# Econ 520 Homework 1

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1. For each of the following experiments, describe the sample space.

   (a) Toss a coin four times.
   The sample space is all possible combinations of heads and tails for tossing one coin four times. For example,

   $$S = \{(H,H,H,H),(H,H,H,T),(H,H,T,H),...,(T,T,T,T)\}$$

   for a sample space consisting of 16 outcomes.

   (b) Count the number of insect-damaged leaves on a plant.
   The sample space is each possible number of insect damaged leaves. For example,

   $$S = \{1,2,...,n\}$$

   for $n$ leaves on a plant.

   (c) Measure the lifetime (in hours) of a particular brand of light bulb.
   This sample space could be anywhere from zero hours to, in theory, and infinite amount. Hence $S = \{0,1,2,...,\infty\}$

   (d) Record the weights of 10-day-old rats.
   Let $\{x_n\}$ be the weight of the $n^{th}$ rat which will be in a range from $[0,\infty)$. Then, the sample space is $S = \{x_1,...x_n\}$.

   (e) Observe the proportion of defectives in a shipment of electronic components.
   Let $x_i = \frac{i}{n}$ where $n$ is the total number of components and $i$ represents the $i^{th}$ component from $1,..,n$. The, the sample space is $S = \{x_1,x_2,...x_n\}$

2. Suppose we toss two fair coins. Let $A$ be the event "two heads," let $B$ be the event "no heads," and let $C$ be the event "at least one head". Define the sample space and the probability function. Calculate the probabilities of the following events:
Let $x$ denote the event that the flip is a head: $P(x) =$

$$\begin{cases} \frac{1}{4} & x = 0 \\ \frac{1}{2} & x = 1 \\ \frac{1}{4} & x = 2 \end{cases}$$

(a) $A \cup B$:

$$S = \{(H, H), (T, T)\}$$

$$P(A \cup B) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

(b) $A \cap B$:

$$S = \emptyset$$

$$P(a \cap B) = 0$$

(c) $B \cap C$:

$$S = \emptyset$$

$$P(B \cap C) = 0$$

(d) $A^c$:

$$S = \{(H, T), (T, H), (T, T)\}$$

$$P = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$$

3. For events A and B, find formulas for the proabilities of the following vents in terms of the quantities $P(A)$, $P(B)$, or $P(A \cap B)$:

(a) either $A$ or $B$ or both:

$$P(A) + P(B) - P(A \cap B)$$

(b) either $A$ or $B$ but not both:

$$P(A) + P(B) - 2P(A \cap B)$$

(c) at least one of $A$ or $B$:

$$P(A) + P(B) - P(A \cap B)$$

(d) at most one of $A$ or $B$:

$$1 - p(A \cap B)$$