

PROPORTIONS



W What department are you matriculated in?

Epi

Health Services

Global Health

Biostat

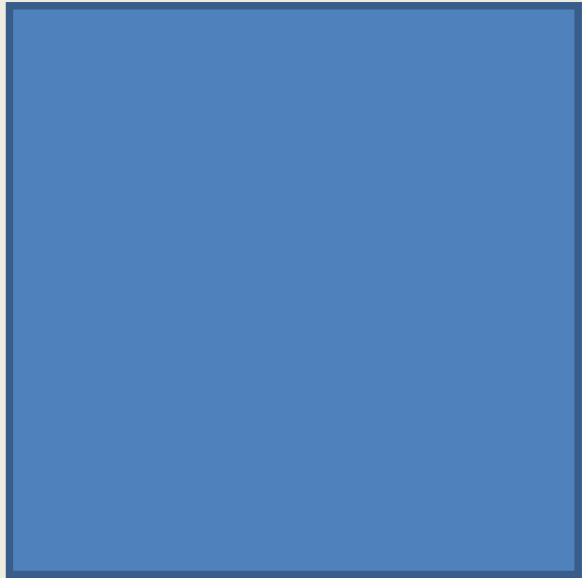
EOHS

Something else

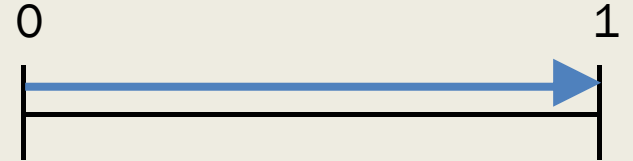
Where is she going with this?

- What is a proportion?
- Fractions
- Other ways we represent and manipulate proportions
- Proportions and other mathematical constructs in epidemiology

What is a proportion?

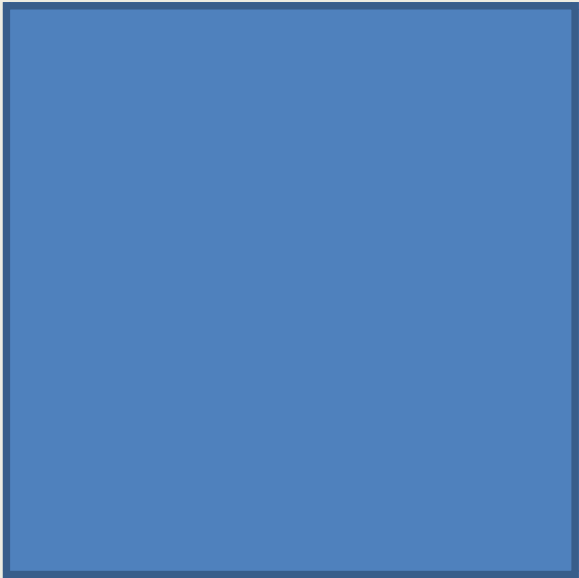


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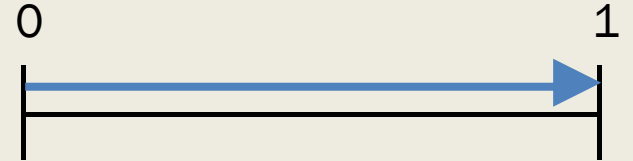


What is a proportion?

a part, share, or number considered in comparative relation to a whole

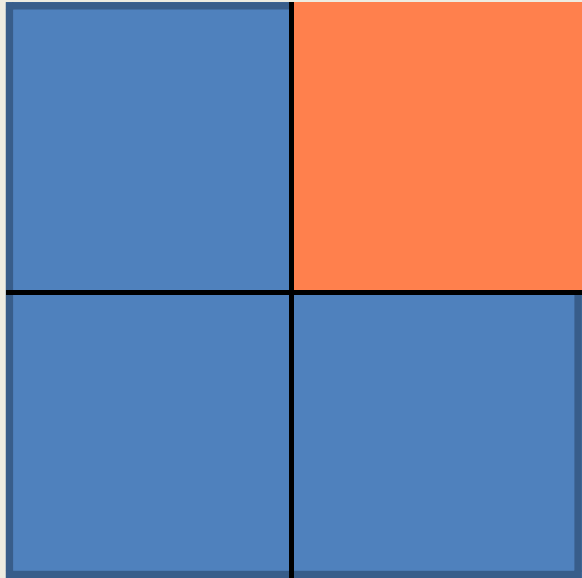


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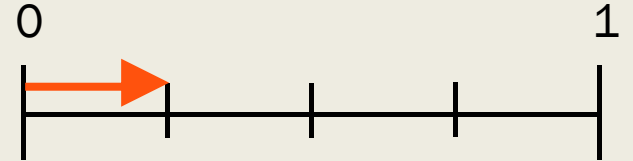


What is a proportion?

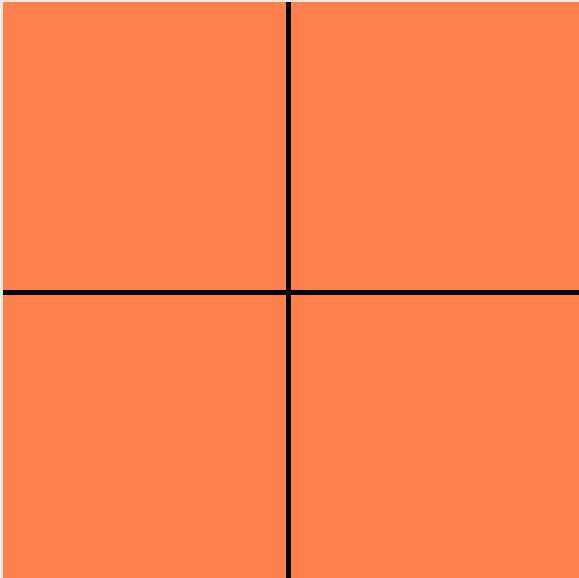
a part, share, or number considered in comparative relation to a whole



$$\frac{1}{4}$$



Fractions



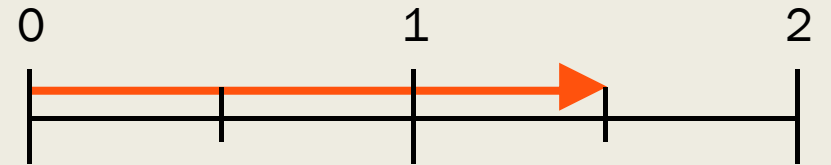
$$\frac{4}{4} = 1$$



Fractions



$$2 = \frac{3}{2}$$



Fraction Terminology

$$\frac{1}{4} = \frac{\textit{numerator}}{\textit{denominator}}$$

Manipulating fractions

- Comparing

[PRO-TIP] Matching denominators will make this easier

- Addition & subtraction

[RULE] Denominators must match, perform operation on numerators

- Multiplication & division

[RULE] Numerator and denominator operate separately

Adding & subtracting fractions

- Same denominator? **YES!**
 - *Ignore the denominator, work through the numerator*

$$\frac{2}{8} + \frac{5}{8}$$

Adding & subtracting fractions

- Same denominator? **YES!**
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$$\frac{2}{8} + \frac{5}{8} = \frac{7}{8}$$

Adding & subtracting fractions

- Same denominator? **YES!**
 - *Ignore the denominator, work through the numerator*

$$\frac{2}{8} + \frac{5}{8} = \frac{7}{8}$$

$$\frac{3}{5} + \frac{4}{5} = \frac{7}{5} = 1\frac{2}{5}$$

Adding & subtracting fractions

- Same denominator? **YES!**
 - *Ignore the denominator, work through the numerator*

$$\frac{2}{8} + \frac{5}{8} = \frac{7}{8}$$

$$\frac{3}{5} + \frac{4}{5} = \frac{7}{5} = 1\frac{2}{5}$$

$$\frac{10}{12} - \frac{4}{12} = \frac{6}{12} = \frac{1}{2}$$

Adding & subtracting fractions

- Same denominator? **NO!**

$$\frac{3}{4} + \frac{5}{6}$$

Adding & subtracting fractions

- Same denominator? **NO!**
 1. *Find the least common denominator*

$$\frac{3}{4} + \frac{5}{6}$$

Adding & subtracting fractions

- Same denominator? **NO!**

1. *Find the least common denominator*



Least Common Multiple

$$\frac{3}{4} + \frac{5}{6}$$

Adding & subtracting fractions

- Same denominator? **NO!**

1. *Find the least common denominator*

Least Common Multiple

4: 4, 8, 12, 16, 20...

6: 6, 12, 18, 24, 30...

$$\frac{3}{4} + \frac{5}{6}$$

Adding & subtracting fractions

- Same denominator? **NO!**

1. *Find the least common denominator*

Least Common Multiple

4: 4, 8, 12, 16, 20...

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$$\frac{3}{4} + \frac{5}{6}$$

Adding & subtracting fractions

- Same denominator? **NO!**
 1. Find the least common denominator
 2. Convert fractions

Least Common Multiple

4: 4, 8, 12, 16, 20...

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$$\frac{3}{4} + \frac{5}{6}$$

Adding & subtracting fractions

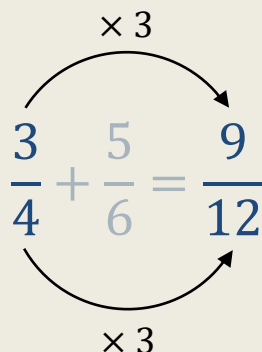
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1. Find the least common denominator
2. Convert fractions

Least Common Multiple

4: 4, 8, 12, 16, 20...

6: 6, 12, 18, 24, 30...

$$\frac{3}{4} + \frac{5}{6} = \frac{9}{12}$$


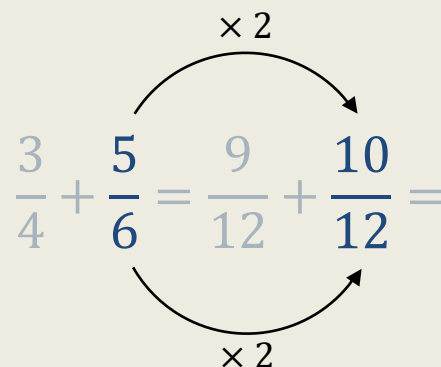
Adding & subtracting fractions

- Same denominator? **NO!**
 1. Find the least common denominator
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Least Common Multiple

4: 4, 8, 12, 16, 20...

6: 6, 12, 18, 24, 30...

$$\frac{3}{4} + \frac{5}{6} = \frac{9}{12} + \frac{10}{12} =$$


Adding & subtracting fractions

■ Same denominator? **NO!**

1. *Find the least common denominator*
2. *Convert fractions*
3. *Add (or subtract or compare) as usual*

Least Common Multiple

4: 4, 8, 12, 16, 20...

6: 6, 12, 18, 24, 30...

$$\frac{3}{4} + \frac{5}{6} = \frac{9}{12} + \frac{10}{12} = \frac{19}{12}$$

Adding & subtracting fractions

■ Same denominator? **NO!**

1. Find the least common denominator
2. Convert fractions
3. Add (or subtract or compare) as usual

Least Common Multiple

4: 4, 8, 12, 16, 20...

6: 6, 12, 18, 24, 30...

$$\frac{3}{4} + \frac{5}{6} = \frac{9}{12} + \frac{10}{12} = \frac{19}{12} = 1\frac{7}{12}$$

Adding & subtracting fractions

- Same denominator?

$$\frac{3}{7} + \frac{2}{3}$$

Adding & subtracting fractions

- Same denominator? **NO!**

$$\frac{3}{7} + \frac{2}{3}$$

Adding & subtracting fractions

- Same denominator? **NO!**

1. *Find the least common denominator*

Least Common Multiple

7: 7, 14, 21, 28, 35...

3: 3, 6, 9, 12, 15, 18, 21...

$$\frac{3}{7} + \frac{2}{3}$$

Adding & subtracting fractions

- Same denominator? **NO!**

1. *Find the least common denominator*

Least Common Multiple

7: 7, 14, 21, 28, 35...

3: 3, 6, 9, 12, 15, 18, 21...

$$\frac{3}{7} + \frac{2}{3}$$

Adding & subtracting fractions

- Same denominator? **NO!**
 1. Find the least common denominator
 2. Convert fractions

Least Common Multiple

7: 7, 14, 21, 28, 35...

3: 3, 6, 9, 12, 15, 18, 21...

$$\frac{3}{7} + \frac{2}{3} = \frac{9}{21} + \frac{14}{21}$$

Adding & subtracting fractions

- Same denominator? **NO!**

1. *Find the least common denominator*
2. *Convert fractions*
3. *Add (or subtract or compare) as usual*

Least Common Multiple

7: 7, 14, 21, 28, 35...

3: 3, 6, 9, 12, 15, 18, 21...

$$\frac{3}{7} + \frac{2}{3} = \frac{9}{21} + \frac{14}{21} = \frac{23}{21} = 1\frac{2}{21}$$

Adding & subtracting fractions

- Same denominator?

$$-\frac{3}{7} - \left(-\frac{4}{4}\right)$$

Adding & subtracting fractions

- Same denominator? **NO!**

1. Find the least common denominator

Least Common Multiple

7: 7, 14, 21, 28, 35...

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$$-\frac{3}{7} - \left(-\frac{4}{4}\right)$$

Adding & subtracting fractions

- Same denominator? **NO!**
 1. Find the least common denominator
 2. Convert fractions

Least Common Multiple

7: 7, 14, 21, 28, 35...

4: 4, 8, 12, 16, 20, 24, 28...

$$-\frac{3}{7} - \left(-\frac{4}{4}\right) = -\frac{12}{28} - \left(-\frac{7}{28}\right)$$

Adding & subtracting fractions

■ Same denominator? **NO!**

1. Find the least common denominator
2. Convert fractions
3. Add (or subtract or compare) as usual

Least Common Multiple

7: 7, 14, 21, 28, 35...

4: 4, 8, 12, 16, 20, 24, 28...

$$-\frac{3}{7} - \left(-\frac{4}{4}\right) = -\frac{12}{28} - \left(-\frac{7}{28}\right) = -\frac{12}{28} + \frac{7}{28} = -\frac{5}{28} = \frac{23}{21} = 1\frac{2}{21}$$

YOUR TURN

1. $1\frac{1}{3} - \frac{2}{3}$

2. $\frac{1}{6} ? \frac{2}{9}$

3. $\frac{3}{4} - \frac{1}{3}$

Fraction exercises

SOLUTION

$$1. \quad 1\frac{1}{3} - \frac{2}{3} = \frac{4}{3} - \frac{2}{3} = \frac{2}{3}$$

$$2. \quad \frac{1}{6} ? \frac{2}{9}$$

$$3. \quad \frac{3}{4} - \frac{1}{3}$$

SOLUTION

$$1. \quad 1\frac{1}{3} - \frac{2}{3} = \frac{4}{3} - \frac{2}{3} = \frac{2}{3}$$

$$2. \quad \frac{1}{6} ? \frac{2}{9} \rightarrow \frac{3}{18} ? \frac{4}{18} \rightarrow \frac{3}{18} < \frac{4}{18}$$

$$3. \quad \frac{3}{4} - \frac{1}{3}$$

SOLUTION

$$1. \quad 1\frac{1}{3} - \frac{2}{3} = \frac{4}{3} - \frac{2}{3} = \frac{2}{3}$$

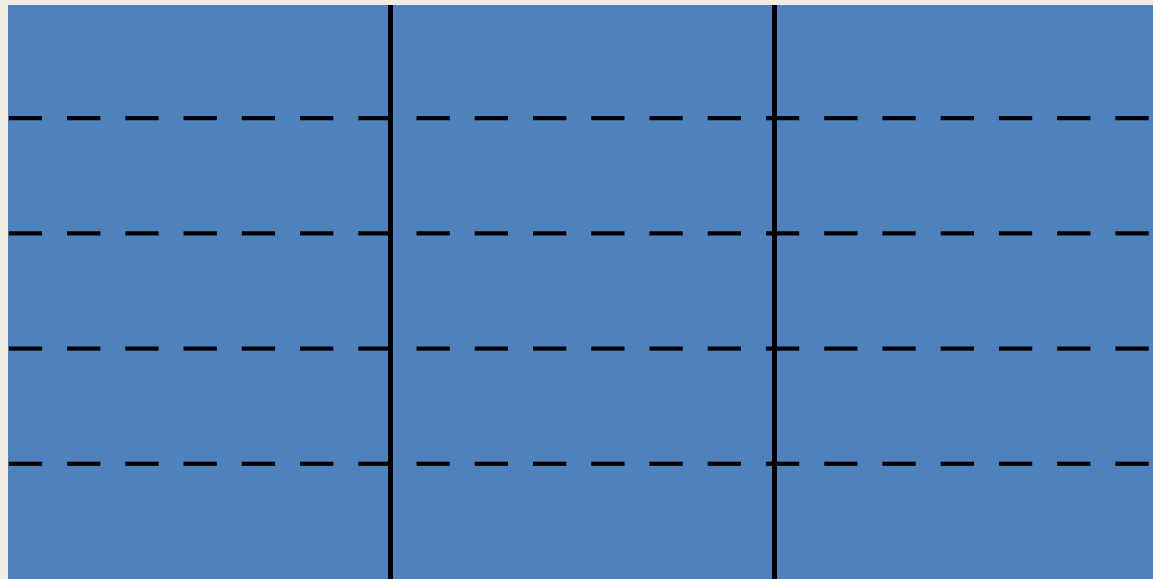
$$2. \quad \frac{1}{6} ? \frac{2}{9} \rightarrow \frac{3}{18} ? \frac{4}{18} \rightarrow \frac{3}{18} < \frac{4}{18}$$

$$3. \quad \frac{3}{4} - \frac{1}{3} = \frac{9}{12} - \frac{4}{12} = \frac{5}{12}$$

Multiplying Fractions

- Numerator by numerator, denominator by denominator

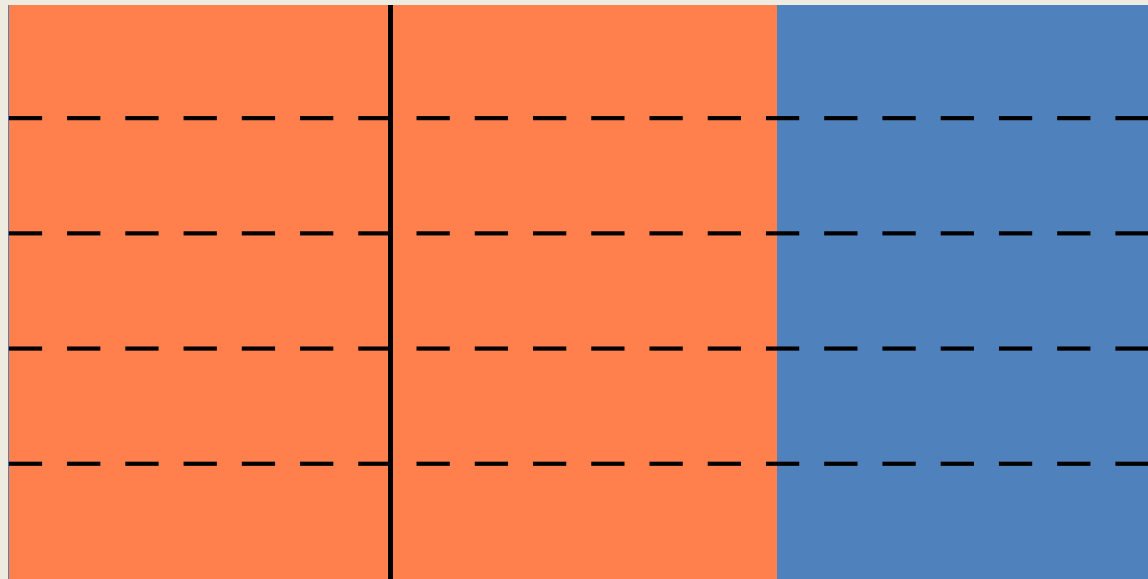
$$\frac{2}{3} \times \frac{2}{5} = ?$$



Multiplying Fractions

- Numerator by numerator, denominator by denominator

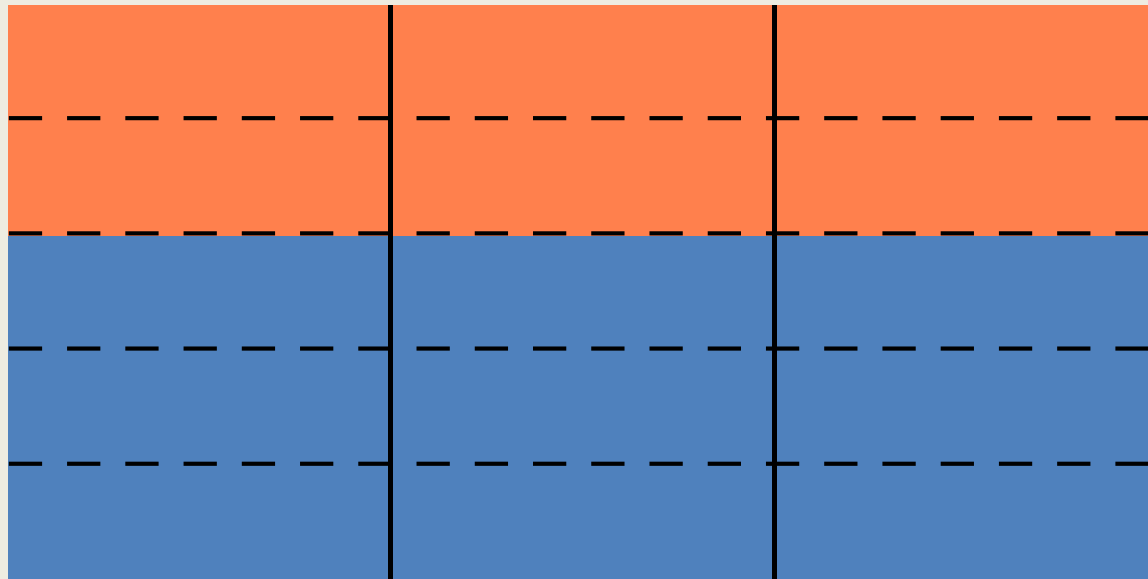
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Multiplying Fractions

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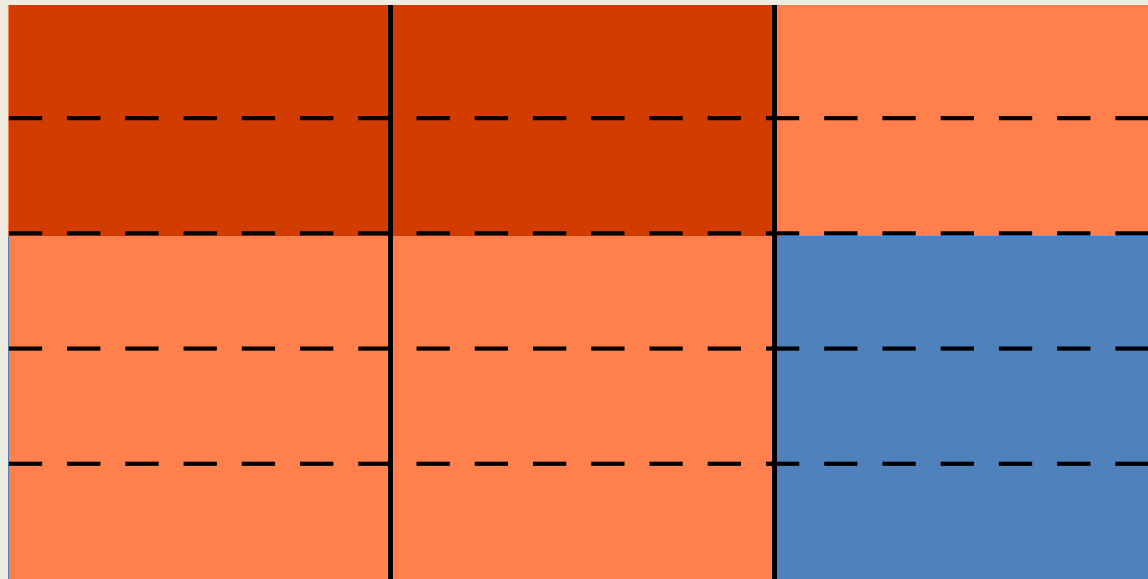
$$\frac{2}{3} \times \frac{2}{5} = ?$$



Multiplying Fractions

- Numerator by numerator, denominator by denominator

$$\frac{2}{3} \times \frac{2}{5} = \frac{4}{15}$$



Multiplying Fractions

- Numerator by numerator, denominator by denominator

$$\frac{2}{3} \times \frac{2}{5} = \frac{2 \times 2}{3 \times 5} = \frac{4}{15}$$

YOUR TURN

$$\frac{3}{4} \times \frac{4}{5} =$$

$$\frac{7}{8} \times \frac{5}{6} =$$

Multiplying fractions

SOLUTION

$$\frac{3}{4} \times \frac{4}{5} = \frac{12}{20}$$

$$\frac{7}{8} \times \frac{5}{6} =$$

SOLUTION

$$\frac{3}{4} \times \frac{4}{5} = \frac{12}{20} = \frac{3}{5}$$

$$\frac{7}{8} \times \frac{5}{6} =$$

SOLUTION

$$\frac{3}{\cancel{4}} \times \frac{\cancel{4}}{5} = \frac{12}{20} = \frac{3}{5} \quad \text{or cross-cancel!}$$

$$\frac{7}{8} \times \frac{5}{6} =$$

SOLUTION

$$\frac{3}{4} \times \frac{4}{5} = \frac{12}{20} = \frac{3}{5}$$

$$\frac{7}{8} \times \frac{5}{6} = \frac{35}{48}$$

Dividing Fractions

- Multiply by the reciprocal

$$\frac{4}{\frac{5}{\frac{2}{3}}}$$

Dividing Fractions

- Multiply by the reciprocal

$$\frac{\frac{4}{5}}{\frac{2}{3}} = \frac{4}{5} \div \frac{2}{3}$$

Dividing Fractions

- Multiply by the reciprocal

$$\frac{\frac{4}{5}}{\frac{2}{3}} = \frac{4}{5} \div \frac{2}{3} = \frac{4}{5} \times \frac{3}{2}$$

Dividing Fractions

- Multiply by the reciprocal

$$\frac{\frac{4}{5}}{\frac{2}{3}} = \frac{4}{5} \div \frac{2}{3} = \frac{4}{5} \times \frac{3}{2} = \frac{12}{10}$$

Dividing Fractions

- Multiply by the reciprocal

$$\frac{4\frac{4}{5}}{2\frac{2}{3}} = \frac{4}{5} \div \frac{2}{3} = \frac{4}{5} \times \frac{3}{2} = \frac{12}{10} = 1\frac{1}{5}$$

YOU TRY

$$\frac{\frac{14}{100}}{\frac{3}{200}}$$

W

$$\frac{\frac{14}{100}}{\frac{3}{200}}$$

$$\frac{10}{3}$$

$$\frac{21}{10000}$$

$$9\frac{1}{3}$$

I don't know

SOLUTION

$$\frac{\frac{14}{\frac{100}{3}}}{\frac{200}{200}} = \frac{14}{100} \times \frac{200}{3} = \frac{2800}{300} = \frac{28}{3} = 9\frac{1}{3}$$

REMINDER: Don't do this!

$$\frac{3}{4} + \frac{5}{6} \neq \frac{3+5}{4+6}$$

$$1\frac{7}{12} \neq \frac{8}{12}$$

$$\frac{3}{4a} + \frac{4}{5b} \neq \frac{7}{4a+5b}$$

$$\frac{X}{Y} + \frac{X}{Z} \neq \frac{X}{Y+Z}$$

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Where is she going with this?

- What is a proportion? ✓
- Fractions ✓
- Other ways represent and manipulate proportions
- Proportions and other mathematical constructs in epidemiology

Other ways to represent proportions

■ Percentage

- *Just a fraction with a denominator equal to 100*

$$\frac{1}{4} = \frac{25}{100} = 25\%$$

$$\frac{3}{2} = \frac{150}{100} = 150\%$$

■ Decimal

- *Just a percentage rendered on the number line*

$$\frac{1}{4} = \frac{25}{100} = 0.25$$

$$\frac{3}{2} = \frac{150}{100} = 1.5$$

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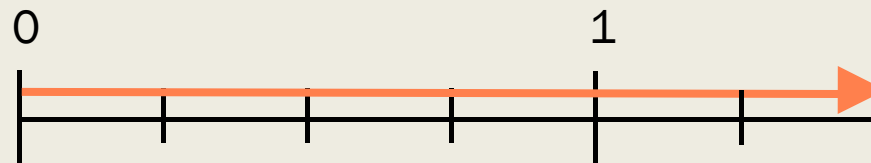
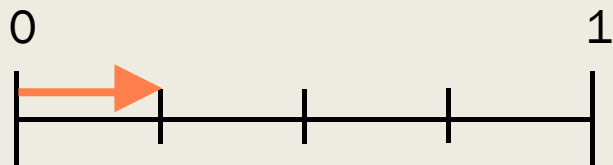
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■ Decimal

- *Just 10-based representation for increments less than 1*

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Percentage \leftrightarrow Decimal

■ Percentage

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■ Decimal

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$$\frac{1}{4} = \frac{25}{100} = 0.25$$

$$\frac{3}{2} = \frac{150}{100} = 1.5$$


$$0.25 \times 100 = 25\%$$

$$1.5 \times 100 = 150\%$$

YOUR TURN

- What percent of the square is shaded red?



W What percent of the square is shaded red?

0.625 1

6.25% 2

62.5% 3

I don't know 4

YOUR TURN

- What percent of the square is shaded red?



$$\frac{5}{8} = 0.625 = 62.5\%$$

YOUR TURN

1. 12 is what percent of 40?
2. $\frac{0.012}{0.4}$
3. Compare: 0.05 ? 0.011
4. Compare: -0.078 ? -0.035

Percents and decimals

SOLUTION

1. 12 is what percent of 40?

$$\frac{12}{40} = 0.3 = 30\%$$

2. $\frac{0.012}{0.4}$

3. Compare: 0.05 ? 0.011

4. Compare: -0.078 ? -0.035

SOLUTION

1. 12 is what percent of 40?

$$\frac{12}{40} = 0.3 = 30\%$$

2. $\frac{0.012}{0.4} = \frac{12}{400} = \frac{3}{100} = 0.03$

3. Compare: 0.05 ? 0.011

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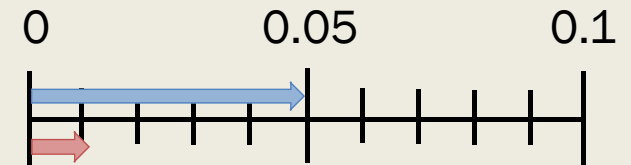
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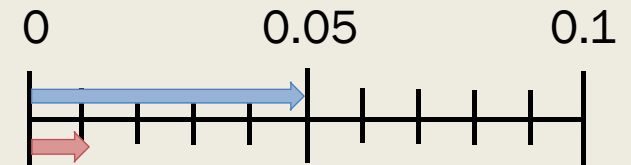
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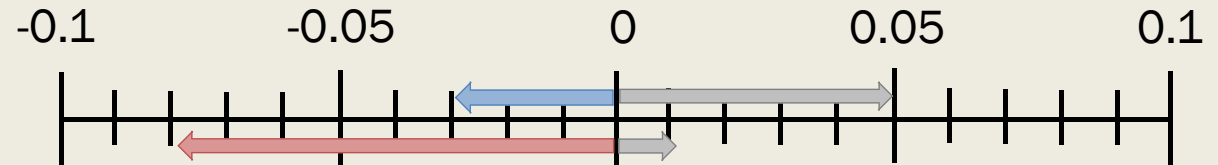
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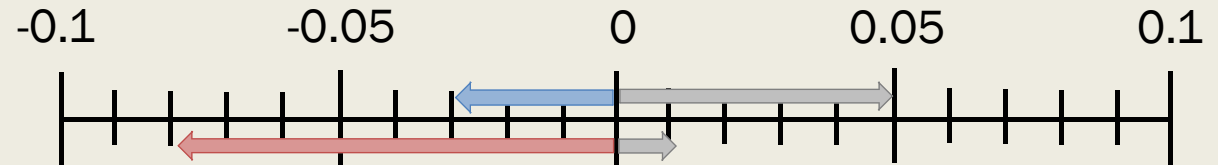
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Mathematical constructs in epi

- Proportion: a part, share, or number considered in comparative relation to a whole
 - *Represented by fraction, percentage or decimal*
 - *These have units!*

Example: Proportion of deaths due to heart disease, 23.5%
- Rate: a measure, quantity, or frequency measured against some other quantity or measure
 - *Represented by fraction, percentage or decimal*
 - *Usually measuring something in relation to time*

Example: mortality rate of heart disease, 193 deaths per 100,000 person-years
- Ratio: the relationship of two comparable amounts
 - *Often represented in the form, number:number (e.g. 3:1)*
 - *BUT also represented by fraction, percentage or decimal*
 - *Unitless!*

Example: Sex ratio at age 60 is 81 males:100 females

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Extra practice

- Fractions

<https://www.khanacademy.org/math/arithmetic/fraction-arithmetic>

- Decimals

<https://www.khanacademy.org/math/arithmetic/arith-decimals>

- Relationship of fractions, decimals and percentages

<https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-fractions-decimals>