Day 2, Session 1: Graphs

Jessica Williams-Nguyen and Brian D. Williamson

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Graphs

Why do we use graphs?

- Describe relationships in the data

- Visualize functions

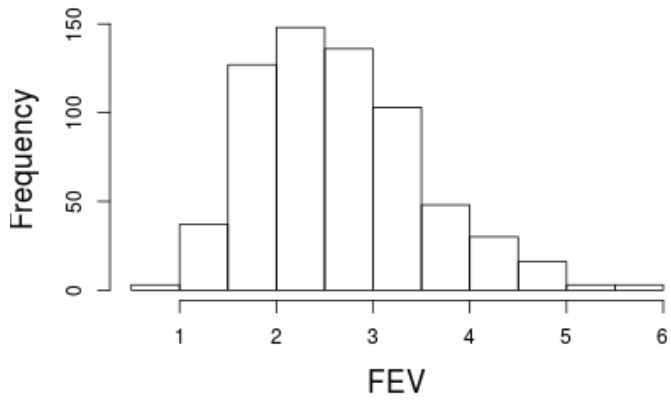
Graphs are very useful in exploratory analyses, or for description. Often a graph (or multiple graphs) provide visual evidence that supports a statistical analysis.

Example data analysis: FEV (from Scott Emerson, MD PhD)

Based on numerous studies, we believe that smoking tends to impair lung function. Much of the data to support this claim arises from studies of long-term adult smokers. A natural question is: can deleterious effects of smoking be detected in children who smoke?

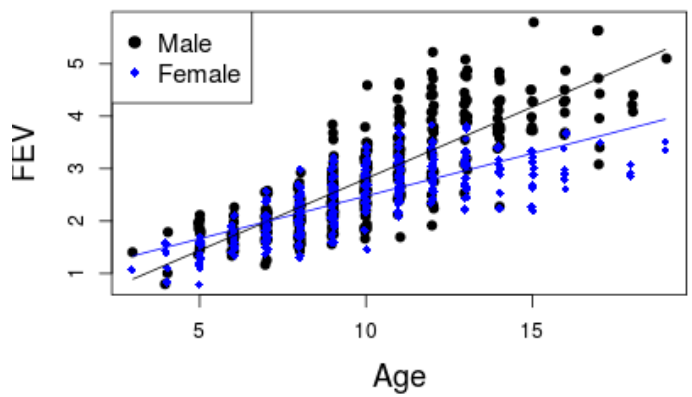
- Data on 654 children seen during routine check-up at pediatric clinic
- Outcome: forced expiratory volume (FEV)
  - measures how much air you can blow out of your lungs in a short period of time
  - higher FEV typically associated with better respiratory function
- Predictor of interest: smoking status
- Other variables: sex, age, height

Common types of graphs in data analysis



(a) Histogram

(b) Boxplot



(c) Scatterplot

What do graphs tell us?

- Histograms: summaries of one-dimensional distributions
  - Counts or frequencies of each occurrence
- Boxplots: summaries of two-dimensional distributions
  - measures of center (typically median)
  - measures of spread (typically inter-quartile range)
- Scatterplots: summaries of two-dimensional distributions
  - Can visualize the whole data
  - Trends in two or more dimensions by using different colors/shapes for strata

Linear trends

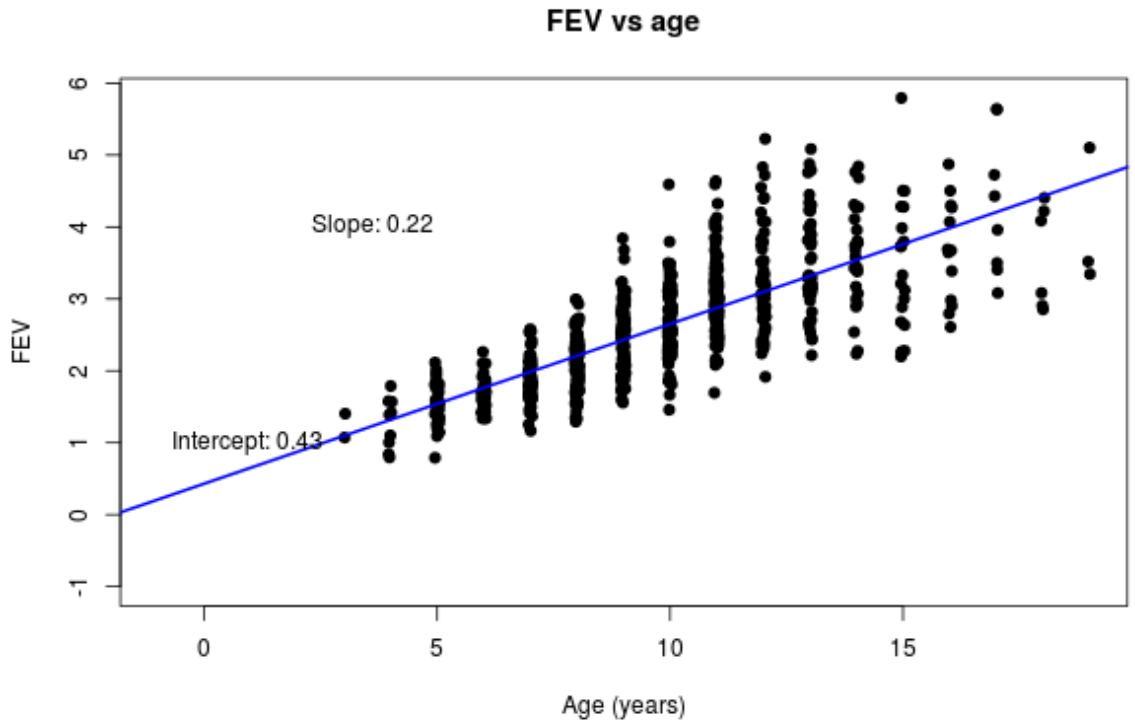
- A common way to describe data—determine if the trend is increasing or decreasing
  - Example: test scores tend to increase with time spent studying
- Lines are easy to compute
  - Only need a point and a slope
  - Two common forms of linear equations

Slope-intercept form

- $y = mx + b$
- Slope: m
  - Rate of change, i.e., how does y change with each one unit change in x?
  - Example: speed, the distance traveled with each unit change in time
- Intercept: b
  - The point where the line crosses the $y-$axis

Example: FEV

Fitting a trend line to a scatter plot of the FEV data helps us understand the association between age and FEV. If we use the *least squares* criterion to fit this line (you'll learn more about this when you cover linear regression), we obtain the plot below:

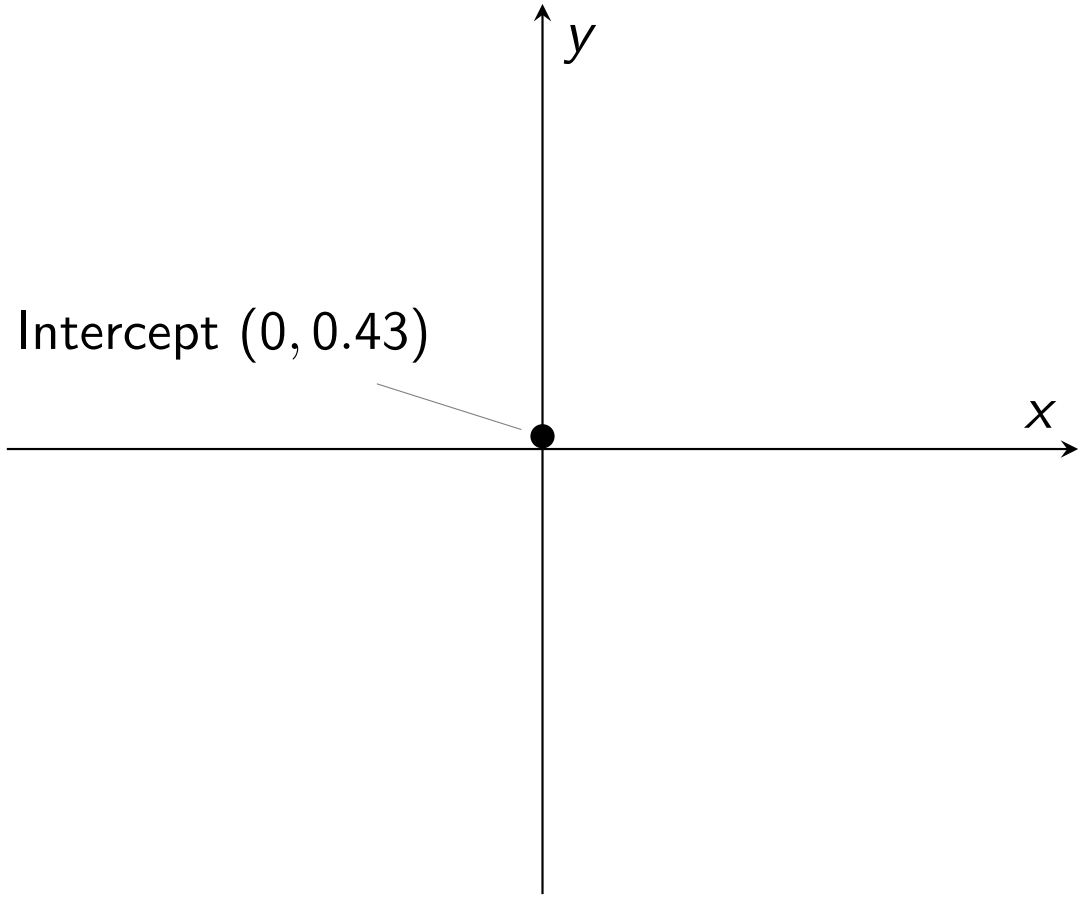


Example: FEV

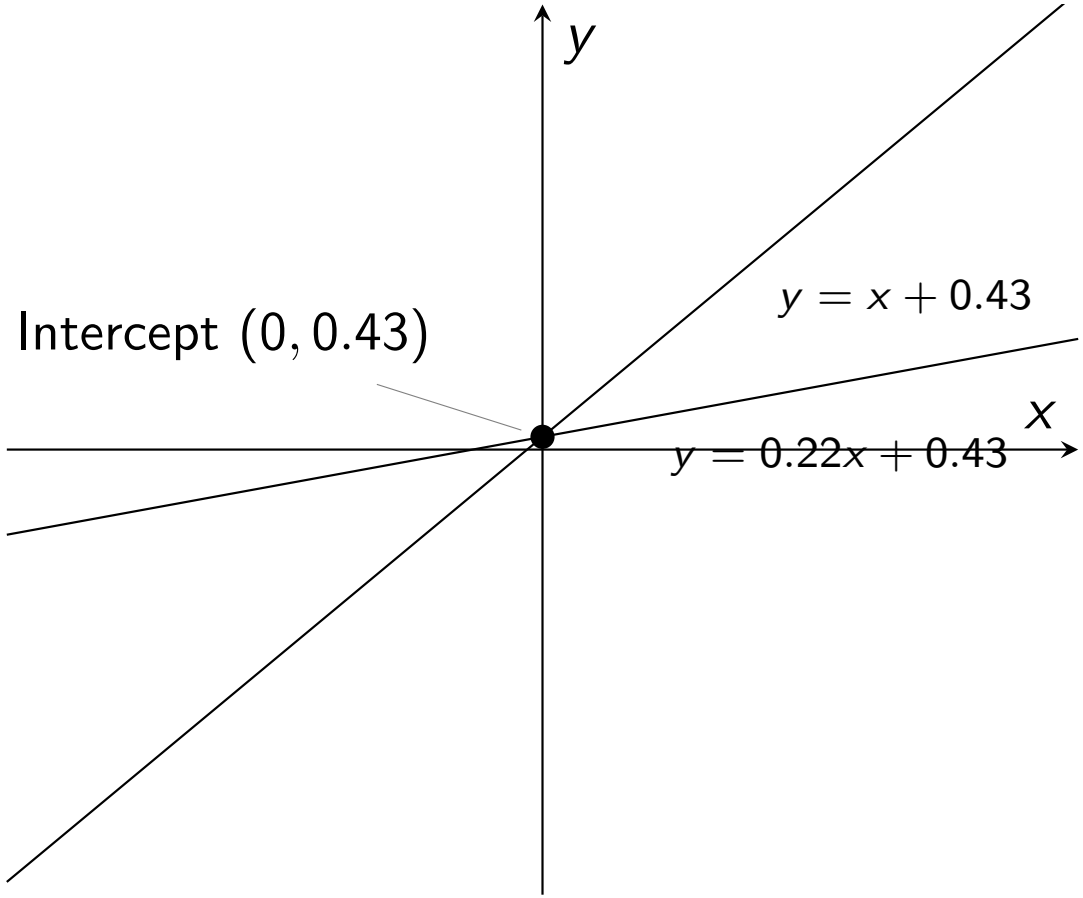
The intercept, 0.43, is interpreted as the mean FEV for children at age zero (does this make sense?). The slope, 0.22, is interpreted as the mean increase in FEV for each one year increase in age, where older children tend to have higher mean FEV.

How do the slope and intercept together determine a line? What happens to the interpretation of our regression coefficients (which is what the slope and intercept are in this example) if either one of these values is different?

Slope-intercept form: determining a line



Slope-intercept form: determining a line



What is the difference between the two slopes? How does this affect our understanding of the association between age and FEV?

Point-slope form

- $y - y_1 = m(x - x_1)$
- Point: (x_1, y_1)
  - A point on the line (can be any point! Even the intercept!)
- Slope: m
  - Same as in slope-intercept form!
- Example: $y - 0.43 = 0.22(x - 0)$ is the same as $y = 0.22x + 0.43$ in slope-intercept form!

Exercise: slopes and intercepts

Consider data examining inflammatory biomarkers and mortality. We are interested in the association between the biomarker fibrinogen (related to inflammation) and prior history of cardiovascular disease (CVD). These data are described here. Your collaborator ran a linear regression and obtained the following equation (using the estimated regression coefficients), where y denotes fibrinogen (ranges from 109 mg/dl to 872 mg/dl) and x denotes presence/absence of prior CVD (0/1):
$y = 14.89x + 319.57$.

1. What is the slope of the line $y = 14.89x + 319.57$?

2. What is the $y-$intercept of the line $y = 14.89x + 319.57$?

3. Does fibrinogen tend to increase or decrease with the presence of prior CVD compared to the absence of prior CVD?

4. What is the slope of the line $y + 1 = 2(x - 1)$?

Solution: slopes and intercepts

1. The equation is in slope-intercept form, so the slope is 14.89
2. The equation is in slope-intercept form, so the intercept is 319.57
3. The slope estimate is positive, so fibrinogen tends to increase with presence of prior CVD
4. The equation is in point-slope form, so the slope is 2
5. The equation is in point-slope form, so the point is $(1, -1)$

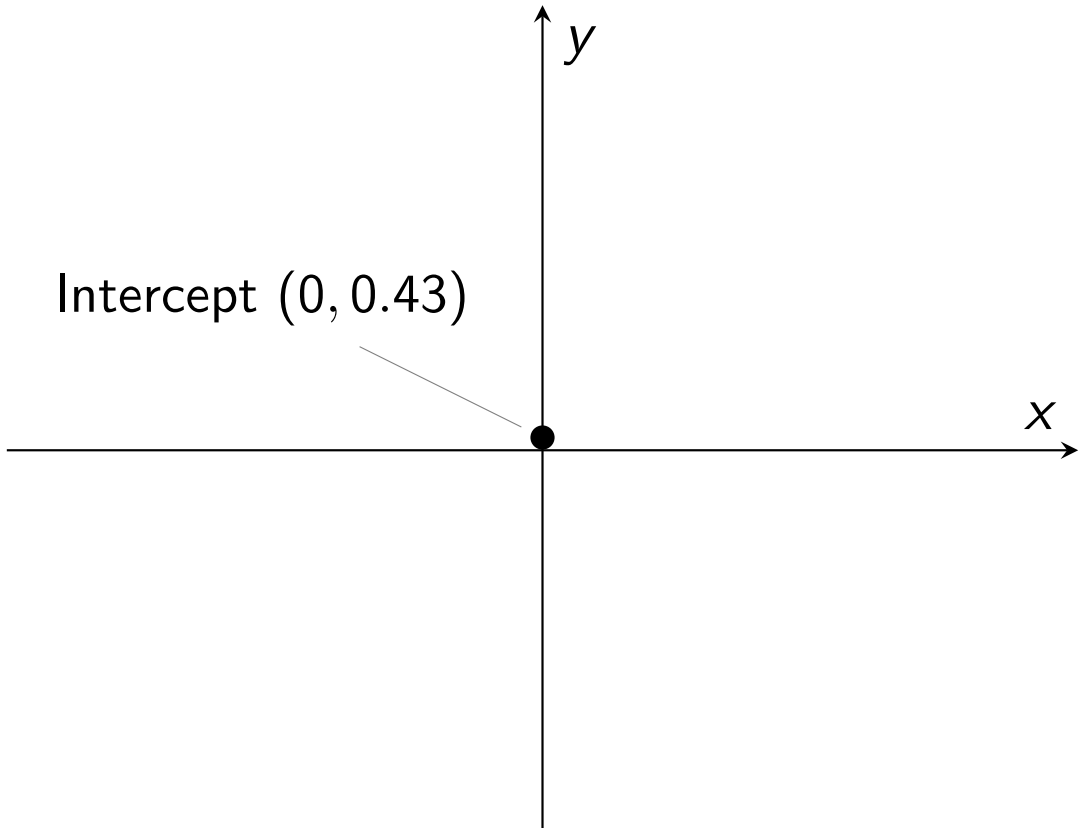
Creating a graph using an equation

- Steps:
  1. Draw axes
  2. Place a point at the $y-$intercept (slope-intercept form) or at the starting point (point-slope form)
  3. Increase x by one unit, increase y by m units, place a new point
  4. Draw a line between the old point and the new point!

Example: creating a graph using an equation

- Equation $y = 0.22x + 0.43$ (from linear regression of FEV on age, in the FEV data)
- Slope: 0.22, Intercept: 0.43

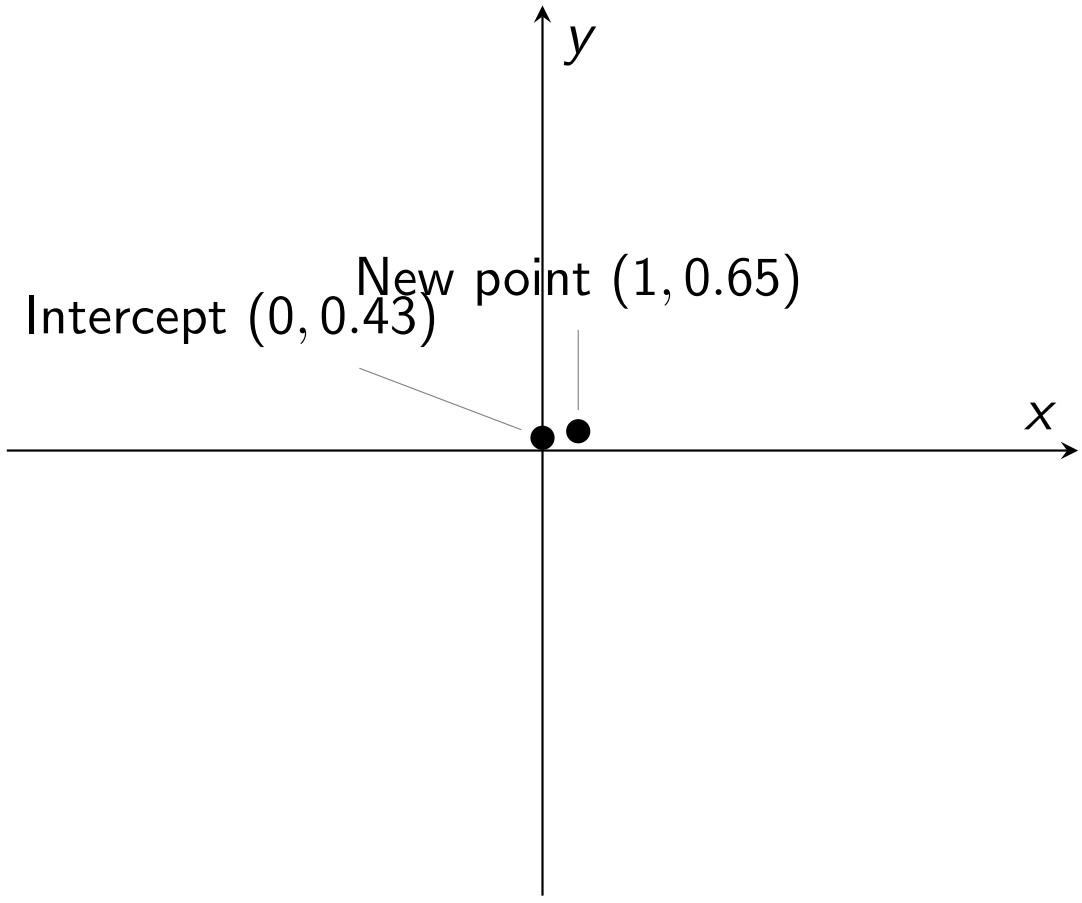
1. Draw a point at $(0, 0.43)$



Example: creating a graph using an equation

- Equation $y = 0.22x + 0.43$
- Slope: 0.22, Intercept: 0.43

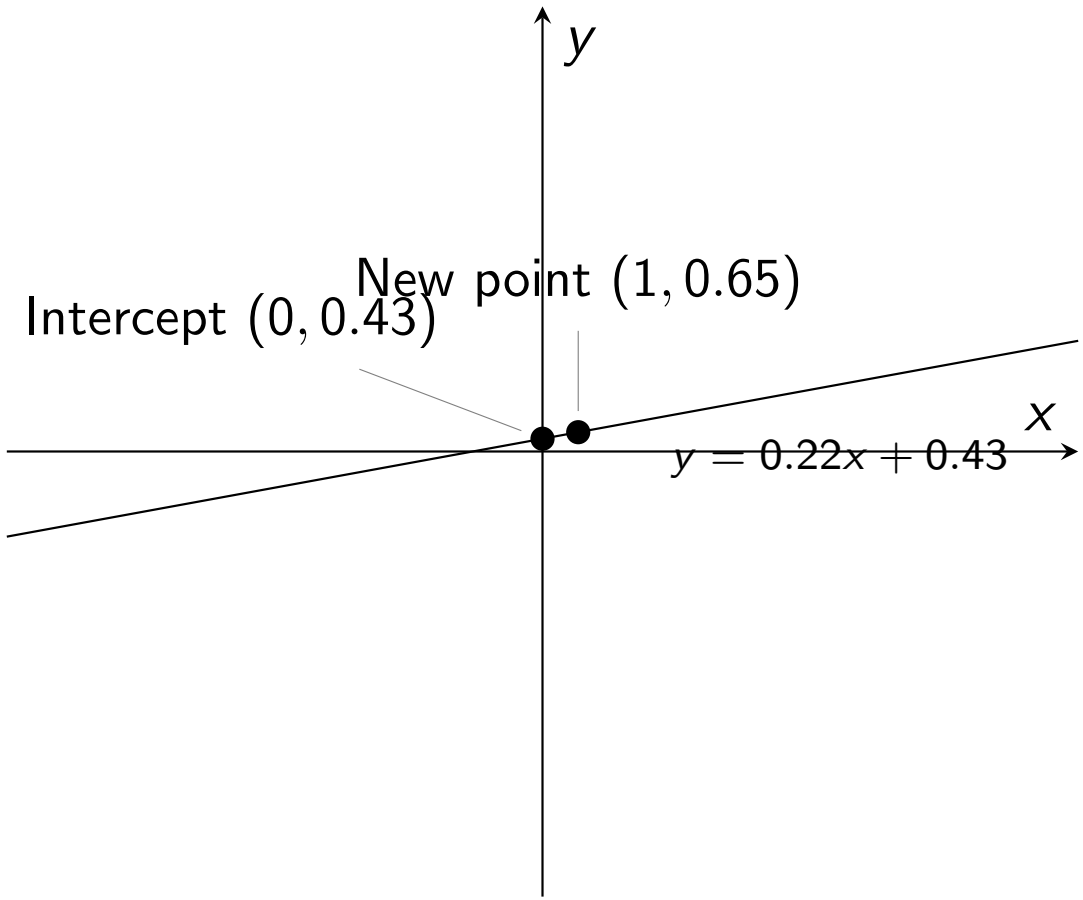
2. Increase x to 1, increase y by 0.22. New point at $(1, 0.65)$



Example: creating a graph using an equation

- Equation $y = 0.22x + 0.43$
- Slope: 0.22, Intercept: 0.43

3. Draw a line!



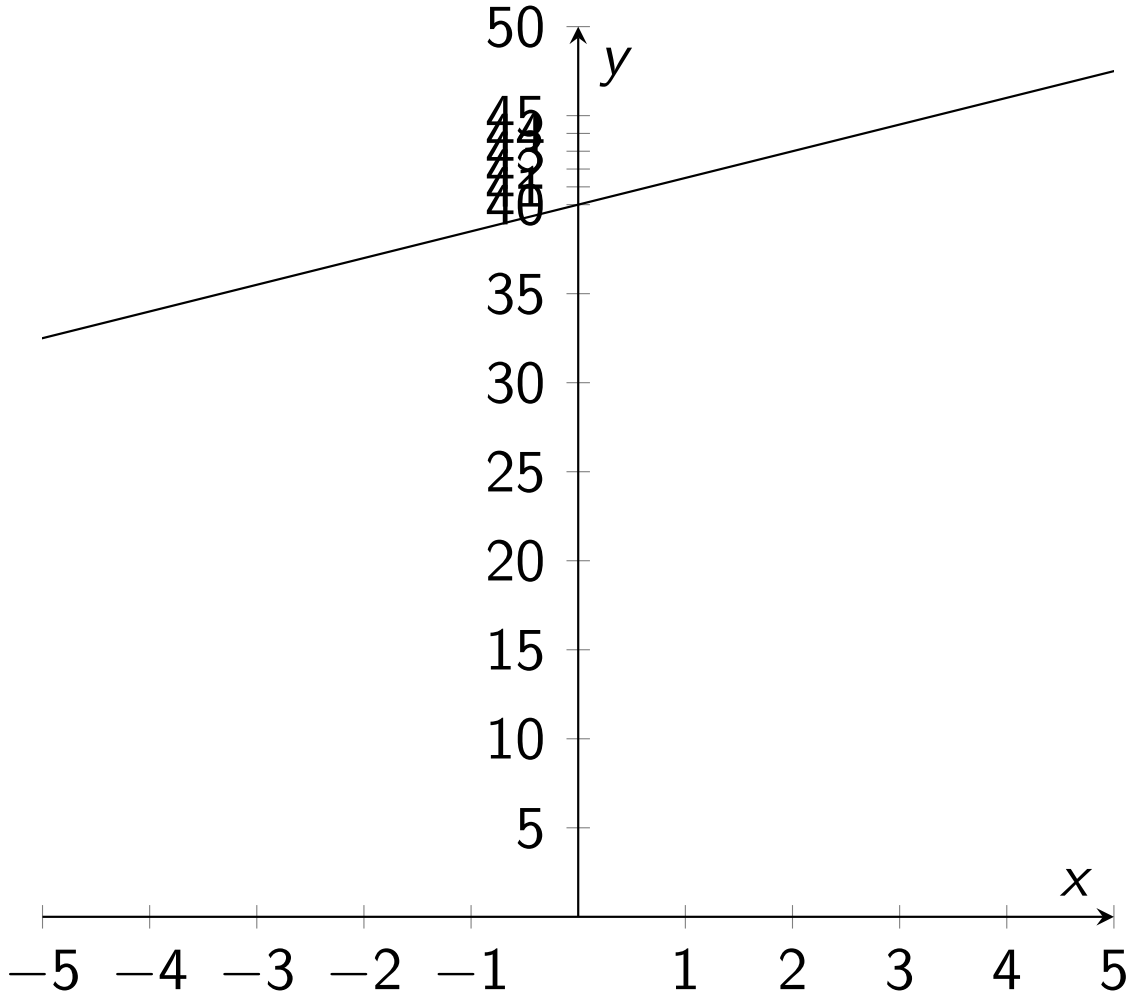
Reading an equation from a graph

- Two options:
  1. Slope-intercept form
     - 1.1 Find the $y-$intercept
     - 1.2 Find the slope: how much does y change with each 1 unit difference in x?
  2. Point-slope form
     - 2.1 Choose any point on the line
     - 2.2 Find the slope: how much does y change with each 1 unit difference in x?

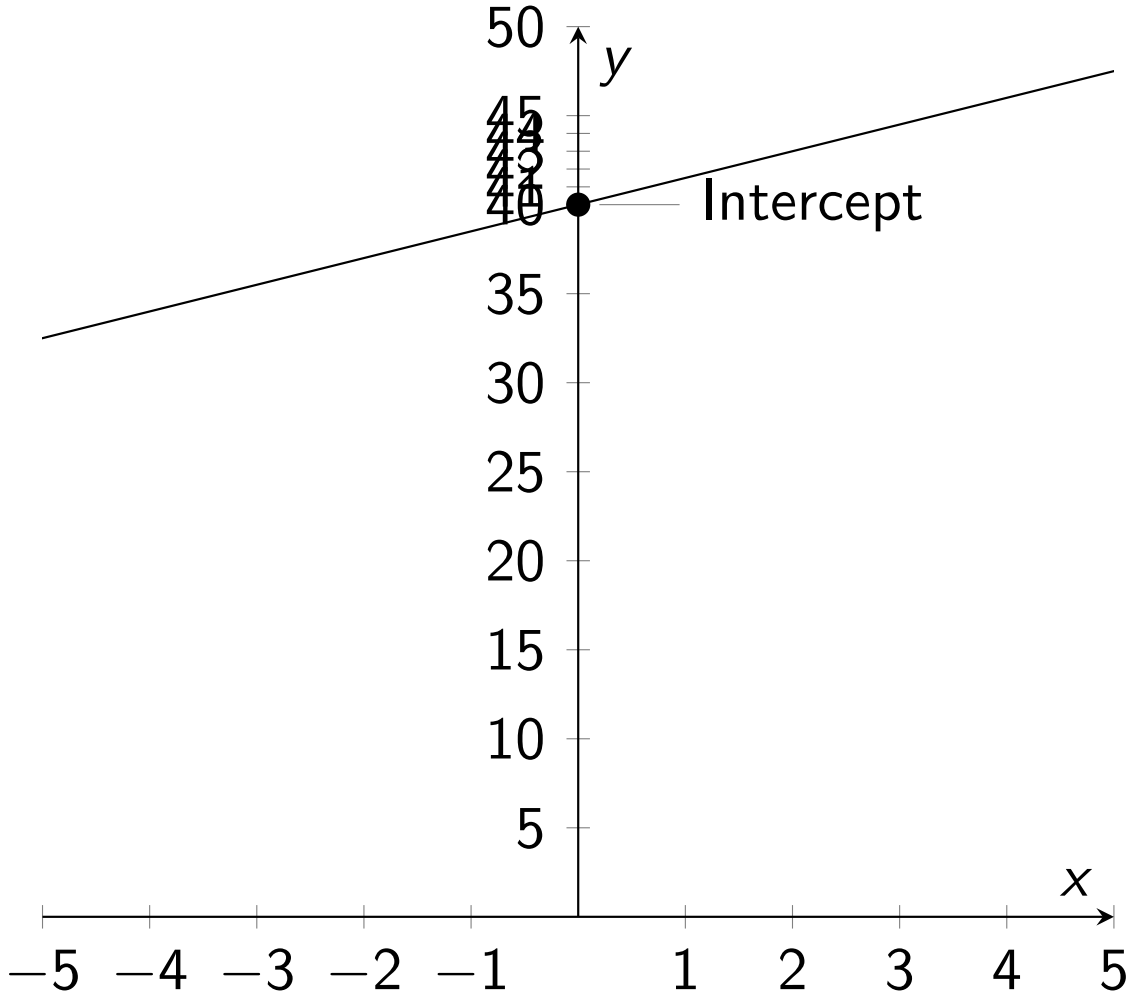
Example: age (years) and height (inches)

Say we have a random sample of 100 pre-school aged-children (age 3–5) from the Seattle area. We expect height to increase with increasing age. We fit the best fitting line to these data to try to better understand these data, and get the plot on the next slide... what are the slope and intercept, and what do they tell us about this association?

Example: reading an equation from a graph

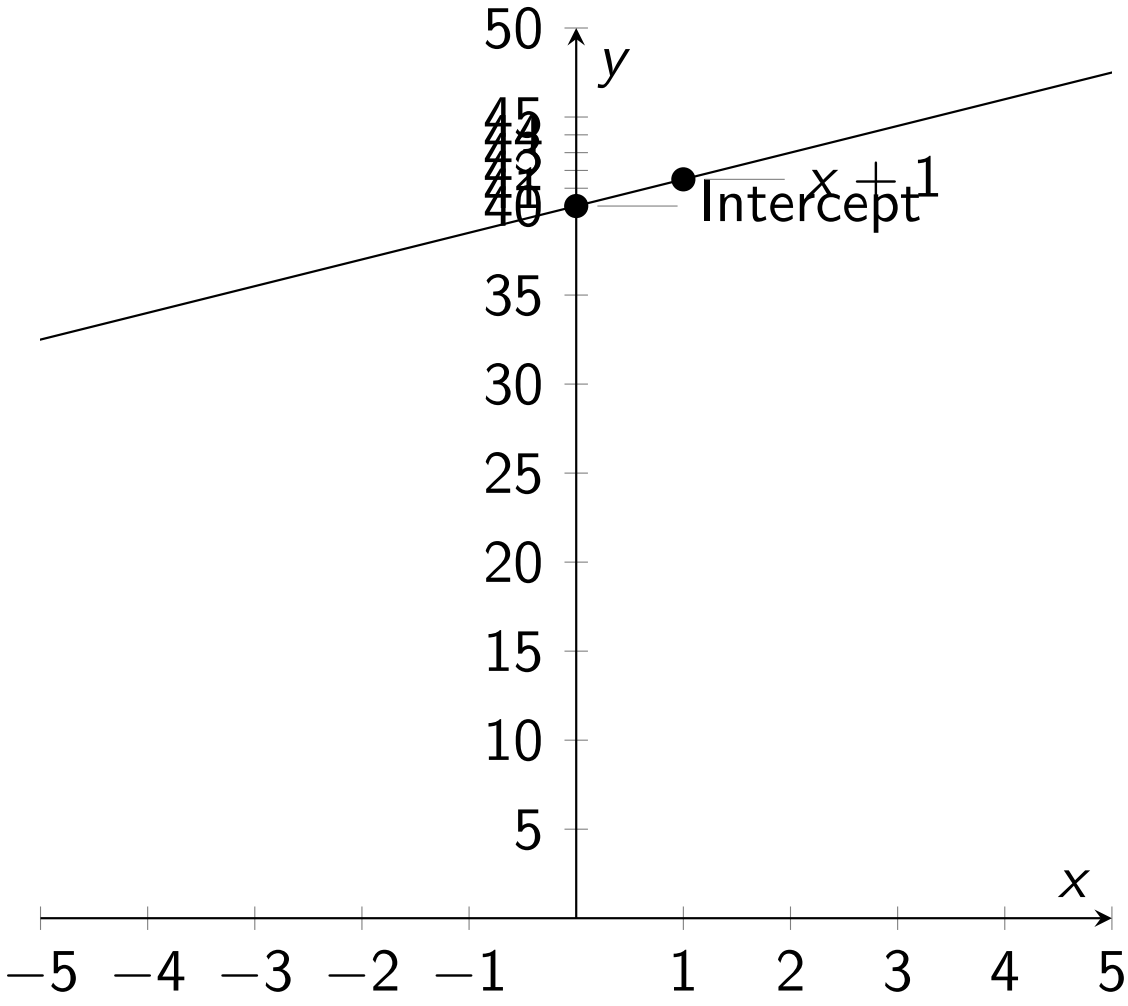


Example: reading an equation from a graph



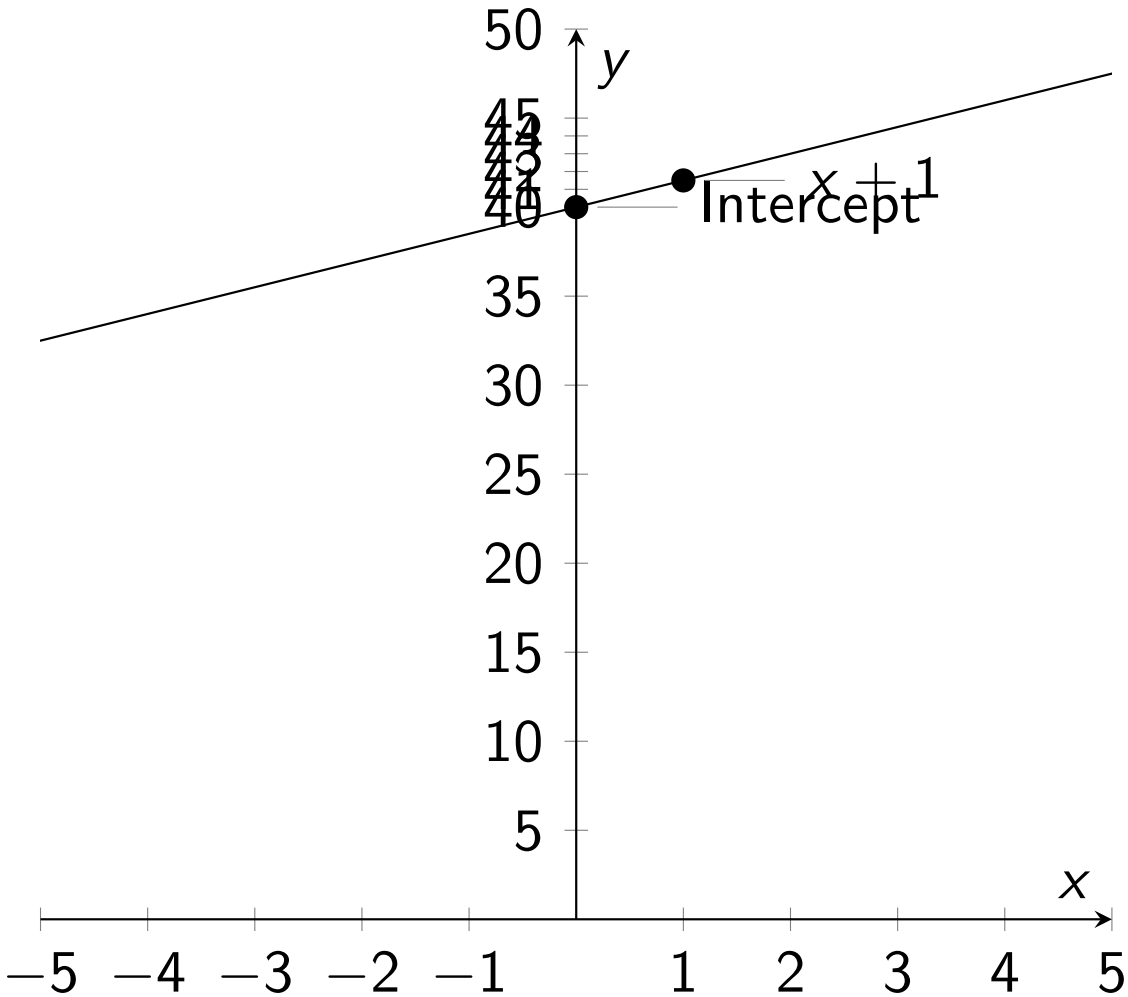
Example: reading an equation from a graph

1. Find the intercept. Here it is 40



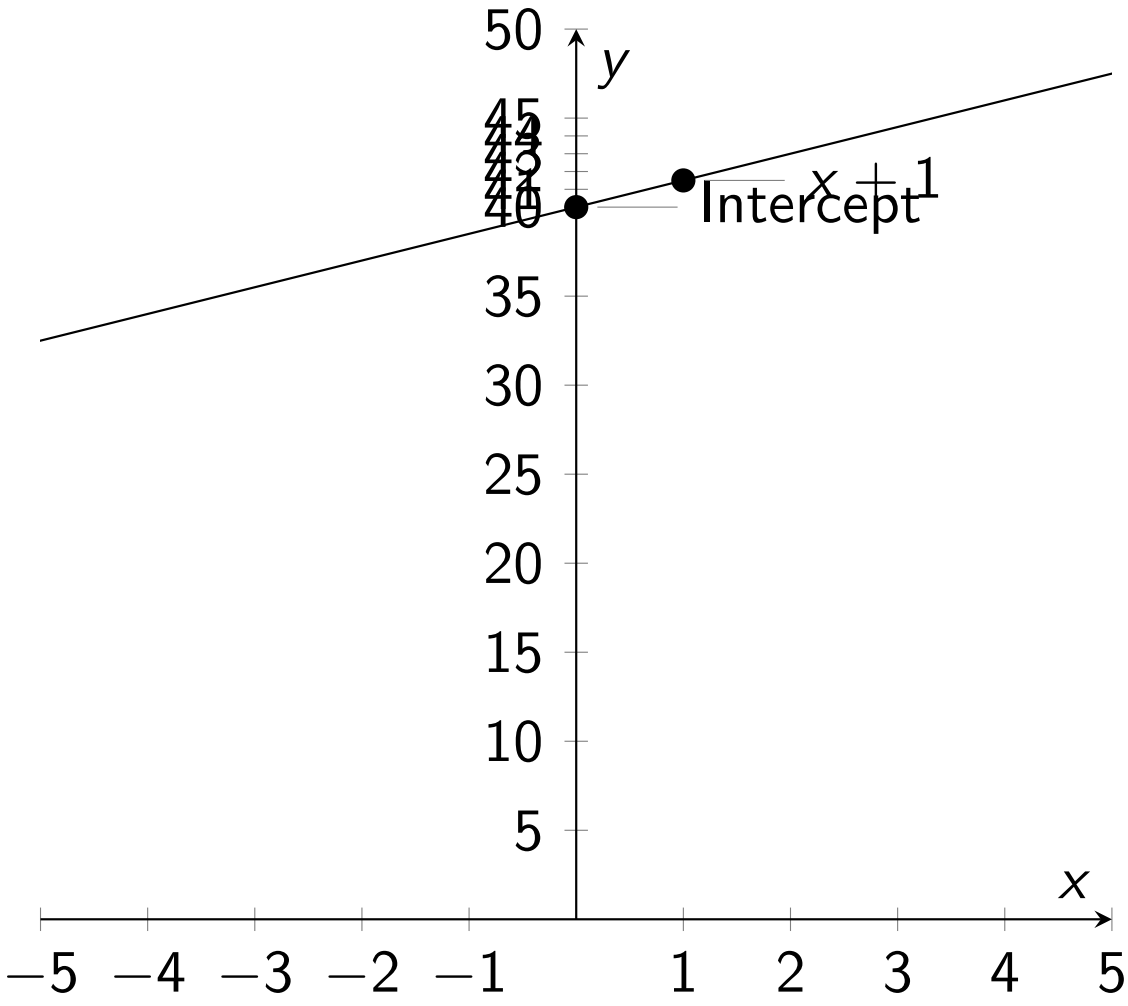
Example: reading an equation from a graph

2. Increase x by one to find the slope. The y value at $x = 1$ is 41.5 (a bit hard to read), so y changed by 1.5



Example: reading an equation from a graph

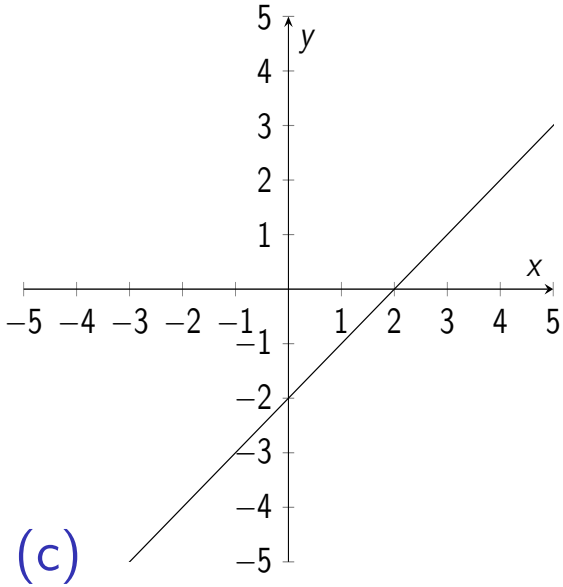
3. Slope-intercept: $y = 1.5x + 40$, point-slope: $y - 40 = 1.5(x - 0)$



Exercise: matching graphs to equations

1. Which is the graph of $y = 2x - 3$?

(a)



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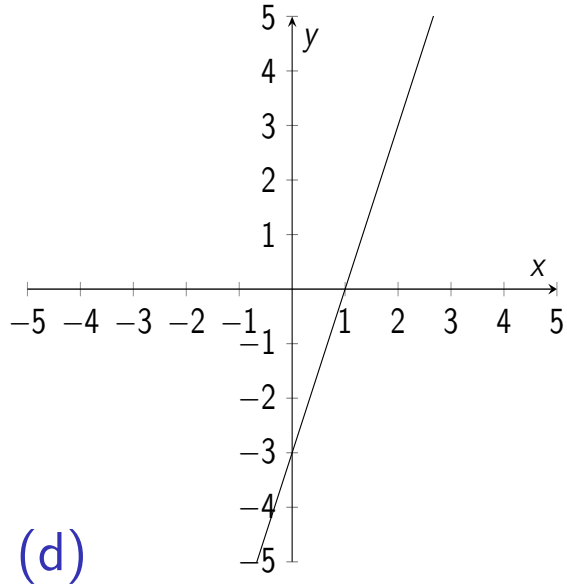

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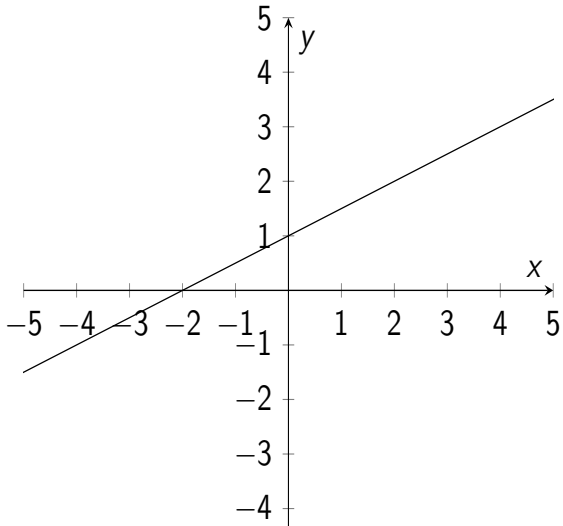
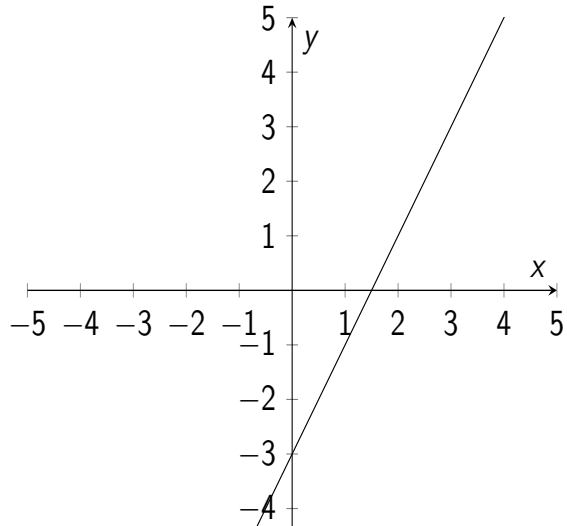


(d)



Solution: matching graphs to equations

(a) Intercept: -2, slope: 1

(b) Intercept: -3, slope: 3

(c) Intercept: 1, slope: 1

(d) Intercept: -3, slope: 2 ✓

Quadratics

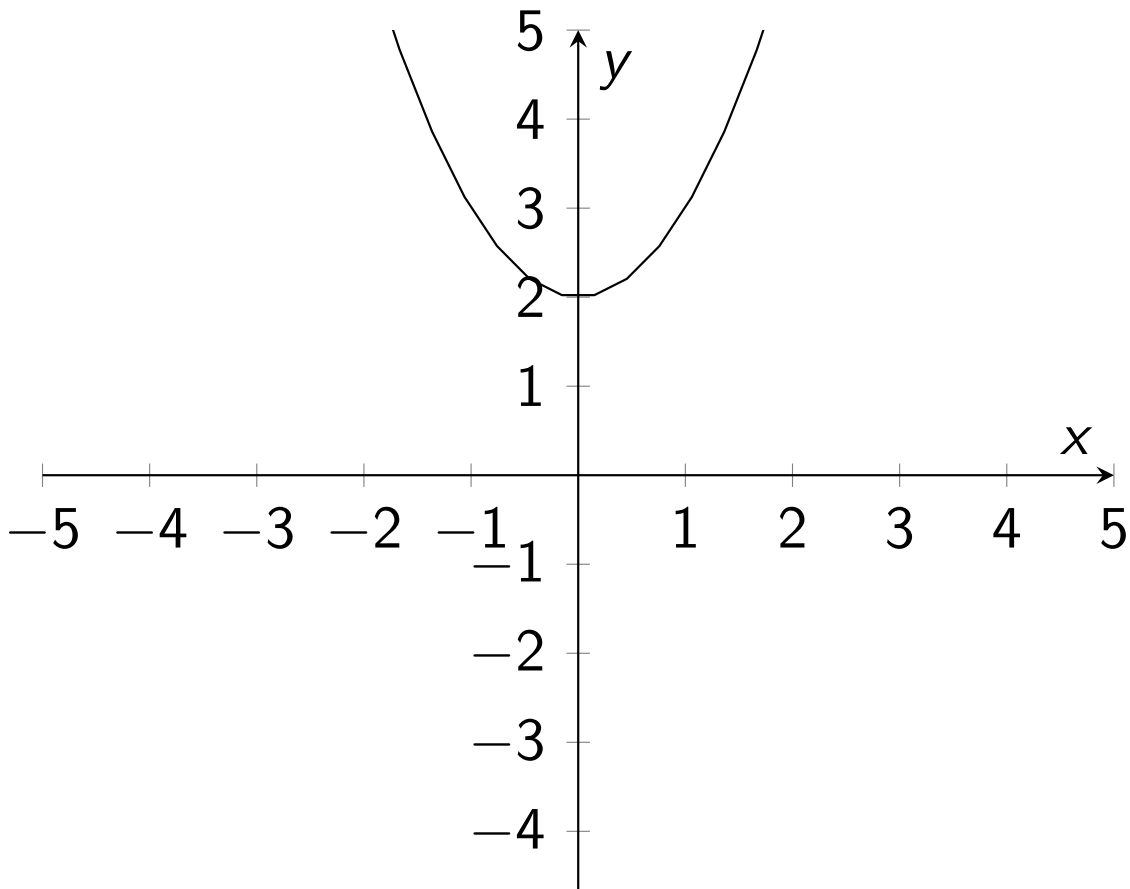
- Sometimes we believe a trend is higher order than linear
  - For example, height might increase more quickly with age at a younger age, and then level off at older ages
- Higher order terms allow more flexibility in modeling
- Quadratics are the natural next step from linear terms, and are shaped like parabolas

Defining a quadratic

- Standard form: $y = ax^2 + bx + c$
- a determines the direction of the tails and the degree of curvature
  - $a > 0$ means the curve faces up (convex)
  - $a < 0$ means the curve faces down (concave)
  - large $|a| > 1$ means steep slope
  - $0 < |a| < 1$ means shallow slope
- b and a together determine the $x-$coordinate of the vertex: $x = -\frac{b}{2a}$
- c controls the height

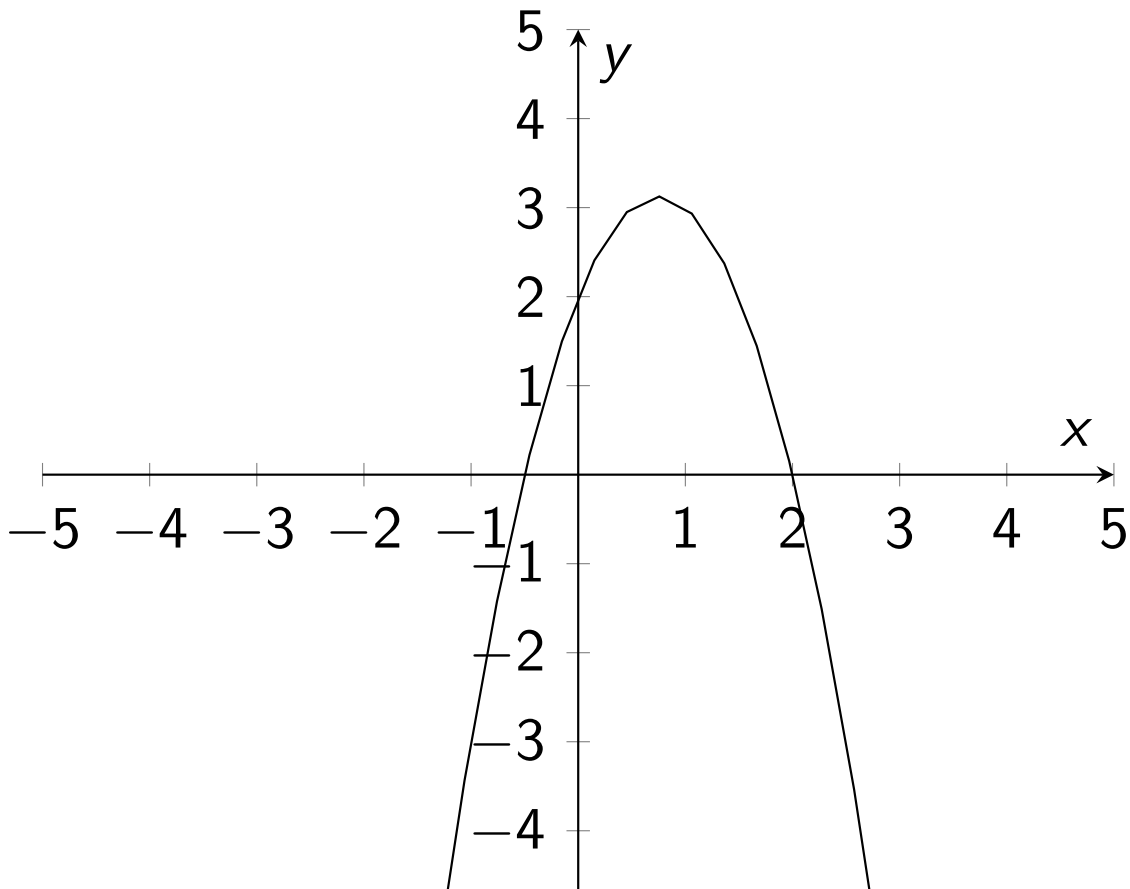
Example: quadratics

- $y = x^2 + 2$
- $a = 1,\ b = 0,\ c = 2$
- Not too steep, convex, and vertex is at $(0, 2)$



Example: quadratics

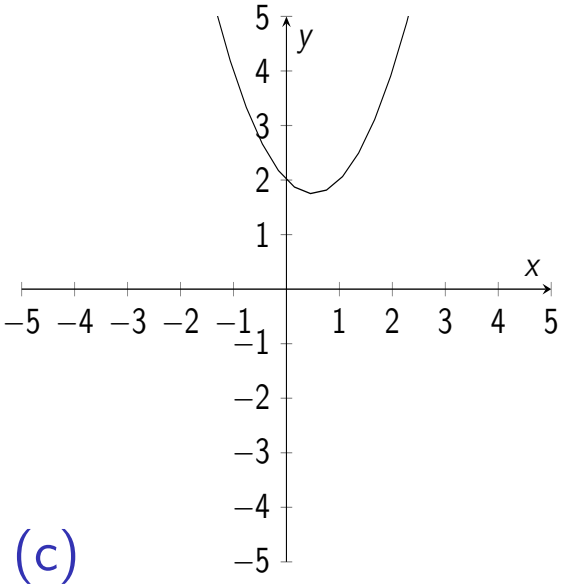
- $y = -2x^2 + 3x + 2$
- $a = -2,\ b = 3,\ c = 2$
- Steeper than before, concave, and vertex is at $(3/4, 50/16)$



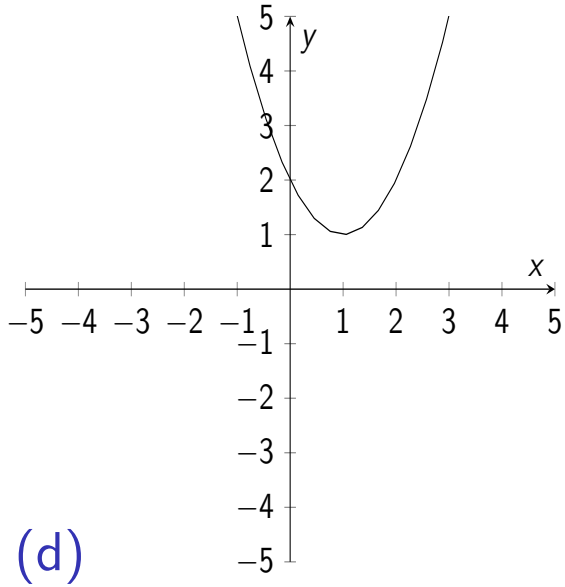
Exercise: quadratics

1. Which is a plausible plot of $x^2 - x + 2$?

(a)



(b)



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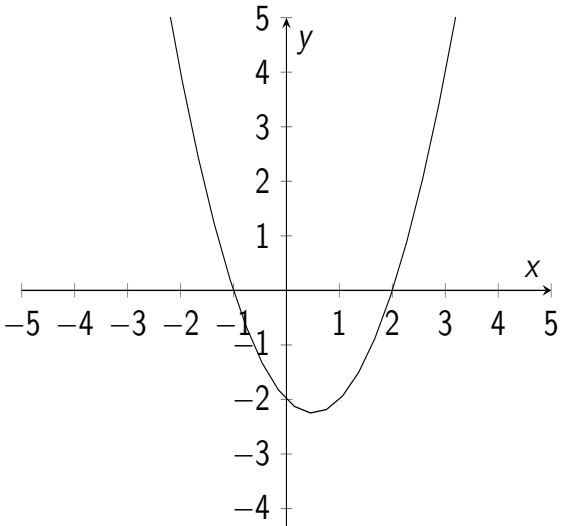

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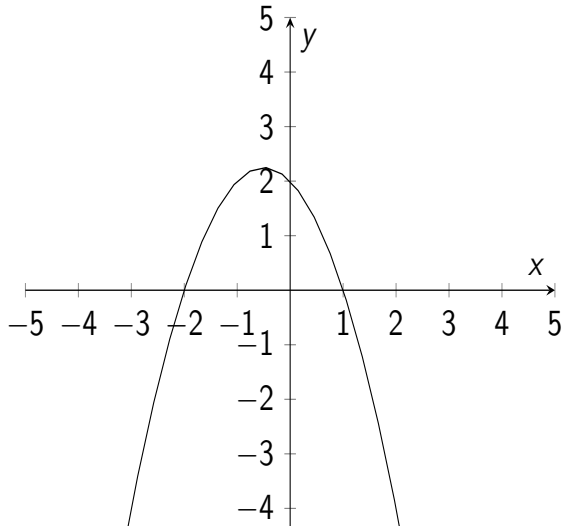


Solution: quadratics

1. $a = 1$, $b = -1$, $c = 2$
   - We are looking for a plot where the vertex has $y-$coordinate near 2
   - This rules out (b)
   - Now we want a plot with $y = 2$ when $x = 1$, ruling out (b) [and (d)]
   - Of the two remaining, (d) is concave, so it has $a < 0$
   - (a) is the solution!

Transforming graphs

In the example of the association between FEV and age, we obtained the line $y = 0.22x + 0.43$; this means that children at age 0 have a mean FEV of 0.43 L/sec, and that mean FEV tends to be 0.22 L/sec higher for each one year increase in age.

How can we make the intercept interpretable? What would the association be if the slope were different, and what would this look like? What would happen if we changed the units on FEV (to, say, mL/sec)?

These are all examples of transformations:

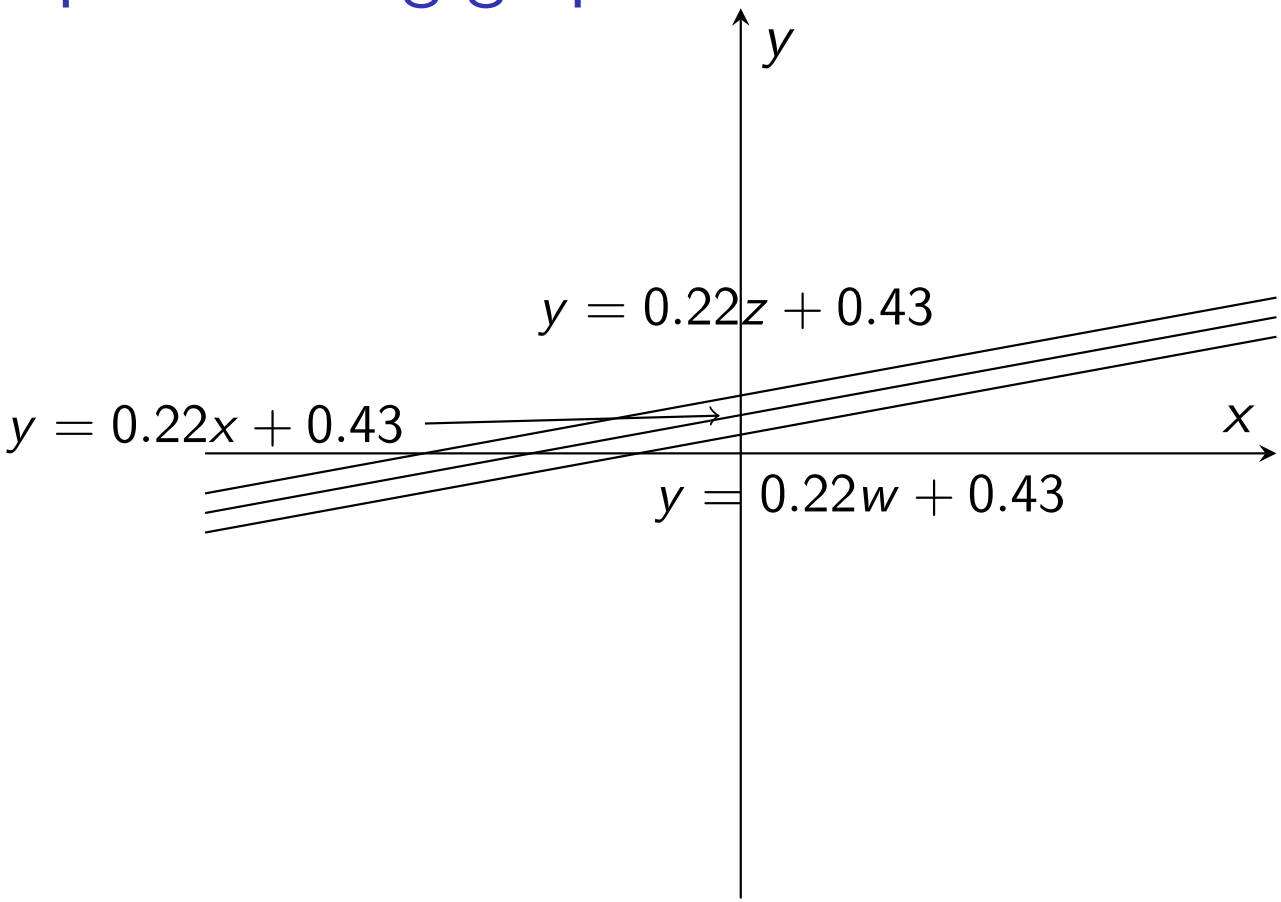
- Once we have one graph, how do we get another?

- Two main types of transformations: shifting and stretching

Shifting graphs

- Sometimes we want to change the interpretation of the intercept
  - FEV data: e.g., we could make the intercept the mean FEV at the average age in the data
- Once we know the properties of a graph, shifting doesn't change much!
- Shift left: add to x, get z
- Shift right: subtract from x, get w
- Shift up: add to intercept
- Shift down: subtract from intercept
- Why? Adding to x: smaller z's now have the same y as x. Subtracting from x: larger w's now have the same y as x.

Example: shifting graphs



Interpretations (from top to bottom):

- mean FEV in -1 year olds is 0.21 L/sec ($z = x + 1$)
- mean FEV in zero year olds is 0.43 L/sec
- mean FEV in one year olds is 0.65 L/sec ($w = x - 1$)

Stretching graphs

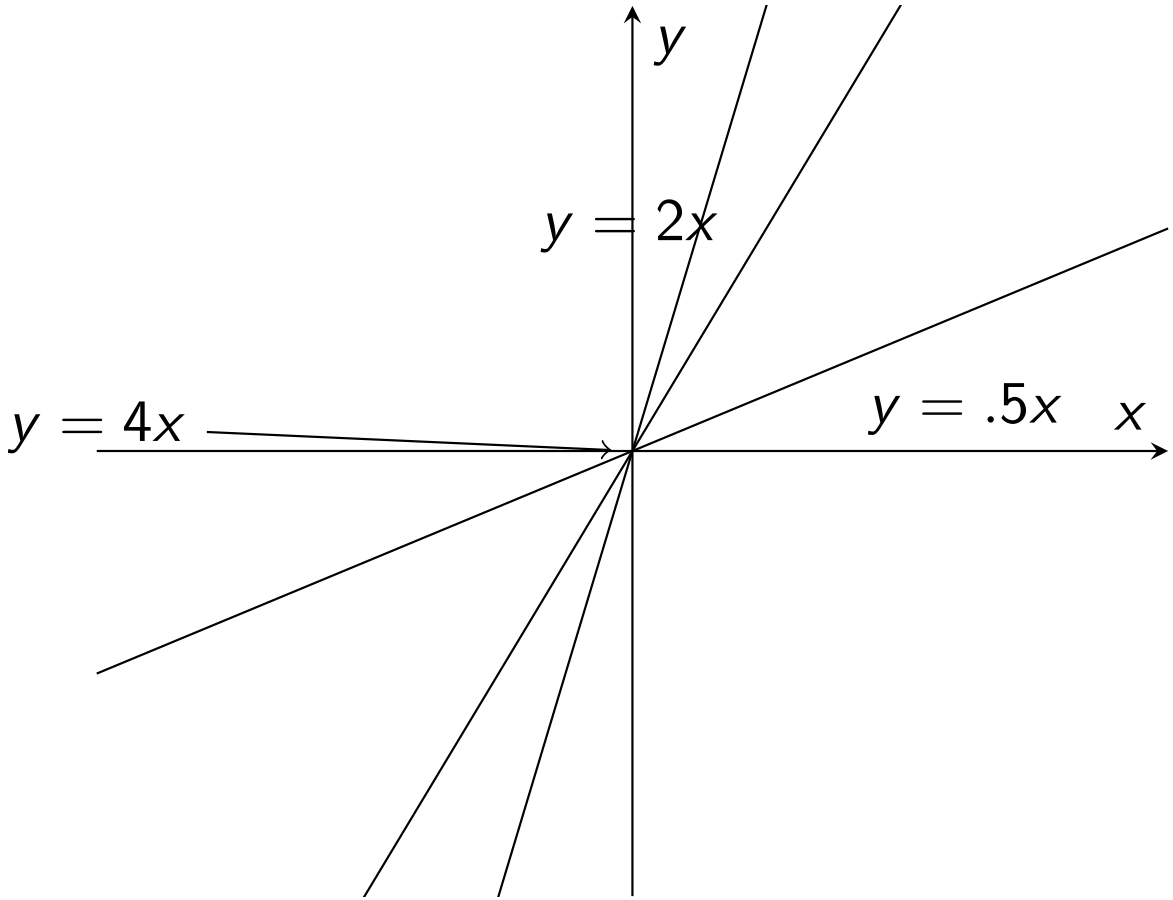
If we change the units on our predictor of interest, we will change the slope (and hence the interpretation). We also might want to see how the graph would change if the slope were larger or smaller, using the same units—this helps to understand the estimated association between our predictor and the outcome.

These are examples of stretching:

- Make the graph steeper, or shrink it: make $|m|$ larger in a linear equation, and make $|a|$ larger in a quadratic equation

- Make the graph shallower, or stretch it: make $|m|$ smaller in a linear equation, and make $|a|$ smaller in a quadratic equation

Example: stretching graphs



Exercise: transforming graphs

1. How do we shift the graph of $y = 2x + 3$ one unit right?

2. How do we transform the graph of $y = 2x + 3$ to have a shallower slope?

3. How do we transform the graph of $y = 2x + 3$ to have a slope of 1 and a $y-$intercept of 4?

Solution: shifting graphs

1. Subtract 1 from x! New equation: $y = 2(x - 1) + 3$

2. Multiply by a number less than 1; for example, take $x/2$. This gives new equation $y = 2(x/2) + 3$, or $y = x + 3$

3. To get a slope of 1, divide x by 2. To make the $y-$intercept 4, shift left by adding $1/2$ to x. New equation: $y = 2 * (x/2 + 1/2) + 3$, or $y = x + 4$

Summary

- Graphs are useful tools to describe relationships in data or visualize functions
- Histograms, boxplots, and scatterplots are common and useful types of graphs
- Linear trends can be described in:
  - Slope-intercept form — $y = mx + b$
  - Point-slope form — $y - y_1 = m(x - x_1)$
- Reading an equation from a graph involves finding the intercept and calculating the slope
- Graphs can be transformed by adding/subtracting from x, or multiplying/dividing x