## Using the uwIntroStats Package

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### Introduction

### Preparing uwIntroStats

Before we can dive in and run any analyses, we first need to install the package. This is done via

```
install.packages("uwIntroStats")
```

Regardless of the graphical user interface (GUI) that you are using, R will prompt you to select a CRAN mirror. It is essentially asking you where you want to download the package files from. Select the mirror closest to you - for us at the University of Washington it is WA(1) or the Fred Hutchinson Cancer Research Center (FHCRC) - and the package will download and say that it has installed. Now each time we open a new R session (whether that is at the command line, a new RGui window, or a new RStudio window) we need to load the package for use.

Five other packages provide a few key functions that the uwIntroStats package uses or adds functionality to. We must install these packages like we did above if we have not installed them previously, and then load uwIntroStats. While the packages do not need to be loaded every time (in fact, some are only used for specific functions) it is good practice to load them for the R session where you need to use uwIntroStats. This makes sure that we can use their other functions while doing analyses.

```
library(Exact)
library(geepack)
library(plyr)
library(sandwich)
library(survival)
library(uwIntroStats)

##
## Attaching package: 'uwIntroStats'
##
## The following object is masked from 'package:base':
##
```

Don't worry about the warning message for now; that will be covered in section 3.2. Last, we load the data, mri that we will be using throughout this document. Information about the dataset can be found at mri.pdf. Since the data is part of the package, we can load it via

```
data(mri)
```

The uwIntroStats package should be used for descriptive statistics, basic plotting (like scatterplots and boxplots), and regression analyses. The following sections will go through examples of these tasks, in addition to pointing out how our package differs from base R and other existing packages. We will assume familiarity with basic data manipulation and statistical tasks (for a refresher, see "An Introduction to R".

### **Descriptive Statistics**

tabulate

##

Descriptive statistics are an important part of any analysis. Often, they are just as important for the statistician or data analysist as they are for the scientific collaborators on a project. For example, descriptive statistics can help identify errors in the data, like observations that are particularly unusual (usually far out

of the expected or observed range). They can also help to identify patterns of missing data, examine the study population, and identify aspects of the data that might lead to statistical issues. Descriptive statistics can also unearth relationships that had not previously been considered for analysis, thereby generating new studies. More information on this, and an outline of an approach to analysing a dataset, "Organizing Your Approach to a Data Analysis", was written by Scott Emerson for this purpose.

### The basics: descrip()

The uwIntroStats package was designed with descriptive statistics in mind. The basic function for descriptive statistics is descrip(). This function takes in an arbitrary number of variables, and by default calculates the number of observations, the number of missing values, the mean, standard deviation, minimum value, maximum value, and the 25th, 50th, and 75th percentiles of each variable. For instance, if we wanted a quick glance at the mri data, we could type

#### descrip(mri)

##		N I	Msng	Mean	Std Dev	Min	25%	Mdn
##	ptid:	735	0	368.0	212.3	1.000	184.5	368.0
##	mridate:	735	0	76423	31896	10192	66642	80992
##	age:	735	0	74.57	5.451	65.00	71.00	74.00
##	male:	735	0	0.4980	0.5003	0.0000	0.0000	0.0000
##	race:	735	0	1.318	0.6659	1.000	1.000	1.000
##	weight:	735	0	159.9	30.74	74.00	138.5	158.0
##	height:	735	0	165.8	9.710	139.0	158.0	165.9
##	packyrs:	735	1	19.60	27.11	0.0000	0.0000	6.500
##	yrsquit:	735	0	9.661	14.10	0.0000	0.0000	0.0000
##	alcoh:	735	0	2.109	4.852	0.0000	0.0000	0.01920
##	physact:	735	0	1.922	2.052	0.0000	0.5538	1.312
##	chf:	735	0	0.05578	0.2297	0.0000	0.0000	0.0000
##	chd:	735	0	0.3347	0.6862	0.0000	0.0000	0.0000
##	stroke:	735	0	0.2367	0.6207	0.0000	0.0000	0.0000
##	diabetes:	735	0	0.1075	0.3099	0.0000	0.0000	0.0000
##	genhlth:	735	0	2.588	0.9382	1.000	2.000	3.000
##	ldl:	735	10	125.8	33.60	11.00	102.0	125.0
##	alb:	735	2	3.994	0.2690	3.200	3.800	4.000
##	crt:	735	2	1.064	0.3030	0.5000	0.9000	1.000
##	plt:	735	7	246.0	65.80	92.00	201.8	239.0
##	sbp:	735	0	131.1	19.66	78.00	118.0	130.0
##	aai:	735	9	1.103	0.1828	0.3171	1.027	1.112
##	fev:	735	10	2.207	0.6875	0.4083	1.745	2.158
##	dsst:	735	12	41.06	12.71	0.0000	32.00	40.00
##	atrophy:	735	0	35.98	12.92	5.000	27.00	35.00
##	whgrd:	735	1	2.007	1.410	0.0000	1.000	2.000
##	numinf:	735	0	0.6109	0.9895	0.0000	0.0000	0.0000
##	volinf:	735	1	3.223	17.36	0.0000	0.0000	0.0000
##	obstime:	735	0	1804	392.3	68.00	1837	1879
##	death:	735	0	0.1810	0.3852	0.0000	0.0000	0.0000
##		75%		Max				
##	ptid:	551.		735.0				
##	mridate:	9139		1.232e+05				
##	age:	78.0		99.00				
##	male:	1.00		1.000				
##	race:	1.00	U	4.000				

```
##
     weight:
                   179.0
                                264.0
##
     height:
                   173.2
                                190.5
##
    packyrs:
                   33.75
                                240.0
##
    yrsquit:
                   18.50
                                56.00
##
      alcoh:
                   1.144
                                35.00
##
    physact:
                   2.513
                                13.81
##
         chf:
                   0.0000
                                1.000
##
         chd:
                   0.0000
                                2.000
##
     stroke:
                   0.0000
                                2.000
##
   diabetes:
                   0.0000
                                1.000
##
    genhlth:
                   3.000
                                5.000
                   147.0
                                247.0
##
         ldl:
##
         alb:
                   4.200
                                5.000
                   1.200
##
         crt:
                                4.000
##
         plt:
                   285.0
                                539.0
##
         sbp:
                   142.0
                                210.0
##
                   1.207
                                1.728
         aai:
##
         fev:
                   2.649
                                4.471
##
                                82.00
        dsst:
                   50.00
##
    atrophy:
                   44.00
                                84.00
##
      whgrd:
                   3.000
                                9.000
##
     numinf:
                   1.000
                                5.000
##
     volinf:
                  0.09420
                                197.0
    obstime:
##
                    2044
                                2159
##
      death:
                   0.0000
                                1.000
```

This call gives us a quick look at the distribution of each variable in the sample, and can be easily exported to a word processing software. This is important, because displaying raw R output in a publication is not usually ideal, and taking the time to format the output is a pain.

Notice that there are a lot of variables measured in these data, and that some interesting phenomena are measured by multiple variables. For instance, smoking is measured by packyrs and yrsquit - if someone has zero pack-years (that is, they have never smoked) then they will also have zero years quit. Similarly, a current smoker of any amount of packs per year will have zero years quit. We can also look at the variables measured through blood samples, like LDL cholesterol (ldl) and get an idea of the range in our sample population. Last, in the variables like male which consist of only two values (sex was measured as male or female in this dataset), the mean tells us the proportion of our sample who was male (or for other binary variables, it will tell us the proportion which is coded as 1). The descrip() function works similarly to summary() in base R, but gives more information and returns it in a format that, as we described above, is easier to export from R.

Another powerful feature of descrip() is its ability to present stratified summaries, or summaries on a subset of the data. For instance, let's calculate descriptive statistics on the age variable within males and females:

#### descrip(mri\$age, strata = mri\$male)

```
##
                        N
                               Msng
                                     Mean
                                                 Std Dev
                                                              Min
                                                                         25%
## mri$age:
                           735
                                    0
                                        74.57
                                                   5.451
                                                               65.00
                                                                          71.00
              All
## mri$age:
                Str
                      0
                           369
                                    0
                                        74.41
                                                   5.258
                                                               65.00
                                                                          71.00
  mri$age:
                      1
                                        74.73
                                                               66.00
                                                                          71.00
##
                Str
                           366
                                                   5.642
##
                         Mdn
                                     75%
                                                Max
## mri$age:
              All
                           74.00
                                      78.00
                                                 99.00
## mri$age:
                Str
                      0
                           73.00
                                      78.00
                                                 91.00
                                      78.00
                                                 99.00
## mri$age:
                Str
                      1
                           74.00
```

```
## Call the summary function, to compare
summary(mri$age)
```

```
## Min. 1st Qu. Median Mean 3rd Qu. Max.
## 65.00 71.00 74.00 74.57 78.00 99.00
```

Here the row for Str 0 corresponds to females (since they are coded as 0 for male), and males are Str 1. If we further wanted to restrict to only those people who were over 75 years old, we could use the subset argument:

```
descrip(mri$age, strata = mri$male, subset = mri$age > 75)
```

##			N	Msng	Mean	Std Dev	Min	25%
## mri\$age:	All		260	0	80.59	4.111	76.00	77.75
## mri\$age:	Str	0	127	0	80.44	3.572	76.00	78.00
## mri\$age:	Str	1	133	0	80.73	4.576	76.00	77.00
##			Mdn	•	75%	Max		
## mri\$age:	All		79.0	00	83.00	99.00		
## mri\$age:	Str	0	79.0	00	83.00	91.00		
## mri\$age:	Str	1	79.0	00	83.00	99.00		

However, the descrip() function can only give us a fixed number of summaries (mean, count, etc). Oftentimes we need only a subset of these, or perhaps different summary measures. For this, we turn to more powerful functions.

### tableStat() and tabulate(): Flexible Functions

The main draw of tableStat() and tabulate() is their ability to supply only a subset of the summary measures from descrip(). These functions focus on display and flexibility. The user supplies the format for the summary statistics to be displayed in, which makes exporting results from a call to tablestat() or tabulate() even easier than it was for descrip(). Of the two, tableStat() is the base, and tabulate() adds some additional formatting and options. For example, let's say that we want the same summary statistics as we would get from descrip(), but we want to control the printout. For this, we use the stat argument to tableStat(). The options are presented in below.

Name	Summary Measure	Name	Summary Measure
"count"	Number of observations on a variable	"sd"	Standard Deviation
"missing"	Number of missing observations	"variance"	Variance
"mean"	Arithmetic mean	"min"	Minimum value
"geometric mean"	Geometric mean	"max"	Maximum value
"median"	50th percentile	"quantiles"	Display quantiles
"probabilities"	Display percentiles	"mn(sd)"	Display Mean (SD) format
"range"	Maximum value - minimum value	"iqr"	Interquartile range (75th - 25th percentiles)
"all"	Display all summary measures	"row%"	Display row percentages
"col%"	Display column percentages	"tot%"	Display percentage of total

The following gets us close to results we could display in a table for a puplication. We want to have

$$N: n; Mean (SD): mn (sd); Range: max - min,$$

which we get via:

```
tableStat(mri$age, stat = "N: @count@; Mean (SD): @mean@ (@sd@); Range: @max@ - @min@")
```

```
## Tabled descriptive statistics by strata
## Call:
## tableStat(variable = mri$age, stat = "N: @count@; Mean (SD): @mean@ (@sd@); Range: @max@ - @mi:
## - NaN denotes strata with no observations
## - NA arises from missing or censored data
##
## Format: N: Cnt; Mean (SD): Mean (SD); Range: Max - Min
##
## ...ALL
## N: 735.0; Mean (SD): 74.57 (5.451); Range: 99.00 - 65.00
```

Note that all of the values enclosed in @ symbols are from Table ?? and are run by tableStat(), while the other values in the string are used for formatting. This function can also take stratification variables; if we want to see the above stratified by sex, we use

```
tableStat(mri$age, strata = mri$male, stat = "N: @count@; Mean (SD): @mean@ (@sd@); Range: @min@ - @max

## Tabled descriptive statistics by strata
## Call:
## tableStat(variable = mri$age, strata = mri$male, stat = "N: @count@; Mean (SD): @mean@ (@sd@);
## - NaN denotes strata with no observations
## - NA arises from missing or censored data
##
## Format: N: Cnt; Mean (SD): Mean (SD); Range: Min - Max
##
## strata.0
## N: 369.0; Mean (SD): 74.41 (5.258); Range: 65.00 - 91.00
```

strata.ALL

Now if we wanted to see this in table form, in perhaps a nicer layout, we could use the tabulate() function. When we loaded the uwIntroStats package at the beginning of this document, recall that we got a warning message from R saying

The following object is masked from 'package:base':

## N: 366.0; Mean (SD): 74.73 (5.642); Range: 66.00 - 99.00

## N: 735.0; Mean (SD): 74.57 (5.451); Range: 65.00 - 99.00

tabulate

This means that our function tabulate() overwrites the function called tabulatte() in the base R package, which takes a numeric vector and counts the number of times each integer occurs in it. Our function does a similar task, but on combinations of strata. The syntax is very similar to the syntax for tableStat(), but rather than explicitly telling tabulate() the strata, it creates the tables with each dimension corresponding to a variable in the order they are entered. For instance, if we want a table of age (in the rows) and male (in the columns), we would write

```
tabulate(mri$age, mri$male)
```

```
##
## Call:
   tabulate(mri$age, mri$male)
                mri$male.0 mri$male.1 mri$male.ALL
##
## mri$age.65
                  2
                               0
                                           2
                  2
                               1
                                           3
## mri$age.66
## mri$age.67
                                          33
                 18
                              15
## mri$age.68
                 20
                              21
                                          41
## mri$age.69
                 19
                              19
                                          38
## mri$age.70
                 30
                              30
                                          60
## mri$age.71
                 37
                              39
                                          76
                              28
  mri$age.72
                 31
                                          59
## mri$age.73
                 27
                              28
                                          55
## mri$age.74
                 29
                              26
                                          55
## mri$age.75
                 27
                              26
                                          53
## mri$age.76
                              17
                                          28
                 11
                                          37
## mri$age.77
                 18
                              19
## mri$age.78
                              19
                                          37
                 18
## mri$age.79
                                          32
                 19
                              13
## mri$age.80
                  8
                              15
                                          23
## mri$age.81
                 10
                              10
                                          20
## mri$age.82
                               5
                 10
                                          15
                               2
## mri$age.83
                 10
                                          12
## mri$age.84
                               6
                  5
                                          11
                               6
## mri$age.85
                  3
                                           9
## mri$age.86
                  5
                               5
                                          10
## mri$age.87
                  4
                               4
                                           8
                  2
                                           3
## mri$age.88
                               1
                  2
                                           5
## mri$age.89
                               3
## mri$age.90
                  0
                               4
                                           4
## mri$age.91
                  2
                               0
                                           2
## mri$age.92
                  0
                               1
                                           1
  mri$age.94
                  0
                               1
                                           1
  mri$age.95
                  0
                               1
                                           1
   mri$age.99
                  0
                               1
                                           1
                             366
                                         735
## mri$age.ALL 369
##
                Point Estimate Test Statistic df 95% CI p-value Warnings
## Chi-squared
                                 26.244
                                                  30
                                                             0.66262
```

By default, tabulate() gives us the count in each combination of the strata, and gives us the overall  $\chi^2$  test statistic, degrees of freedom, and p-value. We can add in other arguments as well. Let's say that we are interested in an odds ratio or a risk ratio - tabulate() can supply these, if we set dispRatios = TRUE. This is where our use of the Exact package comes in, because this package contains some very useful functions for calculating these ratios. However, we supply both the odds ratio and the risk ratio, even though in some cases only one is appropriate. Thus it is up to the user to know which they can use. To display these ratios for male versus chf (an indicator variable of whether the patient had been diagnosed with congestive heart failure prior to MRI), we type

```
tabulate(mri$male, mri$chf, dispRatios = TRUE)
##
## Call:
```

## tabulate(mri\$male, mri\$chf, dispRatios = TRUE)

```
##
                 mri$chf.0 mri$chf.1 mri$chf.ALL
## mri$male.0
                 354
                            15
                                      369
## mri$male.1
                 340
                            26
                                      366
                                      735
## mri$male.ALL 694
                            41
##
               Point Estimate Test Statistic df 95% CI
                                                                    p-value
                               3.2214
                                                                     0.072679
## Chi-squared
## Odds Ratio
                                                   [0.9396, 3.4663]
                                                  [1.0136, 1.6531]
## Risk Ratio
               1.2944
##
               Warnings
## Chi-squared
## Odds Ratio
## Risk Ratio
```

Now we are given a point estimate and a 95% confidence interval for both the odds and risk ratio, in addition to the  $\chi^2$  estimate.

Last, similar to tableStat(), we can identify different statistics to display. The syntax is exactly the same, and the same statistics from Table 1 are valid in tabulate().

### **Plotting**

Plots and descriptive statistics together lead to a much better understanding of the data. In our package, we have implemented two types of plots: boxplots and scatterplots. In doing so, we have attempted to address some concerns with both types of plots.

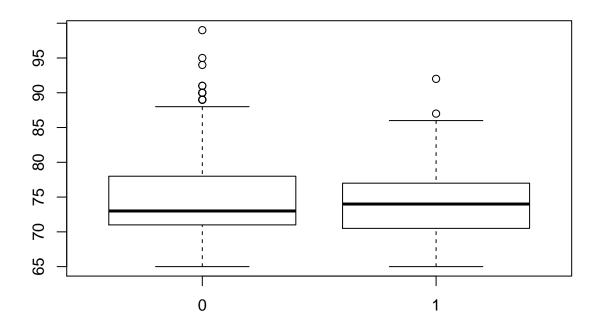
### Boxplots

Boxplots reduce the data down to four summary measures: minimum, maximum, median, and the interquartile range (25th to 75th quantile). They also show "outliers" in the data. We put this in quotes because the definition of "outlier" depends on the function you call, and the software you are running. Even in the base R version, boxplot(), the boxplots can be constructed so that no outliers are shown.

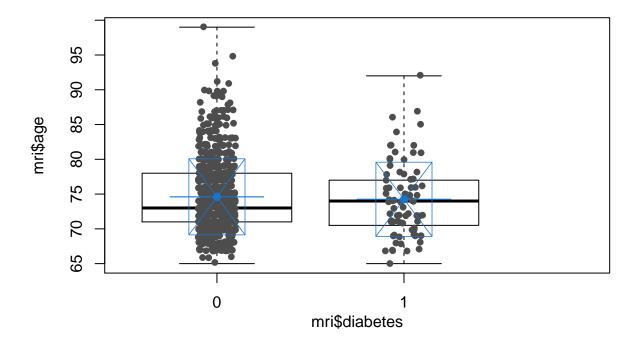
Much of the criticism of boxplots lies both in this determination of outliers and in the dramatic reduction of the data. In our version, <code>bplot()</code>, we have added jittered data onto the boxplots (jittering allows us to see the data, but randomly adds noise so that we can see individual points better), and we have added the mean and standard deviation onto the plot. We can also produce stratified boxplots. For our function, the variable on the y-axis comes first, followed by the variable on the x-axis. This convention, while at odds with base R, is an attempt to make our functions match up with the regression scheme of response followed by predictors.

For our first example, let's create a boxplot showing the age distribution of the diabetes variable:

```
## Base R boxplot
boxplot(mri$age ~ mri$diabetes)
```



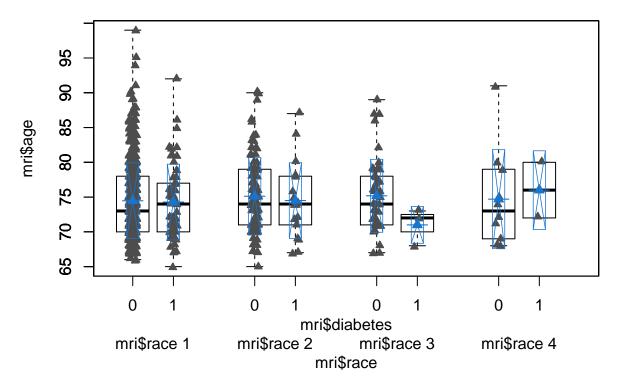
## Our version of boxplot
bplot(mri\$age, mri\$diabetes)



In comparing the two, see that our plot displays the mean line and  $\pm$  standard deviation box in blue and overlays the jittered data on top of the boxplots produced by the base R function. Now if we also wanted to stratify by race, we write

bplot(mri\$age, mri\$diabetes, strata = mri\$race)

### Stratified Boxplot by: mri\$race

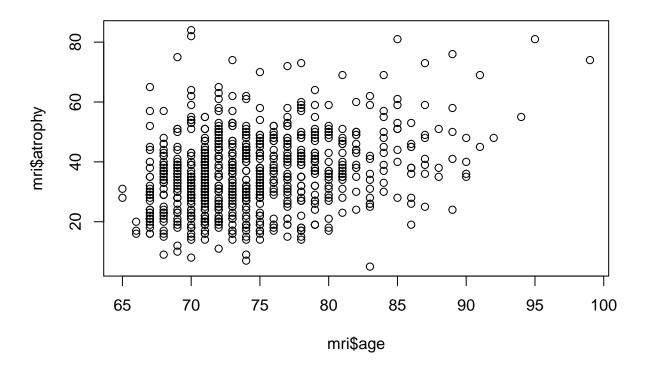


This boxplot breaks down diabetes by race, again showing the mean and standard deviation bars. Note that like normal plots, we can define the axis labels and title by hand.

### **Scatterplots**

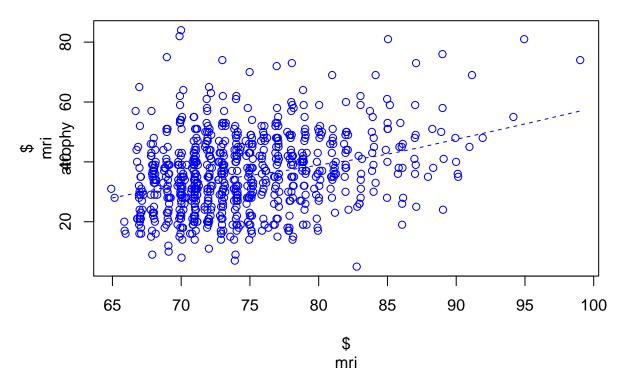
Scatterplots are a very common way to view data. They plot all of the data in one window, which allows the user to get a good overall view. However, many times the data is clumped together in some spots, and it is hard to see all of the points. Thus, similar to our approach in bplot(), our scatter() function jitters the data slightly by default. To illustrate this difference, we consider the distribution of cerebral atrophy by age group. In the base R scatterplot,

plot(mri\$age, mri\$atrophy)



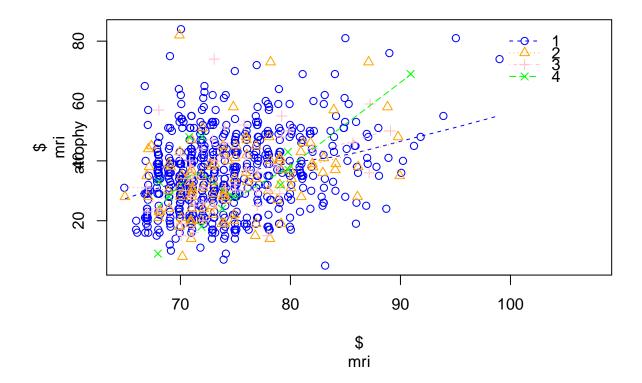
we see that a lot of the values sit on top of each other. In our version (which again uses the Y followed by X convention),

scatter(mri\$atrophy, mri\$age)



the jittered data allows us to see where all of the points lie much better. Also notice that scatter() displayes a loess curve by default. We can also display a line of best fit calculated by least squares by adding plotLSfit = TRUE. Last, we can stratify, which is particularly useful if we want to see differences across a third variable, and scatter() will automatically color the data appropriately:

scatter(mri\$atrophy, mri\$age, strata = mri\$race)



### Transformations of a Variable

In many cases (especially in regression, which we will cover in section 7), it is more useful to model a transformation of a variable than the raw data itself. We present three functions to easily transform a variable. In each case, the function returns a list of: a matrix with the new transformed variable, the type of transformation performed, a vector describing how the variable was created, the name of the created variable, and the original data.

### **Dummy Variables**

Dummy variables are a convenient way of creating indicator variables against some reference. For example, in the mri data, if we wanted to create a dummy variable for race, this would create 3 indicator variables (since there are four levels of race). By default, our dummy() function creates these indicators using the smallest value as the reference.

```
dummy.race <- dummy(mri$race)
head(dummy.race, n = 10)</pre>
```

```
##
          2 vs 1 3 vs 1 4 vs 1
##
     [1,]
     [2,]
                         0
                                 0
##
                1
##
     [3,]
                1
                         0
                                 0
                0
                         0
                                 0
     [4,]
##
```

```
##
     [5,]
                                      0
##
     [6,]
                   0
                            0
                                      1
##
     [7,]
                   0
                            0
                                      0
                                      0
     [8,]
                   1
                            0
##
##
     [9,]
                   0
                            0
                                      0
##
   [10,]
                   0
                            0
                                      0
```

Thus the set of dummy variables for race created above has: an indicator of whether the patient is black or white, an indicator of whether the patient is Asian or white, and an indicator of whether the patient is listed as "other" or white. These variables would allow us, in a regression, to compare each of the races against the white reference group.

### **Linear Splines**

Linear splines create a piecewise linear fit between user specified "knot" points; these are the points in the variable that we allow the slope of the line to change. There are two major ways to parameterize linear splines, and we have implemented both in this package. The first is based on the absolute slope between knots - that is, if we had a variable (like ldl in the mri dataset) that took on values between 11 and 247 and we placed knots at 70 and 150, we would see three columns: the first would show the minimum of 70 and the value of ldl; the second would show the minimum of ldl - 70 and 80; and the last column would show the remaining part of ldl. For example, if ldl = 71 for one patient, we would see 70 1 0. If ldl = 247, we would see 70 80 97.

The second parameterization is denoted by us as "change". This uses the change between knots to specify the slopes. Under this parameterization, we would see, for a value of 115, 115–45–0, which corresponds to: 115 units between the minimum and the value, 45 units between knot 1 (at 70) and the value, and zero units between knot 2 (at 150) and the value. We do not show negative units.

These two parameterizations can be accessed via

```
lspline.ldl <- lspline(mri$ldl, knots = c(70, 150))
head(lspline.ldl, n = 10)</pre>
```

```
##
      $ mri ldl
##
     70
          65
                0
##
     70
          14
                0
##
     70
          45
                0
##
     61
           0
                0
                0
##
     70
          78
##
     70
          80
               13
##
     70
          31
                0
##
     70
          46
                0
##
     70
          54
                0
     70
```

```
lsplineD.ldl <- lsplineD(mri$ldl, knots = c(70, 150))
head(lsplineD.ldl, n = 10)</pre>
```

```
## x:min x: 70 x:150
## 135 65 0
## 84 14 0
## 115 45 0
```

```
##
         61
                  0
                         0
##
       148
                78
                         0
##
       163
                93
                        13
                         0
##
       101
                31
##
       116
                46
                         0
                         0
##
       124
                54
                         0
##
       110
                40
```

### **Polynomials**

Last we come to polynomials. This function creates a polynomial of the specified degree simply by multiplying the variable by itself the correct number of times. If we wanted to create a parabola in age, we type

```
age.parabola <- polynomial(mri$age, degree = 2)
head(age.parabola, n = 10)</pre>
```

```
##
         Linear(ctr) Square(ctr)
##
    [1,]
          -2.5659864
                         6.5842862
##
    [2,]
           6.4340136
                        41.3965311
##
    [3,]
           15.4340136 238.2087760
##
    [4,]
           -2.5659864
                         6.5842862
##
    [5,]
           -4.5659864
                        20.8482318
##
    [6,]
           -2.5659864
                         6.5842862
    [7,]
           0.4340136
##
                         0.1883678
##
    [8,]
           0.4340136
                         0.1883678
##
    [9,]
           -7.5659864
                        57.2441501
  [10,]
           -4.5659864
                        20.8482318
```

The "(ctr)" after each term tells us that polynomial() has automatically centered each term, and by default it centers at the mean of the variable.

### One and Two Sample Functions

Many of our analyses boil down to one-sample or two-sample problems; What is the mean time to death? What is the median home price in Seattle? What is the difference in mean time to death between control and the treatment group? The list goes on. There are many methods of analyzing one-sample relationships, and in our package we have implemented three.

#### Correlation

While correlation is not an excellent source of inference, since it is inherently a feature of the dataset collected, sometimes it is important to know when variables are correlated. In the base R package, there are a few functions to calculate correlation: cor(), var(), cov(), and cov2cor(). Our function, correlate(), computes the correlation matrix between an arbitrary number of variables, and optionally also does this within a stratification variable. This is the real advantage of correlate(), since in base R we would have to manually subset the data into each stratum. If we want to examine the correlation between sex and diabetes, (and then stratify by race), we get

```
## Base R - returns only one number!
cor(mri$male, mri$diabetes)
## [1] 0.1200211
## uwIntroStats version
correlate(mri$male, mri$diabetes)
## Tabled correlation statistics by strata
##
        correlate(mri$male, mri$diabetes)
##
       Method: Pearson
       Data : Pairwise Complete
##
               - NaN denotes strata with no observations
##
              - NA arises from missing data
##
## #### ALL DATA
##
      ## Estimated Correlation Coefficients
##
                     mri$male: mri$diabetes:
##
      mri$male: 1.0000
                                0.1200
## mri$diabetes: 0.1200
                                 1.0000
## Stratify on race
correlate(mri$male, mri$diabetes, strata = mri$race)
## Tabled correlation statistics by strata
## Call:
##
        correlate(mri$male, mri$diabetes, strata = mri$race)
##
       Method: Pearson
##
       Data : Pairwise Complete
##
               - NaN denotes strata with no observations
##
              - NA arises from missing data
##
## #### Stratum
##
      ## Estimated Correlation Coefficients
##
                     mri$male: mri$diabetes:
##
      mri$male: 1.0000
                                0.0941
## mri$diabetes: 0.0941
                                1.0000
##
## #### Stratum
      ## Estimated Correlation Coefficients
##
##
                     mri$male: mri$diabetes:
       mri$male: 1.0000
                                0.2122
##
## mri$diabetes: 0.2122
                                1.0000
##
## ##### Stratum
                  3
      ## Estimated Correlation Coefficients
##
##
                     mri$male: mri$diabetes:
##
      mri$male: 1.0000
                                0.1155
## mri$diabetes: 0.1155
                                1.0000
## #### Stratum
```

```
##
      ## Estimated Correlation Coefficients
##
                      mri$male: mri$diabetes:
       mri$male: 1.0000
##
                                 0.3162
                                 1.0000
## mri$diabetes: 0.3162
##
  ##### ALL DATA
##
      ## Estimated Correlation Coefficients
##
                      mri$male: mri$diabetes:
##
##
       mri$male: 1.0000
                                 0.1200
## mri$diabetes: 0.1200
                                 1.0000
```

Our function takes the pain out of calculating multiple correlations within stratum variables. We also return a correlation matrix rather than a single number, which allows the user to go straight to a covariance matrix by squaring. We also allow calculation of Spearman or Pearson (default) correlation.

#### Point Estimates and Inference

As mentioned above, since correlation is a function of the data gathered, we usually need other methods for our inference. One of the most common tests is the t-test, since we are often interested in the mean and making comparisons between means. Perhaps less common are the Wilcoxon and Mann-Whitney, which use the "rank" of the variables compared.

#### T-tests

##

In the base R package, the t.test() function performs a t-test as you would expect. Our function ttest() improves on this by allowing stratification, calculation of the geometric mean, allowing for presuming unequal variances between samples, and making a cleaner output. For example, a t-test of whether the mean of the ldl variable is equal to 125 yields, in base R and uwIntroStats

```
## base R
t.test(mri$ldl, mu = 125)
##
##
   One Sample t-test
##
## data: mri$ldl
## t = 0.64326, df = 724, p-value = 0.5203
## alternative hypothesis: true mean is not equal to 125
## 95 percent confidence interval:
## 123.3527 128.2528
## sample estimates:
## mean of x
##
   125.8028
## uwIntroStats
ttest(mri$ldl, null.hypoth = 125)
##
## Call:
## ttest(var1 = mri$ldl, null.hypoth = 125)
```

```
## One-sample t-test :
##
## Summary:
   Variable Obs Missing Mean Std. Err. Std. Dev.
##
##
    mri$1d1 735
                     10 126
                                   1.25
                                             33.6 [123, 128]
##
##
  Ho: mean = 125;
##
  Ha:
        mean != 125
## t = 0.6433 , df = 724
## Pr(|T| > t) = 0.520256
```

If instead we wanted a two-sample t-test of whether the difference in mean LDL between males and females were zero, we would get

```
## base R
t.test(mri$ldl[mri$male == 0], mri$ldl[mri$male == 1])
##
##
   Welch Two Sample t-test
##
## data: mri$ldl[mri$male == 0] and mri$ldl[mri$male == 1]
## t = 4.1938, df = 721.23, p-value = 3.084e-05
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
    5.502342 15.188221
## sample estimates:
## mean of x mean of y
  130.9397 120.5944
## uwIntroStats
ttest(mri$ldl, by = mri$male)
##
## Call:
## ttest(var1 = mri$ldl, by = mri$male)
##
## Two-sample t-test allowing for unequal variances :
##
## Summary:
##
                                                                  95% CI
             Group Obs Missing Mean Std. Err. Std. Dev.
##
      mri$male = 1 369
                             4 130.9
                                           1.79
                                                     34.3 [127.4, 134.5]
                                                     32.1 [117.3, 123.9]
      mri$male = 0 366
                             6 120.6
##
                                           1.69
##
        Difference 735
                            10 10.3
                                           2.47
                                                     <NA>
                                                             [5.5, 15.2]
##
   Ho: difference in means = 0 ;
   Ha: difference in means != 0
   t = 4.194, df = 721
##
  Pr(|T| > t) = 3.08428e-05
```

Note that in our package, the default is to presume unequal variances between samples, which the authors believe to usually be the correct course. Also, we don't have to subset the data manually. We also run two-sided tests by default, but others can be specified.

#### Generic Inference

Until now, all of the methods in this section work on multiple samples. However, if we want to produce point estimates, interval estimates, and p-values for an arbitrary functional (mean, geometric mean, proportion, median, quantile, odds) of a variable, we can use the oneSample() function. Base R does not have a function which performs this general inference all in one spot. In the following tests, we will run one- and two-sample t-tests and an exact binomial test, all on the LDL variable. The t-tests run by oneSample() are an alternative to ttest() as described above. The output is not as verbose, but all of the information is the same:

```
## base R
t.test(mri\$ldl, mu = 125)
##
##
    One Sample t-test
##
## data: mri$ldl
## t = 0.64326, df = 724, p-value = 0.5203
## alternative hypothesis: true mean is not equal to 125
## 95 percent confidence interval:
## 123.3527 128.2528
## sample estimates:
## mean of x
##
    125.8028
## ttest
ttest(mri$ldl, null.hypoth = 125)
##
## Call:
## ttest(var1 = mri$ldl, null.hypoth = 125)
##
## One-sample t-test :
##
## Summary:
##
    Variable Obs Missing Mean Std. Err. Std. Dev.
                                                        95% CI
##
     mri$1d1 735
                      10
                          126
                                    1.25
                                              33.6 [123, 128]
##
##
         mean = 125;
  Ho:
        mean != 125
##
   Ha:
##
   t = 0.6433 , df = 724
   Pr(|T| > t) = 0.520256
## oneSample
oneSample("mean", mri$ldl, null.hypothesis = 125)
## Hypothesis test of two-sided alternative that Mean <> 125
## Method: One sample t test
## 10 observations deleted due to missing values
##
                       95% CIlo
                                95% CIhi
                                           Null Hyp
                                                        P two
       n
               Mean
##
      725
               125.8
                                              125.0
                         123.4
                                    128.3
                                                         0.5203
```

However, the true flexibility of the oneSample() function comes into play when we ask about different functionals. If we want inference on the geometric mean (which might make sense since LDL is a biological

variable that may exhibit extreme values due to illness), in base R or with ttest() we have to logarithmically transform our variable and then run the t-test. With oneSample(), we simply run on "geometric mean":

```
## base R
t.test(log(mri\$ldl), mu = log(125))
##
##
    One Sample t-test
##
## data: log(mri$ldl)
## t = -3.0054, df = 724, p-value = 0.002744
## alternative hypothesis: true mean is not equal to 4.828314
## 95 percent confidence interval:
## 4.774290 4.816983
## sample estimates:
## mean of x
##
  4.795636
## ttest
ttest(log(mri$ldl), null.hypoth = log(125))
##
## Call:
## ttest(var1 = log(mri$ldl), null.hypoth = log(125))
## One-sample t-test :
##
## Summary:
        Variable Obs Missing Mean Std. Err. Std. Dev.
##
##
   log(mri$ldl) 735
                          10 4.8
                                      0.0109
                                                 0.293 [4.77, 4.82]
##
## Ho: mean = 4.8283137373023;
## Ha: mean != 4.8283137373023
## t = -3.005, df = 724
## Pr(|T| > t) = 0.00274401
## oneSample
oneSample("geometric mean", mri$ldl, null.hypothesis = 125)
## Hypothesis test of two-sided alternative that GeomMn \iff 125
## Method: One sample t test on log transformed data
## 10 observations deleted due to missing values
##
       n
              {\tt GeomMn}
                       95% CIlo 95% CIhi
                                           Null Hyp
                                                        P two
##
      725
               121.0
                         118.4
                                    123.6
                                              125.0
                                                       2.744e-03
We can also compute an exact binomial test of the proportion of LDL values that are greater than 128:
```

## base R

binom.test(sum(mri\$ldl > 128, na.rm = TRUE), n = sum(!is.na(mri\$ldl)))

```
Exact binomial test
##
## data: sum(mri$ldl > 128, na.rm = TRUE) and sum(!is.na(mri$ldl))
## number of successes = 336, number of trials = 725, p-value =
## 0.05338
## alternative hypothesis: true probability of success is not equal to 0.5
## 95 percent confidence interval:
## 0.4266728 0.5005223
## sample estimates:
## probability of success
                0.4634483
## oneSample
oneSample("prop", mri$ldl > 128, null.hypothesis = 0.5)
## Hypothesis test of two-sided alternative that Pr(Event) <> 0.5
## Method: One sample inference for binomial proportions using exact distribution (LR ordering)
## 10 observations deleted due to missing values
##
              Pr(Event) 95% CIlo
                                  95% CIhi
                                            Null Hyp
                                                         P two
                0.4634
                          0.4267
                                    0.5005
                                              0.5000
                                                        0.05338
##
      725
oneSample("prop", mri$ldl, above = 128, null.hypothesis = 0.5)
## Hypothesis test of two-sided alternative that Pr>128 <> 0.5
## Method: One sample inference for binomial proportions using exact distribution (LR ordering)
## 10 observations deleted due to missing values
##
                                 95% CIhi
              Pr>128
                       95% CIlo
                                           Null Hyp
                                                       P two
##
      725
               0.4634
                         0.4267
                                   0.5005
                                             0.5000
                                                       0.05338
```

The syntax for oneSample() is much more intuitive; we simply enter that we want to test a proportion, and then give the logical value to test or specify the threshold. Also, rather than having to count successes, the total, and dealing with missing values by hand, we can let oneSample() handle everything for us and give output that is clearer to read.

#### Wilcoxon and Mann-Whitney

The most similar function between our package and the base R package is the function to perform Wilcoxon Signed Rank tests and Mann-Whitney-Wilcoxon Ranked Sum tests. Our function, wilcoxon(), adds formatting and variances, and prints the z-score and p-value in addition to the test statistic and p-value. Otherwise, the functions are the same, though we follow our usual syntax of Y followed by X. For example, if we want to test diseased versus healthy (in a made-up dataset), we can get

```
## create the data
cf <- c(1153, 1132, 1165, 1460, 1162, 1493, 1358, 1453, 1185, 1824, 1793, 1930, 2075)
healthy <- c(996, 1080, 1182, 1452, 1634, 1619, 1140, 1123, 1113, 1463, 1632, 1614, 1836)
## base R
wilcox.test(healthy, cf, paired = TRUE)</pre>
```

```
##
## Wilcoxon signed rank test
```

```
##
## data: healthy and cf
## V = 20, p-value = 0.08032
## alternative hypothesis: true location shift is not equal to 0
## uwIntroStats
wilcoxon(cf, healthy, paired = TRUE)
##
##
    Wilcoxon signed rank test
##
            obs sum ranks expected
  positive
             10
                        71
                        20
                               45.5
## negative
              3
## zero
              0
                         0
                                0.0
             13
##
  all
                        91
                               91.0
##
## unadjusted variance
                          204.75
## adjustment for ties
                            0.00
## adjustment for zeroes
                            0.00
  adjusted variance
                          204.75
##
                        HO Ha
## Hypothesized Median 0
                          two.sided
    Test Statistic p-value
## V 71
                    0.080322
```

Note that in the output of wilcoxon()} we have displayed the data calculated by the signed rank test, and given both the test statistic (V) and the z-score (Z).

### Regression

## Z 1.7821

0.037368

Regression is one of the most widely used and easily understood methods of statistical analysis. The base R package has many different functions to perform regression: lm() for linear regression, glm() for generalized linear regression (which allows you to perform logistic regression, for example), coxph() for proportional hazards regression, and geeglm() as (one) method for correlated data regression. Each function has slightly different options, and remembering all of these is a chore. Part of the motivation for regress() is to alleviate this problem. This function provides one interface through which all types of regression can be performed. We also print the output from the regression in a much more understandable manner. Also, as we did in ttest(), we allow the user to calculate and use robust standard error estimates (using the Huber-White sandwich estimator), which adds flexibility. The regress() function does all of this while keeping the same formula syntax as its predecessors: y~x1+x2\*x3 still produces a linear model with coefficients for x1, x2, x3, and the interaction between x2 and x3. The next four subsections deal with aspects of regress() in more detail.

### Basics of regress()

The minimum we need to enter into regress() is a functional, a formula, and a dataset. A functional takes an object and returns a value; for instance, the mean is a functional because it takes a distribution and returns the mean. The allowed functionals are displayed in Table 2. Once we have a functional in mind (for now we choose the mean), we need to decide if we want to use robust standard error estimates. The default in regress() is to use these estimates, since we usually believe that the variances are not truly equal

between groups compared in a regression analysis. The base R functions do presume equal variances between groups, which can lead to conservative (or anti-conservative) inference in some situations. The last change we have made to the usual regression output is displaying F-statistics rather than t-statistics for the test of each variable. We decided to use the F-distribution to make our results line up more easily with classical ANOVA tables. In most cases, the F-statistic is simply the square of the t-statistic. The inference is the same. To see regress() in action, consider testing the association between age and atrophy. The syntax for this computation is the same as that for any of the previous commands you might have used.

```
## base R
summary(lm(atrophy ~ age, data = mri))
##
## Call:
  lm(formula = atrophy ~ age, data = mri)
##
## Residuals:
##
       Min
                1Q
                    Median
                                3Q
                                       Max
   -36.870 -8.589
                    -0.870
                             7.666
                                    51.203
##
##
##
  Coefficients:
##
                Estimate Std. Error t value Pr(>|t|)
##
  (Intercept) -16.06213
                            6.25619
                                     -2.567
                                               0.0104 *
## age
                 0.69798
                            0.08368
                                       8.341 3.64e-16 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 12.36 on 733 degrees of freedom
## Multiple R-squared: 0.08669,
                                    Adjusted R-squared:
## F-statistic: 69.58 on 1 and 733 DF, p-value: 3.635e-16
## uwIntroStats
regress("mean", atrophy ~ age, data = mri)
##
## Call:
  regress(fnctl = "mean", formula = atrophy ~ age, data = mri)
##
## Residuals:
                    Median
                                3Q
##
       Min
                1Q
                                        Max
  -36.870 -8.589
                    -0.870
                             7.666
                                    51.203
##
##
## Coefficients:
##
                    Estimate
                              Naive SE
                                        Robust SE
                                                      95%L
                                                                 95%H
## [1] Intercept
                     -16.06
                                6.256
                                                       -29.22
                                                                 -2.907
                                           6.701
                                          0.09002
                      0.6980
                               0.08368
                                                        0.5213
                                                                  0.8747
##
  [2] age
                                 df Pr(>F)
##
                       F stat
## [1] Intercept
                            5.75 1
                                       0.0168
## [2] age
                           60.12 1 < 0.00005
## Residual standard error: 12.36 on 733 degrees of freedom
## Multiple R-squared: 0.08669,
                                    Adjusted R-squared: 0.08545
## F-statistic: 60.12 on 1 and 733 DF, p-value: 2.988e-14
```

Note that by default, our output is printed in a table, rather than having to call summary.lm(). The inference on the individual coefficients is different because we have used the F-distribution and are using robust standard error estimates, but we get approximately the same p-values. Also, the overall F-statistic is different in our version because we are using the robust standard error estimates. The numbers next to the coefficients tell us which to specify in any post-testing commands we run (see section 8).

### Regression on different functionals

As mentioned above, part of the motivation for the regress() function was to have all types of regression in one function. Thus the other functionals we allow, and their corresponding commands in base R, are displayed in Table 2. Note that we only display the options which lead to a certain type of regression, not the rest of the syntax.

Functional	Type of Regression	Previous command (package)
"mean"	Linear Regression	lm() (stats - base R)
"geometric mean"	Linear Regression on logarithmically transformed Y	lm(), with Y log transformed (stats - base R)
"odds"	Logistic Regression	glm(family = binomial) (stats - base R)
"rate"	Poisson Regression	glm(family = poisson) (stats - base R)
"hazard"	Proportional Hazards Regression	coxph() (survival)

If we enter a functional other than the mean, there is usually a transformation that needs to be applied to the data within the regress() function. In all of the cases listed in Table 2, the transformation is the logarithm. For example, sometimes the logistic regression model is taught as having a "log link", or the poisson regression model is taught as having the "logit link". These both refer to the function of the parameter having logarithms, so that the model is linear in the coefficients on the predictors. Therefore, in each case, we back-transform - by using the exp(x) function, which is equivalent to  $e^x$  - the output of the regression by default, but also display the output that has not been back-transformed. This allows the user to easily see the results in the correct units, while also retaining the ability to compare with other software or handle the un-exponentiated results. If we want to examine the association between LDL and age, but want to use the geometric mean, we get

```
regress("geom", ldl ~ age, data = mri)
```

```
## ( 10 cases deleted due to missing values)
##
##
## Call:
##
   regress(fnctl = "geom", formula = ldl ~ age, data = mri)
##
## Residuals:
##
        Min
                   1Q
                        Median
                                      3Q
                                              Max
## -2.39512 -0.17020 0.03572 0.19342 0.70883
##
## Coefficients:
##
## Raw Model:
##
                                Naive SE
                                            Robust SE
                                                             F stat
                                                                        df
                    Estimate
                                                                1150.70 1
## [1] Intercept
                        4.876
                                  0.1494
                                              0.1437
## [2] age
                     -1.077e-03 1.999e-03
                                             1.932e-03
                                                                   0.31 1
##
                    Pr(>F)
## [1] Intercept
                    < 0.00005
```

```
## [2] age
                      0.5771
##
## Transformed Model:
                                                       F stat
##
                    e(Est)
                              e(95%L)
                                        e(95%H)
                                                                   дf
## [1] Intercept
                      131.1
                                98.87
                                          173.8
                                                          1150.70 1
## [2] age
                                0.9951
                                          1.003
                                                              0.31 1
                      0.9989
                    Pr(>F)
## [1] Intercept
                    < 0.00005
## [2] age
                      0.5771
##
## Residual standard error: 0.2929 on 723 degrees of freedom
     (10 observations deleted due to missingness)
## Multiple R-squared: 0.0004018, Adjusted R-squared:
## F-statistic: 0.3112 on 1 and 723 DF, p-value: 0.5771
```

In the transformed table, we do not display the standard error estimates, since they do not scale appropriately with the transformation. We could compare this to the base R function by log-transforming the ldl variable, running lm(), and back-transforming the results ourselves:

```
## transform the ldl variable
logldl <- log(mri$ldl)
## create the model
mod <- lm(logldl ~ age, data = mri)
## view the coefficients (untransformed)
summary(mod)

##
## Call:
## | m(formula = logldl ~ age, data = mri)</pre>
```

```
## lm(formula = logldl ~ age, data = mri)
##
## Residuals:
##
                  1Q
       Min
                     Median
                                    3Q
                                            Max
## -2.39512 -0.17020 0.03572 0.19342 0.70883
##
## Coefficients:
                Estimate Std. Error t value Pr(>|t|)
##
## (Intercept) 4.875983
                           0.149445 32.627
                                              <2e-16 ***
              -0.001077
                           0.001999 -0.539
                                                0.59
## age
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2929 on 723 degrees of freedom
     (10 observations deleted due to missingness)
## Multiple R-squared: 0.0004018, Adjusted R-squared:
## F-statistic: 0.2906 on 1 and 723 DF, p-value: 0.59
## back-transform the coefficients and CI
mod.sum <- summary(mod)</pre>
## this gives the coefficients
exp(mod.sum$coefficients[,1])
```

```
## (Intercept) age
## 131.1029250 0.9989231
```

```
## this gives the CI
exp(mod.sum$coefficients[,1] - 1.96*mod.sum$coefficients[,2])

## (Intercept) age
## 97.8143254 0.9950173

exp(mod.sum$coefficients[,1] + 1.96*mod.sum$coefficients[,2])

## (Intercept) age
## 175.720447 1.002844
```

First, this is much more work, and requires thought for how to calculate the confidence interval - should we use the convenient approximation of 1.96, or should we use the qnorm() function? Second, we again lose the option of using robust standard error estimates. To use these, we would have to manually code a few more lines, which would increase the likelihood of a small mistake in the code. Our regress() function pulls all of this together in a simple format, leading to much fewer mistakes.

### Correlated data regression

This section does not serve as a primer for when to account for correlated data in your regression model; rather, if you know that you have correlated data, it gives you a method to accounting for it. We will not use the mri data for this example. Instead, we will use the salary data, available as a text file "salary.txt". The documentation for this file is at "salary.doc". These data deal with salaries at the University of Washington, and have multiple records on most participants. In fact, for any current faculty member in 1995 who had been employed by the university for more than one year, there were yearly records dating back to when the faculty member was hired. Thus by nature these data are correlated - in the simplest sense, we expect that the salary for a single faculty member will rise (or at least stay constant) each year. Therefore, we need to use the correlated data apparatus built into regress(). This functionality uses the geeglm() function from the package geepack, but frames the output consistently with the rest of the regress() options and uses the same syntax.

First, let's create the data:

```
salary <- read.table("http://www.emersonstatistics.com/datasets/salary.txt", header = TRUE, stringsAsFa
salary$female <- ifelse(salary$sex == "F", 1, 0)</pre>
```

This code makes sure that the header in the text file is read as variable names, and that the string variables do not get converted to factor variables. We also create an indicator variable of whether the person is female or not. Next, suppose we are interested in the mean salary for females as opposed to males. Since raises are usually calculated as a percentage of current salary, and starting salaries can vary by year, we decide that the year in which a person started is important in determining their salary. For a more in-depth look at this thought process, see any document on determining potential confounding variables (for one, see parts of "Organizing Your Approach to a Data Analysis"). By adding the id argument to regress(), we can account for the correlated data. In the salary dataset, the ID column is named id, and thus we have:

```
## adjusting for correlated data
regress("mean", salary ~ female*year, id = id, data = salary)

##
## Call:
## regress(fnctl = "mean", formula = salary ~ female * year, data = salary,
```

```
##
      id = id
##
## Deviance Residuals:
##
      Min
            1Q
                                  3Q
                    Median
                                           Max
## -3727.9
           -907.4
                     -230.0
                               719.2
                                       7605.5
##
## Coefficients:
##
                     Estimate Std Err
                                          95%L
                                                       95%H
                                                                    Wald
## [1] Intercept
                       -17417
                                 283.6
                                           -17973
                                                        -16861
                                                                      3771.66
                                 528.7
                                                                        60.20
## [2] female
                        4102
                                            3065
                                                         5138
## [3] year
                       255.5
                                 3.536
                                            248.6
                                                        262.5
                                                                      5222.05
## [4] female:year
                      -57.88
                                  6.370
                                           -70.36
                                                        -45.39
                                                                        82.56
                     df Pr(>|W|)
## [1] Intercept
                     1 < 0.00005
## [2] female
                     1 < 0.00005
## [3] year
                     1 < 0.00005
## [4] female:year
                     1 < 0.00005
##
##
  Estimated Scale Parameters:
##
              Estimate Std.err
## (Intercept) 2024610 77997.57
##
  Correlation: Structure = independence
##
## Number of Clusters: 1597
##
##
  Maximum Cluster Size: 20
## without adjusting
regress("mean", salary ~ female*year, data = salary)
##
## Call:
## regress(fnctl = "mean", formula = salary ~ female * year, data = salary)
##
## Residuals:
##
      Min
               1Q Median
                                3Q
                                      Max
## -3727.9 -907.4 -230.0
                            719.2 7605.5
##
## Coefficients:
##
                     Estimate Naive SE Robust SE
                                                       95%L
                                                                 95%H
## [1] Intercept
                       -17417
                                 176.5
                                            167.6
                                                       -17745
                                                                  -17088
## [2] female
                                 420.7
                                            299.5
                        4102
                                                         3515
                                                                   4689
## [3] year
                       255.5
                                 2.021
                                            2.005
                                                        251.6
                                                                   259.5
## [4] female:year
                      -57.88
                                 4.757
                                            3.541
                                                       -64.82
                                                                  -50.94
                                  df Pr(>F)
                        F stat
## [1] Intercept
                         10802.86 1 < 0.00005
                            187.59 1 < 0.00005
## [2] female
## [3] year
                         16235.87 1 < 0.00005
                            267.10 1 < 0.00005
## [4] female:year
##
## Residual standard error: 1423 on 19788 degrees of freedom
## Multiple R-squared: 0.487, Adjusted R-squared: 0.4869
## F-statistic: 7166 on 3 and 19788 DF, p-value: < 2.2e-16
```

Notice that we get some extra information in the output where we adjusted - we see that there are 1597 unique faculty members in the data, and that the longest we have data for is 20 years. Therefore treating the data as independent, like we do in the second call to regress(), will lead to invalid standard error estimates and therefore invalid confidence intervals and inference.

### Multiple-partial F-tests

The major added functionality that regress() brings to the table is the ability to perform multiple-partial F-tests. Some of these are done automatically when certain dummy variables, polynomial variables, or linear splines are entered - if created in a manner similar to our functions above. Others must be specified by the user, using a special function called U().

As an example of the automatic multiple-partial F-test calculation, let's say we run a regression of atrophy on race, modeled as dummy variables. Recall from our work above that the dummy variables created by race have three levels, and each is in reference to the level coded as white. When we run this regression,

```
regress("mean", atrophy ~ dummy(race), data = mri)
```

```
##
## Call:
## regress(fnctl = "mean", formula = atrophy ~ dummy(race), data = mri)
##
## Residuals:
##
       Min
                 1Q
                     Median
                                  3Q
                                          Max
   -31.243
            -9.243
                     -0.462
                               7.757
                                      47.757
##
##
##
  Coefficients:
                                  Naive SE
                                             Robust SE
                                                           95%L
                                                                      95%H
##
                       Estimate
                                                                        37.31
##
   [1] Intercept
                         36.24
                                    0.5408
                                               0.5438
                                                             35.18
##
       dummy(race)
                                    1.379
##
   [2]
                        -1.781
                                               1.334
                                                            -4.401
                                                                        0.8381
          race.2
                                                                        4.005
##
   [3]
          race.3
                         0.2676
                                    1.962
                                               1.904
                                                            -3.470
   [4]
                        -1.493
                                    3.772
                                                            -10.17
                                                                        7.189
##
          race.4
                                               4.422
##
                          F stat
                                     df Pr(>F)
##
                             4441.94 1
                                        < 0.00005
   [1] Intercept
##
       dummy(race)
                                0.65 3
                                           0.5832
  [2]
##
          race.2
                                1.78 1
                                           0.1823
##
   [3]
          race.3
                                0.02 1
                                           0.8882
   [4]
##
          race.4
                                0.11 1
                                           0.7357
##
##
    Dummy terms calculated from race, reference = 1
##
## Residual standard error: 12.93 on 731 degrees of freedom
## Multiple R-squared: 0.002535,
                                      Adjusted R-squared:
## F-statistic: 0.6499 on 3 and 731 DF, p-value: 0.5832
```

notice that the coefficients for each dummy variable are presented in the usual way (though it is up to you to remember what race.2 stands for). However, there is a new line above all of the dummy variable coefficients. First we see that this line does not get a coefficient number - that's because this line is only for the multiple partial F-test. Also, note that the regression coefficients are indented beneath it. This denotes that these dummy variables all belong to the same original variable (race). The test has three degrees of freedom, because there are three dummy variables that it is simultaneously testing. Recall that in an F-test, and in the t-test of normal linear regression in lm(), the null hypothesis for each of the p-values presented in the

regression table is that the regression coefficient is equal to zero. The multiple partial F-test simulataneously tests that all three coefficients are equal to zero. It allows us to declare that there is no significant association between race and atrophy at the 0.05 level, since we have tested all race variables simultaneously and returned a p-value of 0.58.

We also see that after the coefficients table, regress() tells us how the dummy variables were computed.

Now for the user-defined multiple-partial F-tests, we need to be a bit careful. If entered in a call to regress(), the U() function takes in a formula, which can be named, and returns the specified multiple-partial F-test to the regression output. It also adds any new variables to the regression model. As an example of the U() function, if we wanted to add age and male to our model, and we wanted to have a multiple-partial F-test of these two variables, we could add

```
U(~age + male)
```

to the model above.

```
regress("mean", atrophy ~ dummy(race) + U(~age + male), data = mri)
```

```
##
## Call:
  regress(fnctl = "mean", formula = atrophy ~ dummy(race) + U(~age +
##
##
       male), data = mri)
##
##
  Residuals:
##
       Min
                 1Q
                     Median
                                  3Q
                                          Max
##
   -34.120
            -8.331
                     -0.434
                               7.325
                                      53.915
##
##
  Coefficients:
##
                          Estimate
                                    Naive SE
                                               Robust SE
                                                             95%L
                                                                        95%H
##
   [1] Intercept
                           -17.87
                                      6.079
                                                 6.589
                                                              -30.81
                                                                         -4.933
##
       dummy(race)
                                                 1.246
##
  [2]
          race.2
                           -2.109
                                       1.280
                                                              -4.554
                                                                          0.3370
##
  [3]
          race.3
                            0.2664
                                       1.822
                                                              -3.228
                                                                          3.761
                                                 1.780
  [4]
          race.4
                           -2.812
                                      3.503
                                                              -10.30
                                                                          4.671
##
                                                 3.812
##
       U(age + male)
##
   [5]
                            0.6866
                                     0.08134
                                                0.08836
                                                               0.5132
                                                                          0.8601
         age
   [6]
                            5.988
##
         male
                                      0.8867
                                                 0.8895
                                                               4.242
                                                                          7.734
                             F stat
                                       df Pr(>F)
##
                                  7.35 1
##
   [1]
       Intercept
                                             0.0068
##
       dummy(race)
                                  1.14 3
                                             0.3315
##
  [2]
          race.2
                                  2.87 1
                                             0.0909
   [3]
##
          race.3
                                  0.02 1
                                             0.8810
##
   [4]
          race.4
                                  0.54 1
                                             0.4609
##
       U(age + male)
                                 52.66 2
                                          < 0.00005
   [5]
##
                                 60.38 1
                                          < 0.00005
         age
##
   [6]
                                 45.32 1
                                          < 0.00005
         male
##
##
    Dummy terms calculated from race, reference = 1
##
## Residual standard error: 12 on 729 degrees of freedom
## Multiple R-squared: 0.1439, Adjusted R-squared: 0.138
## F-statistic: 21.32 on 5 and 729 DF, p-value: < 2.2e-16
```

Note that again the multiple-partial F-test line is not a coefficient, but that the variables specified in the U() forumla are part of the regression model. If we wanted to make this output a bit more readable, we could give the second multiple-partial F-test a name by adding testnm = within our call to U(), before the formula.

```
regress("mean", atrophy ~ dummy(race) + U(ma = ~age + male), data = mri)
##
## Call:
   regress(fnctl = "mean", formula = atrophy ~ dummy(race) + U(ma = ~age +
##
       male), data = mri)
##
##
  Residuals:
##
       Min
                 1Q
                     Median
                                  3Q
                                          Max
   -34.120
            -8.331
                     -0.434
                               7.325
                                      53.915
##
##
  Coefficients:
##
                                  Naive SE
                                             Robust SE
                                                           95%L
##
                       Estimate
                                                                      95%H
                         -17.87
                                    6.079
                                               6.589
                                                            -30.81
                                                                       -4.933
##
   [1] Intercept
       dummy(race)
##
   [2]
                                                            -4.554
##
          race.2
                        -2.109
                                    1.280
                                               1.246
                                                                        0.3370
   [3]
##
          race.3
                          0.2664
                                    1.822
                                               1.780
                                                            -3.228
                                                                        3.761
##
   [4]
                        -2.812
                                    3.503
                                                            -10.30
                                                                        4.671
           race.4
                                               3.812
##
       ma
                          0.6866
                                                             0.5132
                                                                        0.8601
##
   [5]
         age
                                   0.08134
                                              0.08836
##
   [6]
         male
                          5.988
                                    0.8867
                                               0.8895
                                                             4.242
                                                                        7.734
##
                           F stat
                                      df Pr(>F)
##
   [1] Intercept
                                7.35 1
                                           0.0068
##
       dummy(race)
                                1.14 3
                                           0.3315
                                2.87 1
##
   [2]
          race.2
                                           0.0909
##
   [3]
           race.3
                                0.02 1
                                           0.8810
##
   [4]
                                0.54 1
                                           0.4609
          race.4
##
                               52.66 2
                                         < 0.00005
       ma
##
                               60.38 1
   [5]
                                         < 0.00005
         age
   [6]
                               45.32 1
                                         < 0.00005
##
         male
##
##
    Dummy terms calculated from race, reference = 1
##
## Residual standard error: 12 on 729 degrees of freedom
## Multiple R-squared: 0.1439, Adjusted R-squared:
## F-statistic: 21.32 on 5 and 729 DF, p-value: < 2.2e-16
```

Other than the new name, the output is exactly the same as our original call.

### Post Estimation

After we have created a regression model and run the test, sometimes we want to check parts of our model. Usually, this comes in the form of an ANOVA table testing whether a combination of our coefficients are simultaneously equal to zero. As we saw in the previous section, regress() allows us to run these types of commands within the regression call. However, to check these results - or to avoid using U() within a call to regress() - we can use post estimation commands.

Also, it is sometimes of interest to predict on a new data set. The object created by a call to regress() is like all other regression objects (from calls to lm(), glm(), etc) in that it allows predictions.

### Linear Combinations of Regression Coefficients

A very useful function in STATA is the lincom function. This is a post-testing function which allows the user to specify a linear combination of the regression coefficients to simultaneously test. We have recreated this function in R, and it follows a similar syntax. Recall that the regress() function displays a number next to each coefficient in the coefficients table. These numbers refer to the position of each variable in the call to lincom(). The default null hypothesis is that the linear combination is equal to zero.

This function can take either a vector or a matrix determining the linear combination to test. To have a better idea of how this works, we provide an example. In the previous section, when we introduced multiple-partial F-tests, we ran a regression of atrophy on race (modeled as dummy variables), male, and age. We can perform the same test using lincom():

```
## get the model
mod <- regress("mean", atrophy ~ dummy(race) + age + male, data = mri)
## get the test of age and male
lincom(mod, comb = c(0,0,0,0,1,1))
##
## HO: 1*age+1*male
## Ha: 1*age+1*male
                     ! =
##
      Estimate
                 Std. Err.
                                      T Pr(T > |t|)
                                                            95%L
                                                                        95%H
##
        6.6744
                    0.8942
                                 7.4645
                                             1.0000
                                                          4.9190
                                                                       8.4299
```

This test gives us the same inference as before - that the probability of seeing this event is extrememly small if the null hypothesis is true. Now in some cases we might be interested in combinations that aren't just the raw data. If we want, for example, twice the male coefficient, we can get this easily:

```
lincom(mod, comb = c(0,0,0,0,1,2))
##
## HO: 1*age+2*male
## Ha: 1*age+2*male
                    ! =
##
      Estimate
                 Std. Err.
                                      T Pr(T > |t|)
                                                            95%L
                                                                         95%H
       12.6622
                    1.7814
                                 7.1079
                                              1.0000
                                                          9.1649
##
                                                                      16.1596
```

### Prediction

Prediction with a uRegress object (which is created by a call to regress()) follows the same syntax as prediction for any other regression object. Say we have split the mri data into a training and test set to learn our linear regression model on. First we set a seed, so that our random number generator will always start in the same place and we are guaranteed reproducible results.

```
set.seed(47)
samp <- sample(1:nrow(mri), nrow(mri)/2, replace = FALSE)
mri.train <- mri[samp, ]
mri.test <- mri[-samp,]

modlm <- lm(atrophy ~ age + male + dummy(race), data = mri.train)
modreg <- regress("mean", atrophy ~ age + male + dummy(race), data = mri.train)</pre>
```

```
predslm <- predict(modlm, data = mri.test)</pre>
predsreg <- predict(modreg, data = mri.test)</pre>
head(predslm, n = 5)
##
        719
                  275
                            559
                                      603
                                               420
## 34.67201 46.44437 31.99498 38.41326 38.41326
head(predsreg, n = 5)
                            559
##
        719
                  275
                                      603
                                               420
## 34.67201 46.44437 31.99498 38.41326 38.41326
```

The above code reassures us that the predictions given on a uRegress object are the same as the predictions given on a lm object. The same is true for all of the other types of regression possible with regress().

### **Diagnostics**

Besides the other tools we have already covered that can double as diagnostic tools - scatterplots, boxplots - sometimes it is useful to look at residuals calculated from a regression model. Objects of class uRegress, like all other regression objects, have a function to extract residuals. The function we use is uresiduals(), which can return unstandardized, standardized, studentized, or jackknifed residuals. If we want the residuals from the model regressing age on ldl, it is easy to get both studentized and jackknifed residuals.

```
ldlReg <- regress("mean", age ~ ldl, data = mri)</pre>
student.resid <- uResiduals(ldlReg, "studentized")</pre>
jack.resid <- uResiduals(ldlReg, "jackknife")</pre>
head(student.resid, n = 5)
                        2
                                    3
## -0.4676531
               1.1641617
                           2.8293373 -0.5011149 -0.8294973
head(jack.resid, n = 5)
##
            1
                        2
                                    3
                                                            5
## -0.4674003 1.1644483 2.8431637 -0.5008553 -0.8293181
```

As with prediction, the residuals can be returned from any regression type allowed by regress().