MATH325: Discrete Math 2
Assignment 3

Questions are taken from Rosen, Discrete Mathematics and Applications, 6th edition. When I write "Exercise 5.1.2", I mean "Exercise 2 of section 5.1".

The solutions to Q1–Q3 are provided. Make sure you read it and follow the way I present the solution. For instance note that I justify the computation by specifying the task breakdown. I also summarize by answer with a boxed text.

Not all questions will be graded. For some questions I will only grade the final answer. For some I will grade the explanation and the final answer.

Q1. Exercise 5.1.1.

SOLUTION.

(a) The task T of selecting a math major and a computer science major is the same as performing T_1 followed by T_2 where

$$T_1 = \text{select a math major}$$
$$T_2 = \text{select a computer science major}$$

Hence by the multiplication principle, the number of ways to perform T is the same as the product of the number of ways to perform T_1 and the number of ways to perform T_2, i.e., $325 \cdot 18 = 5850$.

ANSWER: $\boxed{5850}$

(b) The task T of selecting either a math major or a computer science major is the same as performing either T_1 or T_2 where

$$T_1 = \text{select a math major}$$
$$T_2 = \text{select a computer science major}$$

Hence by the addition principle, the number of ways to perform T is the same as the sum of the number of ways to perform T_1 and the number of ways to perform T_2, i.e., $325+18 = 5850$.

ANSWER: $\boxed{343}$

Notes. What I call multiplication principle and addition principle in counting techniques is sometimes also called product rule and sum rule or product principle and sum principle, etc. There's no standard name for these two techniques.

Note that for (a) technically, you should check that your assumptions are indeed correctly, i.e., that T is really performing T_1 followed by T_2 and, *remember this! I said so in class!*, that the number of ways to perform T_2 is independent of the way you perform T_1. For simple cases, you don't have to give any explanation.

And for (b), remember that you have to be really certain that T is really made up of either T_1 and T_2, i.e. *no more and no less*, and furthermore that there are no overlaps, i.e. no task is a T_1 task and a T_2 task, otherwise you'll be double counting. For simple cases, you don't have to give any explanation.

Q2. Exercise 5.1.2.

SOLUTION.

The number of rooms is of course the same as the number of ways of selecting an office from the building. The task T of selecting an office in the building is the same as performing T_1, T_2 where

$$T_1 = \text{select a floor in the build}$$
$$T_2 = \text{select a room on the selected floor}$$

Hence by the multiplication principle, the number of ways to perform T is the same as the product of the number of ways to perform T_1 and the number of ways to perform T_2, i.e., $27 \cdot 37 = 999$.

ANSWER: $\boxed{999}$

Q3. Exercise 5.1.3.

SOLUTION.

(a) The task T of answering the test is the same as filling in 10 blanks:

$$\_\quad \_\quad \_\quad \_\quad \_\quad \_\quad \_\quad \_\quad \_\quad \_$$

where there are four possible answers for each slot. The task T is the same as performing T_1, T_2, ..., T_{10} in sequence (i.e. one following another) where where

$$\begin{aligned} T_1 &= \text{task of filling in blank \#1} \\ T_2 &= \text{task of filling in blank \#2} \\ &\ldots \\ T_{10} &= \text{task of filling in blank \#10} \end{aligned}$$

Hence by the multiplication principle, the number of ways to perform T is the same as the product of the number of ways to perform T_1, T_2, ..., T_{10}, i.e., 4^{10}.

ANSWER: $\boxed{1048576}$

Q4. Exercise 5.1.4.

SOLUTION.

The task T of finding out how many shirts there are, is the same as performing T_1 followed by T_2 followed by T_3 where

$$T_1 = \text{select a color for the shirt}$$
$$T_2 = \text{select a male or female version for the shirt}$$
$$T_3 = \text{select a size for the shirt}$$

Hence by the multiplication principle, the number of ways to perform T is the same as the product of the number of ways to perform T_1 and the number of ways to perform T_2 and the number of ways to perform T_3, i.e., $12 \cdot 2 \cdot 3 = 72$.

ANSWER: $\boxed{72}$

Q5. Exercise 5.1.5.

SOLUTION.

The number of pairs of airlines providing flights from New York to San Francisco via Denver is the same as the number of ways of selecting two flights. The task T of two flights is the same as performing T_1, T_2 where

$$T_1 = \text{is selecting a flight from New York to Denver}$$
$$T_2 = \text{is selecting a flight from Denver to San Francisco}$$

Hence by the multiplication principle, the number of ways to perform T is the same as the product of the number of ways to perform T_1 and the number of ways to perform T_2, i.e., $6 \cdot 7 = 42$.

ANSWER: $\boxed{42}$

The number of these pairs involve more than airline is the same as finding the number of pair that involve only one airline and subtracting this number from all the pairs resulting in the pairs not envolving the same airline. However, the problem never tells you if any of the 6 airlines from N.Y to Denver are the same as the 7 from Denver to San Francisco.

Q6. Exercise 5.1.6.

SOLUTION.

The number of major auto routes providing routes from Boston to Los Angeles via Detroit is the same as the number of ways of selecting two routes. The task T of two routes is the same as performing T_1, T_2 where

$$T_1 = \text{is selecting a route from Boston to Detroit}$$
$$T_2 = \text{is selecting a route from Detroit to Los Angeles}$$

Hence by the multiplication principle, the number of ways to perform T is the same as the product of the number of ways to perform T_1 and the number of ways to perform T_2, i.e., $4 \cdot 6 = 24$.

ANSWER: $\boxed{24}$

Q7. Exercise 5.1.7.

SOLUTION.

The task T of finding 3 letter initials is the same as filling in 3 blanks:

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where there are 26 possible answers for each slot. The task T is the same as performing T_1, T_2, T_3 in sequence (i.e. one following another) where where

$$T_1 = \text{task of filling in blank \#1}$$
$$T_2 = \text{task of filling in blank \#2}$$
$$T_3 = \text{task of filling in blank \#3}$$

Hence by the multiplication principle, the number of ways to perform T is the same as the product of the number of ways to perform T_1, T_2, T_3, i.e., $26^3 = 17,576$.

ANSWER: $\boxed{17,576}$

Q8. Exercise 5.1.8.

SOLUTION.

The task T of finding 3 letter initials without any repating letters is the same as filling in 3 blanks:

$$\_\_ \quad \_\_ \quad \_\_$$

where there are 26 possible answers for the first slot, 25 possible answers for the second slot, 24 possible answers for the third slot. The task T is the same as performing T_1, T_2, T_3 in sequence (i.e. one following another) where where

$$T_1 = \text{task of filling in blank \#1}$$
$$T_2 = \text{task of filling in blank \#2}$$
$$T_3 = \text{task of filling in blank \#3}$$

Hence by the multiplication principle, the number of ways to perform T is the same as the product of the number of ways to perform T_1, T_2, T_3, i.e., $26 \cdot 25 \cdot 24 = 15,600$.

ANSWER: $\boxed{15,600}$

Q9. Exercise 5.1.8.

SOLUTION.

The task T of finding 3 letter initials with the first letter the same is the same as filling in 2 blanks:

$$\text{A} \quad \_\_ \quad \_\_$$

where there are 26 possible answers for each slot. The task T is the same as performing T_1, T_2 in sequence (i.e. one following another) where where

$$T_1 = \text{task of filling in blank \#2}$$
$$T_2 = \text{task of filling in blank \#3}$$

Hence by the multiplication principle, the number of ways to perform T is the same as the product of the number of ways to perform T_1, T_2, i.e., $26^2 = 676$.

ANSWER: $\boxed{676}$

Q10. Exercise 5.1.10.

SOLUTION.

The task T of finding bit strings of length 8 is the same as filling in 8 blanks:

$$\_\_ \quad \_\_ \quad \_\_ \quad \_\_ \quad \_\_ \quad \_\_ \quad \_\_ \quad \_\_$$

where there are 2 possible answers for each slot a 1 or a 0. The task T is the same as performing T_1, T_2, ..., T_8 in sequence (i.e. one following another) where where

$$\begin{aligned} T_1 &= \text{task of filling in blank \#1} \\ T_2 &= \text{task of filling in blank \#2} \\ &\ldots \\ T_8 &= \text{task of filling in blank \#8} \end{aligned}$$

Hence by the multiplication principle, the number of ways to perform T is the same as the product of the number of ways to perform T_1, T_2, ..., T_{10}, i.e., $2^8 = 256$.

ANSWER: $\boxed{256}$

Q11. Exercise 5.1.11.

SOLUTION.

The task T of finding bit strings of length 10 starting and ending with a 1 is the same as filling in 8 blanks:

$$1 \quad \_\_ \quad \_\_ \quad \_\_ \quad \_\_ \quad \_\_ \quad \_\_ \quad \_\_ \quad \_\_ \quad 1$$

where there are 2 possible answers for each slot a 1 or a 0. The task T is the same as performing T_1, T_2, ..., T_8 in sequence (i.e. one following another) where where

$$\begin{aligned} T_1 &= \text{task of filling in blank \#2} \\ T_2 &= \text{task of filling in blank \#3} \\ &\ldots \\ T_8 &= \text{task of filling in blank \#9} \end{aligned}$$

Hence by the multiplication principle, the number of ways to perform T is the same as the product of the number of ways to perform T_1, T_2, ..., T_8, i.e., $2^8 = 256$.

ANSWER: $\boxed{256}$

Q12. Exercise 5.1.12.

SOLUTION.

The task T of finding bit strings of length 6 or less is the same as finding bit strings of length 6, and length 5, and length 4, and length 3, and length 2, and length 1,:

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where there are 2 possible answers for each slot a 1 or a 0. The task T is the same as performing T_1, T_2, ..., T_6 in sequence (i.e. one following another) where where

$$T_1 = \text{task of filling in 6 blanks}$$
$$T_2 = \text{task of filling in 5 blanks}$$
$$T_3 = \text{task of filling in 4 blanks}$$
$$T_4 = \text{task of filling in 3 blanks}$$
$$T_5 = \text{task of filling in 2 blanks}$$
$$T_6 = \text{task of filling in 1 blank}$$

Hence by the addition and multiplication principles, the number of ways to perform T is the same as the sum of product of the number of ways to perform T_1, T_2, ..., T_2, i.e., $2^6 + 2^5 + 2^4 + 2^3 + 2^2 + 2^1 = 126$.

ANSWER: $\boxed{126}$

Q13. Exercise 5.1.13.

SOLUTION.

The task T of finding bit strings of length n or less is the same as finding bit strings of length n, and length n-1, ..., and length 1, and length 0,:

$$\underline{\quad} \quad \underline{\quad} \quad \underline{\ldots} \quad \underline{\quad}$$

...

$$\underline{\quad}$$

empty string

where there is 1 possible answer for each slot a 1. The task T is the same as performing T_1, T_2, ..., T_{n+1} in sequence (i.e. one following another) where where

$$\begin{aligned} T_1 &= \text{task of filling in n blanks} \\ T_2 &= \text{task of filling in n-1 blanks} \\ &\ldots \\ T_5 &= \text{task of filling in 1 blanks} \\ T_n + 1 &= \text{task of filling in 0 blank} \end{aligned}$$

Hence by the addition and multiplication principles, the number of ways to perform T is the same as the sum of product of the number of ways to perform T_1, T_2, ..., T_2, i.e., $1^n + 1^{n-1} + ... + 1^1 + 1^0 = n + 1$.

ANSWER: $\boxed{n+1}$

Q14. Exercise 5.1.14.

SOLUTION.

The task T of finding bit strings of length n begining in a 1 and ending in a 1:

$$\underline{1} \quad \_\_ \quad \_\_ \quad \underline{...} \quad \_\_ \quad \underline{1}$$

...

$$\underline{1} \quad \_\_ \quad \underline{1}$$

$$\underline{1} \quad \underline{1}$$

where there are 2 possible answers for each slot a 1 or a 0. The task T is the same as performing T_1, T_2, ..., T_{n+1} in sequence (i.e. one following another) where where

$$\begin{aligned} T_1 &= \text{task of filling in n blanks} \\ T_2 &= \text{task of filling in n-1 blanks} \\ &... \\ T_5 &= \text{task of filling in 1 blanks} \\ T_n + 1 &= \text{task of filling in 0 blank} \end{aligned}$$

Hence by the addition and multiplication principles, the number of ways to perform T is the same as the sum of product of the number of ways to perform T_1, T_2, ..., T_2, i.e.,

$$2^{n-2} + 2^{n-1} + ... + 2^1 + 2^0 = \sum_{i=0}^{i=n-2} a^i$$

ANSWER: $\boxed{\sum_{i=0}^{i=n-2} a^i}$

Q15. Exercise 5.1.15.

SOLUTION.

The task T of finding strings of length 4 or less of lowercase letters is the same as filling in 4 blanks, and 3 blanks, and 2 blanks, and 1 blank:

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where there are 26 possible answers for each slot. The task T is the same as performing T_1, T_2, T_3 in sequence (i.e. one following another) where where

$$T_1 = \text{task of filling in 4 blanks}$$
$$T_2 = \text{task of filling in 3 blanks}$$
$$T_3 = \text{task of filling in 2 blanks}$$
$$T_4 = \text{task of filling in 1 blank}$$

Hence by the multiplication principle, the number of ways to perform T is the same as the sum of the product of the number of ways to perform T_1, T_2, T_3, T_4, i.e., $26^4+26^3+26^2+26^1 = 475,254$.

ANSWER: $\boxed{475,254}$

Q16. Exercise 5.1.16.

SOLUTION.

The task of finding strings of length 4 or less of with the letter x in it is the same as finding all the possibilities (previous quetion 15) 475,254 and subtracting all the strings without x in them we call this task T.:

where there are 25 possible answers for each slot. The task T is the same as performing T_1, T_2, T_3 in sequence (i.e. one following another) where where

$$T_1 = \text{task of filling in 4 blanks}$$
$$T_2 = \text{task of filling in 3 blanks}$$
$$T_3 = \text{task of filling in 2 blanks}$$
$$T_4 = \text{task of filling in 1 blank}$$

Hence by the multiplication principle, the number of ways to perform T is the same as the sum of the product of the number of ways to perform T_1, T_2, T_3, T_4, i.e., $(26^4 + 26^3 + 26^2 + 26^1) - (25^4 + 25^3 + 25^2 + 25^1) = 68,354$.

ANSWER: $\boxed{68,354}$

Q17. Exercise 5.1.17.

SOLUTION.

The task of finding strings of length 5 of ascii characters contain @ at least once in them is the same as finding all the possibilities of strings of length 5 of ascii characters and subtracting all the strings with @ in them we call this task T it is made of T_1 and T_2.:

The task T_1 where there are 128 possible answers for each slot. The task T_2 where there are 127 possible answers for each slot. The task T_1 is the same as performing $T_1 1$, $T_1 2$, $T_1 3$, $T_1 4$, $T_1 5$ in sequence (i.e. one following another) where where

$$T_1 1 = \text{task of filling in blank 1}$$
$$T_1 2 = \text{task of filling in blank 2}$$
$$T_1 3 = \text{task of filling in blank 3}$$
$$T_1 4 = \text{task of filling in blank 4}$$
$$T_1 5 = \text{task of filling in blank 5}$$

The task T_2 is the same as performing $T_2 1$, $T_2 2$, $T_2 3$, $T_2 4$, $T_2 5$ in sequence (i.e. one following another) where

$$T_2 1 = \text{task of filling in blank 1}$$
$$T_2 2 = \text{task of filling in blank 2}$$
$$T_2 3 = \text{task of filling in blank 3}$$
$$T_2 4 = \text{task of filling in blank 4}$$
$$T_2 5 = \text{task of filling in blank 5}$$

Hence by the multiplication principle, the number of ways to perform T is the same as the sum of the product of the number of ways to perform T_1 - T_2, i.e., $(128^5) - (127^5) = 1321368961$.

ANSWER: $\boxed{1321368961}$