EECS 16B Midterm 1 Review Session

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Disclaimer

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If you want to follow along: head to <Slides location and/or link >

Slides are also posted at @# on Piazza.

HKN Drop-In Tutoring

- HKN has office hours every weekday from **11 AM - 3 PM** and **8 PM - 10 PM** on hkn.mu/ohqueue
- The schedule of tutors can be found at hkn.mu/tutor
  <Itemize the presenter hours here >

LCCDE

Linear Constant-Coefficient Difference Equations

- Discrete LTI systems can be described by LCCDEs in the following form

$$\sum_{k=0}^{N} a_k y(n-k) = \sum_{L=0}^{M} b_L x(n-L)$$

- Taking the Fourier transform on both sides and applying linearity and time-shifting properties results in

$$Y(e^{jw})[\sum_{k=0}^{N} a_k e^{-j\omega k}] = X(e^{jw})[\sum_{L=0}^{N} b_L e^{-j\omega L}]$$

$$H(e^{jw}) = \frac{Y(e^{jw})}{X(e^{jw})} = \frac{\sum_{L=0}^{N} b_L e^{-j\omega L}}{\sum_{k=0}^{N} a_k e^{-j\omega k}}$$

Linear Constant-Coefficient Difference Equations

- LCCDEs for CT LTI systems have the following equations

$$\sum_{n=0}^{N} a_n \frac{d^n}{dt^n} y(t) = \sum_{m=0}^{M} b_m \frac{d^m}{dt^m} x(t)$$

$$H(\omega) = \frac{\sum_{m=0}^{M} b_m (j\omega)^m}{\sum_{n=0}^{N} a_n (j\omega)^n}$$

DT LTI Systems

Discrete Signals and Systems

- A discrete signal is described by a function

$$x : \mathbb{Z} \to \mathbb{R}$$

- A delta function is the unit impulse and any signal can be decomposed into a sum of deltas

$$\delta(n) = \begin{cases} 1 & x = 0 \\ 0 & x \neq 0 \end{cases}$$

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\delta(n-k) \forall x$$

- A discrete system, H, takes an input signal, x, and yields an output signal, y

Linear and Time-Invariant Systems

Linearity

- **Scaling** an input by some amount scales the output by the same amount

$$\alpha x_1(n) \xrightarrow{H} \alpha y_1(n)$$

- **Superposition** of two inputs generates a superposition of two outputs

$$x_1(n) + x_2(n) \xrightarrow{H} y_1(n) + y_2(n)$$

Time-Invariance

- A time invariant system means that a shift in time in the input results in a shift in time in the output

$$x_1(n-N) \xrightarrow{H} y_1(n-N) \quad \forall N \in \mathbb{Z}$$

Impulse Response

- The impulse response is the output of a system for a $\delta(n)$ input

$$x(n) = \delta(n) \xrightarrow{H} y(n) = h(n)$$

Verify that knowing $h(n)$ allows you to determine the output $y(n)$ for any input in an LTI system. *Hint: decompose* $x(n)$

Proof and the Convolutional Sum

Proof Decompose $x(n)$

$$x(n) = \sum_k x(k)\delta(n-k)$$

Recall the definition of the impulse response

$$\delta(n) \xrightarrow{H} h(n)$$

Apply linearity and time-invariance to the definition of the impulse response

$$\sum_k x(k)\delta(n-k) \xrightarrow{H} \sum_k x(k)h(n-k)$$

Convolution

- This is the convolutional sum, and denoted by $(x * h)(n)$
- Note: $(x * h)(n) = (h * x)(n)$

LTI Example Problems

Determine if the system described by the input-output relationship is 1) linear and 2) time-invariant. *Credit: Discussion #1 EE120 Sp20*

1. $y(n) = 7x(n+1)$
2. $y(n) = x(n)x(n-1)$
3. $y(n) = e^{x(n)}$
4. $y(n) = x(-n)$
5. $y(n) = v(n)x(n)$, where v is some fixed signal

LTI Example Solutions

1. $y(n) = 7x(n+1)$
   - Linear: **Yes**
   - Time-invariant: **Yes**
2. $y(n) = x(n)x(n-1)$
   - Linear: **No**, any scaling factor becomes squared at the output
   - Time-invariant: **Yes**
3. $y(n) = e^{x(n)}$
   - Linear: **No**, exponentials are non-linear (verify at $n = 0$)
   - Time-invariant: **Yes**
4. $y(n) = x(-n)$
   - Linear: **Yes**
   - Time-invariant: **No**, $\delta(n-1)$ input results in $\delta(-n-1)$
5. $y(n) = v(n)x(n)$, where v is some fixed signal
   - Linear: **Yes**
   - Time-invariant: **No**, consider when $v(n) = \delta(n)$ for $x(n)$ and $\tilde{x}(n) = x(n-1)$; $y(0) = x(0)$ and $\tilde{y(0)} = x(-1)$

DT System Example Problem

Consider the *linear* DT system H

$$\delta(n-k) \xrightarrow{H} h_k(n) = \alpha^{|k|} u(n-k) \quad \forall |\alpha| < 1$$

Show that the output, $y(n)$, to a general input, $x(n)$ is

$$y(n) = \sum_{k=-\infty}^{+\infty} x(k) h_k(n)$$

Credit: Midterm #1 EE120 Sp20

DT System Example Solution

Decompose $x(n)$

$$x(n) = \sum_{m=-\infty}^{+\infty} x(m)\delta(n-m)$$

Recall the definition of the system H

$$\delta(n-m) \xrightarrow{H} h_m(n)$$

Apply linearity

$$x(n) = \sum_{m=-\infty}^{+\infty} x(m)\delta(n-m) \xrightarrow{H} \sum_{m=-\infty}^{+\infty} x(m)h_m(n) = y(n)$$

Notice the differences in this sum from the usual convolutional sum.

DT System Example Problem

Derive a closed-form expression (no summations) for the output $y(n)$ when the input $x(n) = u(n)$, the unit step

Recall:

$$h_k(n) = \alpha^{|k|} u(n-k) \quad \forall |\alpha| < 1$$

$$y(n) = \sum_{k=-\infty}^{+\infty} x(k) h_k(n)$$

$$\sum_{k=M}^{N} \alpha^k = \begin{cases} \frac{\alpha^{N+1} - \alpha^M}{\alpha - 1} & \alpha \neq 1 \\ N - M + 1 & \alpha = 1 \end{cases}$$

DT System Example Solution

$$y(n) = \sum_{k=-\infty}^{+\infty} x(k) h_k(n)$$

Change the bounds and plug in for $x(k)$ and $h_k(n)$

$$y(n) = \sum_{k=0}^{+\infty} 1 \cdot \alpha^{|k|} u(n-k)$$

$$y(n) = u(n) + \alpha u(n-1) + \alpha^2 u(n-2) + \dots$$

$$y(0) = 1$$

$$y(1) = 1 + \alpha$$

$$y(2) = 1 + \alpha + \alpha^2$$

$$y(n) = \sum_{p=0}^{n} \alpha^p = \frac{\alpha^{n+1} - 1}{\alpha - 1}$$