MATH325: Discrete Math 2
Assignment 9

This assignment involves converting rational functions to power series and partial fractions.

The two most important formulas for converting rational functions to power series are

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$$

and

$$\left(\frac{1}{1-x}\right)^k = \sum_{n=0}^{\infty} \binom{k+n-1}{n} x^n$$

(Well ... actually the second formula include the first ... $k = 1$ in the second gives you the first.) In order to use both, remember that x is a variable. For instance you can think of the first formula as:

$$\frac{1}{1-\text{BLAH}} = \sum_{n=0}^{\infty} \text{BLAH}^n$$

So for instance this is also true

$$\frac{1}{1-12345x} = \sum_{n=0}^{\infty} (12345x)^n = \sum_{n=0}^{\infty} 12345^n x^n$$

Remember that if you want to test the equality above with a specific value for x, you can only use a value for x such that $|12345x| < 1$, i.e., $|x| < 1/12345$. If you go outside this range, the power series will very likely blow up in your face. Also,

$$\left(\frac{1}{1-123x^{567}}\right)^{999} = \sum_{n=0}^{\infty} \binom{999+n-1}{n} (123x^{567})^n = \sum_{n=0}^{\infty} \binom{999+n-1}{n} 123^n x^{567n}$$

Note however that the two 1's must be 1's:

$$\frac{\underline{1}}{\underline{1}-x} = \sum_{n=0}^{\infty} x^n$$

As shown in class if they are not 1's, then you just ... well ... make them 1:

$$\frac{111}{222-333x} = 111\frac{1}{222-333x} = \frac{111}{222}\frac{1}{1-333x/222}$$

In questions where you are asked to read a coefficient, if the value is too big, you can simply tidy up the expression and leave it as it is, i.e., you need not evaluate the expression to get a value. You can leave huge powers, binomial coefficients, etc. alone.

Q1. (a) Rewrite

$$\frac{1}{1-2x}$$

as a power series.

(b) What is the coefficient of x^{1000}?

SOLUTION.

(a)

$$\frac{1}{1-2x} = ???$$

ANSWER: $\boxed{???}$ □

(b)

ANSWER: $\boxed{???}$ □

Q2. (a) Rewrite

$$\frac{1}{5x-3}$$

as a power series.

[HINT: Remember to make the above like $\frac{1}{1-\text{BLAH}}$.]

(b) What is the coefficient of x^{1000}?

SOLUTION.

(a)

$$\frac{1}{5-3x} = ???$$

ANSWER: $\boxed{???}$ $\square$

(b)

ANSWER: $\boxed{???}$ $\square$

Q3. (a) Rewrite

$$\left(\frac{1}{2x-3}\right)^5$$

as a power series.

(b) What is the coefficient of x^3?

SOLUTION.

(a)

$$\left(\frac{1}{2x-3}\right)^5 = ???$$

ANSWER: $\boxed{???}$ □

(b)

ANSWER: $\boxed{???}$ □

Q4. (a) Rewrite

$$\left(\frac{1}{1-4x^2}\right)^5$$

as a power series.

(b) What is the coefficient of x^{1000}?

(c) What is the coefficient of x^{1001}?

SOLUTION.

(a)

$$\left(\frac{1}{1-4x^2}\right)^5 = ???$$

ANSWER: $\boxed{???}$ □

(b)

ANSWER: $\boxed{???}$ □

Q5. (a) Solve for A and B where

$$\frac{1}{(1-2x)(3x-1)} = \frac{A}{1-2x} + \frac{B}{3x-1}$$

(b) Rewrite

$$f(x) = \frac{1}{(1-2x)(3x-1)}$$

as a power series.

(c) What is the coefficient of x^{1000} of $f(x)$?

SOLUTION.

(a) When we multiply

$$\frac{1}{(1-2x)(3x-1)} = \frac{A}{1-2x} + \frac{B}{3x-1} \tag{1}$$

with $(1-2x)(3x-1)$, we get

$$1 = ??? \tag{2}$$

Substituting $x = ???$ into (2), we get

$$\begin{aligned} ??? &= ??? \\ \therefore \quad A &= ??? \end{aligned} \tag{3}$$

Substituting $x = ???$ into (2), we get

$$\begin{aligned} ??? &= ??? \\ \therefore \quad B &= ??? \end{aligned} \tag{4}$$

ANSWER: $\boxed{A = ???, \qquad B = ???}$ □

(b) Substituting (3) and (4) into (1) we get

$$\begin{aligned} f(x) &= \frac{1}{(1-2x)(3x-1)} \\ &= ???\frac{1}{1-2x} + ???\frac{1}{3x-1} \\ &= ??? \\ &= ??? \\ &= ??? \end{aligned}$$

ANSWER: $\boxed{???}$ □

(c)

ANSWER: $\boxed{???}$ □