ALGEBRA & CROSS-TABULATION

Where is she going with this?

- Solving for an unknown quantity
- Variables
- Weighted averages
- Cross-tabulation

Solving for unknown quantity

Problem: $x + 4 = 7$

Solving for unknown quantity

Problem: $x + 4 = 7$

Goal: Isolate the unknown variable

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Goal: Isolate the unknown variable

Process: Do the same operation to both sides

Solving for unknown quantity

Problem: $x + 4 = 7$

Goal: Isolate the unknown variable

Process: Do the same operation to both sides

Solution:

$$x + 4 = 7$$

$$x + 4 - 4 = 7 - 4$$

$$\boldsymbol{x = 3}$$

Solving for unknown quantity

Problem: $3x = 12$

Solving for unknown quantity

Problem: $3x = 12$

Goal: Isolate the unknown variable

Process: Do the same operation to both sides

Solution:

$$3x = 12$$

Solving for unknown quantity

Problem: $3x = 12$

Goal: Isolate the unknown variable

Process: Do the same operation to both sides

Solution:

$$3x = 12$$

$$\frac{3x}{3} = \frac{12}{3}$$

Solving for unknown quantity

Problem: $3x = 12$

Goal: Isolate the unknown variable

Process: Do the same operation to both sides

Solution:

$$3x = 12$$

$$\frac{3x}{3} = \frac{12}{3}$$

$$\boldsymbol{x = 4}$$

EXERCISE

1. $x + 5 = 12.7$
2. $\frac{38}{216} = \frac{x}{1{,}000}$
3. $5(3x - 2) = 35$
4. $\frac{3}{x} = 6$
5. $4x^2 = 100$

EXERCISE

1.

$$x + 5 = 12.7$$

$$x + 5 - 5 = 12.7 - 5$$

$$x = 7.7$$

EXERCISE

1.

$$x + 5 = 12.7$$

$$x + 5 - 5 = 12.7 - 5$$

$$x = 7.7$$

2.

$$\frac{38}{216} = \frac{x}{1{,}000}$$

$$\frac{38}{216} \times 1{,}000 = \frac{x}{1{,}000} \times 1{,}000$$

$$\frac{38{,}000}{216} = x$$

$$x = 176$$

EXERCISE

3.

$$5(3x - 2) = 35$$

$$\frac{5(3x - 2)}{5} = \frac{35}{5}$$

$$3x - 2 = 7$$

$$3x - 2 + 2 = 7 + 2$$

$$3x = 9$$

$$\frac{3x}{3} = \frac{9}{3}$$

$$x = 3$$

EXERCISE

3.

$$5(3x-2)=35$$

$$\frac{5(3x-2)}{5}=\frac{35}{5}$$

$$3x-2=7$$

$$3x-2+2=7+2$$

$$3x=9$$

$$\frac{3x}{3}=\frac{9}{3}$$

$$x=3$$

4.

$$\frac{3}{x}=6$$

$$\frac{3}{x}*x=6*x$$

$$3=6x$$

$$\frac{3}{6}=x$$

$$0.5=x$$

EXERCISE

5.

$$4x^2 = 100$$

$$\sqrt{4x^2} = \sqrt{100}$$

$$2x = 10$$

$$\frac{2x}{2} = \frac{10}{2}$$

$$x = 5$$

EXERCISE

5.

$$4x^2 = 100$$

$$\sqrt{4x^2} = \sqrt{100}$$

$$2x = 10$$

$$\frac{2x}{2} = \frac{10}{2}$$

$$x = 5$$

6.

$$4x^2 = 100$$

$$\frac{4x^2}{4} = \frac{100}{4}$$

$$x^2 = 25$$

$$\sqrt{x^2} = \sqrt{25}$$

$$x = 5$$

Variables

- Variable: a quantity that during a calculation is assumed to vary or be capable of varying in value
  - *Often represented as a letter or a symbol*
  - *For example* $x, y, z, a, b, \beta, \alpha, \theta, \ldots$

Function Notation

$$y = 4x - 3$$

$$f(x) = 4x - 3$$

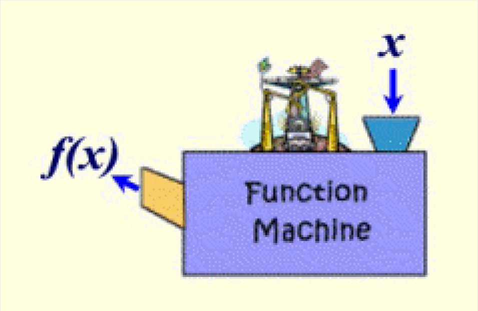


Image: http://www.regentsprep.org/, 2016

Function Notation

$$y = 4x - 3$$

$$f(x) = 4x - 3$$

$$g(x) = 2x - 1$$

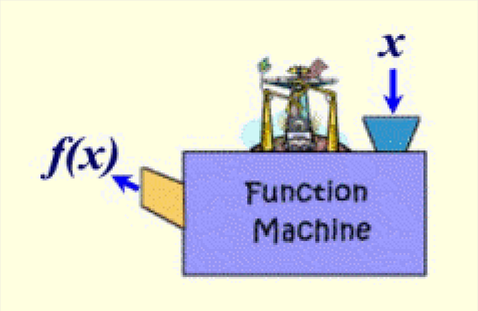


Image: http://www.regentsprep.org/, 2016

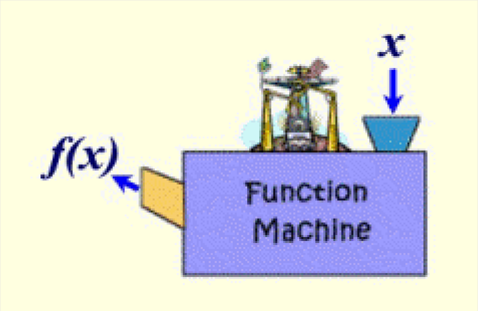
Function Notation

$$y = 4x - 3$$

$$f(x) = 4x - 3$$

$$g(x) = 2x - 1$$

$$h(x) = 5x - 6$$



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Evaluating Algebraic Equations

$$y = 5 + 2x$$

When given values for *x*, obtain *y*

x: {2, 4, 7, 12}

$$y = 5 + 2(2) = 5 + 4 = 9$$

$$y = 5 + 2(4) = 5 + 8 = 13$$

$$y = 5 + 2(7) = 5 + 14 = 19$$

$$y = 5 + 2(12) = 5 + 24 = 29$$

$$f(x) = 5 + 2x$$

When given values for *x*, evaluate the function

x: {2, 4, 7, 12}

$$f(2) = 5 + 2(2) = 5 + 4 = 9$$

$$f(4) = 5 + 2(4) = 5 + 8 = 13$$

$$f(7) = 5 + 2(7) = 5 + 14 = 19$$

$$f(12) = 5 + 2(12) = 5 + 24 = 29$$

EXERCISE

1. Evaluate $y = \frac{2}{3}\alpha + 4\beta$ for $\alpha = 5$ and $\beta = 2$.
2. If $f(x) = x^2 - x + 2$, then $f(5) = ?$
3. Given $f(x) = x^2 - x + 2$, evaluate $f(2a)$.

EXERCISE

1. Evaluate $y = \frac{2}{3}\alpha + 4\beta$ for $\alpha = 5$ and $\beta = 2$.

$$y = \frac{2}{3}(5) + 4(2)$$

$$y = \frac{10}{3} + 8 = \frac{10}{3} + \frac{24}{3} = \frac{34}{3}$$

$$y = 11\frac{1}{3} = 11.333$$

EXERCISE

2. If $f(x) = x^2 - x + 2$, then $f(5) =$...

$$f(5) = 5^2 - 5 + 2$$

$$f(5) = 25 - 5 + 2$$

$$f(5) = 22$$

EXERCISE

3. Given $f(x) = x^2 - x + 2$, evaluate $f(2a)$.

$$f(2a) = (2a)^2 - 2a + 2$$

$$f(2a) = 4a^2 - 2a + 2$$

Average vs. Weighted Average

You're taking a class for which you have received three grades: an 85% on your midterm, a 91% on your project, and a 93% on your final. All count equally toward your final grade. What is your final percentage grade?

$$\frac{85 + 91 + 93}{3} = 89.7$$

Average vs. Weighted Average

You’re taking a class for which you have received three grades: an 85% on your midterm, a 91% on your project, and a 93% on your final. All count equally toward your final grade. What is your final percentage grade?

$$\frac{85 + 91 + 93}{3} = 89.7$$

If instead the midterm and project were worth 30% of your grade and the final was worth 40% of your grade, what would be your final percentage?

Average vs. Weighted Average

Average

$$\frac{85 + 91 + 93}{3} = 89.7$$

Average vs. Weighted Average

Average

$$\frac{85 + 91 + 93}{3} = 89.7$$

$$\left(\frac{1}{3}\right)85 + \left(\frac{1}{3}\right)91 + \left(\frac{1}{3}\right)93 = 89.7$$

Average vs. Weighted Average

Average

$$\frac{85 + 91 + 93}{3} = 89.7$$

$$\left(\frac{1}{3}\right)85 + \left(\frac{1}{3}\right)91 + \left(\frac{1}{3}\right)93 = 89.7$$

Weighted Average

$$\left(\frac{3}{10}\right)85 + \left(\frac{3}{10}\right)91 + \left(\frac{4}{10}\right)93 = 90$$

Average vs. Weighted Average

Average

$$\frac{85 + 91 + 93}{3} = 89.7$$

$$\left(\frac{1}{3}\right)85 + \left(\frac{1}{3}\right)91 + \left(\frac{1}{3}\right)93 = 89.7$$

$$(0.33)85 + (0.33)91 + (0.33)93 = 88.8$$

Weighted Average

$$\left(\frac{3}{10}\right)85 + \left(\frac{3}{10}\right)91 + \left(\frac{4}{10}\right)93 = 90$$

$$(0.3)85 + (0.3)91 + (0.4)93 = 90$$

Average vs. Weighted Average

Average

$$\frac{85 + 91 + 93}{3} = 89.7$$

$$\left(\frac{1}{3}\right)85 + \left(\frac{1}{3}\right)91 + \left(\frac{1}{3}\right)93 = 89.7$$

$$(0.33)85 + (0.33)91 + (0.33)93 = 88.8$$

Weighted Average

$$\left(\frac{3}{10}\right)85 + \left(\frac{3}{10}\right)91 + \left(\frac{4}{10}\right)93 = 90$$

$$(0.3)85 + (0.3)91 + (0.4)93 = 90$$

EXERCISE

For a particular disease, 80% of cases are among children <10-years-old. Among these children, the risk of experiencing serious sequelae is 15%. In older children and adults, the risk of serious sequelae is only 5%. What is the risk of serious sequelae overall for all ages?

EXERCISE

For a particular disease, 80% of cases are among children <10-years-old. Among these children, the risk of experiencing serious sequelae is 15%. In older children and adults, the risk of serious sequelae is only 5%. What is the risk of serious sequelae overall for all ages?

$$(0.8 * 0.15) + (0.2 * 0.05) = 0.13 = 13\%$$

Where is she going with this?

- Solving for an unknown quantity ✓
- Variables ✓
- Weighted averages ✓
- Cross-tabulation

Cross-tabulation

a tool that allows you analyze the relationship between two or more categorical variables

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Other names: frequency table, contingency table

Cross-tabulation

a tool that allows you analyze the relationship between two or more categorical variables

2×2 table

Cross-tabulation

a tool that allows you analyze the relationship between two or more categorical variables

2 × 2 table

Gene variant A		
Gene variant B		

Cross-tabulation

a tool that allows you analyze the relationship between two or more categorical variables

2 × 2 table

	Breast cancer	No breast cancer
Gene variant A		
Gene variant B		

Cross-tabulation

a tool that allows you analyze the relationship between two or more categorical variables

2×2 table

	Breast cancer	No breast cancer
Gene variant A	5	
Gene variant B		

Cross-tabulation

a tool that allows you analyze the relationship between two or more categorical variables

2 × 2 table

	Breast cancer	No breast cancer
Gene variant A	5	
Gene variant B	7	

Cross-tabulation

a tool that allows you analyze the relationship between two or more categorical variables

2 × 2 table

	Breast cancer	No breast cancer
Gene variant A	5	10
Gene variant B	7	

Cross-tabulation

a tool that allows you analyze the relationship between two or more categorical variables

2×2 table

	Breast cancer	No breast cancer
Gene variant A	5	10
Gene variant B	7	30

Cross-tabulation

a tool that allows you analyze the relationship between two or more categorical variables

2 × 2 table

	Breast cancer	No breast cancer
Gene variant A	5	10
Gene variant B	7	30

cell

Cross-tabulation

a tool that allows you analyze the relationship between two or more categorical variables

2 × 2 table

	Breast cancer	No breast cancer	
Gene variant A	5	10	15
Gene variant B	7	30	37
	12	40	

Cross-tabulation

a tool that allows you analyze the relationship between two or more categorical variables

2 × 2 table

	Breast cancer	No breast cancer	
Gene variant A	5	10	15
Gene variant B	7	30	37
	12	40	

margins

Cross-tabulation

a tool that allows you analyze the relationship between two or more categorical variables

2×2 table

	Breast cancer	No breast cancer	
Gene variant A	5	10	15
Gene variant B	7	30	37
	12	40	52

Cross-tabulation

a tool that allows you analyze the relationship between two or more categorical variables

2 × 2 table

	Breast cancer	No breast cancer	
Gene variant A	5 (5/52)	10	15
Gene variant B	7	30	37
	12	40	52

Cross-tabulation

a tool that allows you analyze the relationship between two or more categorical variables

2×2 table

	Breast cancer	No breast cancer	
Gene variant A	5 (0.096)	10	15
Gene variant B	7	30	37
	12	40	52

Cross-tabulation

a tool that allows you analyze the relationship between two or more categorical variables

2 × 2 table

	Breast cancer	No breast cancer	
Gene variant A	5 (9.6%)	10	15
Gene variant B	7	30	37
	12	40	52

Cross-tabulation

a tool that allows you analyze the relationship between two or more categorical variables

2 × 2 table

	Breast cancer	No breast cancer	
Gene variant A	5 (9.6%)	10 (19.2%)	15
Gene variant B	7 (13.5%)	30 (57.7%)	37
	12	40	52

Cross-tabulation

a tool that allows you analyze the relationship between two or more categorical variables

2 × 2 table

	Breast cancer	No breast cancer	
Gene variant A	5 (9.6%)	10 (19.2%)	15 (28.8%)
Gene variant B	7 (13.5%)	30 (57.7%)	37 (71.2%)
	12 (23.1%)	40 (76.9%)	52

Cross-tabulation

a tool that allows you analyze the relationship between two or more categorical variables

2 × 2 table

	Breast cancer	No breast cancer	
Gene variant A	5 (9.6%)	10 (19.2%)	15 (28.8%)
Gene variant B	7 (13.5%)	30 (57.7%)	37 (71.2%)
	12 (23.1%)	40 (76.9%)	52 (100%)

Cross-tabulation

a tool that allows you analyze the relationship between two or more categorical variables

2 × 2 table

	Breast cancer	No breast cancer	
Gene variant A	5 (5/12)	10	15
Gene variant B	7 (7/12)	30	37
	12	40	52

Cross-tabulation

a tool that allows you analyze the relationship between two or more categorical variables

2 × 2 table

	Breast cancer	No breast cancer	
Gene variant A	5 (41.7%)	10	15
Gene variant B	7 (58.3%)	30	37
	12	40	52

Cross-tabulation

a tool that allows you analyze the relationship between two or more categorical variables

2 × 2 table

	Breast cancer	No breast cancer	
Gene variant A	5 (41.7%)	10	15
Gene variant B	7 (58.3%)	30	37
	12 (100%)	40	52

Cross-tabulation

a tool that allows you analyze the relationship between two or more categorical variables

2 × 2 table

	Breast cancer	No breast cancer	
Gene variant A	5 (41.7%)	10 (25%)	15 (28.8%)
Gene variant B	7 (58.3%)	30 (75%)	37 (71.2%)
	12 (100%)	40 (100%)	52 (100%)

Cross-tabulation

a tool that allows you analyze the relationship between two or more categorical variables

2 × 2 table

	Breast cancer	No breast cancer	
Gene variant A	5 (33.3%)	10 (66.7%)	15 (100%)
Gene variant B	7 (18.9%)	30 (81.1%)	37 (100%)
	12 (23.1%)	40 (76.9%)	52 (100%)

EXERCISE

In a particular class, 12 of 27 people with a height greater than 66 inches prefer pizza. Of the 49 students in the class, 10 have a height less than or equal 66 inches and prefer soup.

Construct a frequency table describing the relationship between food preference and height. Use the table to answer these questions. Given that a person prefers soup, is the person more likely to be taller or shorter than 66 inches? Does it seem like pizza/soup preference is related to height?

EXERCISE

In a particular class, 12 of **27 people with a height greater than 66 inches** prefer pizza. Of the 49 students in the class, 10 have a height less than or equal 66 inches and prefer soup.

	Prefers pizza	Prefers soup	
> 66 in			27
≤ 66 in			

EXERCISE

In a particular class, 12 of 27 people with a height greater than 66 inches prefer pizza. Of the 49 students in the class, 10 have a height less than or equal 66 inches and prefer soup.

	Prefers pizza	Prefers soup	
> 66 in	12		27
≤ 66 in			

EXERCISE

In a particular class, 12 of 27 people with a height greater than 66 inches prefer pizza. Of the **49 students in the class**, 10 have a height less than or equal 66 inches and prefer soup.

	Prefers pizza	Prefers soup	
> 66 in	12		27
≤ 66 in			
			49

EXERCISE

In a particular class, 12 of 27 people with a height greater than 66 inches prefer pizza. Of the 49 students in the class, 10 have a height less than or equal 66 inches and prefer soup.

	Prefers pizza	Prefers soup	
> 66 in	12		27
≤ 66 in		10	
			49

EXERCISE

In a particular class, 12 of 27 people with a height greater than 66 inches prefer pizza. Of the 49 students in the class, 10 have a height less than or equal 66 inches and prefer soup.

	Prefers pizza	Prefers soup	
> 66 in	12	15	27
≤ 66 in		10	
			49

EXERCISE

In a particular class, 12 of 27 people with a height greater than 66 inches prefer pizza. Of the 49 students in the class, 10 have a height less than or equal 66 inches and prefer soup.

	Prefers pizza	Prefers soup	
> 66 in	12	15	27
≤ 66 in		10	
		25	49

EXERCISE

In a particular class, 12 of 27 people with a height greater than 66 inches prefer pizza. Of the 49 students in the class, 10 have a height less than or equal 66 inches and prefer soup.

	Prefers pizza	Prefers soup	
> 66 in	12	15	27
≤ 66 in	12	10	
		25	49

EXERCISE

In a particular class, 12 of 27 people with a height greater than 66 inches prefer pizza. Of the 49 students in the class, 10 have a height less than or equal 66 inches and prefer soup.

	Prefers pizza	Prefers soup	
> 66 in	12	15	27
≤ 66 in	12	10	22
	24	25	49

EXERCISE

Given that a person prefers soup, is the person more likely to be taller or shorter than 66 inches?

	Prefers pizza	Prefers soup	
> 66 in	12	15	27
≤ 66 in	12	10	22
	24	25	49

EXERCISE

Given that a person prefers soup, is the person more likely to be taller or shorter than 66 inches?

	Prefers pizza	Prefers soup	
> 66 in	12	15 (15/25)	27
≤ 66 in	12	10 (10/25)	22
	24	25	49

EXERCISE

Given that a person prefers soup, is the person more likely to be taller or shorter than 66 inches?

	Prefers pizza	Prefers soup	
> 66 in	12	15 (0.6)	27
≤ 66 in	12	10 (0.4)	22
	24	25	49

EXERCISE

Given that a person prefers soup, is the person more likely to be taller or shorter than 66 inches? Taller

	Prefers pizza	Prefers soup	
> 66 in	12	15 (0.6)	27
≤ 66 in	12	10 (0.4)	22
	24	25	49

EXERCISE

Does it seem like pizza/soup preference is related to height?

	Prefers pizza	Prefers soup	
> 66 in	12	15	27
≤ 66 in	12	10	22
	24	25	49

EXERCISE

Does it seem like pizza/soup preference is related to height?

	Prefers pizza	Prefers soup	
> 66 in	12 (12/49)	15 (15/49)	27 (27/49)
≤ 66 in	12 (12/49)	10 (10/49)	22 (22/49)
	24 (24/49)	25 (25/49)	49

EXERCISE

Does it seem like pizza/soup preference is related to height?

	Prefers pizza	Prefers soup	
> 66 in	12 (0.245)	15 (0.306)	27 (0.551)
≤ 66 in	12 (0.245)	10 (0.204)	22 (0.449)
	24 (0.490)	25 (0.510)	49

EXERCISE

Does it seem like pizza/soup preference is related to height?

	Prefers pizza	Prefers soup	
> 66 in	12 (24.5%)	15 (30.6%)	27 (55.1%)
≤ 66 in	12 (24.5%)	10 (20.4%)	22 (44.9%)
	24 (49.0%)	25 (51.0%)	49

EXERCISE

Does it seem like pizza/soup preference is related to height?

	Prefers pizza	Prefers soup	
> 66 in	12 (25%)	15 (31%)	27 (55%)
≤ 66 in	12 (25%)	10 (20%)	22 (45%)
	24 (49%)	25 (51%)	49

EXERCISE

Does it seem like pizza/soup preference is related to height? **Maybe! Particularly for those who prefer soup.**

	Prefers pizza	Prefers soup	
> 66 in	12 (24.5%)	15 (30.6%)	27 (55.1%)
≤ 66 in	12 (24.5%)	10 (20.4%)	22 (44.9%)
	24 (49.0%)	25 (51.0%)	49

Cross-tabulation

- Helps us understand the relationship between categorical variables (or those that can be made categorical)
- Good way to summarize data
- Can be useful in organizing diverse pieces of data about the same population