EECS 16B MT1 Redo

Bryan Ngo

2020-03-12

1 CMOS Circuits

$$\begin{array}{c|c|l} V_{in,1} & V_{in,2} & V_o \\ \hline 0\,\mathrm{V} & 0\,\mathrm{V} & V_{DD} \\ V_{DD} & 0 & 0 \\ 0 & V_{DD} & 0 \\ V_{DD} & V_{DD} & 0 \end{array} \tag{1}$$

2 Differential Equations

2.a

$$\frac{d}{dt}M(t) = -rM(t) \tag{2}$$

$$\int \frac{1}{M(t)}\,dM = \int -r\,dt \tag{3}$$

$$\ln|M(t)| = -rt + C \tag{4}$$

$$M(t) = Ce^{-rt} \tag{5}$$

$$M(0) = M_0 = C \tag{6}$$

$$\Rightarrow M(t) = M_0 e^{-rt} \tag{7}$$

2.b

$$\cancel{M_0}e^{-rt} = \frac{1}{2}\cancel{M_0} \tag{8}$$

$$-rt = \ln\left(\frac{1}{2}\right) = -\ln(2) \tag{9}$$

$$t = \frac{\ln(2)}{r} \tag{10}$$

3 Complex Numbers

3.a

3.a.i

$$|1+j| = \sqrt{2} \tag{11}$$
$$\arg(1+j) = \frac{\pi}{4} \tag{12}$$
$$\Rightarrow 1+j = \sqrt{2}e^{j\frac{\pi}{4}} \tag{13}$$

3.a.ii

$$\sqrt{j} = \sqrt{e^{j\frac{\pi}{2}}} = e^{j\frac{\pi}{4}} \tag{14}$$

3.b

3.b.i

$$3e^{j\frac{\pi}{3}} = 3\left(\cos\left(\frac{\pi}{3}\right) + j\sin\left(\frac{\pi}{3}\right)\right) = \frac{3}{2} + j\frac{3\sqrt{3}}{2} \tag{15}$$

3.b.ii

$$-\sqrt{7}e^{j\pi} = \sqrt{7} \tag{16}$$

3.c

3.c.i

Proof.

$$\frac{1}{j} \cdot \frac{j}{j} = \frac{j}{j^2} = -j \tag{17}$$

□

3.c.ii

Proof.

$$\begin{aligned}
\sin(2x) &= \frac{e^{2x} - e^{-2x}}{2j} = \frac{(e^x)^2 - (e^{-x})^2}{2j} && (18)\\
&= \frac{(\cos x + j\sin x)^2 - (\cos x - j\sin x)^2}{2j} && (19)\\
&= \frac{\cancel{\cos^2(x)} + 2j\cos(x)\sin(x) - \cancel{\sin^2(x)} - \cancel{\cos^2(x)} + 2j\cos(x)\sin(x) + \cancel{\sin^2(x)}}{2j} && (20)\\
&= \frac{4j\cos(x)\sin(x)}{2j} = 2\cos(x)\sin(x) && (21)
\end{aligned}$$

□

4 Vector Differential Equation

4.a

To derive $\boldsymbol{D}$,

$$\frac{d}{dt}\boldsymbol{z} = \boldsymbol{D}\boldsymbol{z} \tag{22}$$

$$\frac{d}{dt}\boldsymbol{T}\boldsymbol{x} = \boldsymbol{D}\boldsymbol{T}\boldsymbol{x} \tag{23}$$

$$\frac{d}{dt}\boldsymbol{x} = \underbrace{\boldsymbol{T}^{-1}\boldsymbol{D}\boldsymbol{T}}_{\boldsymbol{A}}\boldsymbol{x} \tag{24}$$

$$\Rightarrow \boldsymbol{D} = \boldsymbol{T}\boldsymbol{A}\boldsymbol{T}^{-1} \tag{25}$$

where $\boldsymbol{T} = \boldsymbol{V}^{-1} = \begin{bmatrix}\boldsymbol{v}_1 & \boldsymbol{v}_2\end{bmatrix}^{-1}$. Thus,

$$\boldsymbol{T} = \begin{bmatrix} j & -j \\ 1 & 1 \end{bmatrix}^{-1} = \frac{1}{2j}\begin{bmatrix} 1 & j \\ -1 & j \end{bmatrix} \tag{26}$$

$$\boldsymbol{D} = \boldsymbol{T}\boldsymbol{A}\boldsymbol{T}^{-1} = \frac{1}{2j}\begin{bmatrix} 1 & j \\ -1 & j \end{bmatrix}\begin{bmatrix} \alpha & -\omega \\ \omega & \alpha \end{bmatrix}\begin{bmatrix} j & -j \\ 1 & 1 \end{bmatrix} \tag{27}$$

$$= \frac{1}{2j}\begin{bmatrix} 1 & j \\ -1 & j \end{bmatrix}\begin{bmatrix} j\alpha - \omega & -j\alpha - \omega \\ j\omega + \alpha & -j\omega + \alpha \end{bmatrix} \tag{28}$$

$$= \frac{1}{2j}\begin{bmatrix} j\alpha - \omega - \omega + j\alpha & \cancel{-j\alpha - \omega + \omega + j\alpha} \\ \cancel{-j\alpha + \omega - \omega + j\alpha} & j\alpha + \omega + \omega + j\alpha \end{bmatrix} \tag{29}$$

$$= -\frac{j}{2}\begin{bmatrix} 2j\alpha - 2\omega & 0 \\ 0 & 2j\alpha + 2\omega \end{bmatrix} = \begin{bmatrix} \alpha + j\omega & 0 \\ 0 & \alpha - j\omega \end{bmatrix} \tag{30}$$

4.b

$$\frac{d}{dt}\boldsymbol{z} = \begin{bmatrix} \alpha + j\omega & 0 \\ 0 & \alpha - j\omega \end{bmatrix}\boldsymbol{z} \tag{31}$$

$$\Rightarrow \boldsymbol{z} = \begin{bmatrix} z_a e^{(\alpha + j\omega)t} \\ z_b e^{(\alpha - j\omega)t} \end{bmatrix} \tag{32}$$

$$\boldsymbol{z}(0) = \boldsymbol{T}\boldsymbol{x}(0) = -\frac{j}{2}\begin{bmatrix} 1 & j \\ -1 & j \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -\frac{j}{2} \\ \frac{j}{2} \end{bmatrix} \tag{33}$$

4.c

$$\boldsymbol{x} = \boldsymbol{T}^{-1}\boldsymbol{z} = \begin{bmatrix} j & -j \\ 1 & 1 \end{bmatrix}\begin{bmatrix} -\frac{j}{2}e^{(\alpha + j\omega)t} \\ \frac{j}{2}e^{(\alpha - j\omega)t} \end{bmatrix} \tag{34}$$

$$x_1 = \frac{1}{2}e^{(\alpha + j\omega)t} + \frac{1}{2}e^{(\alpha - j\omega)t} \tag{35}$$

$$= e^{\alpha t}\left(\frac{e^{j\omega t} + e^{-j\omega t}}{2}\right) \tag{36}$$

$$= e^{\alpha t}\cos(\omega t) \tag{37}$$

5 Transient Identification

5.a $v(t) = V_0 e^{\lambda t}, \lambda < 0$

RC, RCRC. RL

5.b $v(t) = V_0 e^{jt}$

None

5.c $v(t) = V_0 e^{\alpha t} \cos(\omega t), \alpha < 0$

Underdamped RLC

5.d $v(t) = V_1 e^{\alpha t} + V_2 e^{\beta t};\ \alpha, \beta < 0$

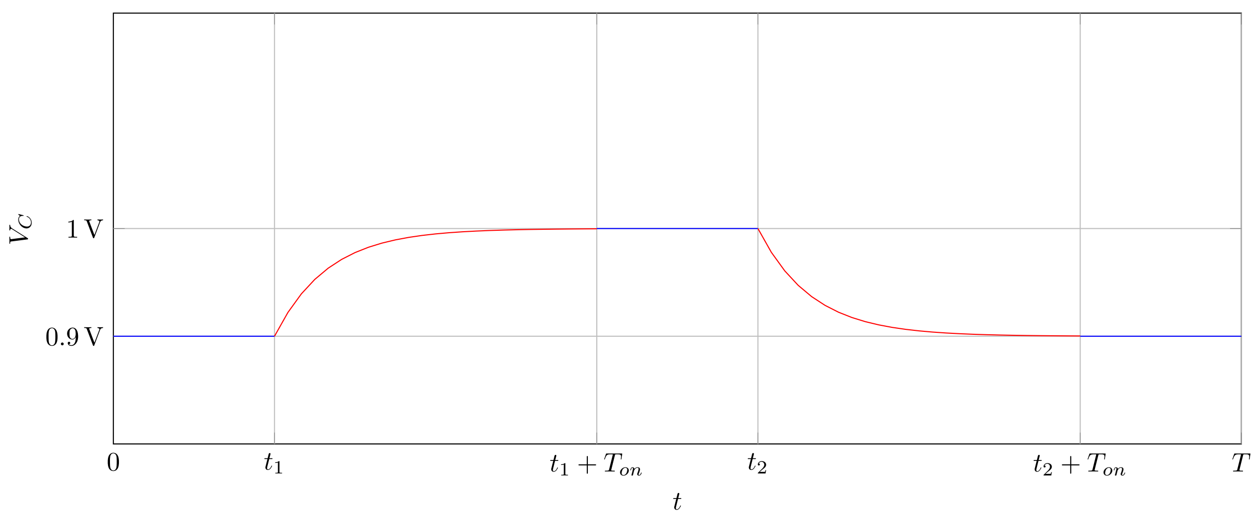
RCRC, overdamped RLC

6 Phasors

	Phasor	Waveform
(a)	$e^{j\frac{\pi}{2}}$	5
(b)	$3e^{j\frac{\pi}{2}}$	6
(c)	1	3
(d)	$e^{-j\frac{\pi}{2}}$	1
(e)	$e^{-j\pi}$	2
(f)	$e^{j\cdot 0} + e^{j\frac{\pi}{2}}$	4

7 Energy

7.a



7.b

Since the $1.0\,\mathrm{V}$ source is only connected during the time $t \in [t_1, t_1 + T_{on}]$, we only need to consider the change in voltage across those points. Using the definition of capacitance,

$$Q = C\Delta V_c = C(1.0 - 0.9) = 0.1C\,\mathrm{C} \tag{38}$$

7.c

Since at any given time *at most* only one transistor with equivalent gate resistance is connected in series with the capacitor, the energy dissipated across the entire circuit is simply the change in energy across the capacitor from $t \in [0, T]$. Since the change in energy of a capacitor is only dependent on voltage, we only concern ourselves with whenever the voltage changes, i.e. $t \in [t_1, t_1 + T_{on}] \cup [t_2, t_2 + T_{on}]$,

$$\begin{aligned}
\Delta E_{tot} &= |\ \Delta E|_{t_1}^{t_1+T_{on}} + \Delta E|_{t_2}^{t_2+T_{on}}\ | && (39)\\
&= \frac{1}{2}C(V(t_1+T_{on})^2 - V(t_1)^2) + \frac{1}{2}C(V(t_2)^2 - V(t_2+T_{on})^2) && (40)\\
&= C(1^2 - 0.9^2) = 0.19C\,\mathrm{J} && (41)
\end{aligned}$$