EECS 16A HW01

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1 Counting Solutions

1.1 A

Given our system of linear equations

$$2x + 3y = 5 \tag{1}$$
$$x + y = 2 \tag{2}$$

we convert it to the augmented matrix

$$\left[\begin{array}{cc|c} 2 & 3 & 5 \\ 1 & 1 & 2 \end{array}\right] \tag{3}$$

Performing the row operation $r_1 - 2r_2 \to r_1$,

$$\left[\begin{array}{cc|c} 0 & 1 & 1 \\ 1 & 1 & 2 \end{array}\right] \tag{4}$$

we see that $\boxed{y = 1}$. Substituting into Equation 2, it is clear that $\boxed{x = 1}$, yielding our *unique* solution.

1.2 B

Converting our system of equations to the augmented matrix

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 2 & 2 & 2 & 5 \end{array}\right] \tag{5}$$

Performing the row operation $r_2 - 2r_1 \to r_2$,

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 1 & 1 & 1 & -1 \end{array}\right] \tag{6}$$

Heuristically, this means that $3 = -1$. As this is a false statement, the system has *no* solutions.

1.3 C

It can be proven that if the system of equations can be described using a parameter t then the system has infinite solutions. This is because the set describes the line of intersection of the planes in $\mathbb{R}^3$. Thus, we only need to parameterize the system. Given the system

$$-y+2z=1 \tag{7}$$
$$2x+z=2 \tag{8}$$

Letting $z=t$, we can first solve for y, leaving us with

$$-y+2t=1 \tag{9}$$
$$-y=1-2t \tag{10}$$
$$y=2t-1 \tag{11}$$

Solving for x,

$$2x+t=2 \tag{12}$$
$$2x=2-t \tag{13}$$
$$x=\frac{2-t}{2} \tag{14}$$

The existence of the parametric line

$$f(t)=\begin{cases}\frac{2-t}{2}\\ 2t-1\\ t\end{cases} \tag{15}$$

proves that there are *infinite* solutions to the system along the line.

1.4 D

Converting our system of equations to the augmented matrix

$$\left[\begin{array}{cc|c} 1 & 2 & 3\\ 2 & -1 & 1\\ 3 & 1 & 4\end{array}\right] \tag{16}$$

Performing the row operation $r_2-2r_1\rightarrow r_2$,

$$\left[\begin{array}{cc|c} 1 & 2 & 3\\ 0 & -5 & -5\\ 3 & 1 & 4\end{array}\right] \tag{17}$$

which yields $\boxed{y=1}$. Substituting into Equation 1, we get $\boxed{x=1}$, which is our *unique* solution.[1]

[1] Equation 3 is redundant, since $(1,1)$ satisfies it too.

1.5 E

Converting our system of equations to the augmented matrix

$$\left[\begin{array}{cc|c} 1 & 2 & 3 \\ 2 & -1 & 1 \\ 1 & -3 & -5 \end{array}\right] \tag{18}$$

Performing the row operations $r_2 - 2r_1 \to r_2$ **and** $r_3 - r_1 \to r_3$,

$$\left[\begin{array}{cc|c} 1 & 2 & 3 \\ 0 & -5 & -5 \\ 0 & -5 & -8 \end{array}\right] \tag{19}$$

The above yields two solutions for y, $y = 1, \frac{8}{5}$. Since $1 \neq \frac{8}{5}$, we reach a logical contradiction and thus there is *no* solution to the system.

2 Filtering Out the Troll

2.1 A

Given our two microphone recordings

$$\mathbf{m}_1 = f_1(\alpha)\mathbf{a} + f_1(\beta)\mathbf{b} \tag{20}$$

$$\mathbf{m}_2 = f_2(\alpha)\mathbf{a} + f_2(\beta)\mathbf{b} \tag{21}$$

where $f_1(\theta) = \cos(\theta)$ and $f_2(\theta) = \sin(\theta)$, plugging in $\alpha = +45 \text{ deg} = \pi/4$ and $\beta = -30 \text{ deg} = -\pi/6$, we obtain

$$\mathbf{m}_1 = \frac{\sqrt{2}}{2}\mathbf{a} + \frac{\sqrt{3}}{2}\mathbf{b} \tag{22}$$

$$\mathbf{m}_2 = \frac{\sqrt{2}}{2}\mathbf{a} - \frac{1}{2}\mathbf{b} \tag{23}$$

2.2 B

Converting our system into an augmented matrix,

$$\left[\begin{array}{cc|c} \frac{\sqrt{2}}{2} & \frac{\sqrt{3}}{2} & \mathbf{m}_1 \\ \frac{\sqrt{2}}{2} & -\frac{1}{2} & \mathbf{m}_2 \end{array}\right] \tag{24}$$

Normalizing $\mathbf{b}$ in the first row gives us

$$\left[\begin{array}{cc|c} \frac{\sqrt{2}}{\sqrt{3}} & 1 & \frac{2}{\sqrt{3}}\mathbf{m}_1 \\ \frac{\sqrt{2}}{2} & -\frac{1}{2} & \mathbf{m}_2 \end{array}\right] \tag{25}$$

Performing the row operation $r_1 + 2r_2 \to r_1$,

$$\left[\begin{array}{cc|c} \sqrt{2}+\frac{\sqrt{2}}{\sqrt{3}} & 0 & \frac{2}{\sqrt{3}}\mathbf{m}_1 + 2\mathbf{m}_2 \\ \frac{\sqrt{2}}{2} & -\frac{1}{2} & \mathbf{m}_2 \end{array}\right] \tag{26}$$

$$\left[\begin{array}{cc|c} \frac{\sqrt{6}+\sqrt{2}}{\sqrt{3}} & 0 & \frac{2}{\sqrt{3}}\mathbf{m}_1 + 2\mathbf{m}_2 \\ \frac{\sqrt{2}}{2} & -\frac{1}{2} & \mathbf{m}_2 \end{array}\right] \tag{27}$$

If we extract row 1, we obtain the equation

$$\frac{\sqrt{6}+\sqrt{2}}{\sqrt{3}}\mathbf{a} = \frac{2}{\sqrt{3}}\mathbf{m}_1 + 2\mathbf{m}_2 \tag{28}$$

Simple algebra leads us to the equation

$$\mathbf{a} = \frac{2}{\sqrt{6}+\sqrt{2}}\mathbf{m}_1 + \frac{2\sqrt{3}}{\sqrt{6}+\sqrt{2}}\mathbf{m}_2 \tag{29}$$

$$\mathbf{a} = \frac{\sqrt{2}}{1+\sqrt{3}}\mathbf{m}_1 + \frac{\sqrt{6}}{1+\sqrt{3}}\mathbf{m}_2 \tag{30}$$

leaving us with $\boxed{u = \dfrac{\sqrt{2}}{1+\sqrt{3}}, v = \dfrac{\sqrt{6}}{1+\sqrt{3}}}$.

2.3 C

The recovered speech is

> *All human beings are born free and equal in dignity and rights.*

-Universal Declaration of Human Rights

prob1

September 5, 2019

1 EECS16A: Homework 1

1.1 Problem 2: Filtering Out The Troll

[1]:
```
import warnings
import wave as wv

import matplotlib.pyplot as plt
import numpy as np
import scipy
import scipy.io.wavfile
from IPython.display import Audio
from scipy import io
from scipy.io.wavfile import read

# For this to work make sure to download m1.wav and m2.wav to the same location␣
↪as this jupyter notebook
warnings.filterwarnings("ignore")
sound_file_1 = "m1.wav"
sound_file_2 = "m2.wav"
```

Let's listen to the recording of the first microphone (it can take some time to load the sound file). Run the cell below, then press the play button to listen.

[2]:
```
Audio(url="m1.wav", autoplay=False)
```

[2]: <IPython.lib.display.Audio object>

And this is the recording of the second microphone (it can take some time to load the sound file). Run the cell below, then press the play button to listen.

[3]:
```
Audio(url="m2.wav", autoplay=False)
```

[3]: <IPython.lib.display.Audio object>

We read the first recording to the variable `corrupt1` and the second recording to `corrupt2`. Treat `corrupt1` and `corrupt2` as the two sound recordings picked up by microphone 1 and microphone 2 respectively.

[4]:
```
rate1, corrupt1 = scipy.io.wavfile.read("m1.wav")
rate2, corrupt2 = scipy.io.wavfile.read("m2.wav")
```

Enter the weights of the two recordings to get the clean speech.
Note: The square root of a number a can be written as `np.sqrt(a)` in IPython.

[5]:
```
# enter the weights u (recording 1) and v (recording 2)
u = np.sqrt(2) / (1 + np.sqrt(3))
v = np.sqrt(6) / (1 + np.sqrt(3))
```

Weighted combination of the two recordings:

[6]:
```
a = u * corrupt1 + v * corrupt2
```

Let's listen to the resulting sound file (make sure your speaker's volume is not very high, the sound may be loud if things go wrong).

[7]:
```
Audio(data=a, rate=rate1)
```

[7]:
```
<IPython.lib.display.Audio object>
```

3 Homework Process and Study Group

I did this homework by myself.