EECS 16A HW09

Bryan Ngo

2019-10-30

1 Thévenin and Norton Equivalent Circuits

In order to find V_{oc}, we must find R_{eq},

$$R_{eq} = R_1 + (R_2 \parallel (R_3 + R_4)) = 5\,\Omega \tag{1}$$

$$I_{eq} = \frac{V}{R_e q} = 2\,\text{A} \tag{2}$$

By KVL, we know that the voltage going through the $R_2 \parallel (R_3 + R_4)$ branch must be $4\,\text{V}$. Then, the current going through $R_3 + R_4$ is $0.66\,\text{A}$. Then, the voltage is

$$V_{oc} = 0.66\,\text{A} \cdot R_3 = 3\,\text{V} \tag{3}$$

Now, finding I_{sc}, we short over R_3. Finding the equivalent current,
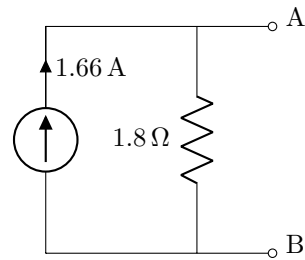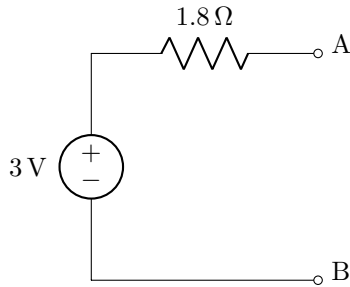
$$R_{eq} = R_1 + (R_2 \parallel R_4) = 4\,\Omega \tag{4}$$

$$I_{eq} = \frac{V}{R_{eq}} = 2.5\,\text{A} \tag{5}$$

By KVL, the voltage of the parallel branch is $2.5\,\text{V}$. Then, the current is

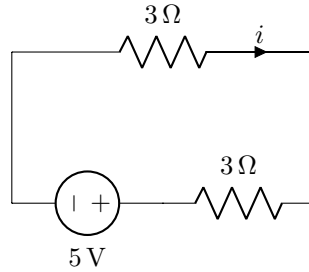$$I_{sc} = \frac{V_{branch}}{R_4} = 1.66\,\text{A} \tag{6}$$

$$R_{Th} = R_N = \frac{V_{oc}}{I_{sc}} = 1.8\,\Omega \tag{7}$$

The Thévenin and Norton equivalents are

2 Superposition

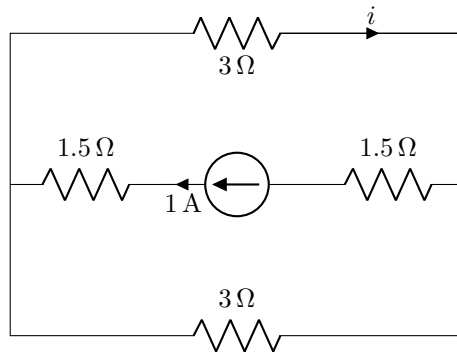Nulling the current source,

The current is simply

$$i = I_{eq} = \frac{V}{R_{eq}} = 0.833\,\text{A} \tag{8}$$
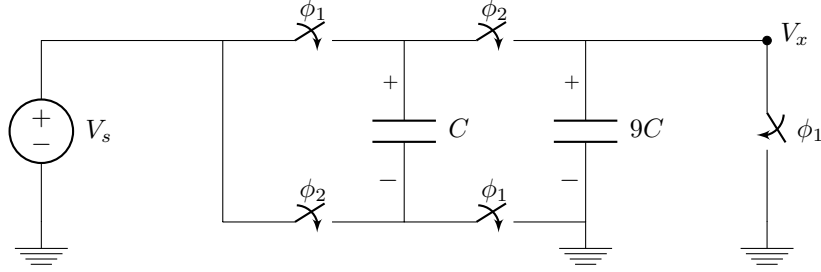
Nulling the voltage source,

Since the circuit is symmetrical along the axis of current flow, it is easy to deduce that half of the current source current constitute i, or $i = 0.5\,\text{A}$.
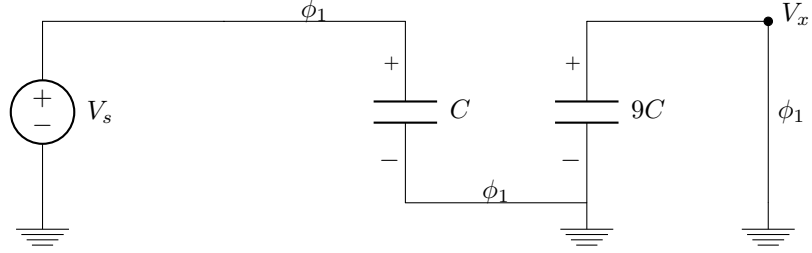
By superposition, the total current is $\sum i = 1.333\,\text{A}$.

3 Charge Sharing

3.a

3.b

3.c

Finding the equivalent capacitance,

$$C_{eq} = C \parallel 9C = \frac{9}{10}C \tag{9}$$

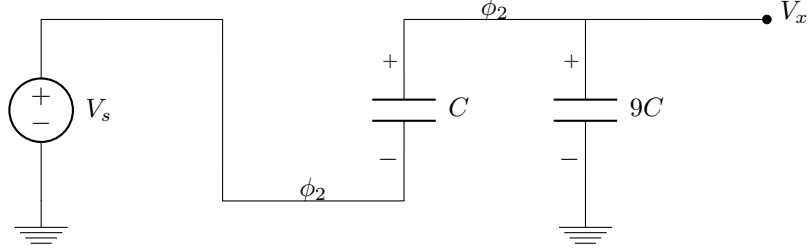The charge on the equivalent capacitor, an thus the separate capacitors is

$$Q_{eq} = \frac{9}{10}CV_s \tag{10}$$

The voltage on C_1 and C_2 is

$$V_{C_1} = \frac{Q_{eq}}{C} = \frac{9}{10}V_s \tag{11}$$

$$V_{C_2} = \frac{Q_{eq}}{9C} = \frac{1}{10}V_s \tag{12}$$

3.d

V_x is nothing more than the voltage across the capacitor, since it can be expressed as the node voltage relative to ground. We can determine the charge on the equivalent capacitor, and thus the separate capacitors, with KVL,

$$V_s + V_{C_{eq}} = 0 \tag{13}$$

$$V_s + \frac{10 Q_{eq}}{9C} = 0 \tag{14}$$

$$Q_{eq} = -\frac{9}{10} C V_s \tag{15}$$

Then, the voltage on V_x is simply

$$V_x = \frac{Q_{eq}}{9C} = -\frac{1}{10} V_s \tag{16}$$

4 Dynamic Random Access Memory

If we assume constant current leakage and capacitance,

$$Q = CV \tag{17}$$

$$\frac{dQ}{dt} = I = C \frac{dV}{dt} \tag{18}$$

$$\int_0^t \frac{I}{C} \, dt = \int_0^t dV \tag{19}$$

$$V(t) = V(0) + \frac{I}{C} t \tag{20}$$

where $V(0) = 1.2 \, \text{V}, C = 28 \times 10^{-15} \, \text{F}$. If we set $t = 1 \, \text{ms}, V(t) = 0.9 \, \text{V}$, we can solve for I,

$$I = C \frac{V(t) - V(0)}{t} = (28 \times 10^{-15} \, \text{F}) \left(\frac{0.9 \text{ V} - 1.2 \text{ V}}{0.001 \text{ s}} \right) = -8.4 \times 10^{-12} \, \text{A} \tag{21}$$
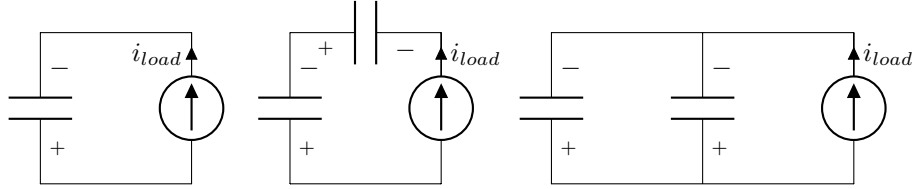
5 Current Sources and Capacitors

Since the capacitors are in series with a current source, the charge on both capacitors is simply $Q = I_s t$. Then, the voltage across C_2 is

$$V = \frac{I_s t}{C_2} \tag{22}$$

6 Super-Capacitors

6.a

6.b

Using the derivation from Problem 4, and taking note of the orientation of the load,

$$V_1(t) = v_{init} - \frac{i_{load}}{C_{sc}} t \tag{23}$$

$$V_2(t) = v_{init} - \frac{2i_{load}}{C_{sc}} t \tag{24}$$

$$V_3(t) = v_{init} - \frac{i_{load}}{2C_{sc}} t \tag{25}$$

6.c

At minimum,

$$v_{min} = v_{init} - \frac{i_{load}}{C_{sc}} t \tag{26}$$

$$= v_{init} - \frac{2i_{load}}{C_{sc}} t \tag{27}$$

$$= v_{init} - \frac{i_{load}}{2C_{sc}} t \tag{28}$$

which means

$$t = \frac{C_{sc}}{i_{load}}(v_{init} - v_{min}) \tag{29}$$

$$= \frac{C_{sc}}{2i_{load}}(v_{init} - v_{min}) \tag{30}$$

$$= \frac{2C_{sc}}{i_{load}}(v_{init} - v_{min}) \tag{31}$$

6.d

Configuration 3 is better than configuration 2 whenever $v_{init} > v_{min}$. Observing the equations tells us that when this is satisfied, Config 3 has *four times* the lifetime of Config 2.

6.e

We can express the change in energy of the capacitor as

$$\Delta E = E - E_0 = \frac{1}{2}(2C_{sc})v_{min}^2 - \frac{1}{2}(2C_{sc})v_{init}^2 = C_{sc}(v_{min}^2 - v_{init}^2) \tag{32}$$

We set the final energy to be at v_{min} rather than 0 because that is the functional range of the capacitor.

7 It's Finally Raining!

7.a

When the tank is full, the capacitance is

$$C = 81\epsilon\frac{h_{tot}\cancel{w}}{\cancel{w}} = 81\epsilon \cdot h_{tot} \tag{33}$$

7.b

The capacitance of the water and air portions are

$$C_{water} = 81\epsilon \cdot h_{h2o} \tag{34}$$

$$C_{air} = \epsilon(h_{tot} - h_{h2o}) \tag{35}$$

$$C_{tank} = C_{air} + C_{water} = \epsilon(80 \cdot h_{h2o} + h_{tot}) \tag{36}$$

7.c

Once more, we use the equation

$$V_C(t) = \frac{I_s}{C_{tank}}t = \frac{I_C}{C_{tank}}t \tag{37}$$

There is no initial voltage since $V(0) = 0\,\text{V}$.

7.d

Ideally, the plot of the voltage-time graph should be a line. Since we know that I_C is constant, the capacitance is simply the amount by which the current is attentuated through the graph. From there, we use the equation from Problem 4.b to solve for the height.

7.e

In the case of a comparator, its output is

$$V_{comp} = \begin{cases} 1 & V_C > V_{ref} \\ -1 & \text{otherwise} \end{cases} \tag{38}$$

This means T_1 occurs at the exact moment

$$V_{ref} = \frac{I_C}{C}T_1 \implies C = \frac{I_C}{V_{ref}}T_1 \tag{39}$$

10   Homework Process and Study Group

I did this homework by myself.