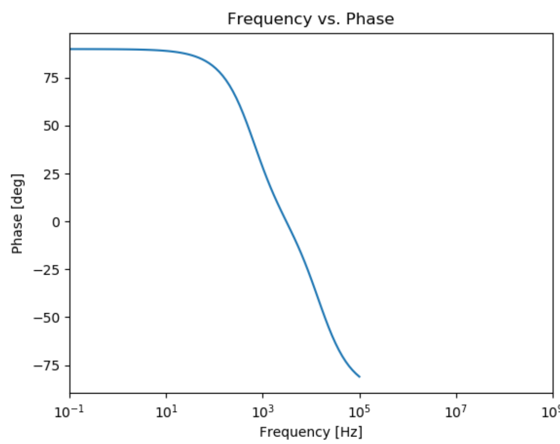
$$V_{out1} = \frac{(R_H + \frac{1}{j\omega C_H}) \parallel \frac{1}{j\omega C_L}}{(R_H + \frac{1}{j\omega C_H}) \parallel \frac{1}{j\omega C_L} + R_L} V_{in} \tag{23}$$

Meaning that

$$V_{out2} = \frac{j\omega R_H C_H}{1 + j\omega R_H C_H} \frac{(R_H + \frac{1}{j\omega C_H}) \parallel \frac{1}{j\omega C_L}}{(R_H + \frac{1}{j\omega C_H}) \parallel \frac{1}{j\omega C_L} + R_L} V_{in} \tag{24}$$

3.c





The numerator becomes zero when $\omega = 0$. The denominator becomes 0 when $\omega = \frac{j}{R_L C_L}, \frac{j}{R_H C_H}$. The maximum magnitude is 0, which makes sense because we cannot amplify the signal with passive elements.

3.d

First, we find K_{mic}, which is

$$\frac{1}{2} V_{pp} = K_{mic} \frac{j\omega \left(1 + \frac{j\omega}{\omega_{z1}}\right)}{\left(1 + \frac{j\omega}{\omega_{p1}}\right)\left(1 + \frac{j\omega}{\omega_{p2}}\right)\left(1 + \frac{j\omega}{\omega_{p3}}\right)} \tag{25}$$

$$\Rightarrow K_{mic} = \frac{\left(1 + \frac{j\omega}{\omega_{p1}}\right)\left(1 + \frac{j\omega}{\omega_{p2}}\right)\left(1 + \frac{j\omega}{\omega_{p3}}\right)}{2j\omega \left(1 + \frac{j\omega}{\omega_{z1}}\right)} V_{pp} \approx (8.6 - 0.5\mathrm{j})\,\mathrm{mV} \tag{26}$$

Then, we plug V_{mic} as the input voltage into the filters,

$$V_{LP} = \frac{2}{1 + \frac{j\omega}{200\pi}} V_{mic} \tag{27}$$

$$V_{BP} = \frac{4.54 \times 10^{-4} j\omega}{\left(1 + \frac{j\omega}{400\pi}\right)\left(1 + \frac{j\omega}{4000\pi}\right)} V_{mic} \tag{28}$$

$$V_{HP} = \frac{\frac{j\omega}{8000\pi}}{1 + \frac{j\omega}{8000\pi}} V_{mic} \tag{29}$$

3.e

With $V_{pp} = 5\,\mathrm{V}$, we want our amplitude of the phasor to be $2.5\,\mathrm{V}$ outside the amplifier. Thus, $G = \frac{2.5}{|V_{out}|}$. At $50\,\mathrm{Hz}$,

$$|V_{LP}(2\pi \cdot 50)| \approx 1.76\,\mathrm{V} \Rightarrow G = 1.42 \tag{30}$$

$$|V_{BP}(2\pi \cdot 50)| \approx 0.136\,\mathrm{V} \Rightarrow G = 18.4 \tag{31}$$

$$|V_{HP}(2\pi \cdot 50)| \approx 12.3\,\mathrm{mV} \Rightarrow G = 203 \tag{32}$$

At $1000\,\mathrm{Hz}$,

$$|V_{LP}(2\pi \cdot 1000)| \approx 0.109\,\mathrm{V} \Rightarrow G = 22.9 \tag{33}$$

$$|V_{BP}(2\pi \cdot 1000)| \approx 0.275\,\mathrm{V} \Rightarrow G = 9.09 \tag{34}$$

$$|V_{HP}(2\pi \cdot 1000)| \approx 0.133\,\mathrm{V} \Rightarrow G = 18.7 \tag{35}$$

At $8000\,\mathrm{Hz}$,

$$|V_{LP}(2\pi \cdot 8000)| \approx 10.6\,\mathrm{mV} \Rightarrow G = 235 \tag{36}$$

$$|V_{BP}(2\pi \cdot 8000)| \approx 58.8\,\mathrm{mV} \Rightarrow G = 42.5 \tag{37}$$

$$|V_{HP}(2\pi \cdot 1000)| \approx 0.380\,\mathrm{V} \Rightarrow G = 6.57 \tag{38}$$

4 Mystery Microphone

4.a

The microphone is most sensitive at frequencies $3 \times 10^2\,\mathrm{Hz}$—$5 \times 10^3\,\mathrm{Hz}$. The microphone is least sensitive mostly everywhere else, particularly $10\,\mathrm{Hz}$—$70\,\mathrm{Hz}$.

4.b

You would have the best time hearing the mid-range frequencies, since the mic is most responsive to those frequencies, and have a harder time hearing the lower-end and higher-end frequencies.

4.c

I would apply an amplification block for frequencies $10\,\mathrm{Hz}$—$70\,\mathrm{Hz}$ with an amplifier gain of 12.5. I would also apply an amplification block for frequencies above $10^4\,\mathrm{Hz}$ with an amplifier gain of 1.67.

6 Homework Process and Study Group

1. I used lecture note 2020-02-13 and Note 5.
2. I worked on this homework by myself.
3. I worked on this homework in one sitting.
4. 4 hours.