

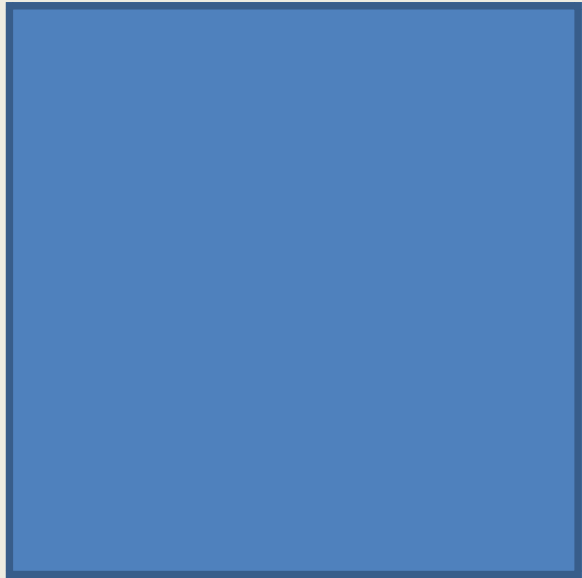
# PROPORTIONS



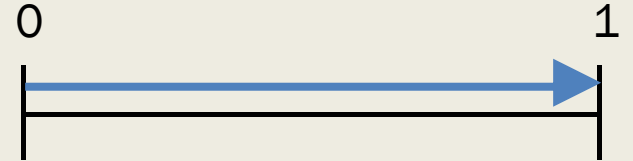
# Where is she going with this?

- What is a proportion?
- Fractions
- Other ways we represent and manipulate proportions
- Proportions and other mathematical constructs in epidemiology

# What is a proportion?

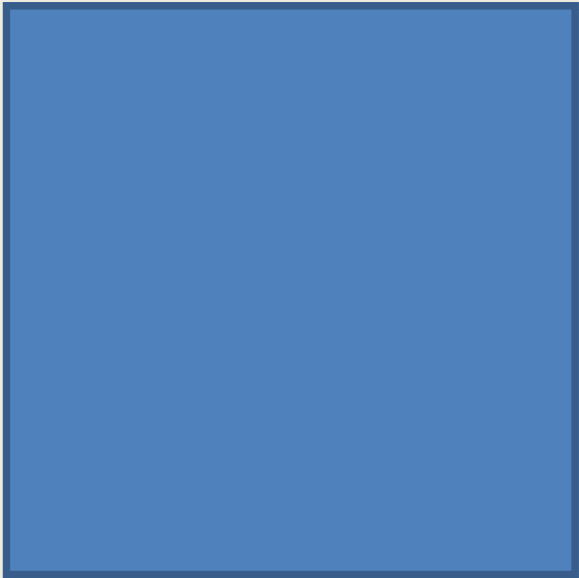


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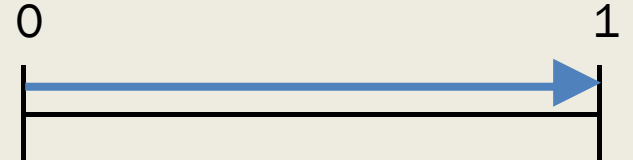


# What is a proportion?

a part, share, or number considered in comparative relation to a whole

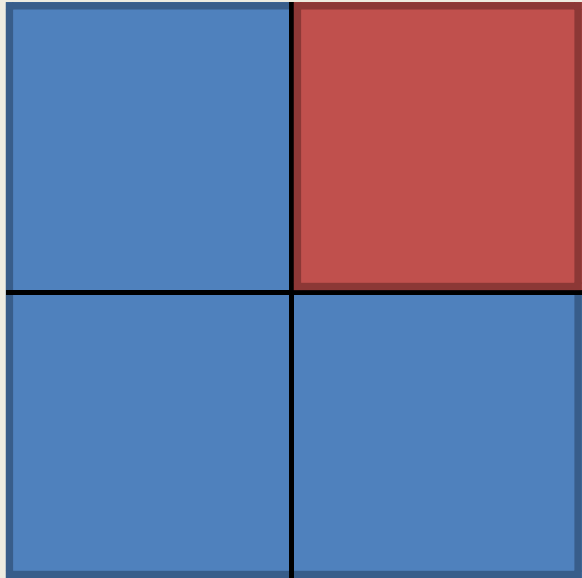


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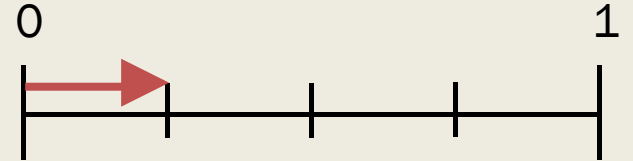


# What is a proportion?

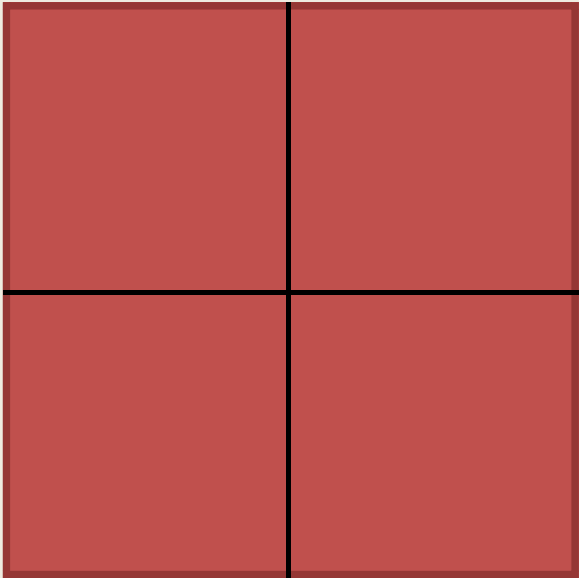
a part, share, or number considered in comparative relation to a whole



$$\frac{1}{4}$$



# Fractions



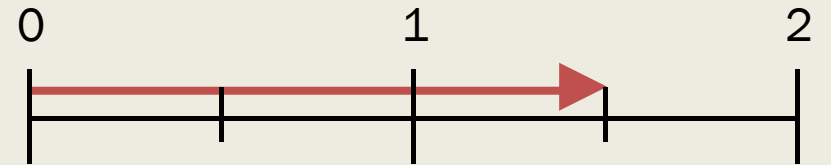
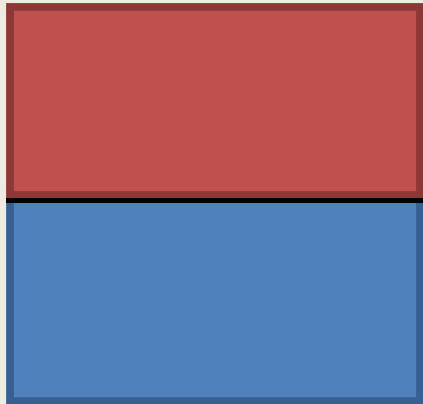
$$\frac{4}{4} = 1$$



# Fractions



$$2 = \frac{3}{2}$$



# Fraction Terminology

$$\frac{1}{4} = \frac{\textit{numerator}}{\textit{denominator}}$$



# Manipulating fractions

- Comparing

*[PRO-TIP] Matching denominators will make this easier*

- Addition & subtraction

*[RULE] Denominators must match, perform operation on numerators*

- Multiplication & division

*[RULE] Numerator and denominator operate separately*

# Adding & subtracting fractions

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- Same denominator?

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- Same denominator? **YES!**

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- Same denominator? **YES!**
  - *Ignore the denominator, work through the numerator*

$$\frac{2}{8} + \frac{5}{8} = \frac{7}{8}$$

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$$\frac{2}{8} + \frac{5}{8} = \frac{7}{8}$$

$$\frac{3}{5} + \frac{4}{5} = \frac{7}{5} = 1\frac{2}{5}$$

# Adding & subtracting fractions

- Same denominator? **YES!**
  - *Ignore the denominator, work through the numerator*

$$\frac{2}{8} + \frac{5}{8} = \frac{7}{8}$$

$$\frac{3}{5} + \frac{4}{5} = \frac{7}{5} = 1\frac{2}{5}$$

$$\frac{10}{12} - \frac{4}{12} = \frac{6}{12} = \frac{1}{2}$$

# Adding & subtracting fractions

- Same denominator? **NO!**

$$\frac{3}{4} + \frac{5}{6}$$



# Adding & subtracting fractions

- Same denominator? **NO!**
  1. *Find the least common denominator*

$$\frac{3}{4} + \frac{5}{6}$$

# Adding & subtracting fractions

- Same denominator? **NO!**

1. *Find the least common denominator*



Least Common Multiple

$$\frac{3}{4} + \frac{5}{6}$$

# Adding & subtracting fractions

- Same denominator? **NO!**

1. *Find the least common denominator*

**Least Common Multiple**

4: 4, 8, 12, 16, 20...

6: 6, 12, 18, 24, 30...

$$\frac{3}{4} + \frac{5}{6}$$

# Adding & subtracting fractions

- Same denominator? **NO!**

1. *Find the least common denominator*

## Least Common Multiple

4: 4, 8, 12, 16, 20...

6: 6, 12, 18, 24, 30...

$$\frac{3}{4} + \frac{5}{6}$$

# Adding & subtracting fractions

- Same denominator? **NO!**

1. *Find the least common denominator*
2. *Convert fractions*

## Least Common Multiple

4: 4, 8, 12, 16, 20...

6: 6, 12, 18, 24, 30...

$$\frac{3}{4} + \frac{5}{6}$$

# Adding & subtracting fractions

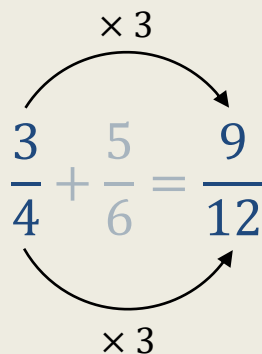
- Same denominator? **NO!**

1. *Find the least common denominator*
2. *Convert fractions*

## Least Common Multiple

4: 4, 8, 12, 16, 20...

6: 6, 12, 18, 24, 30...

$$\frac{3}{4} + \frac{5}{6} = \frac{9}{12}$$


# Adding & subtracting fractions

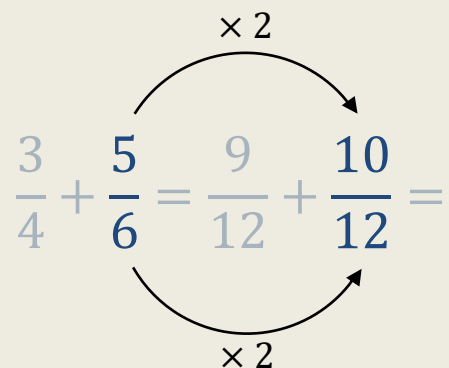
- Same denominator? **NO!**

1. Find the least common denominator
2. Convert fractions

## Least Common Multiple

4: 4, 8, 12, 16, 20...

6: 6, 12, 18, 24, 30...

$$\frac{3}{4} + \frac{5}{6} = \frac{9}{12} + \frac{10}{12} =$$


# Adding & subtracting fractions

■ Same denominator? **NO!**

1. *Find the least common denominator*
2. *Convert fractions*
3. *Add (or subtract or compare) as usual*

## Least Common Multiple

4: 4, 8, 12, 16, 20...

6: 6, 12, 18, 24, 30...

$$\frac{3}{4} + \frac{5}{6} = \frac{9}{12} + \frac{10}{12} = \frac{19}{12}$$



# Adding & subtracting fractions

■ Same denominator? **NO!**

1. *Find the least common denominator*
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## Least Common Multiple

4: 4, 8, 12, 16, 20...

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$$\frac{3}{4} + \frac{5}{6} = \frac{9}{12} + \frac{10}{12} = \frac{19}{12} = 1\frac{7}{12}$$

# Adding & subtracting fractions

- Same denominator?

$$\frac{3}{7} + \frac{2}{3}$$

# Adding & subtracting fractions

- Same denominator? **NO!**

$$\frac{3}{7} + \frac{2}{3}$$

# Adding & subtracting fractions

- Same denominator? **NO!**

1. *Find the least common denominator*

Least Common Multiple

7: 7, 14, 21, 28, 35...

3: 3, 6, 9, 12, 15, 18, 21...

$$\frac{3}{7} + \frac{2}{3}$$

# Adding & subtracting fractions

- Same denominator? **NO!**

1. *Find the least common denominator*

## Least Common Multiple

7: 7, 14, 21, 28, 35...

3: 3, 6, 9, 12, 15, 18, 21...

$$\frac{3}{7} + \frac{2}{3}$$

# Adding & subtracting fractions

- Same denominator? **NO!**
  1. Find the least common denominator
  2. Convert fractions

## Least Common Multiple

7: 7, 14, 21, 28, 35...

3: 3, 6, 9, 12, 15, 18, 21...

$$\frac{3}{7} + \frac{2}{3} = \frac{9}{21} + \frac{14}{21}$$

# Adding & subtracting fractions

- Same denominator? **NO!**

1. *Find the least common denominator*
2. *Convert fractions*
3. *Add (or subtract or compare) as usual*

## Least Common Multiple

7: 7, 14, 21, 28, 35...

3: 3, 6, 9, 12, 15, 18, 21...

$$\frac{3}{7} + \frac{2}{3} = \frac{9}{21} + \frac{14}{21} = \frac{23}{21} = 1\frac{2}{21}$$

# Adding & subtracting fractions

- Same denominator?

$$-\frac{3}{7} - \left(-\frac{4}{4}\right)$$



# Adding & subtracting fractions

- Same denominator? **NO!**

1. *Find the least common denominator*

## Least Common Multiple

7: 7, 14, 21, 28, 35...

4: 4, 8, 12, 16, 20, 24, 28...

$$-\frac{3}{7} - \left(-\frac{4}{4}\right)$$

# Adding & subtracting fractions

- Same denominator? **NO!**

1. Find the least common denominator
2. Convert fractions

## Least Common Multiple

7: 7, 14, 21, 28, 35...

4: 4, 8, 12, 16, 20, 24, 28...

$$-\frac{3}{7} - \left(-\frac{4}{4}\right) = -\frac{12}{28} - \left(-\frac{7}{28}\right)$$

# Adding & subtracting fractions

- Same denominator? **NO!**
  1. Find the least common denominator
  2. Convert fractions
  3. Add (or subtract or compare) as usual

## Least Common Multiple

7: 7, 14, 21, 28, 35...

4: 4, 8, 12, 16, 20, 24, 28...

$$-\frac{3}{7} - \left(-\frac{4}{4}\right) = -\frac{12}{28} - \left(-\frac{7}{28}\right) = -\frac{12}{28} + \frac{7}{28} = -\frac{5}{28} = \frac{23}{21} = 1\frac{2}{21}$$

# EXERCISE

1.  $1\frac{1}{3} - \frac{2}{3}$

2.  $\frac{1}{6} ? \frac{2}{9}$

3.  $\frac{3}{4} - \frac{1}{3}$

# EXERCISE

1.  $1\frac{1}{3} - \frac{2}{3} = \frac{4}{3} - \frac{2}{3} = \frac{2}{3}$

2.  $\frac{1}{6} ? \frac{2}{9}$

3.  $\frac{3}{4} - \frac{1}{3}$

# EXERCISE

$$1. \quad 1\frac{1}{3} - \frac{2}{3} = \frac{4}{3} - \frac{2}{3} = \frac{2}{3}$$

$$2. \quad \frac{1}{6} ? \frac{2}{9} \rightarrow \frac{3}{18} ? \frac{4}{18} \rightarrow \frac{3}{18} < \frac{4}{18}$$

$$3. \quad \frac{3}{4} - \frac{1}{3}$$

# EXERCISE

$$1. \quad 1\frac{1}{3} - \frac{2}{3} = \frac{4}{3} - \frac{2}{3} = \frac{2}{3}$$

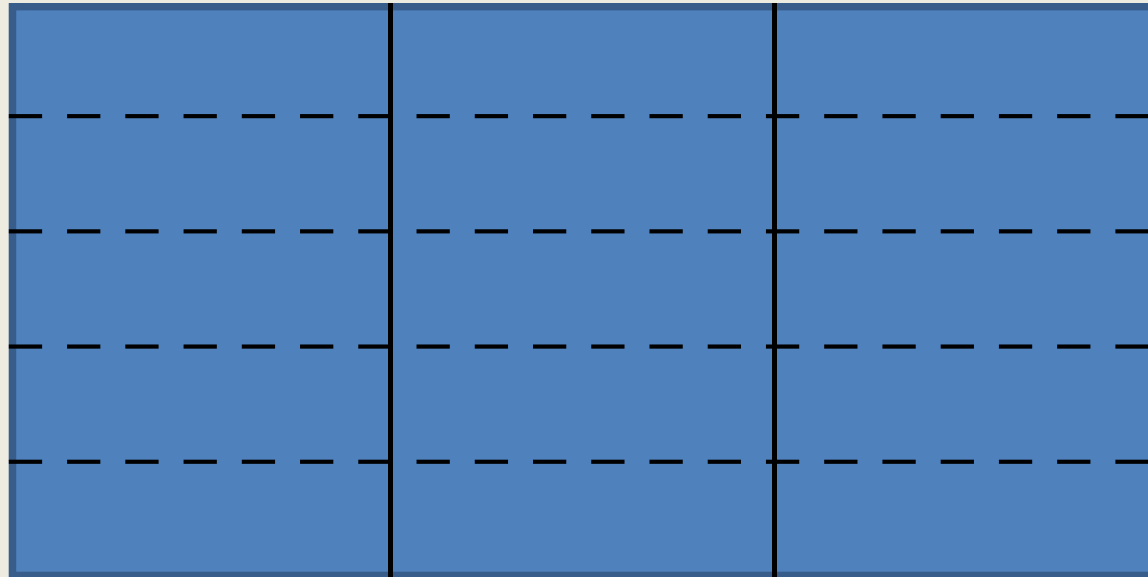
$$2. \quad \frac{1}{6} ? \frac{2}{9} \rightarrow \frac{3}{18} ? \frac{4}{18} \rightarrow \frac{3}{18} < \frac{4}{18}$$

$$3. \quad \frac{3}{4} - \frac{1}{3} = \frac{9}{12} - \frac{4}{12} = \frac{5}{12}$$

# Multiplying Fractions

- Numerator by numerator, denominator by denominator

$$\frac{2}{3} \times \frac{2}{5} = ?$$

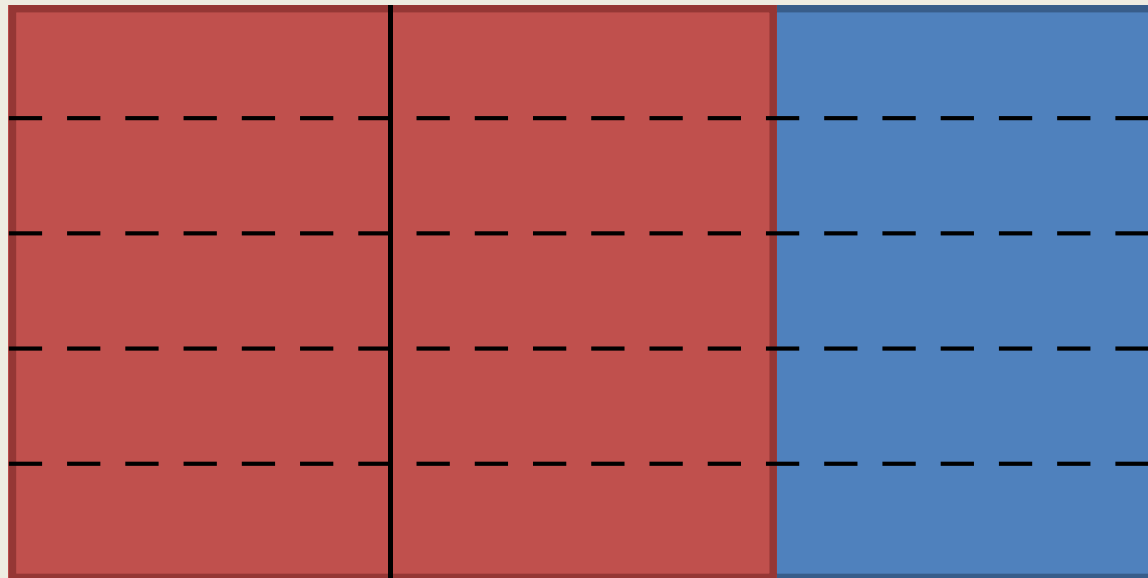




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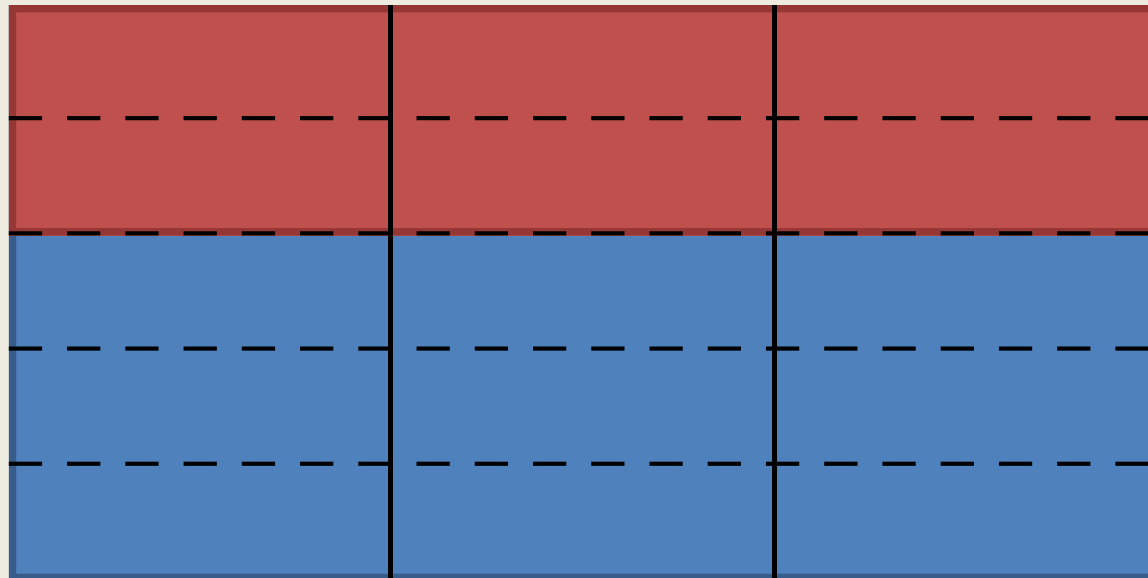
$$\frac{2}{3} \times \frac{2}{5} = ?$$



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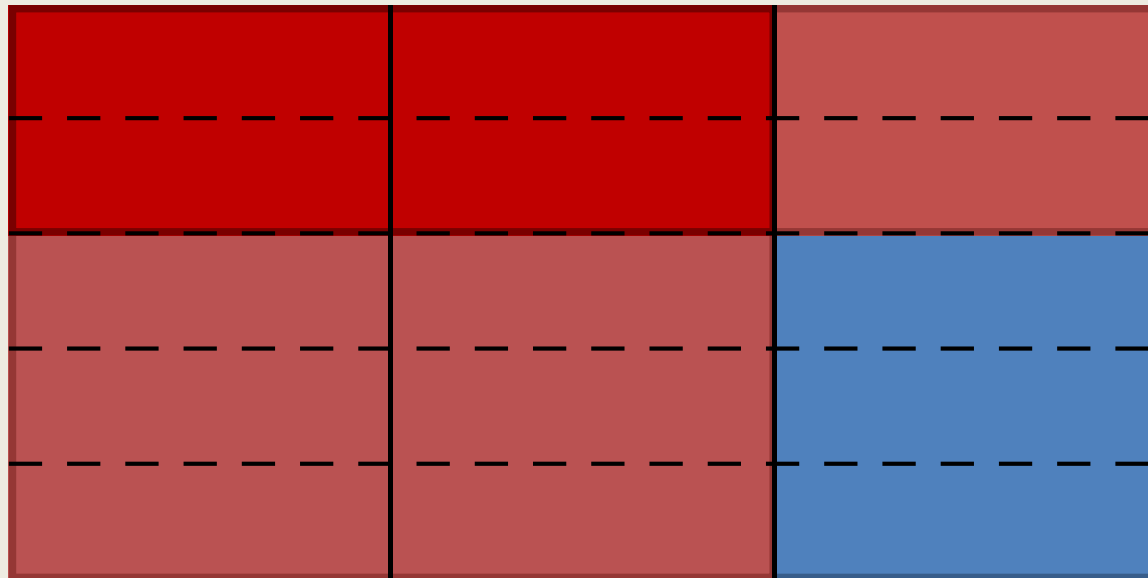
$$\frac{2}{3} \times \frac{2}{5} = ?$$



# Multiplying Fractions

- Numerator by numerator, denominator by denominator

$$\frac{2}{3} \times \frac{2}{5} = \frac{4}{15}$$



# Multiplying Fractions

- Numerator by numerator, denominator by denominator

$$\frac{2}{3} \times \frac{2}{5} = \frac{2 \times 2}{3 \times 5} = \frac{4}{15}$$

# EXERCISE

- Numerator by numerator, denominator by denominator

$$\frac{3}{4} \times \frac{4}{5} =$$

$$\frac{7}{8} \times \frac{5}{6} =$$

# EXERCISE

- Numerator by numerator, denominator by denominator

$$\frac{3}{4} \times \frac{4}{5} = \frac{12}{20}$$

$$\frac{7}{8} \times \frac{5}{6} =$$

# EXERCISE

- Numerator by numerator, denominator by denominator

$$\frac{3}{4} \times \frac{4}{5} = \frac{12}{20} = \frac{3}{5}$$

$$\frac{7}{8} \times \frac{5}{6} =$$

# EXERCISE

- Numerator by numerator, denominator by denominator

$$\frac{3}{\cancel{4}} \times \frac{\cancel{4}}{5} = \frac{12}{20} = \frac{3}{5}$$

or cross-cancel!

$$\frac{7}{8} \times \frac{5}{6} =$$



# EXERCISE

- Numerator by numerator, denominator by denominator

$$\frac{3}{4} \times \frac{4}{5} = \frac{12}{20} = \frac{3}{5}$$

$$\frac{7}{8} \times \frac{5}{6} = \frac{35}{48}$$

# Dividing Fractions

- Multiply by the reciprocal

$$\frac{4\frac{5}{5}}{\frac{2}{3}}$$

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$$\frac{\frac{4}{5}}{\frac{2}{3}} = \frac{4}{5} \div \frac{2}{3} = \frac{4}{5} \times \frac{3}{2}$$

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- Multiply by the reciprocal

$$\frac{\frac{4}{5}}{\frac{2}{3}} = \frac{4}{5} \div \frac{2}{3} = \frac{4}{5} \times \frac{3}{2} = \frac{12}{10}$$

# Dividing Fractions

- Multiply by the reciprocal

$$\frac{\frac{4}{5}}{\frac{2}{3}} = \frac{4}{5} \div \frac{2}{3} = \frac{4}{5} \times \frac{3}{2} = \frac{12}{10} = 1\frac{1}{5}$$

# EXERCISE

$$\frac{\frac{14}{100}}{3} = \frac{14}{200}$$

# EXERCISE

$$\frac{\frac{14}{100}}{\frac{3}{200}}$$

$$= \frac{14}{100} \times \frac{200}{3}$$

$$= \frac{2800}{300}$$

$$= \frac{28}{3} = 9\frac{1}{3}$$



# REMINDER: Don't do this!

$$\frac{3}{4} + \frac{5}{6} \neq \frac{3+5}{4+6}$$

$$1\frac{7}{12} \neq \frac{8}{12}$$

$$\frac{3}{4a} + \frac{4}{5b} \neq \frac{7}{4a+5b}$$

$$\frac{X}{Y} + \frac{X}{Z} \neq \frac{X}{Y+Z}$$

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# Where is she going with this?

- What is a proportion? ✓
- Fractions ✓
- Other ways represent and manipulate proportions
- Proportions and other mathematical constructs in epidemiology

# Other ways to represent proportions

## ■ Percentage

- *Just a fraction with a denominator equal to 100*

$$\frac{1}{4} = \frac{25}{100} = 25\%$$

$$\frac{3}{2} = \frac{150}{100} = 150\%$$

## ■ Decimal

- *Just a percentage rendered on the number line*

$$\frac{1}{4} = \frac{25}{100} = 0.25$$

$$\frac{3}{2} = \frac{150}{100} = 1.5$$

# Other ways to represent proportions

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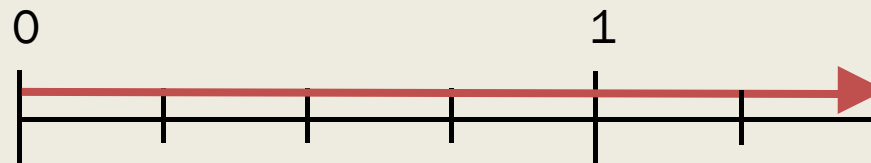
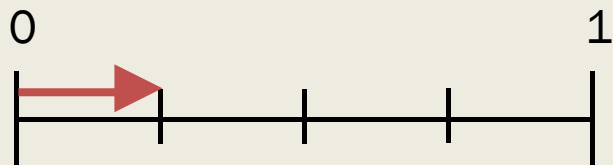
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# Percentage $\leftrightarrow$ Decimal

## ■ Percentage

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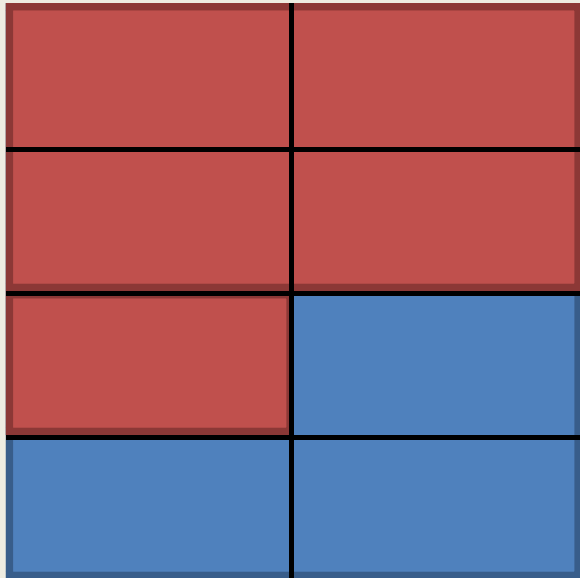
$$\frac{3}{2} = \frac{150}{100} = 1.5$$


$$0.25 \times 100 = 25\%$$

$$1.5 \times 100 = 150\%$$

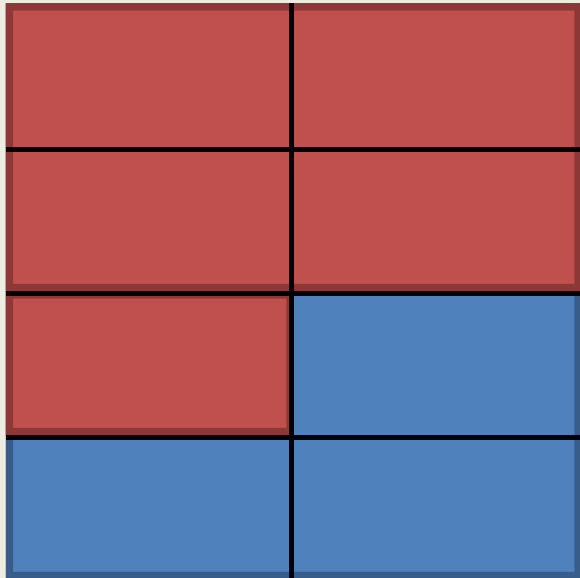
# EXERCISE

- What percent of the square is shaded red?



# EXERCISE

- What percent of the square is shaded red?



$$\frac{5}{8} = 0.625 = 62.5\%$$

# EXERCISE

1. 12 is what percent of 40?

2.  $\frac{0.012}{0.4}$

3. Compare:  $0.05$  ?  $0.011$

4. Compare:  $-0.078$  ?  $-0.035$

# EXERCISE

1. 12 is what percent of 40?

$$\frac{12}{40} = 0.3 = 30\%$$

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# EXERCISE

1. 12 is what percent of 40?

$$\frac{12}{40} = 0.3 = 30\%$$

2.  $\frac{0.012}{0.4} = \frac{12}{400} = \frac{3}{100} = 0.03$

3. Compare: 0.05 ? 0.011

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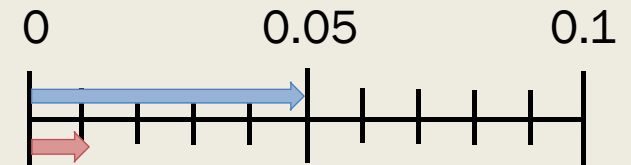
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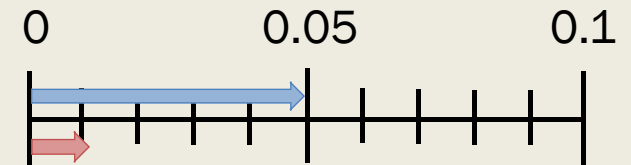
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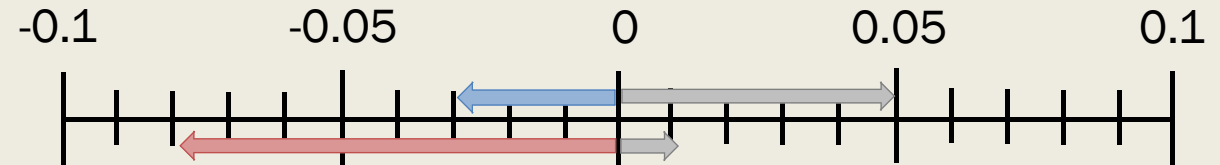
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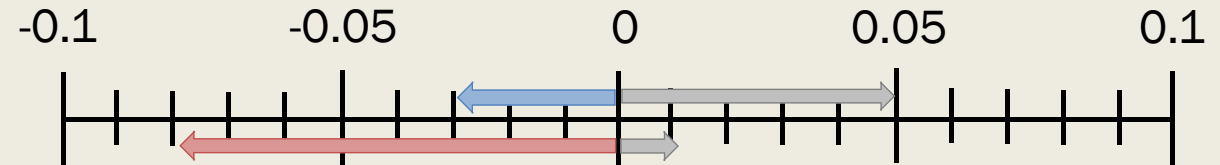
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# Mathematical constructs in epi

- Proportion: a part, share, or number considered in comparative relation to a whole
  - *Represented by fraction, percentage or decimal*
  - *These have units!*

*Example: Proportion of deaths due to heart disease, 23.5%*
- Rate: a measure, quantity, or frequency measured against some other quantity or measure
  - *Represented by fraction, percentage or decimal*
  - *Usually measuring something in relation to time*

*Example: mortality rate of heart disease, 193 deaths per 100,000 person-years*
- Ratio: the relationship of two comparable amounts
  - *Often represented in the form, number:number (e.g. 3:1)*
  - *BUT also represented by fraction, percentage or decimal*
  - *Unitless!*

*Example: Sex ratio at age 60 is 81 males:100 females*

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# Mathematical constructs in epi

- Proportion: a part, share, or number considered in comparative relation to a whole
  - *Represented by fraction, percentage or decimal*
  - *These have units!*

*Example: Proportion of deaths due to heart disease, 23.5%*
- Rate: a measure, quantity, or frequency measured against some other quantity or measure
  - *Represented by fraction, percentage or decimal*
  - *Usually measuring something in relation to time*

*Example: mortality rate of heart disease, 193 deaths per 100,000 person-years*
- Ratio: the relationship of two comparable amounts
  - *Often represented in the form, number:number (e.g. 3:1)*
  - *BUT also represented by fraction, percentage or decimal (e.g.  $3/1 = 3$ )*
  - *Unitless!*

*Example: Sex ratio at age 60 is 81 males:100 females*