ECON 520 Homework 11

Due date: November 14, 2018

1. Let $X_1, X_2, \ldots, X_n$ be independent random variables, all with a binomial $\mathcal{B}(2, p)$ distribution.
   (a) Find the maximum likelihood estimator for p.
   (b) Is the maximum likelihood estimator the minimum variance unbiased estimator?
   (c) Let $n = 100$, $\sum_{i=1}^{n} x_i = 40$, and $\sum_{i=1}^{n} x_i^2 = 48$. Calculate the MLE.

2. Let $X_1, X_2, \ldots, X_n$ represent a random sample from a Poisson distribution with parameter λ.
   (a) Find the MLE for λ and its asymptotic distribution.
   (b) Is the MLE for λ unbiased? Consistent? Explain.
   (c) Suppose we are interested in the probability of a count of zero, i.e. $P(X = 0)$. Let θ represent $P(X = 0)$. Find the MLE for θ and its asymptotic ditribution.

3. Let $X_1, \ldots, X_n$ be i.i.d. from a population with mean μ and variance $\sigma^2 < \infty$, and consider the sample mean $\bar{X}_n = n^{-1} \sum_{n=1}^{n} X_i$. Using asymptotic theory, derive the approximate distributions for:
   (a) $\exp(\bar{X}_n)$
   (b) $\ln(\bar{X}_n)$
   (c) $\bar{X}_n^3$

4. Suppose a random sample is available from a gamma distribution with parameters α and β. Find the method of moments estimators for α and β.

5. Show that the priors in the following cases are conjugate priors:
   (a) $X_1, \ldots, X_n$ is a random sample from the Binomial(p, k) distribution with probability p and size k. Assume that k is known. The prior for p is a Beta distribution with parameters α and β.
   (b) $X_1, \ldots, X_n$ is a random sample from the uniform distribution on $[0, \theta]$. The prior for θ is
   $$f(\theta) = ba^b\theta^{-(b+1)} \cdot 1(\theta \geq a).$$
   (c) $X_1, \ldots, X_n$ is a random sample from the exponential distribution with density $f(x; \lambda) = \lambda \exp(-\lambda x)$ for $x > 0$. The prior for λ is a Gamma distribution with parameters α and γ.