# EECS 16B Midterm 1 Review Session

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# Disclaimer

This is an unofficial review session and HKN is not affiliated with this course. Although some of the presenters may be course staff, the material covered in the review session may not be an accurate representation of the topics covered in and difficulty of the exam.

If you want to follow along: head to <Slides location and/or link >

Slides are also posted at @# on Piazza.

# HKN Drop-In Tutoring

- HKN has office hours every weekday from **11 AM - 3 PM** and **8 PM - 10 PM** on hkn.mu/ohqueue
- The schedule of tutors can be found at hkn.mu/tutor
  <Itemize the presenter hours here >

# DT LTI Systems

## Discrete Signals and Systems

- A discrete signal is described by a function

$$x : \mathbb{Z} \to \mathbb{R}$$

- A delta function is the unit impulse and any signal can be decomposed into a sum of deltas

$$\delta(n) = \begin{cases} 1 & x = 0 \\ 0 & x \neq 0 \end{cases}$$

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\delta(n-k) \forall x$$

- A discrete system, $H$, takes an input signal, $x$, and yields an output signal, $y$

# Linear and Time-Invariant Systems

## Linearity

- **Scaling** an input by some amount scales the output by the same amount

$$\alpha x_1(n) \xrightarrow{H} \alpha y_1(n)$$

- **Superposition** of two inputs generates a superposition of two outputs

$$x_1(n) + x_2(n) \xrightarrow{H} y_1(n) + y_2(n)$$

## Time-Invariance

- A time invariant system means that a shift in time in the input results in a shift in time in the output

$$x_1(n-N) \xrightarrow{H} y_1(n-N) \quad \forall N \in \mathbb{Z}$$

# Impulse Response

- The impulse response is the output of a system for a $\delta(n)$ input

$$x(n) = \delta(n) \xrightarrow{H} y(n) = h(n)$$

Verify that knowing $h(n)$ allows you to determine the output $y(n)$ for any input in an LTI system. *Hint: decompose* $x(n)$

# Proof and the Convolutional Sum

<u>Proof</u> Decompose $x(n)$

$$x(n) = \sum_k x(k)\delta(n-k)$$

Recall the definition of the impulse response

$$\delta(n) \xrightarrow{H} h(n)$$

Apply linearity and time-invariance to the definition of the impulse response

$$\sum_k x(k)\delta(n-k) \xrightarrow{H} \sum_k x(k)h(n-k)$$

<u>Convolution</u>

- This is the convolutional sum, and denoted by $(x * h)(n)$
- Note: $(x * h)(n) = (h * x)(n)$

# LTI Example Problems

Determine if the system described by the input-output relationship is 1) linear and 2) time-invariant. *Credit: Discussion #1 EE120 Sp20*

1. $y(n) = 7x(n+1)$
2. $y(n) = x(n)x(n-1)$
3. $y(n) = e^{x(n)}$
4. $y(n) = x(-n)$
5. $y(n) = v(n)x(n)$, where $v$ is some fixed signal

# LTI Example Solutions

1. $y(n) = 7x(n+1)$
   - Linear: **Yes**
   - Time-invariant: **Yes**
2. $y(n) = x(n)x(n-1)$
   - Linear: **No**, any scaling factor becomes squared at the output
   - Time-invariant: **Yes**
3. $y(n) = e^{x(n)}$
   - Linear: **No**, exponentials are non-linear (verify at $n = 0$)
   - Time-invariant: **Yes**
4. $y(n) = x(-n)$
   - Linear: **Yes**
   - Time-invariant: **No**, $\delta(n-1)$ input results in $\delta(-n-1)$
5. $y(n) = v(n)x(n)$, where $v$ is some fixed signal
   - Linear: **Yes**
   - Time-invariant: **No**, consider when $v(n) = \delta(n)$ for $x(n)$ and $\tilde{x}(n) = x(n-1)$; $y(0) = x(0)$ and $\tilde{y(0)} = x(-1)$

# DT System Example Problem

Consider the *linear* DT system H

$$\delta(n-k) \xrightarrow{H} h_k(n) = \alpha^{|k|} u(n-k) \quad \forall |\alpha| < 1$$

Show that the output, $y(n)$, to a general input, $x(n)$ is

$$y(n) = \sum_{k=-\infty}^{+\infty} x(k) h_k(n)$$

*Credit: Midterm #1 EE120 Sp20*

## DT System Example Solution

Decompose $x(n)$

$$x(n) = \sum_{m=-\infty}^{+\infty} x(m)\delta(n-m)$$

Recall the definition of the system $H$

$$\delta(n-m) \xrightarrow{H} h_m(n)$$

Apply linearity

$$x(n) = \sum_{m=-\infty}^{+\infty} x(m)\delta(n-m) \xrightarrow{H} \sum_{m=-\infty}^{+\infty} x(m)h_m(n) = y(n)$$

Notice the differences in this sum from the usual convolutional sum.

# DT System Example Problem

Derive a closed-form expression (no summations) for the output $y(n)$ when the input $x(n) = u(n)$, the unit step

Recall:

$$h_k(n) = \alpha^{|k|} u(n-k) \quad \forall |\alpha| < 1$$

$$y(n) = \sum_{k=-\infty}^{+\infty} x(k) h_k(n)$$

$$\sum_{k=M}^{N} \alpha^k = \begin{cases} \frac{\alpha^{N+1} - \alpha^M}{\alpha - 1} & \alpha \neq 1 \\ N - M + 1 & \alpha = 1 \end{cases}$$

# DT System Example Solution

$$y(n) = \sum_{k=-\infty}^{+\infty} x(k) h_k(n)$$

Change the bounds and plug in for $x(k)$ and $h_k(n)$

$$y(n) = \sum_{k=0}^{+\infty} 1 \cdot \alpha^{|k|} u(n-k)$$

$$y(n) = u(n) + \alpha u(n-1) + \alpha^2 u(n-2) + \dots$$

$$y(0) = 1$$

$$y(1) = 1 + \alpha$$

$$y(2) = 1 + \alpha + \alpha^2$$

$$y(n) = \sum_{p=0}^{n} \alpha^p = \frac{\alpha^{n+1} - 1}{\alpha - 1}$$