Bluffing Interest: a Mathematical Model for the Love Story of Benedick and Beatrice

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1 Abstract

In the 2015 book *Modeling Love Dynamics*[2], the authors provided a model for one type of romantic 'bluffing' (see also[1]), temporarily changing appeal, and mentioned, without example, another type of bluffing, artificially changing one's interest. No example was provided for this sort of bluffing, and the authors said it was 'hard to imagine' a scenario in which this could be accomplished in the middle of a love story. We examine a story by Shakespeare that provides such an example: Beatrice and Benedick in *Much Ado About Nothing*. We provide a mathematical model that mimics the feelings shown in the play and use it as an argument in favor of a secure attachment style in the face of arguable evidence.

Introduction and Background for Love Dynamics

In *Modeling Love Dynamics*[2], Rinaldi et al. demonstrate different classes of dynamic models for love stories, with applications to famous love stories. In brief, Love dynamical models use sets of differential equations to model the romantic interest felt between subjects across a "love story". These can be thought of as mathematical models where the relationship is defined within 'love space', with negative and positive opinions mapping to negative or positive numbers on the axes, thus a successful love story will be associated with both feelings fairly equally positive.

In this paper we follow this that framework, focusing on the love story of Beatrice and Benedick from Shakespeare's *Much Ado About Nothing*[3]. The reader is encouraged to read the books and papers from Rinaldi, et al cited below for a complete justification for the model used here. The story of Beatrice and Benedick presented in the play begins when they already know and dislike eachother. Their friends then fool them each into believing that the other is secretly in love with them. By the end of the play, their false belief leads them to genuinely fall in love.

The unique circumstances of this story allow us to explore the effects for temporary bluffing on the love dynamics. The effect is similar to that described in a 2015 paper on Cyrano de Bergerac[1], with the difference that, instead of bluffing appeal, this story bluffs interest.

The Story of Beatrice and Benedick

Following *Modeling Love Dynamics*, we will examine this love story by looking at specific moments or scenes within the play. We'll refer to them by act and scene number, beginning with Act 1, Scene 1. In our graphs later we mark these important moments in the timeline as P0-P5, and details are showing below.

P0: Act 1, Scene 1: Beatrice talks negatively about Benedick, he arrives with the prince and they immediately start bickering. Both claim that they are completely uninterested in romance of any sort. Beatrice leaves, and the Prince swears to Benedick that "I shall see thee, ere I die, look pale with love."

Interpretation: Both begin with negative feelings toward each other.

P1 Act 2, Scene 1: Benedick dances with Beatrice in disguise and they continue to mock each other. Benedick later avoids Beatrice. She hints that they were in love in the past. After she leaves, the Prince plots to trick them into falling in love.

Interpretation: Benedick's feelings have worsened as they've interacted.

P2 Act 2, Scene 3: The Prince and his coconspirators makes sure that Benedick overhears them as they falsely claim that Beatrice is desperately in love with Benedick, but refuses to show it because he is so cruel. Benedick is convinced, and resolves to love her in return. Beatrice is sent to tell him dinner is ready, and Benedick convinces himself her hostile behaviour is a further sign that she is in love.

Interpretation: Benedick's perception of Beatrice's feelings toward him changes.

P3 Act 3, Scene 1: Hero and Margaret trick Beatrice in a similar manner, and she resolves to return Benedick's (fictitious) love.

Interpretation:Beatrice's perception of Benedick's love changes.

P4Act 4, Scene 1: Beatrice and Benedick stay behind after the dramatic failure of the first wedding scene, and they confess their love for each other. Beatrice then gets Benedick to agree to challenge his friend to a duel.

Interpretation: The ruse has worked, and their feelings have entered the positive area.

P5 Act 5, Scene 3: Benedick proposes to Beatrice outside of Hero and Claudio's second (apparently successful) attempt at a wedding, when the Prince and his co-conspirators reveal that they were fooled. They then reaffirm their love and agree to marriage.

Interpretation: After removing the bluffing coefficients, the love has reached a new and positive stable state.

The Model

Now we're ready to construct the set of dynamic equations that will define our model. The general model is given by

$$\dot{x_1} = -\alpha_1 x_1 + R_1^L(x_2) + \gamma_1 A_2$$

$$\dot{x_2} = -\alpha_2 x_2 + R_2^L(x_1) + \gamma_2 A_1.$$

Here x_1 is the love of Benedick to Beatrice, x_2 is the love of Beatrice to Benedick. In the model, α_i $(i = 1, 2)$ is the forgetting coefficient, and represents the rate at which feelings drop in intensity. Specifically, it is the inverse of the half-life of any specific emotion. γ_i is the reaction of individual i to the appeal A_j of individual j, which is the individual's overall feeling about every trait of the other individual, positive or negative. The function R_i^L $(i = 1, 2)$ is the reaction to love and is defined as

$$R_1^L = \frac{e^{x_2+B_2} - e^{-x_2+B_2}}{e^{x_2+B_2}/R_1^+ - e^{-x_2+B_2}/R_1^-}$$
$$R_2^L = \frac{e^{x_1+B_1} - e^{-x_1+B_1}}{e^{x_1+B_1}/R_2^+ - e^{-x_1+B_1}/R_2^-}$$

Where R_i^+ and R_i^- are the positive and negative limits of the function, which represent physical limits on feeling based on the other person's love. The parameter B_i is the bluffing coefficient, which represents the false perception of the other person's love interest, which is a central idea of the story.

We'll adopt specific values/range for the above parameters, giving psychological justifications for our choices. Since both have held onto their feelings of dislike for one another over some time, we set the forgetting coefficients (α) as a conservative value of 0.5 for Benedick and 0.4 for Beatrice. In terms of our equations, this is the inverse of the half life of a memory, so that, in the case of Benedick it will take $1/.5 = 2$ time periods for the intensity of feeling about a specific incident to be reduced by half(We discuss the nature of the time periods used below). Her lower score is assigned since she brings up Benedick unprompted in the first scene, while he doesn't ask about her til she appears. For reference, most people would seem to average about 1, and this value must be positive.

Since the reaction to appeal is held constant, we only need to determine the full sum once, without undue concern for specific factors within that sum. Both praise the others looks while disliking the others behaviours, so we assume the sum of the appeal is actually fairly mild altogether, somewhere between $-0.5 < \gamma_i A_j < 0.5$. Both were set to a pessimistic $\gamma_i A_j = -0.2$ for our initial conditions. Anything over 1 represents extreme interest, with anything under -1 extreme dislike.

Since neither Benedick nor Beatrice is especially vindictive, we set the upper bound on their reaction to love as $R^+ = 2$ while the lower bound is $R^- = -0.5$. Within the bounds of our model, then, any number beyond these will signify a significant attachment/dislike.

Lastly, we need to assign the bluffing coefficients B_i. Since they are both characterized as being deeply, madly in love with one another, we assume the maximum of B_i is 2, but the gradual change of B_i may follow different profiles according to the scenes.

Dynamic equations like these have an implicit time component, and this is important to define here, where the bluffing coefficients will change at specific times within the story. The exact time across which the play occurs is not specified exactly, but seems to be somewhere between three weeks to a few months. There are 16 scenes in the play across 5 acts, so Without a specific timeline given we have made the simplifying assumption that each scene in the play averages out to 1 unit of time elapsed within the love story, so $t = 6$ corresponds to the end of the sixth scene, for instance, which is when Benedick is fooled (P2). This unit of time might correspond to a value between 1 and 7 days within the timeline of the play.

Results

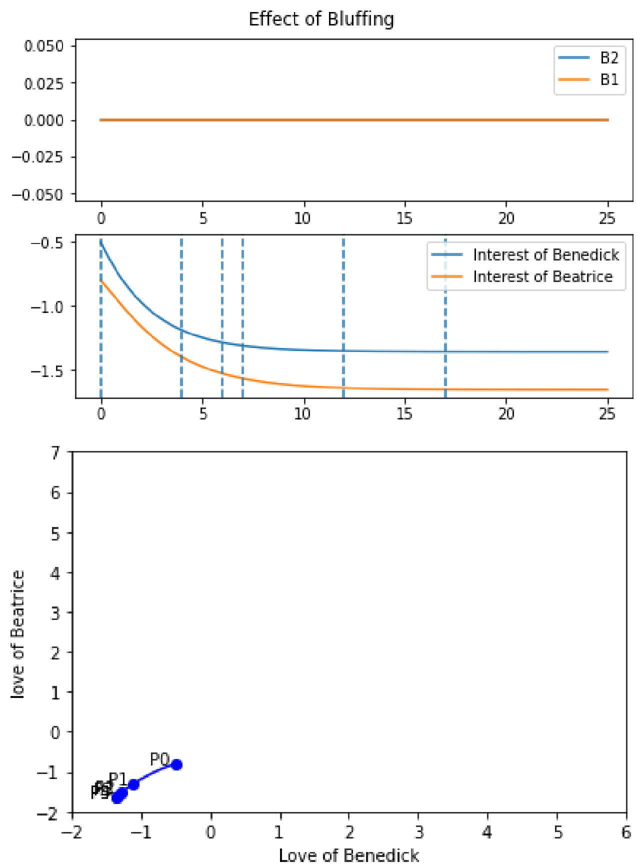

Figure 1: The predicted trajectory in love space of Benedick and Beatrice through the duration of the play, without the bluff

Figure 1 shows what our model predicts the trajectory of the story would have been without the intervention by the prince. As we expect, they quickly reach a negative equilibrium. Note that they we do not start them at that equilibrium, since they have been apart which means their feelings have been moving back toward a neutral state (0,0). Now we can look at the change bluffing makes.

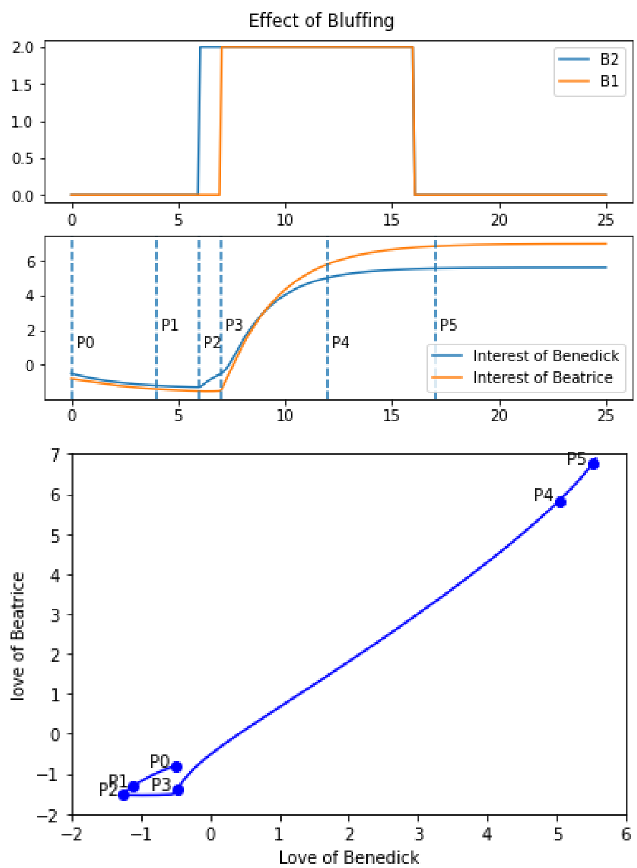

Figure 2: The trajectory in love space of Benedick and Beatrice through the duration of the play, using a discrete jump in bluffing.

Figure 2 shows the trajectory of the relationship throughout the timeline of the play assuming a discrete jump in B. Starting at P0 when the play opens, the trajectory is mutually negative until P2, when Benedick is fooled, then P3, when Beatrice is fooled, where they begin to move in the positive direction and continue to increase even after P5, where they are told the truth. In this model we used an immediate full shift of the bluffing coefficient from 0 to 2 in order to clearly show the effect. This reflects the story as presented, where neither Beatrice nor Benedick question the truth of the bluff after the scene in which they are fooled. Note how well this follows the actual events of the play: mild dislike at the start turns toward genuine irritation until they fall for the lie, at which point they rapidly change their point of view and almost immediately change toward the positive, which remains stable even after the bluff is removed.

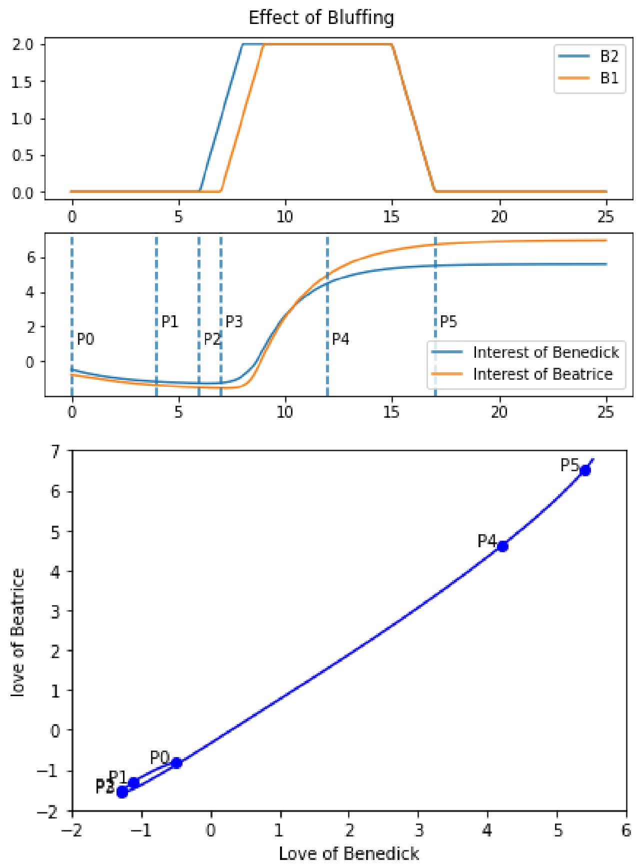

Figure 3: The same trajectory assuming a continuous increase in the bluffing coefficients. Note that P1 and P2 have merged.

A perhaps more realistic approach would be to give a linear or quadratic increase, both of which were attempted. The graph for the linear version is shown Figure 3, and shares all the relevant properties with the graph of a quadratic change. Of special note is the location P2/P3. Since these two events happen across two sequential scenes in the play, the increase we used was not sufficient to produce a change in the graph until after both events had occurred. Once it reaches a point sufficient to trigger the change, the model acts much the same as figure 2. This shouldn't be surprising, since we've already seen that the bluffing coefficient is unnecessary once the positive equilibrium is approached.

To help understand why this is the case, let's look at Figures 4 through 7. These show the phase portrait of the system at the start of the play and at P3, P4 and P5. The red lines represent trajectories that go toward the negative stable point, while green lines go toward the positive stable point.

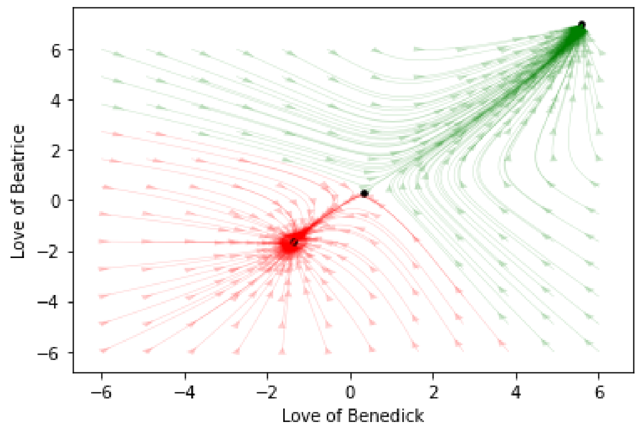

Figure 4: the phase portrait at the beginning of the play

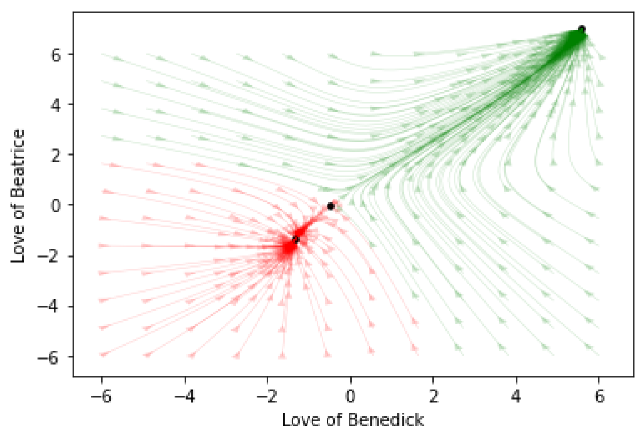

Figure 5: the phase portrait at P3 in the linear increase model

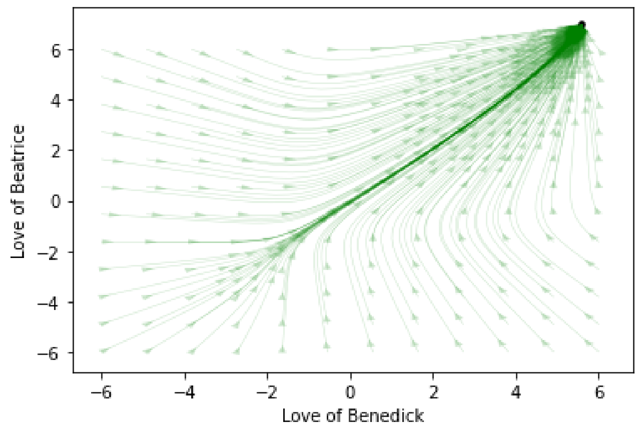

Figure 6: The phase portrait at the height of the charade

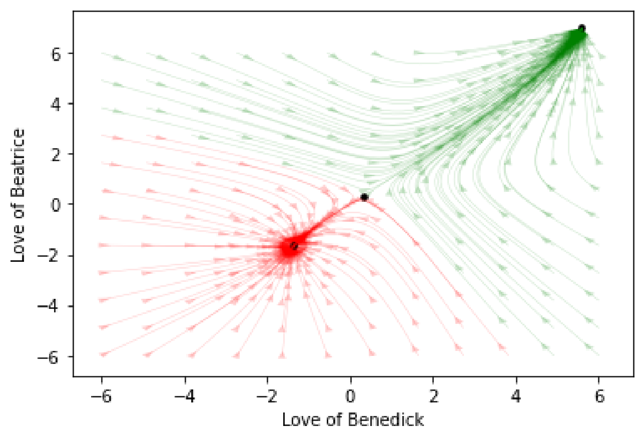

Figure 7: the phase portrait at the end of the Play

You'll note that the negative node and the saddle are gone in figure 6, so we're left with only the positive node. To explain this, we perform a bifurcation analysis[4] on both bluffing coefficients. Figure 8 shows a bifurcation diagram for the bluffing coefficients, which shows that once B_1 and B_2 increase past about 1, there is only one fixed point for the system, the positive node.

We can see that our assumed values were much higher than necessary to produce the desired effect, which shows that our model is robust enough to accept some changes in it's parameters.

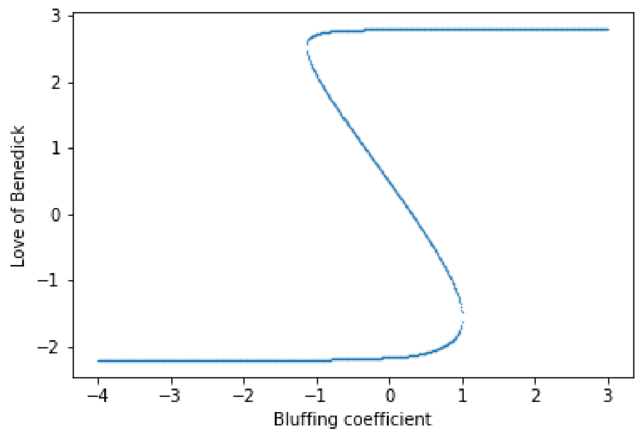

Figure 8: Location of fixed points as the bluffing coefficient increases

Conclusion

We have managed to create a model using reasonable assumptions about the characters' psychological profile which supports the supposition that Beatrice and Benedick have a secure attachment style. This has also allowed us to demonstrate a situation that the authors of "Modeling Love Dynamics" called "difficult to imagine."(page 42). We have NOT shown that they must have a secure attachment style, as there is not enough textual evidence to support that, and it may be possible to model a system in which their attachment style is insecure.

References

[1] Sergio Rinaldi, Pietro Landi, and Fabio Della Rossa. "Temporary Bluffing Can Be Rewarding in Social Systems: The Case of Romantic Relationships". In: *Journal of Mathematical Sociology* 39 (July 2015), pp. 203–220.

[2] Sergio Rinaldi et al. *Modeling love dynamics*. Vol. 89. World Scientific, 2015.

[3] William Shakespeare. *Much Ado About Nothing*. Penguin, 2005.

[4] Steven H. Strogatz. *Nonlinear Dynamics and Chaos: With Applications to Physics, Biology, Chemistry and Engineering*. Westview Press, 2000.