ECON 520 Homework 14

Due date: November 30, 2018

1. Suppose that we observe m iid Bernoulli(θ) random variables, denoted by $Y_1 \dots Y_m$. Show that the LRT of $H_0 : \theta \leq \theta_0$ versus $H_1 : \theta > \theta_0$ will reject H_0 if $\sum_{i=1}^{m} Y_i > b$.

2. Let X be a random variable whose pmf under H_0 and H_1 is given by

x	1	2	3	4	5	6	7
$f(x \mid H_0)$	0.01	0.01	0.01	0.01	0.01	0.01	0.94
$f(x \mid H_1)$	0.06	0.05	0.04	0.03	0.02	0.01	0.79

   Use the Neyman-Pearson Lemma to find the most powerful test for H_0 versus H_1 with size $\alpha = 0.04$. Compute the probability of Type II Error for this test.

3. Let $X_1 \dots X_n$ be iid $N(\theta, \sigma^2)$, where θ_0 is a specified value of θ and σ^2 is unknown. We are interested in testing $H_0 : \theta = \theta_0$ versus $H_1 : \theta \neq \theta_0$.

   (a) Show that the test rejects H_0 when $\mid \bar{X} - \theta_0 \mid > t_{n-1,\frac{\alpha}{2}} \sqrt{\frac{S^2}{n}}$ is a test of size α.

   (b) Show that the test in part (a) can be derived as an LRT.

4. Let $X_1 \dots X_n$ be a random sample from a $N(\mu, \sigma^2)$ population.

   (a) If μ is unknown and σ^2 is known, show that $Z = \sqrt{n}\frac{(\bar{X} - \mu_0)}{\sigma}$ is a Wald statistic for testing $H_0 : \mu = \mu_0$.

   (b) If σ^2 is unknown and μ is known, find a Wald statistic for testing $H_0 : \sigma = \sigma_0$.