2

$$x_i'\hat{\beta} - x_i'\hat{\beta}_{(i)} = \frac{x_i'\left(X_{(i)}'X_{(i)}\right)^{-1}x_i}{1 + x_i'\left(X_{(i)}'X_{(i)}\right)^{-1}x_i}\left(y_i - x_i'\hat{\beta}_{(i)}\right)$$
$$= \frac{x_{1i}^2 + \frac{1}{5}x_{2i}^2}{1 + x_{1i}^2 + \frac{1}{5}x_{2i}^2}\left(y_i - x_i'\hat{\beta}_{(i)}\right).$$

If the size of x_i is fixed, which means $x_{1i}^2 + x_{2i}^2 = C$ for some constant C, then $x_{1i}^2 + \frac{1}{5}x_{2i}^2$ is maximized when $x_{1i}^2 = C$ and $x_{2i}^2 = 0$, and is minimized if $x_{1i}^2 = 0$ and $x_{2i}^2 = C$. Therefore, the impact is larger if x_{1i} is relatively more deviated from the origin, and is smaller if x_{2i} is relatively more deviated from the origin.

4

Let $X_n(w) : [0,1] \to \mathbb{R}$ be as $X_n(w) = \begin{cases} k+1 & \text{if } \frac{n-K+1}{k+1} > w > \frac{n-K}{k+1} \\ 0 & \text{otherwise} \end{cases}$ where $k = \max\left\{k \in \mathbb{N}^+ : \frac{k(k+1)}{2} \leq n\right\}$ and $K = \frac{k(k+1)}{2}$.

- $\{X_n\}$ converges to 0 in probability because given any small positive ϵ,

$$\Pr\{|X_n(w) - 0| < \epsilon\} = 1 - \frac{1}{k+1} \to 0 \qquad \text{as } n \to \infty.$$

- $\{X_n\}$ does not converges to 0 in rth moment for any $r \geq 1$, as

$$E\{|X_n(w) - 0|^r\}^{\frac{1}{r}} = \left(k + 1\frac{1}{k+1}\right)^{\frac{1}{r}} = 1 \nrightarrow 0.$$

- $\{X_n\}$ does not converges to 0 almost surely, because if we fix some $w \in (0,1)$, given any positive ϵ for any N, there exists some $\tilde{k}$ and $\tilde{n} > N$ such that

$$\frac{\tilde{n} - \tilde{K} + 1}{\tilde{k} + 1} > w > \frac{\tilde{n} - \tilde{K}}{\tilde{k} + 1}$$

  which implies $|X_{\tilde{n}}(w) - 0| = \left|\tilde{k} + 1\right| > \epsilon$, which means $X_n(w) \nrightarrow 0$. Therefore, $Pr\{\lim_{n\to\infty} X_n(w) = 0\} = 0 \neq 1$.

5

(a) The CDF for a constant c is

$$F_c(t) = \Pr(c \leq t) = \begin{cases} 0 & \text{if } c > t \\ 1 & \text{if } c \leq t \end{cases}.$$

(b) The condition: for each $t \in \mathbb{R}$,

$$F_n(t) \to F_c(t) \qquad \text{as } n \to \infty.$$

(c) For any $\epsilon > 0$,

1. $F_n(c+\epsilon) \to F_c(c+\epsilon) = 1$,
2. $F_n(c-\epsilon) \to F_c(c-\epsilon) = 0$.

(d)

$$\begin{aligned}\Pr\left(\left|X_n-c\right|\leq\epsilon\right)&=\Pr\left(-\epsilon\leq X_n-c\leq\epsilon\right)\\&=\Pr\left(c-\epsilon\leq X_n\leq c+\epsilon\right)\\&=\Pr\left(X_n\leq c+\epsilon\right)-\Pr\left(X_n<c-\epsilon\right)\\&=\Pr\left(X_n<c+\epsilon\right)-\Pr\left(X_n<c-\epsilon\right)+\Pr\left(X_n=c+\epsilon\right)\\&=F_n\left(c+\epsilon\right)-F_n\left(c-\epsilon\right)+\Pr\left(X_n=c+\epsilon\right).\end{aligned}$$

(e) Given any ϵ,

$$\begin{aligned}\Pr\left(\left|X_n\left(w\right)-c\right|\right)&=F_n\left(c+\epsilon\right)-F_n\left(c-\epsilon\right)+\Pr\left(X_n=c+\epsilon\right)\\&\to1+\Pr\left(X_n=c+\epsilon\right).\end{aligned}$$

Notice that

$$\begin{aligned}\Pr\left(X_n=c+\epsilon\right)&=\Pr\left(X_n\leq c+\epsilon\right)-\Pr\left(X_n<c+\epsilon\right)\\&\leq\Pr\left(X_n\leq c+\epsilon\right)-\Pr\left(X_n<c+\frac{\epsilon}{2}\right)\\&\to1-1\\&=0.\end{aligned}$$

Therefore,

$$\Pr\left(\left|X_n\left(w\right)-c\right|\right)\to1$$

which implies X_n converges to c in probability.

6

(a)

$$\begin{pmatrix}A & C\\ B' & D\end{pmatrix}\begin{pmatrix}W & Y\\ X' & Z\end{pmatrix}=\begin{pmatrix}AW+CX' & AY+CZ\\ B'W+DX' & B'Y+DZ\end{pmatrix}$$

If this equals to $\begin{pmatrix}I & 0\\ 0' & I\end{pmatrix}$, then we have

$$\begin{aligned}AW+CX'&=I\quad(1)\\B'W+DX'&=0'\quad(2)\\AY+CZ&=0\quad(3)\\B'Y+DZ&=I\quad(4)\end{aligned}$$

By (2) and (3), we have

$$\begin{aligned}X'&=-D^{-1}\left(B'W\right)\\Y&=-A^{-1}\left(CZ\right)\end{aligned}$$

Substitue this into (1)

$$\begin{aligned}AW-CD^{-1}\left(B'W\right)&=I\\\left(A-CD^{-1}B'\right)W&=I\\\left(A-CD^{-1}B'\right)^{-1}&=W.\end{aligned}$$

and

$$X'=-D^{-1}B'\left(A-CD^{-1}B'\right)^{-1}.$$

Similarly, in (4) we have

$$\begin{aligned}
-B'A^{-1}(CZ) + DZ &= I \\
\left(D - B'A^{-1}C\right) Z &= I \\
\left(D - B'A^{-1}C\right)^{-1} &= Z.
\end{aligned}$$

and

$$Y = -A^{-1}C\left(D - B'A^{-1}C\right)^{-1}.$$

(b) By the definition of inverse matrix, if

$$AA^{-1} = I,$$

then

$$A^{-1}A = I.$$

(c) Using the relationship implied in (b), we have

$$\begin{aligned}
WA + YB' &= I \\
X'C + ZD &= I,
\end{aligned}$$

which implies

$$\begin{aligned}
W &= A^{-1} - YB'A^{-1} \\
Z &= D^{-1} - X'CD^{-1}.
\end{aligned}$$

Using the results in (1), we have

$$Y = -A^{-1}C\left(D - B'A^{-1}C\right)^{-1},$$

therefore,

$$W = A^{-1} + A^{-1}C\left(D - B'A^{-1}C\right)^{-1} B'A^{-1}.$$

Similarly, the results in (1) implies

$$X' = -D^{-1}B'\left(A - CD^{-1}B'\right)^{-1},$$

and therefore

$$Z = D^{-1} + D^{-1}B'\left(A - CD^{-1}B'\right)^{-1} CD^{-1}.$$