Day 2, Session 1: Graphs

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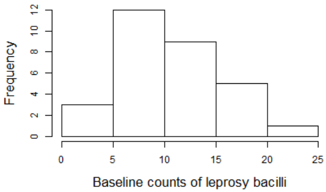
Graphs

- Why do we use graphs?
  - Describe relationships in the data
  - Visualize functions
- Graphs are very useful in exploratory analyses, or for description

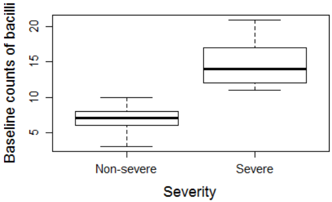
Example data analysis

- Data on 30 patients with leprosy
- Counts of leprosy bacilli measured at baseline and at a further time point
- Three treatments and an indicator of severity of the leprosy

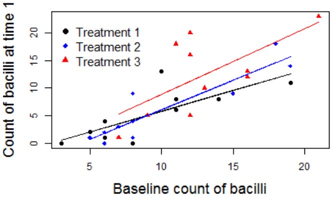
Common types of graphs in data analysis



(a) Histogram



(b) Boxplot



(c) Scatterplot

What do graphs tell us?

- Histograms: summaries of one-dimensional distributions
  - Counts or frequencies of each occurrence
- Boxplots: summaries of two-dimensional distributions
  - measures of center (typically median)
  - measures of spread (typically inter-quartile range)
- Scatterplots: summaries of two-dimensional distributions
  - Can visualize the whole data
  - Trends in two or more dimensions by using different colors/shapes for strata

Linear trends

- A common way to describe data (think linear regression!)
- Lines are easy to compute
  - Only need a point and a slope
  - Two common forms of linear equations

Slope-intercept form

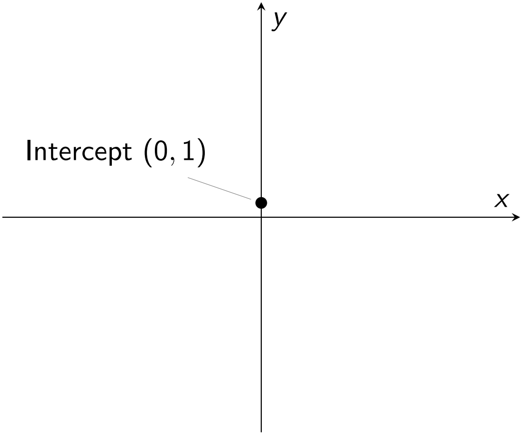
- $y = mx + b$

- Slope: m
  - Rate of change, i.e. how does y change with each one unit change in x?

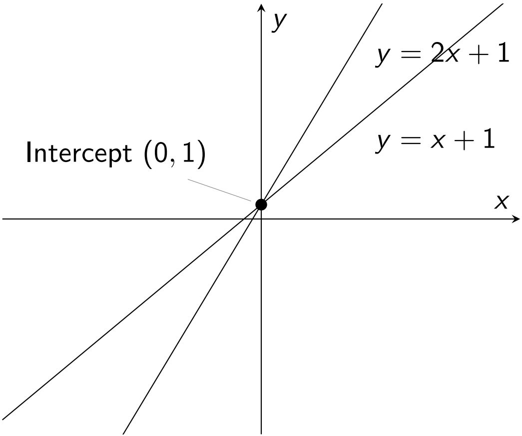
  - Example: speed, the distance traveled with each unit change in time

- Intercept: b
  - The point where the line crosses the $y-$axis

Slope-intercept form: determining a line



Slope-intercept form: determining a line



Point-slope form

- $y - y_1 = m(x - x_1)$
- Point: (x_1, y_1)
  - A point on the line (can be any point! Even the intercept!)
- Slope: m
  - Same as in slope-intercept form!
- Example: $y - 1 = 2(x - 0)$ is the same as $y = 2x + 1$ in slope-intercept form!

Exercise: slopes and intercepts

1. What is the slope of the line $y = 2x - 3$?
2. What is the $y-$intercept of the line $y = 2x - 3$?
3. What is the slope of the line $y + 1 = 2(x - 1)$?
4. What point did we use to create the line $y + 1 = 2(x - 1)$?

Solution: slopes and intercepts

1. The equation is in slope-intercept form, so the slope is 2
2. The equation is in slope-intercept form, so the intercept is -3
3. The equation is in point-slope form, so the slope is 2
4. The equation is in point-slope form, so the point is $(1, -1)$

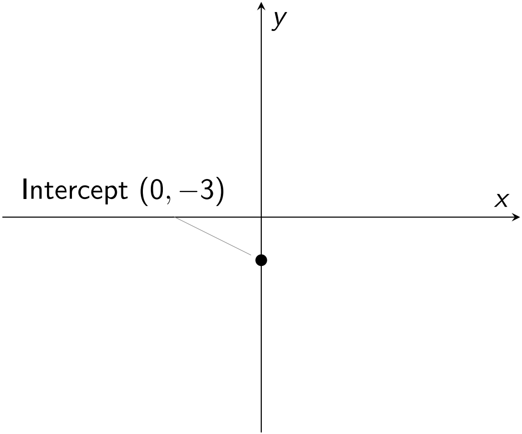
Creating a graph using an equation

- Steps:
  1. Draw axes
  2. Place a point at the $y-$intercept (slope-intercept form) or at the starting point (point-slope form)
  3. Increase x by one unit, increase y by m units, place a new point
  4. Draw a line between the old point and the new point!

Example: creating a graph using an equation

- Equation $y = 2x - 3$
- Slope: 2, Intercept: -3

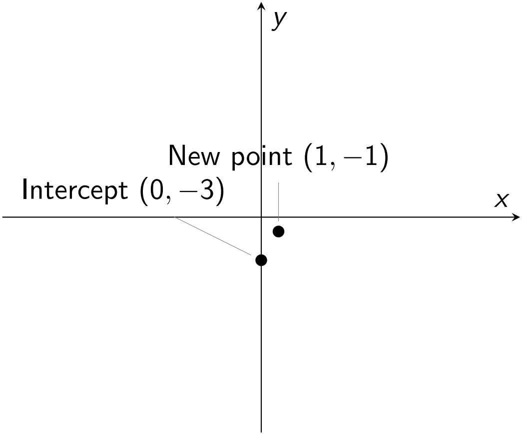
1. Draw a point at $(0, -3)$



Example: creating a graph using an equation

- Equation $y = 2x - 3$
- Slope: 2, Intercept: -3

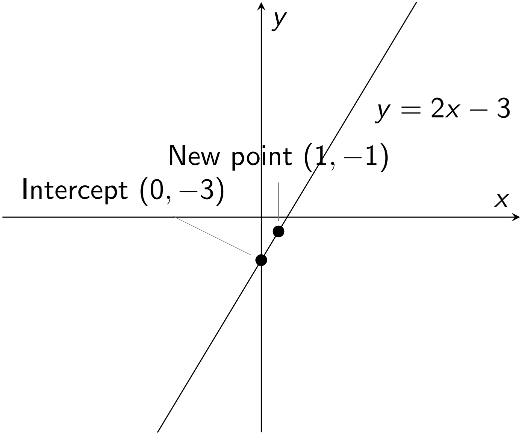
2. Increase x to 1, increase y by 2. New point at $(1, -1)$



Example: creating a graph using an equation

- Equation $y = 2x - 3$
- Slope: 2, Intercept: -3

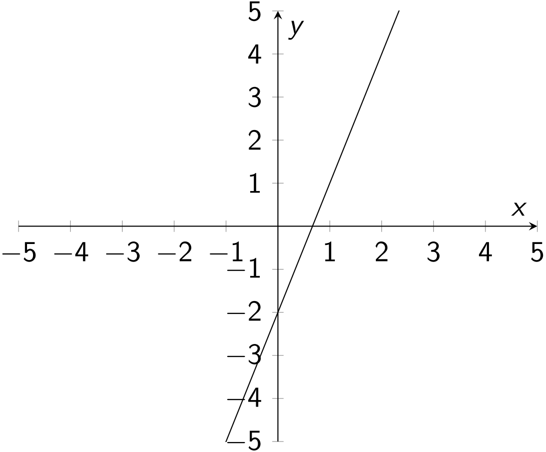
3. Draw a line!



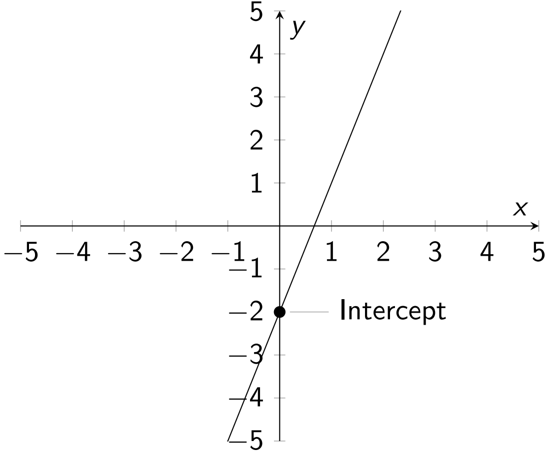
Reading an equation from a graph

- Two options:
  1. Slope-intercept form
     1.1 Find the $y-$intercept
     1.2 Find the slope: how much does y change with each 1 unit difference in x?
  2. Point-slope form
     2.1 Choose any point on the line
     2.2 Find the slope: how much does y change with each 1 unit difference in x?

Example: reading an equation from a graph

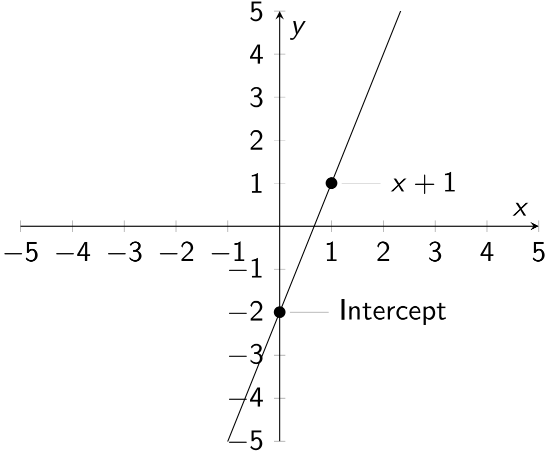


Example: reading an equation from a graph



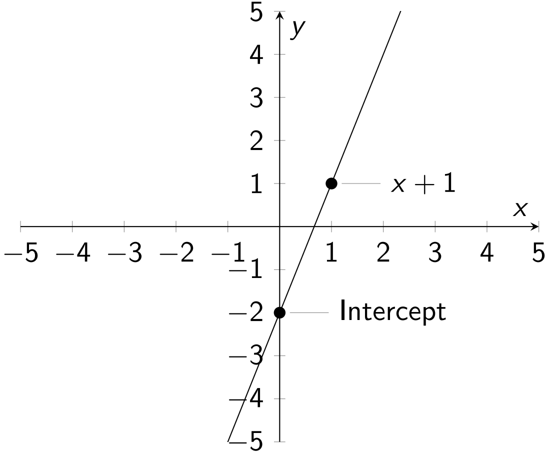
Example: reading an equation from a graph

1. Find the intercept. Here it is -2



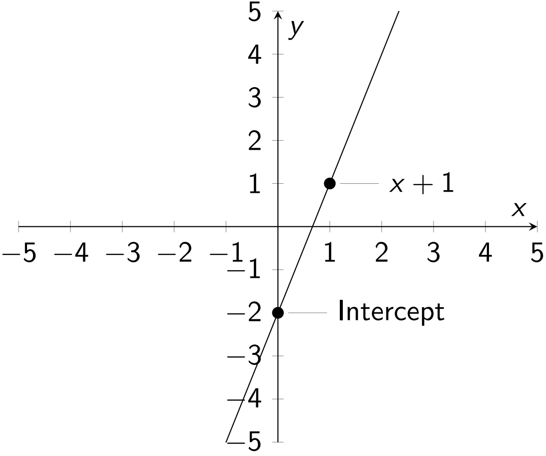
Example: reading an equation from a graph

2. Increase x by one to find the slope. The y value at $x = 1$ is 1, so y changed by 3



Example: reading an equation from a graph

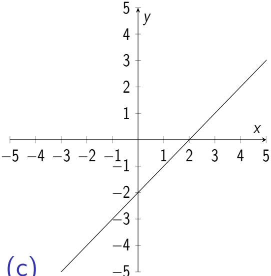
3. Slope-intercept: $y = 3x - 2$, point-slope: $y + 2 = 3(x - 0)$



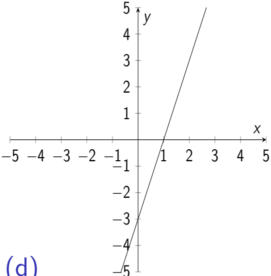
Exercise: matching graphs to equations

1. Which is the graph of $y = 2x - 3$?

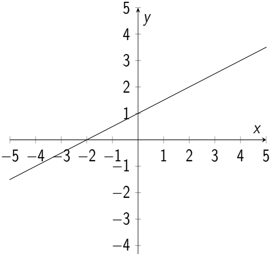
(a)



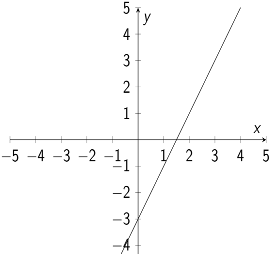
(b)



(c)



(d)



Solution: matching graphs to equations

(a) Intercept: -2, slope: 1

(b) Intercept: -3, slope: 3

(c) Intercept: 1, slope: 1

(d) Intercept: -3, slope: 2 $\checkmark$

Quadratics

- Sometimes we believe a trend is higher order than linear
- Higher order terms allow more flexibility in modeling
- Quadratics are the natural next step from linear terms, and are shaped like parabolas

Defining a quadratic

- Standard form: $y = ax^2 + bx + c$

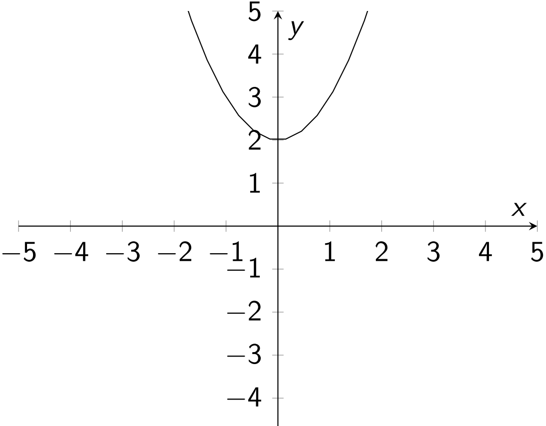
- a determines the direction of the tails and the degree of curvature
  - $a > 0$ means the curve faces up (convex)
  - $a < 0$ means the curve faces down (concave)
  - large $|a| > 1$ means steep slope
  - $0 < |a| < 1$ means shallow slope

- b and a together determine the $x-$coordinate of the vertex: $x = -\frac{b}{2a}$

- c controls the height

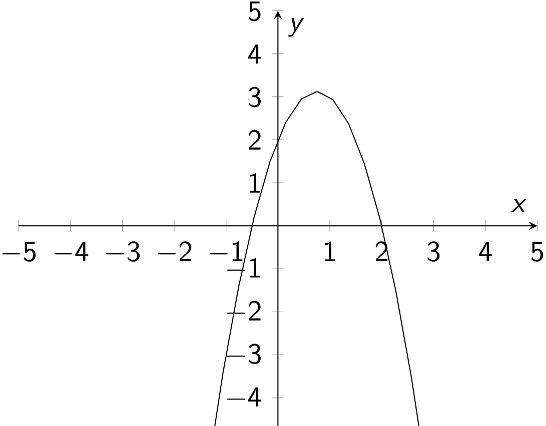
Example: quadratics

- $y = x^2 + 2$
- $a = 1,\ b = 0,\ c = 2$
- Not too steep, convex, and vertex is at $(0, 2)$



Example: quadratics

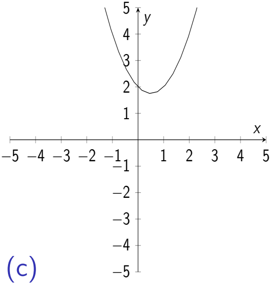
- $y = -2x^2 + 3x + 2$
- $a = -2$, $b = 3$, $c = 2$
- Steeper than before, concave, and vertex is at $(3/4, 50/16)$



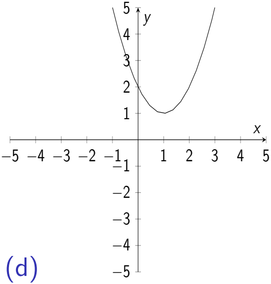
Exercise: quadratics

1. Which is a plausible plot of $x^2 - x + 2$?

(a)



(b)



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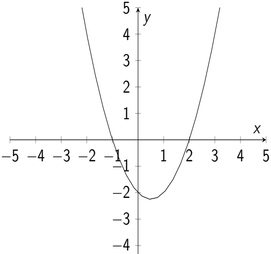
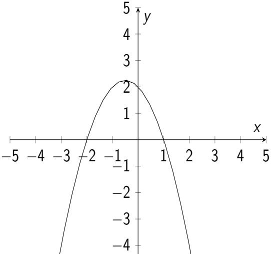


(d)



Solution: quadratics

1. $a = 1$, $b = -1$, $c = 2$
   - We are looking for a plot where the vertex has $y-$coordinate near 2
   - This rules out (b)
   - Now we want a plot with $y = 2$ when $x = 1$, ruling out (b) [and (d)]
   - Of the two remaining, (d) is concave, so it has $a < 0$
   - (a) is the solution!

Transforming graphs

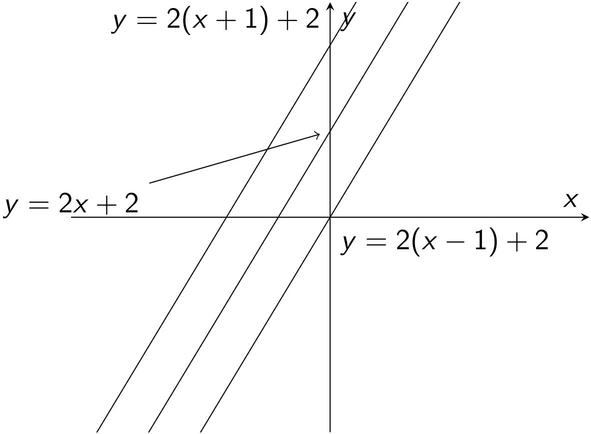
- Once we have one graph, how do we get another?

- Two main types of transformations: shifting and stretching

Shifting graphs

- Sometimes we want to change the interpretation of the intercept
- Once we know the properties of a graph, shifting doesn't change much!
- Shift left: add to x
- Shift right: subtract from x
- Shift up: add to intercept
- Shift down: subtract from intercept
- Why? Adding to x: smaller x's now have the same y. Subtracting from x: larger x's now have the same y.

Example: shifting graphs

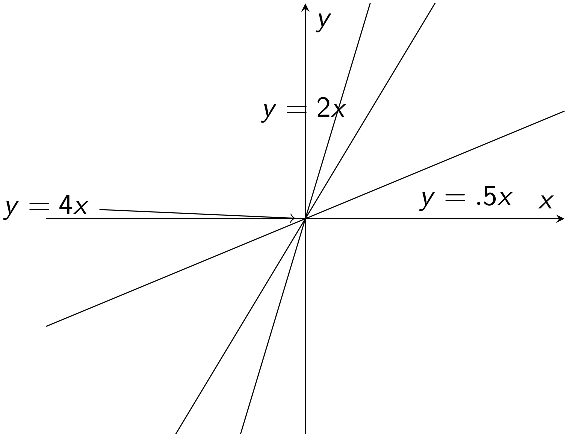


Stretching graphs

- Make the graph steeper, or shrink it: make $|m|$ larger in a linear equation, and make $|a|$ larger in a quadratic equation

- Make the graph shallower, or stretch it: make $|m|$ smaller in a linear equation, and make $|a|$ smaller in a quadratic equation

Example: stretching graphs



Exercise: transforming graphs

1. How do we shift the graph of $y = 2x + 3$ one unit right?
2. How do we transform the graph of $y = 2x + 3$ to have a shallower slope?
3. How do we transform the graph of $y = 2x + 3$ to have a slope of 1 and a $y-$intercept of 4?

Solution: shifting graphs

1. Subtract 1 from x! New equation: $y = 2(x - 1) + 3$

2. Multiply by a number less than 1; for example, take $x/2$. This gives new equation $y = 2(x/2) + 3$, or $y = x + 3$

3. To get a slope of 1, divide x by 2. To make the $y-$intercept 4, shift left by adding $1/2$ to x. New equation: $y = 2 * (x/2 + 1/2) + 3$, or $y = x + 4$

Summary

- Graphs are useful tools to describe relationships in data or visualize functions

- Histograms, boxplots, and scatterplots are common and useful types of graphs

- Linear trends can be described in:
  - Slope-intercept form — $y = mx + b$
  - Point-slope form — $y - y_1 = m(x - x_1)$

- Reading an equation from a graph involves finding the intercept and calculating the slope

- Graphs can be transformed by adding/subtracting from x, or multiplying/dividing x