Day 1, Session 1: Order of operations and negative numbers

Brian D. Williamson

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Example: kidney stones (from Katie Kerr's BIOST 514, Autumn 2014)

A study compared the success rates of two different procedures for removing kidney stones: open surgery (OS) and percutaneous nephrolithotomy (PCNL), a minimally invasive technique. Here are the numbers of procedures that were successful or not at getting rid of patients kidney stones, by type of procedure. The data are separated by patients with small kidney stones and large kidney stones.

Patients with Small Stones:

	OS	PCNL
Success	81	234
Failure	6	36

Patients with Large Stones:

	OS	PCNL
S	192	55
F	71	25

Example: kidney stones

Combined data:

	OS	PCNL
S	273	289
F	77	61

We can ask a few questions using these data:

- What is the percent of kidney stones successfully removed for each procedure?

- What is the percent of small kidney stones successfully removed? The percent of large kidney stones? Which type of stones seem easier to treat?

- What is the percent of successful procedures of each type for small kidney stones only, and for large stones only?

Example: kidney stones

To solve these questions, we will need a variety of tools:

- fraction manipulation—percentages are fractions!

- the order of operations—make sure that we get correct denominators for the fractions

- reading 2x2 tables—get the correct numbers for each question

- parsing sentences—which information is relevant?

We will cover each of these tools in more detail!

Example: statistics in medical research (from Katie Kerr's BIOST 514, Autumn 2014)

Does involving a statistician to help with statistical methods improve the chance that a medical research paper will be published?

A study of a random sample of papers submitted to two medical journals found that 135 of 190 papers that lacked statistical assistance were rejected by the journal editors (without even being circulated for peer review). In contrast 293 of the 514 papers with statistical help were rejected without review.

(Note: a paper is "submitted" when it is sent to a journal for consideration for publication. Not all submitted papers are accepted by the journal editors and published in the journal.)

Example: statistics in medical research

To find out if involving a statistician helps:

- Determine the proportion of papers submitted to these journals that include help from a statistician.

- Give a 95% confidence interval (CI) for this proportion, and interpret it.

- Determine the proportion of papers rejected without review when a statistician is and is not involved.

- Give a 95% CI for this proportion, and interpret it.

- Is this an observational study or a randomized trial?

- Write a paragraph summarizing the conclusions of your analysis, making sure to comment on the ability of the study to address the question of interest and on any important limitations of the study.

Example: statistics in medical research

These questions also involve a variety of tools:

- fraction manipulation

- parsing sentences

- interpreting results

Outline for Session 1

- Order of operations

- Negative numbers

- Fractions

- Algebra

Evaluating expressions

- Example expression: $3(1 + 2) + 5$

- How do we evaluate the above expression? In other words:
    - Which terms to we compute first?

    - Are there rules for evaluating expressions?

Order of operations

- Rules for evaluating expressions:
  1. Parentheses
  2. Exponents
  3. Multiplication and division
  4. Addition and subtraction

- A handy memory device: PEMDAS — Please Excuse My Dear Aunt Sally

Example: order of operations in action!

- Example from slide 2: $3(1 + 2) + 5$

- This notation is equivalent to $3 \times (1 + 2) + 5$

- Apply PEMDAS:

	Current Expression
1. Parentheses: add 1 and 2	$3(3) + 5$
2. Exponents: none	$3(3) + 5$
3. Multiplication: multiply 3 and 3	$9 + 5$
4. Division: none	$9 + 5$
5. Addition: add 9 and 5	14
6. Subtraction: none	

- The final answer is 14!

Example: order of operations with exponents!

- Expression: $\dfrac{(2^2 + 5)^2}{3 \times 3} + 5$

- Apply PEMDAS:

	Current Expression

1. Parentheses: $2^2 + 5$.
   Need to apply PEMDAS again! $\quad\dfrac{(2^2+5)^2}{3\times 3} + 5$

   1.1 Parentheses: none $\qquad\qquad\qquad \dfrac{(2^2+5)^2}{3\times 3} + 5$

   1.2 Exponents: $2^2 = 4$ $\qquad\qquad\quad \dfrac{(4+5)^2}{3\times 3} + 5$

   1.3 Multiplication/division: none $\quad \dfrac{(4+5)^2}{3\times 3} + 5$

   1.4 Addition/subtraction: $4 + 5 = 9$ $\quad \dfrac{(9)^2}{3\times 3} + 5$

2. Exponents: $9^2 = 81$ $\qquad\qquad\qquad \dfrac{81}{3\times 3} + 5$
3. Multiplication: $3 \times 3 = 9$ $\qquad\qquad\quad \dfrac{81}{9} + 5$
4. Division: $81/9 = 9$ $\qquad\qquad\qquad\quad 9 + 5$
5. Addition/subtraction: $9 + 5 = 14$!

Order of operations: nesting

- Earlier, we needed to apply PEMDAS a second time, within the evaluation of the parentheses

- This is common!

- Apply PEMDAS as many times as necessary within each sub-expression, like $(2^2 + 5)$ in the previous example

Exercise: order of operations

Kidney stone data:

Patients with Small Stones:				Patients with Large Stones:				Combined data:		
	OS	PCNL			OS	PCNL			OS	PCNL
Success	81	234		S	192	55		S	273	289
Failure	6	36		F	71	25		F	77	61

Try to work out these examples alone or in pairs:

1. $273/(273+77)$ and $289/(289+61)$. What proportions do these correspond to?
2. What is the percent of small kidney stones successfully removed? Large kidney stones?
3. Which type of kidney stones seem easier to treat?
4. PEMDAS example: $\{(2+1)^2+1\}^2$

Note on exercises

- Solutions for the exercises are usually in the slides

- However, please attempt them first without looking at the solution!

Solution: order of operations

1. These correspond to the proportion of kidney stones successfully removed in OS and PCNL, respectively. We find the proportions using PEMDAS:
   - Parentheses: $273 + 77 = 350$
   - Division: $273/350 = 0.78$, or 78% (OS)
   - Parentheses: $289 + 61 = 350$
   - Division: $289/350 = 0.82$, or 82% (PCNL)

2. We first have to find the numbers of small and large kidney stones successfully removed, and then find the total number of both small and large kidney stones operated on.
   - Small kidney stones removed: $81 + 234$; Large: $192 + 55$
   - Total small stones operated on: $81 + 234 + 6 + 36$; Large: $192 + 55 + 71 + 25$
   - Proportion small: $(81 + 234)/(81 + 234 + 6 + 36) = 0.88$, which is 88%
   - Proportion large: $(192 + 55)/(192 + 55 + 71 + 25) = 0.72$, which is 72%

Solution: order of operations

3. Based on the numbers calculated in (1) and (2), it seems that small kidney stones are easier to treat.

4. $[(2+1)^2 + 1]^2 = 100$. PEMDAS:
   - Parentheses: $(2+1)^2 + 1$. Nested PEMDAS:
     - Parentheses: $2 + 1$. Nested PEMDAS — Addition: $2 + 1 = 3$
     - Exponent: $3^2 = 9$
     - Addition: $9 + 1 = 10$
   - Exponent: $10^2 = 100$

Negative numbers: what are they?

- Ways to think about negative numbers:
  - A positive number subtracted from zero

  - Opposites of positive numbers: $-4 + 4 = 0$

  - Movement left on the number line

- When might we see them?
  - Comparing two proportions

  - Decreasing trend (e.g., lung capacity for smokers vs. non-smokers)

The number line
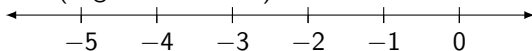
- Movement on the number line:
  - Right (positive numbers):
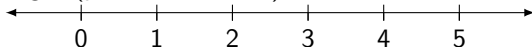
  Increasing

  - Left (negative numbers):

  Decreasing

- Example: move left 3, starting at 100?
  - Subtract 3
  - Add negative 3

Properties of negative numbers

- Represent opposites of positive numbers, or movement left on the number line

- Subtraction = adding a negative number

- The product of two negatives is a positive

- Movement left on the numberline $\rightarrow$ smaller numbers

Negative fractions

- First, positive fractions: for the same numerator, a larger denominator makes a smaller number, e.g. $1/4 < 1/2$

- Negatives are opposites: think of zero as a mirror

- So for the same numerator (a negative number), a larger denominator makes a less negative number, e.g. $-1/2 < -1/4$

Example: two negatives make a positive

- Expression: -1×-1

- Answer: 1!

- Why?
  - -1 is a negative number

  - Negative numbers mean opposites; the opposite of -1 is 1

Example: ordering negative numbers

- Expression: $-3 \underline{} -2$

- Answer: $-3 < -2$

- Why?
  - Negative numbers are left motion on number line! -3 is further left than -2

Exercise: negative numbers

- Try to work out the following examples by yourself or in pairs:
  1. $-5.2 - (-11.3)$

  2. $-5 - 6$

  3. $(-1) \times (-5) + (-3)$

  4. $-\dfrac{3}{7} - \left(-\dfrac{1}{4}\right)$

Solution: negative numbers

1. $-5.2 - (-11.3) = 6.1$
   - $-(-11.3) = -1 \times (-11.3) = 11.3$
   - $-5.2 + 11.3 = 11.3 - 5.2 = 6.1$

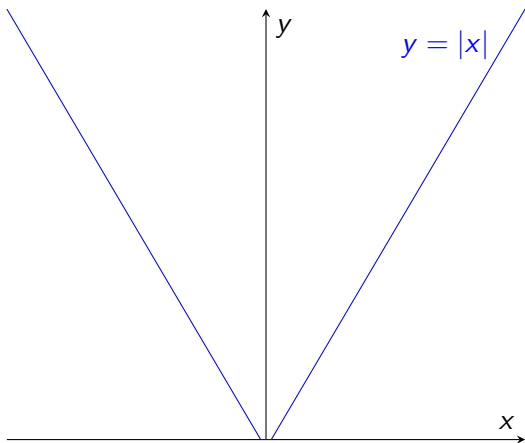2. $-5 - 6 = -11$

3. $(-1) \times (-5) + (-3) = 2$
   - $-1 \times (-5) = 5$
   - $5 + (-3) = 5 - 3 = 2$

4. $-\dfrac{3}{7} - \left(-\dfrac{1}{4}\right) = -5/28 \approx -0.17$
   - $-\dfrac{3}{7} - \left(-\dfrac{1}{4}\right) = \dfrac{3}{7} + \dfrac{1}{4}$
   - $-\dfrac{3}{7} + \dfrac{1}{4} = -12/28 + 7/28 = -5/28$

Related concepts: absolute value

- Magnitudes: how "large" is a number, with no direction
  - Examples: speed (how fast an object is moving), length

- Symbol for absolute value is $|\cdot|$

$y = |x|$

Example: absolute value of a positive number

- Expression: $|4|$

- Answer is 4! Positive numbers already measure size, with no direction

Example: absolute value of a negative number

- Expression: $|-4|$

- Answer is 4!

- Why?
  - Negatives are opposites of positives
  - Absolute value has no direction
  - 4 and -4 are equally far away from zero

Related concepts: negative numbers and inequalities

- Expression from before: $-3 < -2$

- What happens if we multiply both sides by -1?

- Negatives are opposites: signs change and inequality flips, yielding $2 < 3$

Back to negative fractions

- Recall that for a fixed negative numerator, a larger denominator means a less negative number

- Absolute value makes this more clear!

- Example: -1/2, -1/4
    1. $|-1/2| = 1/2$, $|-1/4| = 1/4$

    2. We already know that $1/4 < 1/2$

    3. Multiply both sides by -1, yielding (with inequality rules) $-1/2 < -1/4$

Exercise: absolute value, negative numbers

- Try to work out the following examples by yourself or in pairs:

  1. $|-5|$ and $|5|$

  2. Is $|-5| < 4$?

  3. Is $-15 > -14$?

  4. Is $-(3+1) \times 5 < -(4+1) \times 3$?

Solution: absolute value, negative numbers

1. $|-5| = |5| = 5$

2. $|-5| = 5$, and $5 > 4$; answer is no

3. -15 is further from 0 than -14; also, $14 < 15$. Hence $-15 < -14$

4. Two ways to solve this:
   - $-(3+1) \times 5 = -1 \times (4) \times 5 = -20$, and $-(4+1) \times 3 = -1 \times (5) \times 3 = -15$. So $-20 < -15$

   - If $-(3+1) \times 5 < -(4+1) \times 3$, then $(3+1) \times 5 > (4+1) \times 3$. But this means $20 > 15$, which is true!

Summary

- The order of operations is a recipe for solving expressions

- A handy memory tool is PEMDAS: Parentheses, Exponents, Multiplication/Division, Addition/Subtraction

- Negative numbers decrease as we move away from zero (left on the number line)

- Absolute value measures the magnitude of a number — how far away from zero is it?