BIOST 311: Intepreting coefficients after transformations

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1 Transformations of the predictor

Interpret β_0, β_1 in the following models:

1. $E[\text{FEV}|\left(\text{Age} - \overline{Age}\right)] = \beta_0 + \beta_1 \left(\text{Age} - \overline{Age}\right)$

   Answer:
   β_0: average FEV (l/sec) among subjects with average (in the dataset) age

$$\begin{aligned} E[\text{FEV}|\text{Age} = \overline{Age}] &= E[\text{FEV}|\left(\text{Age} - \overline{Age}\right) = \overline{Age} - \overline{Age}] \\ &= E[\text{FEV}|\left(\text{Age} - \overline{Age}\right) = 0] \\ &= \beta_0 + \beta_1(0) \\ &= \beta_0 \end{aligned}$$

   β_1: difference in average FEV (l/sec) comparing two groups that differ in age by one year

$$\begin{aligned} &E[\text{FEV}|\text{Age} = x+1] - E[\text{FEV}|\text{Age} = x] \\ &= E[\text{FEV}|\left(\text{Age} - \overline{Age}\right) = x + 1 - \overline{Age}] - E[\text{FEV}|\left(\text{Age} - \overline{Age}\right) = x - \overline{Age}] \\ &= \left[\beta_0 + \beta_1\left(x + 1 - \overline{Age}\right)\right] - \left[\beta_0 + \beta_1\left(x - \overline{Age}\right)\right] \\ &= \left[\beta_0 + \beta_1(x) + \beta_1(1) + \beta_1(-\overline{Age})\right] - \left[\beta_0 + \beta_1(x) + \beta_1(-\overline{Age})\right] \\ &= \beta_1 \end{aligned}$$

2. $E[\text{FEV}|\left(\frac{\text{Age}}{10}\right)] = \beta_0 + \beta_1\left(\frac{\text{Age}}{10}\right)$

Answer:
β_0: average FEV (l/sec) among newborns

$$\begin{aligned}
E[\text{FEV}|\text{Age}=0] &= E[\text{FEV}|\left(\frac{\text{Age}}{10}\right) = \left(\frac{0}{10}\right)] \\
&= E[\text{FEV}|\left(\frac{\text{Age}}{10}\right) = 0] \\
&= \beta_0 + \beta_1(0) \\
&= \beta_0
\end{aligned}$$

β_1: difference in average FEV (l/sec) comparing two groups that differ in age by one decade (10 years)

$$\begin{aligned}
&E[\text{FEV}|\text{Age}=x+10] - E[\text{FEV}|\text{Age}=x] \\
&= E[\text{FEV}|\left(\frac{\text{Age}}{10}\right) = \left(\frac{x+10}{10}\right)] - E[\text{FEV}|\left(\frac{\text{Age}}{10}\right) = \left(\frac{x}{10}\right)] \\
&= \left[\beta_0 + \beta_1\left(\frac{x+10}{10}\right)\right] - \left[\beta_0 + \beta_1\left(\frac{x}{10}\right)\right] \\
&= \left[\beta_0 + \beta_1\left(\frac{x}{10}\right) + \beta_1\left(\frac{10}{10}\right)\right] - \left[\beta_0 + \beta_1\left(\frac{x}{10}\right)\right] \\
&= \beta_1
\end{aligned}$$

3. $E[\text{FEV}|\left(\frac{\text{Age}-3}{10}\right)] = \beta_0 + \beta_1 \left(\frac{\text{Age}-3}{10}\right)$

Answer: β_0: average FEV (l/sec) among 3 year olds

$$\begin{aligned}
E[\text{FEV}|\text{Age}=3] &= E[\text{FEV}|\left(\frac{\text{Age}-3}{10}\right) = \left(\frac{3-3}{10}\right)] \\
&= E[\text{FEV}|\left(\frac{\text{Age}-3}{10}\right) = 0] \\
&= \beta_0 + \beta_1(0) \\
&= \beta_0
\end{aligned}$$

β_1: difference in average FEV (l/sec) comparing two groups that differ in age by one decade (10 years)

$$\begin{aligned}
&E[\text{FEV}|\text{Age}=x+10] - E[\text{FEV}|\text{Age}=x] \\
&= E[\text{FEV}|\left(\frac{\text{Age}-3}{10}\right) = \left(\frac{x+10-3}{10}\right)] - E[\text{FEV}|\left(\frac{\text{Age}-3}{10}\right) = \left(\frac{x-3}{10}\right)] \\
&= \left[\beta_0 + \beta_1\left(\frac{x+10-3}{10}\right)\right] - \left[\beta_0 + \beta_1\left(\frac{x-3}{10}\right)\right] \\
&= \left[\beta_0 + \beta_1\left(\frac{x-3}{10}\right) + \beta_1\left(\frac{10}{10}\right)\right] - \left[\beta_0 + \beta_1\left(\frac{x-3}{10}\right)\right] \\
&= \beta_1
\end{aligned}$$

2 Transformations of the outcome

Interpret β_0, β_1 in the following models:

1. $E[(\text{FEV} \times 60)|\text{Age}] = \beta_0 + \beta_1\text{Age}$

   Answer:
   By multiplying FEV by 60, we've changed the units from l/sec to l/min:

$$\begin{aligned}
\text{FEV}\frac{\text{liters}}{\text{sec}} &= \frac{\text{FEV liters}}{\text{sec}} \times \frac{60 \text{ sec}}{1 \text{ min}} \\
&= (\text{FEV} \times 60)\frac{\text{liters}}{\text{min}} \\
\text{Example: } 30\frac{\text{liters}}{\text{sec}} &= 180\frac{\text{liters}}{\text{min}}
\end{aligned}$$

   Then, the rest of our interpretation is the same as without the transformation, except we have new units...

   β_0: average FEV (l/min) among newborns

$$\begin{aligned}
E[(\text{FEV} \times 60)|\text{Age} = 0] &= \beta_0 + \beta_1(0) \\
&= \beta_0
\end{aligned}$$

   β_1: difference in average FEV (l/min) comparing two groups that differ in age by one year

$$\begin{aligned}
&E[(\text{FEV} \times 60)|\text{Age} = x + 1] - E(\text{FEV} \times 60)|\text{Age} = x] \\
&= [\beta_0 + \beta_1\,(x + 1)] - [\beta_0 + \beta_1\,(x)] \\
&= [\beta_0 + \beta_1(x) + \beta_1(1)] - [\beta_0 + \beta_1(x)] \\
&= \beta_1
\end{aligned}$$

2. $E[\left(\frac{\text{FEV}}{3.78541}\right)|\text{Age}] = \beta_0 + \beta_1\text{Age}$ (Hint: 1 gallon = 3.78541 liters)

Answer:
By dividing FEV by 3.78541, we've changed the units from l/sec to gal/sec:

$$
\begin{aligned}
\text{FEV}\frac{\text{liters}}{\text{sec}} &= \frac{\text{FEV liters}}{\text{sec}} \times \frac{1 \text{ gallon}}{3.78541 \text{ liters}} \\
&= (\text{FEV} \div 3.78541)\frac{\text{gallons}}{\text{sec}} \\
\text{Example: } 30\frac{\text{liters}}{\text{sec}} &= 7.925\frac{\text{gallons}}{\text{sec}}
\end{aligned}
$$

Then, the rest of our interpretation is the same as without the transformation, except we have new units...

β_0: average FEV (gal/sec) among newborns

$$
\begin{aligned}
E[(\text{FEV} \times 60)|\text{Age} = 0] &= \beta_0 + \beta_1(0) \\
&= \beta_0
\end{aligned}
$$

β_1: difference in average FEV (gal/sec) comparing two groups that differ in age by one year

$$
\begin{aligned}
&E[(\text{FEV} \div 3.78541)|\text{Age} = x + 1] - E(\text{FEV} \div 3.78541)|\text{Age} = x] \\
&= [\beta_0 + \beta_1\,(x + 1)] - [\beta_0 + \beta_1\,(x)] \\
&= [\beta_0 + \beta_1(x) + \beta_1(1)] - [\beta_0 + \beta_1(x)] \\
&= \beta_1
\end{aligned}
$$