

# SOLVING WORD PROBLEMS



# Where is she going with this?

- Units
- Steps for solving word problems
- Example
- Lots of exercises!

# Using Units

If you know that  $\frac{2}{3}$  of those with a disease seek care and  $\frac{2}{5}$  of those who seek care are hospitalized, what fraction of those with a disease are hospitalized?

$$\frac{2 \cancel{\text{ seek care}}}{3 \text{ diseased}} \times \frac{2 \text{ hospitalized}}{5 \cancel{\text{ seek care}}} = \frac{4 \text{ hospitalized}}{15 \text{ diseased}}$$

# Using Units

To calculate a ratio (RECALL: These are unitless!)

$$\begin{aligned} & \frac{\frac{14 \text{ diseased}}{100 \text{ at risk}}}{\frac{3 \text{ diseased}}{200 \text{ at risk}}} \\ &= \frac{14 \cancel{\text{ diseased}}}{100 \cancel{\text{ at risk}}} \times \frac{200 \cancel{\text{ at risk}}}{3 \cancel{\text{ diseased}}} \\ &= \frac{2800}{300} \\ &= 7 \end{aligned}$$

# Using Units

To solve for an unknown

$$\frac{38 \text{ dead}}{216 \text{ ill}} = \frac{x}{1,000 \text{ ill}}$$

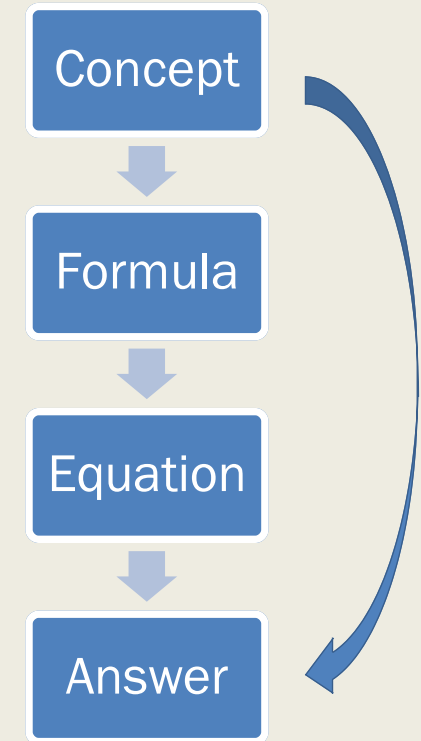
$$\frac{38 \text{ dead}}{216 \cancel{\text{ ill}}} \times 1,000 \cancel{\text{ ill}} = \frac{x}{1,000 \cancel{\text{ ill}}} \times 1,000 \cancel{\text{ ill}}$$

$$\frac{38,000}{216} \text{ dead} = x$$

$$x = 176 \text{ dead}$$

# Problem Solving

1. Read the whole problem
2. Identify the question
3. Break down the quantity being asked for
4. Identify useful information
5. Review remaining information
6. Calculate
7. Reflect on your answer



# Example: Word Problem

- In a population of 500,000 how many deaths would you expect in a year?

Age (years)	Persons (in 1000's)	Deaths (in 1000's)
Under 5	198	1
5 - 19	580	4
20 - 44	601	1.3
45 - 64	396	4.6
65 and over	185	11.4
All ages	1,960	18.7

Average annual deaths in a population

1. Read the whole problem
2. Identify the question
3. Break down the quantity being asked for
  - *Expected deaths in a year*
4. Identify useful information
5. Review remaining information
6. Calculate
$$\left( \frac{18.7 \text{ deaths}}{1,960 \text{ persons}} \right) * 500,000 \text{ persons}$$
7. Reflect on your answer

# Example: Word Problem

- A manufacturer sells products for \$5 per unit. Some retailers add \$1 per unit to the price for consumers. Fixed costs are constant at \$3,000 regardless of the number of units of product involved. Total cost is equal to the sum of fixed costs and variable costs. In this company, variable costs are estimated to be \$2 per unit. What is the breakeven point (i.e. how many units must be sold so that the total costs equal total revenues) for the manufacturer?

1. Read the whole problem
2. Identify the question
3. Break down the quantity being asked for
  - $Costs = Revenue$
  - $Costs = Fixed + (Variable * Units)$
  - $Revenue = Price * Units$
  - $F + (V * U) = P * U$
4. Identify useful information
5. Review remaining information
6. Calculate
7. Reflect on your answer



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1. Read the whole problem
2. Identify the question
3. Break down the quantity being asked for
  - $F + (V * U) = P * U$
4. Identify useful information
  - $F = \$3,000$
  - $V = \$2$
  - $P = \$5$
5. Review remaining information
6. Calculate
  - $\$3,000 + \$2U = \$5U$
7. Reflect on your answer

# Example: Word Problem

- A manufacturer sells products for \$5 per unit. Some retailers add \$1 per unit to the price for consumers. Fixed costs are constant at \$3,000 regardless of the number of units of product involved. Total cost is equal to the sum of fixed costs and variable costs. In this company, variable costs are estimated to be \$2 per unit. What is the breakeven point (i.e. how many units must be sold so that the total costs equal total revenues)?

1. Read the whole problem
2. Identify the question
3. Break down the quantity being asked for
4. Identify useful information
5. Review remaining information
6. Calculate
  - $\$3,000 + \$2U = \$5U$
  - $\$3,000 + \$2U - \$2U = \$5U - \$2U$
  - $\$3,000 = \$3U$
  - $\frac{\$3,000}{\$3} = \frac{\$3U}{\$3}$
  - $1,000 = U$
7. Reflect on your answer

# EXERCISE

The following data were obtained for children 6 months through 4 years from the Second NHANES survey, conducted February 1976-February 1980, among 27,801 persons in the United States.

Blood lead (micrograms/ deciliter)	No. examined	% with history of eating unusual substances
≥30	117	16.2
20-29	503	14.1
<20	1,752	5.2

How many children examined with blood lead levels greater than or equal to 20 mcg/dl had a history of eating unusual substances?

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GOAL:  $x$  children

	Pica	No pica
> 30	<i>a</i>	<i>b</i>
20-29	<i>c</i>	<i>d</i>
<20	<i>e</i>	<i>f</i>

$$x = a + c$$

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How many children examined with blood lead levels greater than or equal to 20 mcg/dl had a history of eating unusual substances?

GOAL:  $x$  children

	Pica	No pica	
> 30	$a = 0.162 * 117$	$b$	117
20-29	$c = 0.141 * 503$	$d$	503
<20	$e$	$f$	

$$x = a + c$$

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	Pica	No pica	
$> 30$	19	$b$	117
20-29	71	$d$	503
$< 20$	$e$	$f$	

$$x = a + c$$

$$x = 19 + 71 = 90 \text{ children}$$



# EXERCISE

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<20	1,752	5.2

How many children examined with blood lead levels greater than or equal to 20 micrograms/deciliter had a history of eating unusual substances?

$$\#Eating = \#Examined * \%Eating$$

$$\#Eat_{\geq 20} = \#Eat_{20-29} + \#Eat_{\geq 30}$$

$$\#Eat_{20-29} = \#Ex_{20-29} * \%Eat_{20-29}$$

$$\#Eat_{\geq 30} = \#Ex_{\geq 30} * \%Eat_{\geq 30}$$

$$\#Eat_{\geq 20} = 503 * 0.141 + 117 * 0.162$$

$$\#Eat_{\geq 20} = 90$$

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What proportion of children examined had a history of eating unusual substances?

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	Pica	No pica	
> 30	19	$b$	117
20-29	71	$d$	503
<20	$e$	$f$	
	$m1$	$m2$	total

$$m1 = 19 + 71 + e$$

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GOAL:  $x\%$  children

	Pica	No pica	
> 30	19	$b$	117
20-29	71	$d$	503
<20	$e =$ (1752*0.052)	$f$	1752
	$m1$	$m2$	total

$$m1 = 19 + 71 + e$$

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	Pica	No pica	
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20-29	71	$d$	503
<20	91	$f$	1752
	$m1$	$m2$	total

$$m1 = 19 + 71 + e$$

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GOAL:  $x\%$  children

	Pica	No pica	
> 30	19	$b$	117
20-29	71	$d$	503
<20	91	$f$	1752
	$m1$	$m2$	total

$$m1 = 19 + 71 + 91$$

$$m1 = 181$$

# EXERCISE

The following data were obtained for children 6 months through 4 years from the Second NHANES survey, conducted February 1976-February 1980, among 27,801 persons in the United States.

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What proportion of children examined had a history of eating unusual substances?

GOAL:  $x\%$  children

	Pica	No pica	
> 30	19	$b$	117
20-29	71	$d$	503
<20	91	$f$	1752
	$m1$	$m2$	total

$$m1 = 19 + 71 + 91$$

$$m1 = 181 \text{ children}$$



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The following data were obtained for children 6 months through 4 years from the Second NHANES survey, conducted February 1976-February 1980, among 27,801 persons in the United States.

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What proportion of children examined had a history of eating unusual substances?

GOAL:  $x\%$  children

	Pica	No pica	
> 30	19	$b$	117
20-29	71	$d$	503
<20	91	$f$	1752
	181	$m2$	total

$$x\% = 181/\text{total}$$

$$x\% = 181/(117 + 503 + 1752)$$

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<20	1,752	5.2

What proportion of children examined had a history of eating unusual substances?

GOAL:  $x\%$  children

	Pica	No pica	
> 30	19	$b$	117
20-29	71	$d$	503
<20	91	$f$	1752
	181	$m2$	2372

$$x\% = 181/\text{total}$$

$$x\% = 181/(117 + 503 + 1752)$$

$$x\% = 181/2372 = 0.076 = 7.6\%$$

# EXERCISE

What proportion of children examined had a history of eating unusual substances?

$$\%Eat_{tot} = \frac{\#Eat_{tot}}{\#Ex_{tot}}$$

Blood lead level (micrograms/d eciliter)	No. examined	% with history of eating unusual substances
≥30	117	16.2
20-29	503	14.1
<20	1,752	5.2

$$\#Ex_{tot} = \#Ex_{\geq 30} + \#Ex_{20-29} + \#Ex_{<20} = 117 + 503 + 1752 = 2372$$

$$\%Eat_{tot} = \frac{\#Ex_{\geq 30}}{\#Ex_{tot}} (\%Eat_{\geq 30}) + \frac{\#Ex_{20-29}}{\#Ex_{tot}} (\%Eat_{20-29}) + \frac{\#Ex_{<20}}{\#Ex_{tot}} (\%Eat_{<20})$$

$$\%Eat_{tot} = \frac{117}{2372} (0.162) + \frac{503}{2372} (0.141) + \frac{1752}{2372} (0.052) = 0.076 = 7.6\%$$

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Blood lead (micrograms/ deciliter)	No. examined	% with history of eating unusual substances
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What proportion of children with a history of eating unusual substances had a blood lead level  $\geq 30$  micrograms/deciliter?

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What proportion of children with a history of eating unusual substances had a blood lead level  $\geq 30$  micrograms/deciliter?

GOAL:  $x\%$  children

	Pica	No pica	
$> 30$	19	$b$	117
20-29	71	$d$	503
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	181	$m2$	2372

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What proportion of children with a history of eating unusual substances had a blood lead level  $\geq 30$  micrograms/deciliter?

GOAL:  $x\%$  children

	Pica	No pica	
$> 30$	19	$b$	117
20-29	71	$d$	503
$< 20$	91	$f$	1752
	181	$m_2$	2372

$$x\% = 19/181$$

$$x\% = 0.105 = 10.5\%$$

# EXERCISE

- What proportion of children with a history of eating unusual substances had a blood lead level  $\geq 30$  micrograms/deciliter?

$$\%Eat_{\geq 30} = \frac{\#Eat_{\geq 30}}{\#Eat_{tot}}$$

$$\%Eat_{\geq 30} = \frac{\#Eat_{\geq 30}}{\#Eat_{\geq 30} + \#Eat_{\geq 30} + \#Eat_{\geq 30}}$$

$$= \frac{\#Ex_{\geq 30}(\%Eat_{\geq 30})}{\#Ex_{\geq 30}(\%Eat_{\geq 30}) + \#Ex_{20-29}(\%Eat_{20-29}) + \#Ex_{<20}(\%Eat_{<20})}$$

$$= \frac{117(0.162)}{117(0.162) + 503(0.141) + 1752(0.052)} = 0.105 = 10.5\%$$

Blood lead level (micrograms/d eciliter)	No. examined	% with history of eating unusual substances
$\geq 30$	117	16.2
20-29	503	14.1
$< 20$	1,752	5.2

# EXERCISE

The following is from a newspaper article on seat belts:

*Of the 649 people who died in traffic accidents in Washington last year, 55 percent were not wearing seat belts. In those same fatal crashes, 73 percent of people who were belted in survived without serious injury.*

Can you identify the percent of people who survived these crashes without serious injury and were *not* belted in? If so, calculate it. If not, what quantity would you need to calculate this number?



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	Died	Survived	
Belt	<i>a</i>	<i>b</i>	<i>m3</i>
No belt	<i>c</i>	<i>d</i>	<i>m4</i>
	<i>m1</i>	<i>m2</i>	

$$x\% = d/m2$$

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	Died	Survived	
Belt	$(0.55) \cdot 649$	$b$	$m3$
No belt	$c$	$d$	$m4$
	649	$m2$	

$$x\% = d/m2$$

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	Died	Survived	
Belt	$(0.55) \cdot 649$	$(0.73) \cdot m_3$	$m_3$
No belt	$(0.45) \cdot 649$	$d$	$m_4$
	649	$m_2$	

$$x\% = d/m_2$$

NOT ENOUGH INFORMATION

# EXERCISE

The rate of suicide among American physicians, relative to the corresponding rate in the population as a whole, varies by gender. The rate in physicians is 1.5 times higher than the rate in American men in the general population, whereas rate in physicians is 3.0 times higher than the rate among American women in the general population. It turns out that the rate of suicide in male and female physicians is identical. For American men and women in general, how much higher is the rate of suicide in men compared to women?

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Goal:  $\frac{\text{Suicide Rate}_{\text{men}}}{\text{Suicide Rate}_{\text{women}}}$

We have:  $\frac{\text{Suicide Rate}_{\text{physicians}}}{\text{Suicide Rate}_{\text{men}}}, \frac{\text{Suicide Rate}_{\text{physicians}}}{\text{Suicide Rate}_{\text{women}}}$

$$\frac{\text{Suicide Rate}_{\text{men}}}{\cancel{\text{Suicide Rate}_{\text{physicians}}}} \times \frac{\cancel{\text{Suicide Rate}_{\text{physicians}}}}{\text{Suicide Rate}_{\text{women}}} = \frac{1}{1.5} \times 3.0 = \frac{3.0}{1.5} = 2$$

# EXERCISE

The rate of suicide among American physicians, relative to the corresponding rate in the population as a whole, varies by gender. Among men, the rate in physicians is 1.5 times higher, whereas among women the corresponding relative rate is 3.0. It turns out that the rate of suicide in American male and female physicians is identical. For American men and women in general, what is the relative rate of suicide in men compared to women?

$$\frac{\text{Suicide Rate}_{\text{American men}}}{\text{Suicide Rate}_{\text{American women}}}$$

$$S_{P.M.} = S_{P.F.} = S_P$$

$$\frac{S_P}{S_{A.M.}} = 1.5 \rightarrow S_{A.M.} = \frac{S_P}{1.5}$$

$$\frac{S_P}{S_{A.F.}} = 3.0 \rightarrow S_{A.F.} = \frac{S_P}{3.0}$$

$$\frac{S_{A.M.}}{S_{A.F.}} = \frac{\frac{S_P}{1.5}}{\frac{S_P}{3.0}} = \frac{S_P}{1.5} \times \frac{3.0}{S_P} = 2$$