

Bernard and Thomas 1990

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Time Series Review

Random Walk with Drift

The authors in this paper examine a model they refer to as a random walk with trend. However, given the setup in equation (1) on page 312, it appears they refer to what others have called a random walk with drift. Though a drift is different than a trend, it appears that a random walk with drift is used to describe a trend, and can even be derived into the typical trend form dependent on time t .

From Shumway and Stoffer's *Time Series Analysis and Its Applications* page 11-12: ¹

A model for analyzing trend is the random walk with drift model given by:

$$x_t = \delta + x_{t-1} + w_t$$

> for $t = 1, 2, \dots$, with initial condition $x_0 = 0$ and where w_t is white noise. The constant δ is called the *drift*, and when $\delta = 0$ the equation is simply called a random walk. The term random walk comes from the fact that, when $\delta = 0$, the value of the time series at time t is the value of the series at time $t - 1$ plus a completely random movement determined by w_t . Note that the above equation may be rewritten as a cumulative sum of white noise variates. That is:

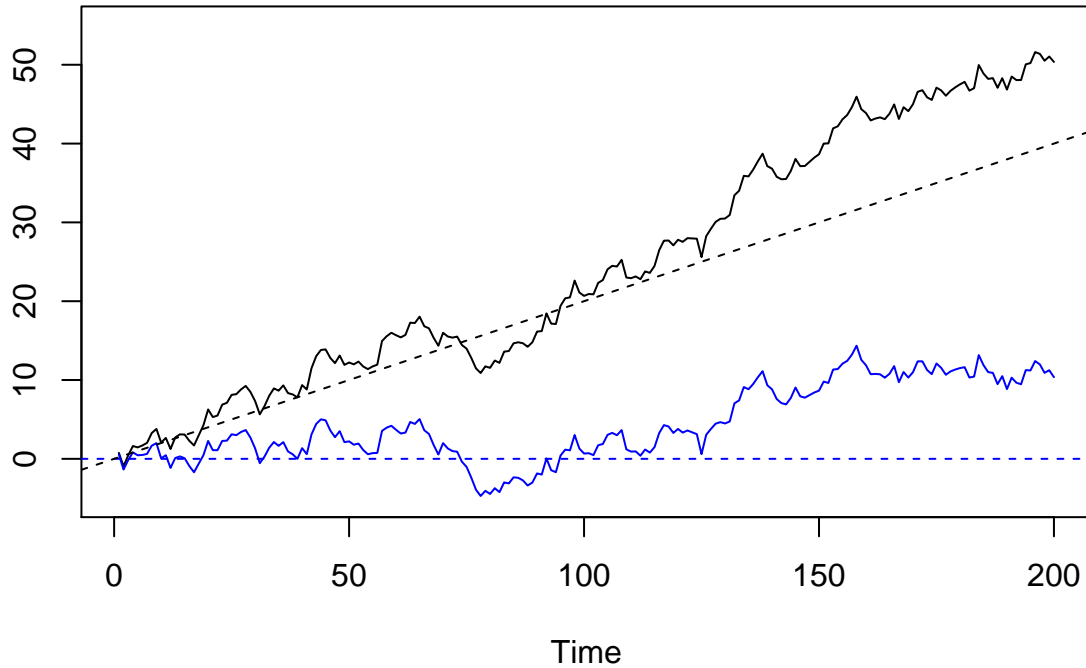
$$x_t = \delta t + \sum_{j=1}^t w_j$$

for $t = 1, 2, \dots$

```
set.seed(154)
w = rnorm(200); x = cumsum(w)
wd = w + .2; xd = cumsum(wd)
plot.ts(xd, ylim = c(-5, 55), main = 'random walk', ylab = '')
lines(x, col=4); abline(h=0, col=4, lty=2); abline(a=0, b=.2, lty=2)
```

¹Available for download at <https://www.stat.pitt.edu/stoffer/tsa4/tsa4.pdf>

random walk



Question: Why is inclusion of a drift a good specification for the context of earnings models such as the one in this paper?

Seasonality

Time series datasets can contain a seasonal component.

This is a cycle that repeats over time, such as monthly or yearly. This repeating cycle may obscure the signal that we wish to model when forecasting, and in turn may provide a strong signal to our predictive models.

Model for season data²:

$$x_t = S_t + W_t$$

Where S_t is a seasonal component that varies slowly from one quarter to the next according to a random walk:

$$S_t = S_{t-4} + Z_t \implies S_t - S_{t-4} = Z_t$$

With W_t and Z_t being uncorrelated white noise processes.

Question: Why is seasonality important to consider?

²<https://www.stat.berkeley.edu/~gido/Removal%20of%20Trend%20and%20Seasonality.pdf>

Hypothesis Development

Key Question

What explains the post-earnings announcement drift?

Background

- Paper investigates possibility that stock prices fail to reflect the full implications of future earnings
- Prices do not adequately reflect the extent to which earnings deviate from a seasonal random walk model
- Perhaps reactions of prices to future earnings are predictable

Question: Why is the last statement at odds with the random walk framework described above?

Main Results and Contribution

Key Findings:

- Stock prices are partially influenced by *naive expectations* allow for accurate predicting of three-day reaction to future earnings announcements given only current information about historical time series
- There is a positive relation between unexpected earnings for quarter t and post-announcement drift for quarter $t + 1$
- There is a negative relation between unexpected earnings and abnormal returns around announcement of earnings for quarter $t + 4$

Contribution

- (1) Gives signs and magnitudes of reactions to subsequent earnings announcements to the historical autocorrelation structure of earnings (could help sort out cause of post-announcement drift)
- (2) Evidence given in this paper creates added obstacles to contentions that drift might ultimately be explained by errors in the methodology used to estimate expected returns

The nature of the evidence is also distinct in an important way from that in the growing body of other studies that question semi-strong or weak-form market efficiency. Previous studies make vague predictions. By linking what appears to be the elimination of discrepancy between prices and fundamentals to prespecified information events, this study gives evidence that a market-efficiency anomaly is rooted in a failure of information to flow completely into price

Question: Does this paper assume market efficiency or inefficiency?

Development

Table 1 shows the time-series behavior frequently demonstrated by many studies: First 3 autocorrelations are positive, though diminishing, and the fourth is negative.

Consider the behavior of stock prices in a market where earnings expectations fail to reflect the pattern described in Table 1. Use a seasonal random walk with trend.

Equation 1:

$$\mathbb{E}^M[Q_t] = \delta + Q_{t-4}$$

Together with an earnings response coefficient λ , we have abnormal return described by:

Equation 2:

$$AR_t = \lambda(Q_t - \mathbb{E}^M[Q_t]) = \lambda(Q_t - Q_{t-4} - \delta)$$

The abnormal return reflects a component equal to the current detrended seasonal difference in earnings.

Hypothesis 1: *If the prices reflect an earnings expectation described by a seasonal random walk with trend, there should be positive but declining associations between the abnormal return at the announcement of quarter t earnings and the detrended seasonal differences in earnings for quarters $t-1$, $t-2$, and $t-3$. There should be a negative association between the abnormal return at the announcement of quarter t earnings and the detrended seasonal difference in earnings for quarter $t-4$.*

Question: What is the difference between the two models Hypothesis 1 is comparing?