EECS 16A HW10

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1 Op-Amp in Negative Feedback

1.a

In an ideal op-amp, $u_+ = u_-$, so $u_+ - u_- = 0$.

1.b

v_x can be treated as the voltage divided from v_{out} across R_1, or

$$v_x = v_{out} \frac{R_1}{R_1 + R_2} \tag{1}$$

1.c

Performing KCL at the node between R_1, R_2, $I_{R_2} - I_{R_1} - I_{u_-} = 0$. Since the current into the terminals of an ideal op-amp are always zero, $I_{R_2} = I_{R_1}$. Then, by the fact that $v_s = v_x$

$$I_{R_1} = I_{R_2} = \frac{v_x}{R_1} = \frac{v_s}{R_1} \tag{2}$$

1.d

Given the equation from 1.b,

$$v_x = v_{out} \frac{R_1}{R_1 + R_2} \tag{3}$$

$$\Rightarrow v_{out} = v_x \frac{R_1 + R_2}{R_1} = v_x \left(1 + \frac{R_2}{R_1}\right) \tag{4}$$

$$= v_s \left(1 + \frac{R_2}{R_1}\right) \tag{5}$$

1.e

The current $i_L = \frac{v_{out}}{R}$. By KCL, $i_{\text{VDD}} + \cancel{i_{a+}} + \cancel{i_{a-}} - i_{\text{VSS}} - i_L = 0$,

$$\begin{cases} i_{\text{VDD}} - i_{\text{VSS}} = \frac{v_{out}}{R} & v_{out} > 0 \\ i_{\text{VSS}} - i_{\text{VDD}} = \frac{v_{out}}{R} & v_{out} < 0 \end{cases} \tag{6}$$

1.f

Given the fundamental op-amp equation

$$\begin{aligned} v_{out} &= A(v_s - v_x) \qquad (7)\\ &= A\left(v_s - v_{out}\frac{R_1}{R_1 + R_2}\right) \qquad (8)\\ v_{out}\left(1 + A\frac{R_1}{R_1 + R_2}\right) &= Av_s \qquad (9)\\ &= \frac{Av_s}{1 + A\frac{R_1}{R_1+R_2}} \cdot \frac{R_1 + R_2}{R_1 + R_2} \qquad (10)\\ &= v_s\frac{A(R_1 + R_2)}{R_2 + R_1(A+1)} \qquad (11) \end{aligned}$$

Plugging into the equation for v_x from 1.b,

$$v_x = v_s\frac{A\cancel{(R_1 + R_2)}}{R_2 + R_1(A+1)}\frac{R_1}{\cancel{R_1 + R_2}} = v_s\frac{AR_1}{R_2 + R_1(A+1)} \tag{12}$$

This means $v_x < v_s$ for any finite gain, independent of R.

1.g

$$\lim_{A\to\infty} v_{out} = \lim_{A\to\infty} v_s\frac{A(R_1 + R_2)}{R_2 + R_1(A+1)} = v_s\frac{R_1 + R_2}{R_1} = v_s\left(1 + \frac{R_2}{R_1}\right) \tag{13}$$

$$\lim_{A\to\infty} v_x = \lim_{A\to\infty} v_s\frac{AR_1}{R_2 + R_1(A+1)} = v_s\frac{R_1}{R_1} = v_s \tag{14}$$

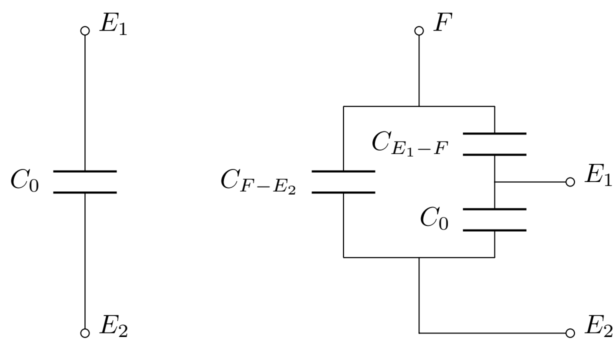
1.h

Solving for $G_{min} = 1.98$, or the smallest gain,

$$\begin{aligned} G_{min} = \frac{v_{out}}{v_s} &= \frac{A(R_1 + R_2)}{R_2 + R_1(A+1)} = 1.98 \qquad (15)\\ A(R_1 + R_2) &= 1.98(R_2 + AR_1 + R_1) \qquad (16)\\ A(R_1 + R_2) - 1.98AR_1 &= 1.98(R_1 + R_2) \qquad (17)\\ A &= \frac{1.98(R_1 + R_2)}{0.02R_1 + R_2} \qquad (18) \end{aligned}$$

2 Capacitive Touchscreen

2.a



2.b

$$C_0 = \epsilon \frac{w_1 \cdot d_2}{t_1} = 4.43 \times 10^{-14}\,\text{F} \tag{19}$$

$$C_{F-E_2} = \epsilon \frac{(w_2 - w_1) \cdot d_2}{t_2} = 2.215 \times 10^{-14}\,\text{F} \tag{20}$$

$$C_{E_1-F} = \epsilon \frac{w_1 \cdot d_1}{t_2 - t_1} = 4.43 \times 10^{-13}\,\text{F} \tag{21}$$

2.c

Trivially, the capacitance between E_1, E_2 with no finger is C_0. We can calculate the equivalent capacitance between E_1, E_2 when a finger is present as

$$C_0 + (C_{F-E_2} \parallel C_{E_1-F}) \tag{22}$$

Since $C_{F-E_2}, C_{E_1-F} > 0$, the equivalent capacitance when a finger is present *must* be greater than C_0, with a difference of $C_{F-E_2} \parallel C_{E_1-F} \approx 2.110 \times 10^{-14}\,\text{F}$.

3 Homework Process and Study Group

I did this homework by myself.