EECS 16B HW08

Bryan Ngo

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1 SVD

$$\boldsymbol{A} = \begin{bmatrix} -1 & 1 & 5 \\ 3 & 1 & -1 \\ 2 & -1 & 4 \end{bmatrix} \tag{1}$$

1.a

$$[\boldsymbol{A}]^\top \boldsymbol{A} = \begin{bmatrix} -1 & 3 & 2 \\ 1 & 1 & -1 \\ 5 & -1 & 4 \end{bmatrix} \begin{bmatrix} -1 & 1 & 5 \\ 3 & 1 & -1 \\ 2 & -1 & 4 \end{bmatrix} = \begin{bmatrix} 14 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 42 \end{bmatrix} \tag{2}$$

$$\boldsymbol{A}\,[\boldsymbol{A}]^\top = \begin{bmatrix} -1 & 1 & 5 \\ 3 & 1 & -1 \\ 2 & -1 & 4 \end{bmatrix} \begin{bmatrix} -1 & 3 & 2 \\ 1 & 1 & -1 \\ 5 & -1 & 4 \end{bmatrix} = \begin{bmatrix} 27 & -7 & 17 \\ -7 & 11 & 1 \\ 17 & 1 & 21 \end{bmatrix} \tag{3}$$

1.b

Using $[\boldsymbol{A}]^\top \boldsymbol{A}$,

$$\lambda = \{42 \geqslant 14 \geqslant 3\} \tag{4}$$

$$\boldsymbol{v} = \{\hat{\boldsymbol{k}}, \hat{\boldsymbol{\imath}}, \hat{\boldsymbol{\jmath}}\} \tag{5}$$

Thus, our $\boldsymbol{u}_i$ is

$$\boldsymbol{u}_1 = \frac{1}{\sqrt{42}} \boldsymbol{A} \boldsymbol{v}_1 = \frac{1}{\sqrt{42}} \begin{bmatrix} 5 \\ -1 \\ 4 \end{bmatrix} \tag{6}$$

$$\boldsymbol{u}_2 = \frac{1}{\sqrt{14}} \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix} \tag{7}$$

$$\boldsymbol{u}_3 = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \tag{8}$$

So our SVD is

$$\boldsymbol{A} = \frac{1}{\sqrt{42}} \begin{bmatrix} 5 \\ -1 \\ 4 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} + \frac{1}{\sqrt{14}} \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} + \frac{1}{\sqrt{3}} \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \tag{9}$$

2 Rank 1 Decomposition

2.a

$\boldsymbol{C}_1$ has rank 2, so we need 2 rank 1 matrices to represent it.

$$\boldsymbol{C}_1 = \frac{1}{\sqrt{8}} \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \\ 1 \\ -1 \\ 1 \\ -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} + \frac{1}{\sqrt{8}} \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \\ -1 \\ 1 \\ -1 \\ 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{10}$$

2.b

$$\boldsymbol{C}_2 = \frac{1}{2} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix} \tag{11}$$

2.c

2.c.i

$$\boldsymbol{F}_{\text{CH}} = \frac{1}{\sqrt{3}} \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \end{bmatrix} + \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \end{bmatrix} \tag{12}$$

2.c.ii

$$\boldsymbol{F}_{\text{CH}} = \frac{1}{\sqrt{3}} \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \end{bmatrix} \tag{13}$$

3 Open-Loop Control of SIXT33N

$$u[t] = \frac{v^* + \beta}{\theta} \tag{14}$$

$$v_L[t] = d_L[t+1] - d_L[t] = \theta_L u_L[t] - \beta_L \tag{15}$$

$$v_R[t] = d_R[t+1] - d_R[t] = \theta_R u_R[t] - \beta_R \tag{16}$$

3.a

Inductively finding $d_L[1], d_R[1]$,

$$d_L[1] = v_L[0] + \cancelto{0}{d_L[0]} = \theta_L u_L[0] - \beta_L = 255\theta_L - \beta_L \tag{17}$$

$$d_L[t+1] = v_L[t] + d_L[t] = \theta_L u_L[t] - \beta_L + t(255\theta_L - \beta_L) \tag{18}$$

$$d_L[t_p] = t_p(255\theta_L - \beta_L) \tag{19}$$

$$d_R[t_p] = t_p(255\theta_R - \beta_R) \tag{20}$$

3.b

$$\delta[t_p] = t_p(255\theta_L - 255\theta_R - \beta_L + \beta_R) \tag{21}$$

If $\theta_L = \theta_R$ and $\beta_L = \beta_R$, $\delta[t_p] = 0$, so the car does not turn after the pulse. Similarly, if $\theta_L \neq \theta_R$ and $\beta_L \neq \beta_R$, $\delta[t_p] \neq 0$ except in a few degenerate cases, so the car will generally be turning after the pulse.

3.c

By induction,

$$\delta[t_p+1] = d_L[t_p+1] - d_R[t_p+1] \tag{22}$$

$$= \left(\theta_L \frac{v^* + \beta_L}{\theta_L} - \beta_L\right) - \left(\theta_R \frac{v^* + \beta_R}{\theta_R} - \beta_R\right) + \delta_0 = \delta_0 \tag{23}$$

$$\Rightarrow \lim_{t\to\infty} \delta[t] = \delta_0 \tag{24}$$

The car does not seem to deviate from δ_0.

3.d

$$\delta[t_p+1] = \left((\theta_L + \Delta\theta_L)\frac{v^* + \beta_L}{\theta_L} - (\beta_L + \Delta\beta_L)\right) - \left((\theta_R + \Delta\theta_R)\frac{v^* + \beta_R}{\theta_R} - (\beta_R + \Delta\beta_R)\right) + \delta_0 \tag{25}$$

$$= \left(\frac{\Delta\theta_L(v^* + \beta_L)}{\theta_L} - \Delta\beta_L\right) - \left(\frac{\Delta\theta_R(v^* + \beta_R)}{\theta_R} - \Delta\beta_R\right) + \delta_0 \tag{26}$$

$$\Rightarrow \delta[t]|_{t>t_p} = t \cdot \delta[t_p+1] \tag{27}$$

The car seems to deviate from δ_0, assuming nonzero deviance factors.

4 Closed-Loop Control of SIXT33N

$$d_L[k+1] - d_L[k] = v^* - k_L\delta[k] \tag{28}$$

$$d_R[k+1] - d_R[k] = v^* - k_R\delta[k] \tag{29}$$

$$u_L[k] = \frac{v^* + \beta_L}{\theta_L} - k_L\frac{\delta[k]}{\theta_L} \tag{30}$$

$$u_R[k] = \frac{v^* + \beta_R}{\theta_R} + k_R\frac{\delta[k]}{\theta_R} \tag{31}$$

4.a

$$\delta[k+1] = d_L[k+1] - d_R[k+1] \tag{32}$$
$$= \cancel{v^*} - k_L\delta[k] + d_L[k] - \cancel{v^*} - k_R\delta[k] - d_R[k] \tag{33}$$
$$= (1 - k_L - k_R)\delta[k] \tag{34}$$

4.b

The eigenvalue is $\lambda = 1 - k_L - k_R$. $\lambda \in (-1, 0] \cup [0, 1)$ is functionally identical to $\lambda \in (-1, 1)$ because $0 \in (-1, 1)$. Finding the values of k_L, k_R,

$$1 - k_L - k_R \in (-1, 1) \tag{35}$$
$$-k_L - k_R \in (-2, 0) \tag{36}$$
$$k_L + k_R \in (0, -2) \tag{37}$$

4.c

Substituting u_L, u_R into their respective distance equations,

$$\delta[k+1] = \left((\theta_L + \Delta\theta_L)\left(\frac{v^* + \beta_L - k_L\delta[k]}{\theta_L}\right) - (\beta_L + \Delta\beta_L)\right) - \left((\theta_R + \Delta\theta_R)\left(\frac{v^* + \beta_R + k_R\delta[k]}{\theta_R}\right) - (\beta_R + \Delta\beta_R)\right) + \delta[k] \tag{38}$$

$$= \left(\Delta\theta_L\left(\frac{v^* + \beta_L - k_L\delta[k]}{\theta_L}\right) - k_L\delta[k] - \Delta\beta_L\right) - \left(\Delta\theta_R\left(\frac{v^* + \beta_R - k_R\delta[k]}{\theta_R}\right) + k_R\delta[k] - \Delta\beta_R\right) + \delta[k] \tag{39}$$

$$= \underbrace{\left(\Delta\theta_L\left(\frac{v^* + \beta_L}{\theta_L}\right) - \Delta\beta_L\right) - \left(\Delta\theta_R\left(\frac{v^* + \beta_R}{\theta_R}\right) - \Delta\beta_R\right)}_{c} + \underbrace{\left(1 - k_L - k_R + \frac{\Delta\theta_L k_L}{\theta_L} + \frac{\Delta\theta_R k_R}{\theta_R}\right)}_{\lambda}\delta[k] \tag{40}$$

This is better than open-loop control because we reduce the difference by an exponential amount rather than having it explode to infinity.

6 Homework Process and Study Group

1. I used Lecture 9B notes.
2. I worked on this homework by myself.
3. I worked on this homework in one sitting,
4. Questions 3 and 4 are cumbersome.
5. 3 hours.