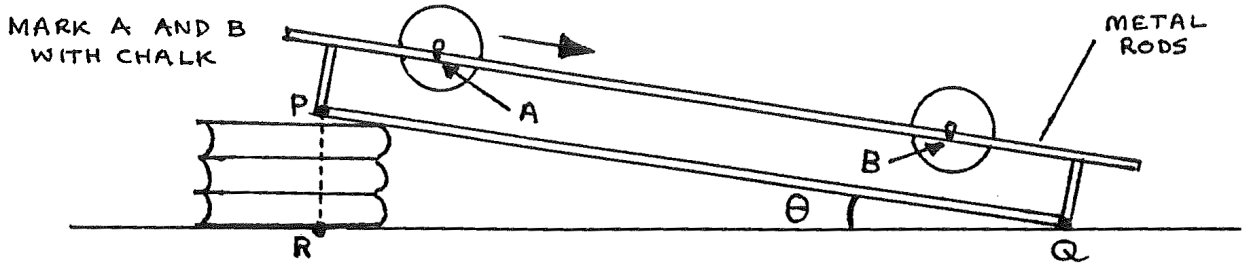
B3-1: The Moment of Inertia of a Cylinder



Apparatus

2 mounted rails; 2 different cylinders with axles; metre rule; micrometer screw gauge; stack of books; stop watch; piece of chalk; triple beam balance; graph paper

Procedure

1. Set up the apparatus as shown above with PR less than 4 cm. Measure $s = AB$ and measure PQ. Record s and PQ.
2. Measure and record PR. Calculate $\sin\theta = \frac{PR}{PQ}$.
3. Place a cylinder at A. Record the time, t, for the cylinder, starting from rest, to roll from A to B.
4. Determine the linear acceleration, a, of the cylinder using your readings of s and t.
5. Increase PR and repeat steps 2, 3, and 4. Increase PR three more times, repeating steps 2, 3, and 4 to obtain five sets of readings.
6. Measure the axle diameter and find the axle radius, r_a. Find the cylinder radius, r. Measure the mass, M, of the cylinder and axle.
7. Repeat steps 2 to 6 for the second cylinder.

Observations

For each cylinder:

M =_____kg

r_a =_____m

r =_____m

Tabulate:

PR (m)	PQ (m)	$\sin\theta = \frac{PR}{PQ}$	s (m)	t (s)	a (ms^{-2})

Theory

The cylinder loses potential energy (PE) and gains kinetic energy (KE) as it moves from A to B. Conservation of energy requires:

$$\text{PE lost} = \text{KE gained}$$
$$Mgh = Mgs\,(\sin\theta)$$

Ignoring friction this becomes the KE of the cylinder where the total KE is:

$$KE = \text{KE (linear)} + \text{KE (rotational)}$$

Therefore:

$$Mgs\,(\sin\theta) = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

Substitute $v^2 = 2as$ and $\omega = \frac{v}{r_a}$:

$$a = \left(\frac{Mgr_a^2}{Mr_a^2 + I}\right)(\sin\theta) \qquad \text{(check this yourself!)}$$

Analysis

1. Plot a graph of a against $\sin\theta$ for each cylinder on the same sheet of graph paper. Find the gradient of each line.
2. Given that $a = \left(\frac{Mgr_a^2}{Mr_a^2+I}\right)\sin\theta$ find I for each cylinder.
3. From theory $I = \frac{1}{2}Mr^2$ where $r = \text{cylinder radius}$. Calculate I using this to check your value from 2 above. Give the % error for your value from 2.
4. If $I = Mk$, find the radius of gyration, k, for each cylinder.
5. Calculate the torque necessary to steadily accelerate each cylinder from rest to an angular velocity of 30 radians/s in 2s.