

Exercise 3.2

Algebraic Proofs

1. Prove $q(\bar{a} + \bar{b}) = q\bar{a} + q\bar{b}$

$$q(\bar{a} + \bar{b})$$

$$= q(a_1 + b_1, a_2 + b_2, \dots, a_n + b_n) \quad \text{Def. 3.3}$$

$$= (qa_1 + qb_1, qa_2 + qb_2, \dots, qa_n + qb_n) \quad \text{Dist. Prop. Mult.}$$

$$= q\bar{a} + q\bar{b}. \quad \text{Def. 3.3}$$

2. Prove $(p+q)\bar{a} = p\bar{a} + q\bar{a}$ (Provided in text.)

$$(p+q)\bar{a}$$

$$= ((p+q)a_1, (p+q)a_2, \dots, (p+q)a_n) \quad \text{Def. 3.3}$$

$$= (pa_1 + qa_1, pa_2 + qa_2, \dots, pa_n + qa_n) \quad \text{Dist. Prop. Mult.}$$

$$= p\bar{a} + q\bar{a}. \quad \text{Def. 3.4}$$

3. Prove $p(q\bar{a}) = (pq)\bar{a}$

$$p(q\bar{a})$$

$$= p(qa_1, qa_2, \dots, qa_n) \quad \text{Def. 3.4}$$

$$= p(qa_1 + q\theta, qa_2 + q\theta, \dots, qa_n + q\theta) \quad \text{Ident. Prop. Vect. Add.}$$

$$= pq(\bar{a} + \theta)$$

Dist 1 Prop Vect./Scalar Mult.

$$= pq(\bar{a})$$

$$= (pq)\bar{a}.$$

4. Prove $1\bar{a} = \bar{a}$

$$1\bar{a}$$

$$= (1a_1, 1a_2, \dots, 1a_n)$$

$$= (a_1, a_2, \dots, a_n)$$

$$= \bar{a}.$$

Def. 3.4

Ident. Prop. Scalar Mult.

Def. 3.4