231B: Spin statistics and multi-electron atoms

Quiz 3, Winter 2020
(Dated: February 11, 2020)

1. For a potential $V(x)$ with only two bound states, one at -10 eV and the other at -2 eV, what is the ground-state energy of two ideal fermions in the well, and what is its spin state?

2. What is the energy and degeneracy of the first excited state for the previous problem?

3. For 4 ideal fermions in the well above, what is the ionization potential?

4. Write the Hartree-Fock ground-state wavefunction for two electrons in a He atom in terms of its atomic orbitals and spin states.

5. Write the Hamiltonian for the three electrons in the Li atom.

6. When adding two angular momenta, each with $j = 1$, list all the allowed values of J and M, the quantum numbers for their sum, and give the total number of states.

7. In the previous problem, write the formula relating the $|J, M>$ state at the top of the ladder with its equivalent representation in terms of the product of individual states, $|m_1, m_2>$.

8. The ground-state Hartree-Fock energy for He is -77.9 eV. What statement can you make with absolute confidence about the ionization potential of He?

9. The Hartree-Fock eigenvalue for the 1s orbital in He is -24.9 eV. Compare this with previous question to give the error in Koopmans' theorem.

10. The allowed terms for the three p-electrons in N are $^2P, ^2D$, and ^4S. Give the order of their energies (say which is lowest, in the middle, and highest).