

Geometric Proofs

1. Prove $q(\bar{a} + \bar{b}) = q\bar{a} + q\bar{b}$

$$q(|\bar{a}| + |\bar{b}|)$$

$$= q(\sqrt{a_1^2 + a_2^2 + \dots + a_n^2} + \sqrt{b_1^2 + b_2^2 + \dots + b_n^2})$$

$$= q\sqrt{a_1^2 + a_2^2 + \dots + a_n^2} + q\sqrt{b_1^2 + b_2^2 + \dots + b_n^2}$$

$$= q|\bar{a}| + q|\bar{b}|.$$

2. Prove $(p+q)\bar{a} = p\bar{a} + q\bar{a}$

$$(p+q)|\bar{a}| = p|\bar{a}| + q|\bar{a}|$$

$$= (p+q)\sqrt{a_1^2 + a_2^2 + \dots + a_n^2}$$

$$= p\sqrt{a_1^2 + a_2^2 + \dots + a_n^2} + q\sqrt{a_1^2 + a_2^2 + \dots + a_n^2}$$

$$= p|\bar{a}| + q|\bar{a}|.$$

3. Prove $p(q\bar{a}) = (pq)\bar{a}$

$$p|q\bar{a}|$$

$$= p\sqrt{(qa_1)^2 + (qa_2)^2 + \dots + (qa_n)^2}$$

$$= pq\sqrt{a_1^2 + a_2^2 + \dots + a_n^2}$$

$$= (pq)|\bar{a}|.$$

4. Prove $|\bar{a}| = \bar{|a|}$

$$|\bar{a}|$$

$$= \sqrt{a_1^2 + a_2^2 + \dots + a_n^2}$$

$$= \sqrt{a_1^2 + a_2^2 + \dots + a_n^2}$$

$$= |\bar{a}|.$$