

Day 1, Session 1: Order of operations and negative numbers

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EPI/BIOST Bootcamp 2018

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Learning objectives

By the end of this session, you should be able to

- **use** the order of operations to answer public health questions
- **determine** the relative ordering of two numbers
- **use** algebra and fractions to answer public health questions

Example: kidney stones (from Katie Kerr's BIOST 514, Autumn 2014)

A study compared the success rates of two different procedures for removing kidney stones: open surgery (OS) and percutaneous nephrolithotomy (PCNL), a minimally invasive technique. Here are the numbers of procedures that were successful or not at getting rid of patients' kidney stones, by type of procedure. The data are separated by patients with small kidney stones and large kidney stones.

Patients with Small Stones:

	OS	PCNL
Success	81	234
Failure	6	36

Patients with Large Stones:

	OS	PCNL
S	192	55
F	71	25

Example: kidney stones

		OS	PCNL
Combined data:	S	273	289
	F	77	61

We can ask a few questions using these data:

- What is the **percentage** of kidney stones successfully removed for each procedure?

Example: kidney stones

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- What is the **percentage** of kidney stones successfully removed for each procedure?
- What is the percentage of **small kidney stones** successfully removed? The percentage of **large kidney stones**? **Which type of stones seem easier to treat?**

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- What is the percentage of **small kidney stones** successfully removed? The percentage of **large kidney stones**? **Which type of stones seem easier to treat?**
- What is the percentage of successful procedures of each type for small kidney stones only, and for large stones only?

Example: kidney stones

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- **reading 2x2 tables**: get the correct numbers for each question

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- **fraction manipulation**: percentages are fractions!
- **the order of operations**: make sure that we get correct denominators for the fractions
- **reading 2x2 tables**: get the correct numbers for each question
- **parsing sentences**: which information is relevant?

We will cover each of these tools in more detail!

Example: statistics in medical research (from Katie Kerr's BIOS T 514, Autumn 2014)

Does involving a statistician to help with statistical methods improve the chance that a medical research paper will be published?

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(Note: a paper is “submitted” when it is sent to a journal for consideration for publication. Not all submitted papers are accepted by the journal editors and published in the journal.)

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- Is this an **observational study or a randomized trial?**

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To find out if involving a statistician helps:

- Determine the **proportion** of papers submitted to these journals that include help from a statistician.
- Give a 95% **confidence interval (CI)** for this proportion, and **interpret it**.
- Determine the **proportion** of papers rejected without review when a **statistician is and is not involved**.
- Give a 95% CI for this proportion, and interpret it.
- Is this an **observational study or a randomized trial**?
- Write a paragraph summarizing the conclusions of your analysis, making sure to comment on the ability of the study to address the question of interest and on any important limitations of the study.

Example: statistics in medical research

These questions also involve a variety of tools:

- fraction manipulation
- parsing sentences
- interpreting results

Outline for Session 1

- Order of operations
- Negative numbers
- Fractions
- Algebra

Evaluating expressions

- Example expression: $3(1 + 2) + 5$
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- How do we evaluate the above expression? In other words:
 - Which terms do we compute first?
 - Are there rules for evaluating expressions?

Order of operations

- Rules for evaluating expressions:
 1. Parentheses
 2. Exponents
 3. Multiplication and division (evaluate left to right)
 4. Addition and subtraction (evaluate left to right)

Order of operations

- Rules for evaluating expressions:
 1. Parentheses
 2. Exponents
 3. Multiplication and division (evaluate left to right)
 4. Addition and subtraction (evaluate left to right)
- A handy memory device: PEMDAS – Please Excuse My Dear Aunt Sally

Example: order of operations in action!

- Example from slide 10: $3(1 + 2) + 5$
- This notation is equivalent to $3 \times (1 + 2) + 5$

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- Apply PEMDAS:

1. Parentheses: add 1 and 2

<u>Current Expression</u>
$3(3) + 5$

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- Apply PEMDAS:

1. Parentheses: add 1 and 2
2. Exponents: none

Current Expression

$$3(3) + 5$$

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- Apply PEMDAS:

1. Parentheses: add 1 and 2

2. Exponents: none

3. Multiplication: multiply 3 and 3

Current Expression

$3(3) + 5$

$3(3) + 5$

$9 + 5$

Example: order of operations in action!

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- This notation is equivalent to $3 \times (1 + 2) + 5$

- Apply PEMDAS:

1. Parentheses: add 1 and 2

2. Exponents: none

3. Multiplication: multiply 3 and 3

4. Division: none

Current Expression

$3(3) + 5$

$3(3) + 5$

$9 + 5$

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- Apply PEMDAS:

1. Parentheses: add 1 and 2

2. Exponents: none

3. Multiplication: multiply 3 and 3

4. Division: none

5. Addition: add 9 and 5

Current Expression

$3(3) + 5$

$3(3) + 5$

$9 + 5$

$9 + 5$

14

Example: order of operations in action!

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- Apply PEMDAS:

1. Parentheses: add 1 and 2

2. Exponents: none

3. Multiplication: multiply 3 and 3

4. Division: none

5. Addition: add 9 and 5

6. Subtraction: none

Current Expression

$3(3) + 5$

$3(3) + 5$

$9 + 5$

$9 + 5$

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Example: order of operations in action!

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- This notation is equivalent to $3 \times (1 + 2) + 5$

- Apply PEMDAS:

1. Parentheses: add 1 and 2

2. Exponents: none

3. Multiplication: multiply 3 and 3

4. Division: none

5. Addition: add 9 and 5

6. Subtraction: none

Current Expression

$3(3) + 5$

$3(3) + 5$

$9 + 5$

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14

- The final answer is 14!

Example: order of operations with exponents!

- Expression: $\frac{(2^2 + 5)^2}{3 \times 3} + 5$

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- Apply PEMDAS:

1. Parentheses: $2^2 + 5$.

Current Expression

$$\frac{(2^2+5)^2}{3 \times 3} + 5$$

Example: order of operations with exponents!

- Expression: $\frac{(2^2 + 5)^2}{3 \times 3} + 5$

- Apply PEMDAS:

1. Parentheses: $2^2 + 5$.

Need to apply PEMDAS again!

Current Expression

$$\frac{(2^2+5)^2}{3 \times 3} + 5$$

Example: order of operations with exponents!

- Expression: $\frac{(2^2 + 5)^2}{3 \times 3} + 5$

- Apply PEMDAS:

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Need to apply PEMDAS again!

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Current Expression

$$\frac{(2^2+5)^2}{3 \times 3} + 5$$

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Example: order of operations with exponents!

- Expression: $\frac{(2^2 + 5)^2}{3 \times 3} + 5$

- Apply PEMDAS:

1. Parentheses: $2^2 + 5$.

Need to apply PEMDAS again!

- 1.1 Parentheses: none

- 1.2 Exponents: $2^2 = 4$

Current Expression

$$\frac{(2^2+5)^2}{3 \times 3} + 5$$

$$\frac{(2^2+5)^2}{3 \times 3} + 5$$

$$\frac{(4+5)^2}{3 \times 3} + 5$$

Example: order of operations with exponents!

- Expression: $\frac{(2^2 + 5)^2}{3 \times 3} + 5$

- Apply PEMDAS:

1. Parentheses: $2^2 + 5$.

Need to apply PEMDAS again!

- 1.1 Parentheses: none

- 1.2 Exponents: $2^2 = 4$

- 1.3 Multiplication/division: none

Current Expression

$$\frac{(2^2+5)^2}{3 \times 3} + 5$$

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- Expression: $\frac{(2^2 + 5)^2}{3 \times 3} + 5$

- Apply PEMDAS:

1. Parentheses: $2^2 + 5$.

Need to apply PEMDAS again!

- 1.1 Parentheses: none

- 1.2 Exponents: $2^2 = 4$

- 1.3 Multiplication/division: none

- 1.4 Addition/subtraction: $4 + 5 = 9$

Current Expression

$$\frac{(2^2+5)^2}{3 \times 3} + 5$$

$$\frac{(2^2+5)^2}{3 \times 3} + 5$$

$$\frac{(4+5)^2}{3 \times 3} + 5$$

$$\frac{(4+5)^2}{3 \times 3} + 5$$

$$\frac{(9)^2}{3 \times 3} + 5$$

Example: order of operations with exponents!

- Expression: $\frac{(2^2 + 5)^2}{3 \times 3} + 5$

- Apply PEMDAS:

1. Parentheses: $2^2 + 5$.

Need to apply PEMDAS again!

- 1.1 Parentheses: none

- 1.2 Exponents: $2^2 = 4$

- 1.3 Multiplication/division: none

- 1.4 Addition/subtraction: $4 + 5 = 9$

2. Exponents: $9^2 = 81$

Current Expression

$$\frac{(2^2+5)^2}{3 \times 3} + 5$$

$$\frac{(2^2+5)^2}{3 \times 3} + 5$$

$$\frac{(4+5)^2}{3 \times 3} + 5$$

$$\frac{(4+5)^2}{3 \times 3} + 5$$

$$\frac{(9)^2}{3 \times 3} + 5$$

$$\frac{81}{3 \times 3} + 5$$

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- 1.1 Parentheses: none

- 1.2 Exponents: $2^2 = 4$

- 1.3 Multiplication/division: none

- 1.4 Addition/subtraction: $4 + 5 = 9$

2. Exponents: $9^2 = 81$

3. Multiplication: $3 \times 3 = 9$

Current Expression

$$\frac{(2^2+5)^2}{3 \times 3} + 5$$

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$$\frac{81}{3 \times 3} + 5$$

$$\frac{81}{9} + 5$$

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Need to apply PEMDAS again!

- 1.1 Parentheses: none

- 1.2 Exponents: $2^2 = 4$

- 1.3 Multiplication/division: none

- 1.4 Addition/subtraction: $4 + 5 = 9$

2. Exponents: $9^2 = 81$

3. Multiplication: $3 \times 3 = 9$

4. Division: $81/9 = 9$

Current Expression

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- 1.2 Exponents: $2^2 = 4$

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- 1.4 Addition/subtraction: $4 + 5 = 9$

2. Exponents: $9^2 = 81$

3. Multiplication: $3 \times 3 = 9$

4. Division: $81/9 = 9$

5. Addition/subtraction: $9 + 5 = 14$!

Current Expression

$$\frac{(2^2+5)^2}{3 \times 3} + 5$$

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$$\frac{(4+5)^2}{3 \times 3} + 5$$

$$\frac{(4+5)^2}{3 \times 3} + 5$$

$$\frac{(9)^2}{3 \times 3} + 5$$

$$\frac{81}{3 \times 3} + 5$$

$$\frac{81}{9} + 5$$

$$9 + 5$$

Order of operations: nesting

- Earlier, we needed to apply PEMDAS a second time, within the evaluation of the parentheses
- This is common!
- Apply PEMDAS as many times as necessary within each sub-expression, like $(2^2 + 5)$ in the previous example

Exercise: order of operations

Kidney stone data:

Patients with Small Stones:

Patients with Large Stones:

Combined data:

	OS	PCNL
Success	81	234
Failure	6	36

	OS	PCNL
S	192	55
F	71	25

	OS	PCNL
S	273	289
F	77	61

Try to work out these examples alone or in pairs:

1. $273/(273+77)$ and $289/(289+61)$. What proportions do these correspond to?
2. What is the percentage of small kidney stones successfully removed? Large kidney stones?
3. Which type of kidney stones seem easier to treat?
4. PEMDAS example: $\{(2 + 1)^2 + 1\}^2$

Solution: order of operations

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 - Division: $273/350 = 0.78$, or 78% (OS)
 - Parentheses: $289 + 61 = 350$

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 - Parentheses: $289 + 61 = 350$
 - Division: $289/350 = 0.82$, or 82% (PCNL)

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2. We first have to find the numbers of small and large kidney stones successfully removed, and then find the total number of both small and large kidney stones operated on.

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 - Small kidney stones removed: $81 + 234$; Large: $192 + 55$

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 - Small kidney stones removed: $81 + 234$; Large: $192 + 55$
 - Total small stones operated on: $81 + 234 + 6 + 36$; Large: $192 + 55 + 71 + 25$
 - Proportion small:
 $(81 + 234)/(81 + 234 + 6 + 36) = 315/357 \approx 0.88$, or 88%

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 - Proportion small:
 $(81 + 234)/(81 + 234 + 6 + 36) = 315/357 \approx 0.88$, or 88%
 - Proportion large:
 $(192 + 55)/(192 + 55 + 71 + 25) = 247/343 \approx 0.72$, or 72%

Solution: order of operations

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4. $\{(2 + 1)^2 + 1\}^2 = 100$. PEMDAS:

Solution: order of operations

3. Based on the numbers calculated in (1) and (2), it seems that small kidney stones are easier to treat.
4. $\{(2 + 1)^2 + 1\}^2 = 100$. PEMDAS:
 - Parentheses: $(2 + 1)^2 + 1$. Nested PEMDAS:
 - Parentheses: $2 + 1$. Nested PEMDAS – Addition: $2 + 1 = 3$
 - Exponent: $3^2 = 9$
 - Addition: $9 + 1 = 10$

Solution: order of operations

3. Based on the numbers calculated in (1) and (2), it seems that small kidney stones are easier to treat.
4. $\{(2 + 1)^2 + 1\}^2 = 100$. PEMDAS:
- Parentheses: $(2 + 1)^2 + 1$. Nested PEMDAS:
 - Parentheses: $2 + 1$. Nested PEMDAS – Addition: $2 + 1 = 3$
 - Exponent: $3^2 = 9$
 - Addition: $9 + 1 = 10$
 - Exponent: $10^2 = 100$

Negative numbers: what are they?

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 - A positive number subtracted from zero
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 - Movement left on the number line
- When might we see them?
 - Comparing two proportions
 - Decreasing trend (e.g., lung capacity for smokers vs. non-smokers)

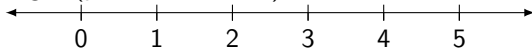
The number line

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The number line

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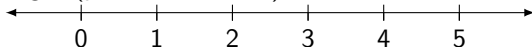
- Right (positive numbers):



The number line

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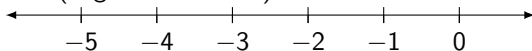
- Right (positive numbers):



Increasing



- Left (negative numbers):



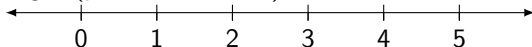
Decreasing



The number line

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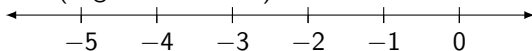
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Decreasing

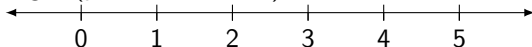


- Example: move left 3, starting at 100?

The number line

- Movement on the number line:

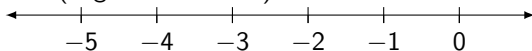
- Right (positive numbers):



Increasing



- Left (negative numbers):



Decreasing

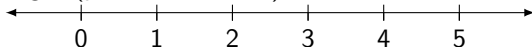


- Example: move left 3, starting at 100?
 - Subtract 3

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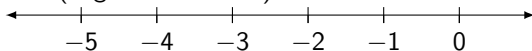
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Increasing



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Decreasing



- Example: move left 3, starting at 100?
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 - Add negative 3

Properties of negative numbers

- Represent opposites of positive numbers, or movement left on the number line

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- Subtraction = adding a negative number
- The product of two negatives is a positive
- Movement left on the number line \rightarrow smaller numbers

Negative fractions

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- Negatives are opposites: think of zero as a mirror
- So for the same numerator (a negative number), a larger denominator makes a less negative number, e.g., $-1/2 < -1/4$

Example: two negatives make a positive

- Expression: -1×-1

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- Why?
 - -1 is a negative number
 - Negative numbers mean opposites; the opposite of -1 is 1

Example: ordering negative numbers

- Expression: -3 ____ -2

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Example: ordering negative numbers

- Expression: -3 ____ -2
- Answer: $-3 < -2$
- Why?
 - Negative numbers are left motion on number line! -3 is further left than -2

Exercise: negative numbers

- Try to work out the following examples by yourself or in pairs:

1. $-5.2 - (-11.3)$

2. $-5 - 6$

3. $(-1) \times (-5) + (-3)$

4. $-\frac{3}{7} - \left(-\frac{1}{4}\right)$

Solution: negative numbers

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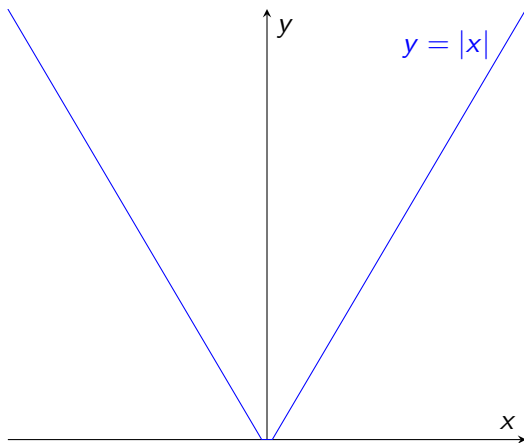
- $-\frac{3}{7} - \left(-\frac{1}{4}\right) = -\frac{3}{7} + \frac{1}{4}$
- $-\frac{3}{7} + \frac{1}{4} = -12/28 + 7/28 = -5/28$

Related concepts: absolute value

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 - Examples: speed (how fast an object is moving), length
- Symbol for absolute value is $|\cdot|$



Example: absolute value of a positive number

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- Answer is 4! Positive numbers already measure size, with no direction

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 - 4 and -4 are equally far away from zero

Related concepts: negative numbers and inequalities

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- What happens if we multiply both sides by -1 ?
- Negatives are opposites: signs change and inequality flips, yielding $2 < 3$

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 1. $|-1/2| = 1/2$, $|-1/4| = 1/4$
 2. We already know that $1/4 < 1/2$
 3. Multiply both sides by -1 , yielding (with inequality rules)
 $-1/2 < -1/4$

Exercise: absolute value, negative numbers

- Try to work out the following examples by yourself or in pairs:

1. $|-5|$ and $|5|$

2. Is $|-5| < 4$?

3. Is $-15 > -14$?

4. Is $-(3 + 1) \times 5 < -(4 + 1) \times 3$?

Solution: absolute value, negative numbers

1. $|-5| = |5| = 5$
2. $|-5| = 5$, and $5 > 4$; answer is no
3. -15 is further from 0 than -14 ; also, $14 < 15$. Hence $-15 < -14$
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 $-(4+1) \times 3 = -1 \times (5) \times 3 = -15$. So $-20 < -15$

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 - $-(3+1) \times 5 = -1 \times (4) \times 5 = -20$, and $-(4+1) \times 3 = -1 \times (5) \times 3 = -15$. So $-20 < -15$
 - If $-(3+1) \times 5 < -(4+1) \times 3$, then $(3+1) \times 5 > (4+1) \times 3$. But this means $20 > 15$, which is true!

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- The order of operations is a recipe for solving expressions
- A handy memory tool is PEMDAS: Parentheses, Exponents, Multiplication/Division, Addition/Subtraction
- Negative numbers decrease as we move away from zero (left on the number line)
- Absolute value measures the magnitude of a number – how far away from zero is it?