ECON 520 Homework 10

Due date: November 7, 2018

1. X has a binomial distribution with parameters $N = 1$ and $p = 1/2$. Y, which is independent of X, has a normal distribution with mean μ and variance 1. Consider the estimator for μ of the form $W_1 = Y + 2X - 1$.

   (a) Is W_1 unbiased?

   (b) What is the variance of W_1?

   (c) Compute the mean squared error of W_1.

   (d) Consider the estimator $W_2 = E[W_1|Y]$. Is W_2 unbiased? How does its variance compare to that of W_1?

2. X_1 and X_2 are independent normally distributed random variables with mean μ and variances σ_1^2 and σ_2^2 respectively. The variances are known and we are interested in estimating the means. Consider estimators of the form $W_{\lambda,\delta} = \lambda X_1 + \delta X_2$. Find the minimum variance unbiased estimator in this class of estimators.

3. Let X and Y have the following joint probability density:

$$f_{X,Y}(x, y) = \frac{1}{2\mu^2} \exp\left(-\frac{x}{\mu} - \frac{y}{2\mu}\right).$$

   for $x > 0$, $y > 0$, and $\mu > 0$.

   (a) Are X and Y independent? What are their marginal densities?

   (b) Find the Cramer-Rao bound for unbiased estimators of μ.

   (c) What is the MLE? Is it minimum variance unbiased?

4. Consider the density

$$f_Y(y) = \frac{2y}{\theta^2},$$

   where $0 < y < \theta, \theta > 0$. Assume that we observe a random sample of observations $Y_1, \ldots, Y_n$ from this distribution.

   (a) Compute the method of moments estimator for θ based on the first moment only.

   (b) Compute the variance of this estimator.

   (c) Compute the Fisher information for estimating θ in the sample (hint: the Fisher information is $I(\theta) = nE[(\frac{\partial}{\partial\theta} \ln f_Y(Y))^2]$).

   (d) Contrast your results in (b) and (c).

5. Suppose we have a random sample from the Poisson (λ) distribution. The probability density function for any X_i is

$$f_X(x) = \frac{e^{-\lambda}\lambda^x}{x!},$$

   for $0 \leq x < \infty$ and $x = 1, 2, 3, \ldots$. Both the mean and variance are equal to λ.

(a) Compute the MLE for λ for a sample size equal to n.

(b) Is the MLE unbiased?

(c) Suggest two method of moment estimators for λ based on the mean and variance.

(d) Compute the Fisher information for estimating λ.

(e) What does your result in (d) suggest for a method for weighting the two method of moment estimators in part (c) to obtain the 'best' linear combination?