EECS 16A HW08

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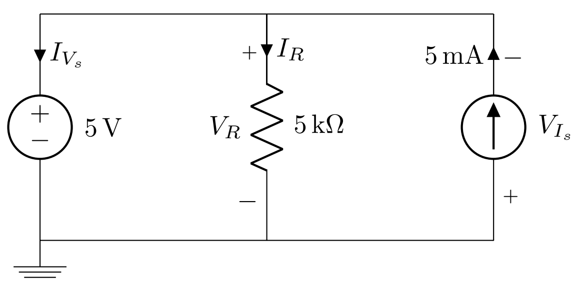
1 Power Analysis



1.a

The power equations are

$$P_R = I_R V_R \tag{1}$$
$$P_{V_s} = I_{V_s} V_s \tag{2}$$
$$P_{I_s} = I_s V_{I_s} \tag{3}$$

Using superposition, we can easily determine the voltage V_R. Setting $I_s = 0$, KVL means that $V_R = V_s$. Setting $V_s = 0$, the short circuit means there is no voltage across the resistor, meaning that the total voltage across the resistor is V_s. KVL also tells us that $V_{I_s} = -V_s$. The current through our resistor is $I_R = \frac{V_R}{R} = \frac{V_s}{R}$. KCL tells us that $I_{V_s} = I_s - I_R = I_s - \frac{V_s}{R}$. This means our final power equations are

$$P_R = \frac{V_s^2}{R} \tag{4}$$
$$P_{V_s} = \left(I_s - \frac{V_s}{R}\right) V_s \tag{5}$$
$$P_{I_s} = I_s(-V_s) \tag{6}$$

1.b

Calculating the power draw of each element,

$$P_R = \frac{V_s^2}{R} = \frac{25\ \text{V}^2}{5\ \Omega} = 5\,\text{mW} \tag{7}$$

$$P_{V_s} = \left(I_s - \frac{V_s}{R}\right) V_s = (4\,\text{mA})(5\,\text{V}) = 20\,\text{mW} \tag{8}$$

$$P_{I_s} = I_s(-V_s) = (5\,\text{mA})(-5\,\text{V}) = -25\,\text{mW} \tag{9}$$

1.c

Solving for the new current,

$$I_s = \frac{40\ \text{mW}}{-5\ \text{V}} = -8\,\text{mA} \tag{10}$$

Recalculating our power draws,

$$P_R = \frac{V_s^2}{R} = \frac{25\ \text{V}^2}{5\ \Omega} = 5\,\text{mW} \tag{11}$$

$$P_{V_s} = \left(I_s - \frac{V_s}{R}\right) V_s = (-9\,\text{mA})(5\,\text{V}) = -45\,\text{mW} \tag{12}$$

$$P_{I_s} = I_s(-V_s) = (-8\,\text{mA})(-5\,\text{V}) = 40\,\text{mW} \tag{13}$$

2 Maximum Power Transfer

2.a

The voltage across R_L is

$$V_L = I_L R_L \tag{14}$$

In order to maximize the voltage drop, $R_L = \infty\ \Omega$, i.e. an open circuit. Intuitively, this makes sense because an infinite electric potential barrier implies infinite resistance to current flow. This means that

$$V_L = \infty\ \text{V} \tag{15}$$

$$I_L = 0\,\text{A} \tag{16}$$

P_L is indeterminate since we get $P_L = 0 \cdot \infty$.

2.b

The current through R_L is

$$I_L = \frac{V_L}{R_L} \tag{17}$$

If we take a limit,

$$\lim_{R_L \to 0+} I_L = \lim_{R_L \to 0+} \frac{V_L}{R_L} = \infty \tag{18}$$

So the resistance for maximum current is exactly $0\,\Omega$, i.e. a short circuit. This means that

$$V_L = 0\,\mathrm{V} \tag{19}$$
$$I_L = I_L \tag{20}$$
$$P_L = 0 \cdot I_L = 0\,\mathrm{W} \tag{21}$$

2.c

The power through R_L is

$$P_{R_L} = I_L V_L \tag{22}$$

Using equivalent resistance and the voltage divider, we can determine I_L, V_L to be

$$I_L = \frac{V_s}{R_s + R_L} \tag{23}$$
$$V_L = V_s \frac{R_L}{R_S + R_L} \tag{24}$$

Plugging in, our new power equation is

$$P_{R_L} = \frac{V_s^2 R_L}{(R_s + R_L)^2} \tag{25}$$

Taking the derivative with respect to R_L on both sides and setting to zero,

$$\frac{dP_{R_L}}{dR_L} = \cancel{V_s^2}((R_s + R_L)^{-2} - 2R_L(R_s + R_L)^{-3}) = 0 \tag{26}$$
$$\frac{1}{(R_s + R_L)^2} = \frac{2R_L}{(R_s + R_L)^3} \tag{27}$$
$$R_s + R_L = 2R_L \tag{28}$$
$$R_L = R_s \tag{29}$$

Calculating our values,

$$V_L = V_s \frac{R_L}{R_S + R_L} = (100\,\mu\mathrm{V}) \frac{50\ \Omega}{100\ \Omega} = 50\,\mu\mathrm{V} \tag{30}$$
$$I_L = \frac{V_s}{R_s + R_L} = \frac{100\ \mu\mathrm{V}}{2 \cdot 50\ \Omega} = 1\,\mu\mathrm{A} \tag{31}$$
$$P_L = \frac{V_s^2 R_L}{(R_s + R_L)^2} = \frac{(100\ \mu\mathrm{V})^2 \cdot 50\ \Omega}{(100\ \Omega)^2} = 50\,\mathrm{pW} \tag{32}$$

2.d

The optimal value of any antenna's ESR will always be exactly equal to that of the transmitter's ESR.

3 Cell Phone Battery

3.a

Assuming the typical power draw of 0.3 W and capacity of 2.770 A h, we find the current draw at 3.8 V,

$$I = \frac{P}{V} = \frac{3}{38}\ \mathrm{A} \tag{33}$$

Then, using the equation for current and solving for time,

$$t = \frac{Q}{I} \approx 35.09\,\mathrm{h} \tag{34}$$

3.b

Converting to coulombs,

$$\frac{2.770\ \mathrm{A\,h}}{1} \cdot \frac{3600\ \mathrm{s}}{1\ \mathrm{h}} = 9972\,\mathrm{A\,s} = 9972\,\mathrm{C} \tag{35}$$

Converting to elementary charges,

$$\frac{9972\ \mathrm{C}}{1} \cdot \frac{1\ e}{1.602\,176\,634 \times 10^{-19}\ \mathrm{C}} = 6.224\,032\,85 \times 10^{22}\ e \tag{36}$$

3.c

Using the equation for power and integrating with respect with time,

$$\int P\,dt = U = IVt = QV = (9972\,\mathrm{C})(3.8\,\mathrm{V}) = 37\,893.6\,\mathrm{J} \tag{37}$$

3.d

First, we can convert the price per kilowatt-hour into joules to simplify calculation,

$$\frac{\$0.12}{1000\ \mathrm{W\,h}} \cdot \frac{1\ \mathrm{h}}{3600\ \mathrm{s}} \approx \$3.33 \times 10^{-8}\,\mathrm{J}^{-1} \tag{38}$$

Then we simply multiply that value by the total energy of our battery, multiplied by the number of days,

$$\text{Price} = \$3.33 \times 10^{-8}\,\mathrm{J}^{-1} \cdot 37\,893.6\,\mathrm{J} \cdot 31 \approx \$0.04 \tag{39}$$

3.e

Using the equation from Problem 2,

$$P_{R_{\text{bat}}} = \frac{25R_{\text{bat}}}{(0.2\ \Omega + R_{\text{bat}})^2} \tag{40}$$

Our dissipated power is

$$1\,\text{m}\Omega \Rightarrow \frac{25 \cdot 1\ \text{m}\Omega}{(201\ \text{m}\Omega)^2} \approx 618.80\,\text{mW} \tag{41}$$

$$1\,\Omega \Rightarrow \frac{25 \cdot 1\ \Omega}{(1.2\ \Omega)^2} \approx 17.36\,\text{W} \tag{42}$$

$$10\,\text{k}\Omega \Rightarrow \frac{25 \cdot 10\ \text{k}\Omega}{(0.2\ \Omega + 10\ \text{k}\Omega)^2} \approx 2.5\,\text{mW} \tag{43}$$

Using the equation for energy and rearranging for time such that $t = \frac{U}{P}$, we can use $U = 37\,893.6\,\text{J}$ and the given power draws,

$$t_{0.1} = 6.12 \times 10^4\,\text{s} \tag{44}$$

$$t_1 = 2.18 \times 10^3\,\text{s} \tag{45}$$

$$t_{10000} = 1.51 \times 10^7\,\text{s} \tag{46}$$

4 Measuring Voltage and Current

4.a

First, we calculate the equivalent resistance of R_2, R_{VM},

$$R_{eq} = R_2 \parallel R_{\text{VM}} \approx 199.96\,\Omega \tag{47}$$

Then, since the voltmeter is in parallel with R_2, the voltage measured across it should be the same as the voltage measured across R_{eq}, which is

$$V_{R_{eq}} = V_S \frac{R_{eq}}{R_1 + R_{eq}} \approx 3.33\,\text{V} \tag{48}$$

4.b

Repeating with $R_1 = R_2 = 10\,\text{M}\Omega$,

$$R_{eq} = R_2 \parallel R_{\text{VM}} = 0.91\,\text{M}\Omega \tag{49}$$

$$V_{R_{eq}} = V_S \frac{R_{eq}}{R_1 + R_{eq}} \approx 0.42\,\text{V} \tag{50}$$

The voltmeter is no longer reliable because ideally, the voltmeter should measure 2.5 V, but since the voltmeter's resistance is now within an order of magnitude of the circuit elements' resistance, it is changing the behavior of the circuit.

4.c

The theoretical value of the voltage across R_2, i.e. v_{out}, should be 2.5 V due to KVL. Using the equation for percent change,

$$-0.1 \leq \frac{v_{\text{meas}} - v_{\text{out}}}{v_{\text{out}}} \leq 0.1 \tag{51}$$

$$-0.1 v_{\text{out}} \leq v_{\text{meas}} - v_{\text{out}} \leq 0.1 v_{\text{out}} \tag{52}$$

$$2.25 \leq v_{\text{meas}} \leq 2.75 \tag{53}$$

Thus, we have two bounds for which to find the resistance for. Solving for the lower bound,

$$2.25 \leq 5 \frac{R_1 \parallel 10^6}{R_1 + R_1 \parallel 10^6} \tag{54}$$

$$0.45 R_1 + 0.45(R_1 \parallel R_{\text{VM}}) \leq R_1 \parallel R_{\text{VM}} \tag{55}$$

$$0.45 R_1 \leq 0.55 \left(\frac{1}{R_1} + 10^{-6} \right)^{-1} \tag{56}$$

$$\frac{11}{9R_1} - \frac{1}{R_1} \geq 10^{-6} \tag{57}$$

Doing the same for the upper bound,

$$2.75 \geq 5 \frac{R_1 \parallel 10^6}{R_1 + R_1 \parallel 10^6} \tag{58}$$

$$0.55 R_1 + 0.55(R_1 \parallel R_{\text{VM}}) \geq R_1 \parallel R_{\text{VM}} \tag{59}$$

$$0.55 R_1 \geq 0.45 \left(\frac{1}{R_1} + 10^{-6} \right)^{-1} \tag{60}$$

$$\frac{9}{11R_1} - \frac{1}{R_1} \leq 10^{-6} \tag{61}$$

Through some simple graphing, we find that the largest value of R_1 that satisfies these constraints is $R_2 = \frac{2 \times 10^6}{9}\ \Omega$.

4.d

The current through R_1 theoretically is $I = \frac{V}{R} = 5\,\text{mA}$. With the voltmeter connected, the current through is (ignoring units) V_{VM},

$$V_{\text{VM}} = 5 \frac{(1 \parallel 10^6)\ \Omega}{1\ \text{k}\Omega + (1 \parallel 10^6)\ \Omega} \approx 4.995\,\text{mA} \tag{62}$$

4.e

We can construct a similar inequality to the one from Subproblem 4.c,

$$4.5 \times 10^{-3} \leq I_{\text{meas}} \leq 5.5 \times 10^{-3} \tag{63}$$

Our new lower bound is

$$4.5 \times 10^{-3} \leq 5\frac{1 \parallel 10^6}{R_1 + (1 \parallel 10^6)} \tag{64}$$

$$4.5 \times 10^{-3}(R_1 + (1 \parallel 10^6)) \leq 5(1 \parallel 10^6) \tag{65}$$

$$R_1 \leq \frac{991}{9}(1 \parallel 10^6) \approx 110.11\,\Omega \tag{66}$$

For the upper bound,

$$5.5 \times 10^{-3} \geq 5\frac{1 \parallel 10^6}{R_1 + (1 \parallel 10^6)} \tag{67}$$

$$5.5 \times 10^{-3}(R_1 + (1 \parallel 10^6)) \geq 5(1 \parallel 10^6) \tag{68}$$

$$R_1 \geq \frac{9989}{11}(1 \parallel 10^6) \approx 908.09\,\Omega \tag{69}$$

meaning our maximum value of R_1 is about $909.09\,\Omega$.

7 Homework Process and Study Group

I did this homework by myself.