OPTIMAL CAPITAL STRUCTURE UNDER CORPORATE AND PERSONAL TAXATION*

Harry DeANGELO

University of Washington, Seattle, WA 98195, USA

Ronald W MASULIS

University of California, Los Angeles, CA 90024, USA

Securities and Exchange Commission, Washington, DC 20549, USA

In this paper, a model of corporate leverage choice is formulated in which corporate and differential personal taxes exist and supply side adjustments by firms enter into the determination of equilibrium relative prices of debt and equity The presence of corporate tax shield substitutes for debt such as accounting depreciation, depletion allowances, and investment tax credits is shown to imply a market equilibrium in which each firm has a unique interior optimum leverage decision (with or without leverage-related costs) The optimal leverage model yields a number of interesting predictions regarding cross-sectional and time-series properties of firms' capital structures Extant evidence bearing on these predictions is examined

1. Introduction

In his recent 'Debt and Taxes', Merton Miller (1977) argues that in a world of differential personal taxes, (i) the marginal personal tax disadvantage of debt combined with (ii) supply side adjustments by firms will override the corporate tax advantage of debt and drive market prices to an equilibrium implying leverage irrelevancy to any given firm Miller's pathbreaking contribution raises the following interesting questions

> Do (i) and (ii) continue to imply leverage irrelevancy under more realistic assumptions about the corporate tax code or in the presence of

bankruptcy, agency, or other leverage-related costs? Alternatively, will these conditions imply a unique interior optimum leverage decision for each firm?

Can Miller's model be generalized to yield testable hypotheses regarding the determinants of firm and industry level leverage structures?

In this paper, we extend Miller's analysis to address these questions We show that Miller's irrelevancy theorem is extremely sensitive to realistic and simple modifications in the corporate tax code Specifically, the existence of non-debt corporate tax shields such as depreciation deductions or investment tax credits is sufficient to overturn the leverage irrelevancy theorem In our model, these realistic tax code features imply a unique interior optimum leverage decision for each firm in market equilibrium after all supply side adjustments are taken into account Importantly, the existence of a unique interior optimum does not require the introduction of bankruptcy, agency, or other leverage-related costs On the other hand, with any of these leverage costs present, each firm will also have a unique interior optimum capital structure regardless of whether non-debt shields are available Moreover, market prices will capitalize personal and corporate taxes in such a way as to make bankruptcy costs a significant consideration in a tax benefit–leverage cost tradeoff This last point is of critical interest because it mitigates Miller's 'horse and rabbit stew' criticism of tax benefit–leverage cost models of optimal capital structure

Our model also yields a number of testable hypotheses regarding the cross-sectional and time-series properties of firms' capital structures Most interestingly, our model predicts that firms will select a level of debt which is negatively related to the (relatively easily measured) level of available tax shield substitutes for debt such as depreciation deductions or investment tax credits We conclude the paper with a brief survey of existing empirical evidence which is relevant to the theories developed in earlier sections

2. Elements of the model

We employ a two-date state-preference model in which firms make leverage decisions and individuals make portfolio decisions at $t=0$ before the true state of nature prevailing at $t=1$ is known At $t=0$, value-maximizing firms package their state-contingent $t=1$ earnings into debt and equity vectors of state-contingent before personal tax dollars for sale to individuals The corporate tax code treats debt charges as deductible in calculating the corporate tax bill Firms also have deductible non-cash charges (e g accounting depreciation and depletion allowances) as well as tax credits The personal tax code is heterogeneous in that applicable personal tax rates differ

both across debt and equity income across investors For a given investor, the personal tax code treats equity income more favorably than debt income At $t=0$, utility maximizing investors select portfolios of firms' debt and equity securities which are optimal for their risk-preferences and personal tax status

2 1 Aggregate demand for debt and equity under differential personal taxation

For simplicity, we assume the following heterogeneous personal tax code [1] For each investor ι, let τ_{PD}^ι and τ_{PE}^ι represent constant marginal personal tax rates on debt and equity income The personal tax code is equity biased as the tax rate on debt income exceeds that on equity income $\tau_{PD}^\iota > \tau_{PE}^\iota \geqq 0$ for all investors Equivalently stated, debt and equity are differentially taxed so that the state-contingent after-personal tax cash flow per unit of state s equity income exceeds that per unit of state s debt income $(1-\tau_{PE}^\iota) > (1-\tau_{PD}^\iota)$ for all ι

Let $P_D(s)$ and $P_E(s)$ denote the current (time $t=0$) market prices per unit (per $t=1$ dollar) of before personal tax debt and equity income to be delivered in state s [2] Define $(1-\tau_{PD}^\iota)/P_D(s)$ and $(1-\tau_{PE}^\iota)/P_E(s)$ as individual ι's after-personal tax yields on state s debt and equity Under our proportional tax code, utility maximization requires investor ι to adjust his holdings of state s debt and equity claims to maximize his portfolio's after-personal tax yield As a consequence, investor ι will choose to hold state s debt over equity if $(1-\tau_{PD}^\iota)/P_D(s) > (1-\tau_{PE}^\iota)/P_E(s)$ Similarly, investor ι will choose to hold state s equity over debt if $(1-\tau_{PD}^\iota)/P_D(s) < (1-\tau_{PE}^\iota)/P_E(s)$ And investor ι will be indifferent to holding either state s debt or equity if after-personal tax yields are equal

Let τ_c denote the cross-sectionally constant corporate tax rate For simplicity, we assume that investors are differentially taxed so that at least

[1]Many other (simple and complex) personal tax codes imply demand curves as in figs 1 and 2 and thus would also suffice for our conclusions See DeAngelo–Masulis (1980) for a discussion of these alternative personal tax codes and associated clientele effects

[2]In the standard (i e , no differential personal tax) model, the single price law of markets requires $P_D(s)=P_E(s)$ for market equilibrium because all state s claims (regardless of whether they are labelled debt, equity, etc) are perfect substitutes to all investors In this standard case, $P_D(s)\neq P_E(s)$ is inconsistent with equilibrium because no one will be willing to hold the higher priced asset or, if unlimited shorting is possible, a pure arbitrage wealth pump will be available However, with differential personal taxes, debt and equity are no longer perfect substitutes to all investors the debt or equity labelling implies different tax treatment for different investors This assumes, of course, that personal tax arbitrage schemes cannot be devised to remove the differential treatment Thus, $P_D(s)\neq P_E(s)$ is perfectly consistent with (and will generally obtain in) market equilibrium Our usage of the state-preference pricing rule with differential personal taxation is analytically identical to the usage in Litzenberger–Van Horne (1977) and DeAngelo–Masulis (1980)

one investor is in each of the following mutually exclusive and exhaustive personal tax brackets [3]

Bracket B 1

$$(1 - \tau_{PD}^i) > (1 - \tau_{PE}^i)(1 - \tau_c),$$

Bracket B 2

$$(1 - \tau_{PD}^i) = (1 - \tau_{PE}^i)(1 - \tau_c),$$

Bracket B 3

$$(1 - \tau_{PD}^i) < (1 - \tau_{PE}^i)(1 - \tau_c)$$

The effect of differential personal taxes on the aggregate demand for debt is most easily understood for the special case in which all individuals in the economy are risk-neutral and believe that each state s will occur with probability $\pi(s)$ Let $\bar{P}_E$ and $\bar{P}_D$ be the current market prices of before-personal tax expected equity and debt cash flow In this case, the markets for state-contingent equity claims must set prices to equate all before personal tax *expected* yields so that $\pi(s)/P_E(s) = 1/\bar{P}_E$ for all s [4] Similarly, before personal tax expected yields on all debt claims must be equated so that $\pi(s)/P_D(s) = 1/\bar{P}_D$ for all s Thus, with homogeneous beliefs and risk neutrality, we can examine individuals' debt–equity demand decisions and firms' debt–equity supply decisions by examining only two markets one for before personal tax *expected* cash flow to equity (with current unit price $\bar{P}_E$) and one for before personal tax *expected* cash flow to debt (with current unit price $\bar{P}_D$)

Market prices $\bar{P}_D$ and $\bar{P}_E$ will establish marginal investors denoted by μ, with tax rates τ_{PD}^μ and τ_{PE}^μ, for whom after-personal tax expected yields on debt and equity are equated

$$\frac{(1 - \tau_{PD}^\mu)\pi(s)}{P_D(s)} = \frac{(1 - \tau_{PD}^\mu)}{\bar{P}_D} = \frac{(1 - \tau_{PE}^\mu)}{\bar{P}_E} = \frac{(1 - \tau_{PE}^\mu)\pi(s)}{P_E(s)} \quad \text{for all } s$$

By definition, these marginal investors are indifferent between taking their next dollar of income in the form of debt or equity As noted in figs 1 and 2,

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[3]The easiest way to interpret this simple proportional tax code is to consider Miller's special case in which equity income is not taxed $\tau_{PE}^i = 0$ for all i Bracket B 1 investors have relatively low personal tax rates on debt income, B 2 investors have intermediate tax rates, and B 3 investors have relatively high tax rates — B 1 $\tau_{PD}^i < \tau_c$, B 2 $\tau_{PD}^i = \tau_c$, B 3 $\tau_{PD}^i > \tau_c$

[4]Among all state-contingent equity securities, every risk-neutral investor whose tax bracket dictates an equity purchase will plunge in the equity claim with the highest *expected* yield Since all claims must be held in equilibrium, they must be priced to have the same expected yield As noted in footnote 2, $1/\bar{P}_E \neq 1/\bar{P}_D$ is perfectly consistent with equilibrium

for prices $\bar{P}_D \geqq \bar{P}_E$, zero units of expected before personal tax debt cash flow are demanded because the after tax expected yield on equity exceeds that on debt for all investors For these prices, no marginal investors exist For relative prices $\bar{P}_E > \bar{P}_D > \bar{P}_E (1 - \tau_c)$, positive quantities of expected debt cash flow will be demanded by investors in bracket B 1 and associated marginal tax rates satisfy $(1 - \tau_{PD}^\mu) > (1 - \tau_{PE}^\mu)(1 - \tau_c)$ As $\bar{P}_D$ is lowered relative to $\bar{P}_E$, larger quantities of before personal tax expected debt cash flow are demanded in the aggregate and implied marginal personal tax rates change correspondingly At prices $\bar{P}_D = \bar{P}_E (1 - \tau_c)$, investors in bracket B 1 demand only debt and the marginal investors who are indifferent between expected equity and expected debt income are in bracket B 2 with tax rates $(1 - \tau_{PD}^\mu)$ $= (1 - \tau_{PE}^\mu)(1 - \tau_c)$ And $\bar{P}_D < \bar{P}_E (1 - \tau_c)$ implies that investors in brackets B 1 and B 2 demand only debt and marginal investors are in bracket B 3 with $(1 - \tau_{PD}^\mu) < (1 - \tau_{PE}^\mu)(1 - \tau_c)$ Introduction of the aggregate debt curve in figs 1 and 2 endogenously determines the market-clearing relative prices and the tax bracket of marginal investors

2 2 Firm valuation under differential personal taxation

In this section, we examine firms' leverage choice problems and lay the groundwork for later sections' derivations of aggregate supply curves

To understand how leverage affects firm value, we must first characterize the effects of leverage on before-personal tax cash flows to debt and equity [5] For a given firm, define the following (state-contingent where noted) variables

$X(s) \equiv$ state s earnings before interest and taxes,

$B \equiv$ face value of debt which is assumed fully deductible in calculating the corporate tax bill (capital structure decision variable),

$\Delta \equiv$ corporate tax deductions resulting from non-cash charges such as accounting depreciation,

$\Gamma \equiv$ dollar value of tax credits,

$\tau_c \equiv$ statutory marginal corporate tax rate,

$\theta \equiv$ statutory maximum fraction of gross tax liability which can be shielded by tax credits [6]

For notational simplicity, let earnings $X(s)$ be monotone increasing in s over the set of possible states $[0, \bar{s}]$ with $0 \leqq X(0) \leqq X(\bar{s}) < \infty$ Segment $[0, \bar{s}]$

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[5] To emphasize that the interaction of corporate and personal tax code provisions alone implies a unique interior optimum leverage decision in our model, we do not consider default costs until section 5

[6] For the investment tax credit, Congress has specified $\theta = 0 5$ (with some exceptions for public utilities, airlines, and railroads) but recently raised θ to 0 9

into sub-intervals so that the state-contingent before personal tax, but after corporate tax cash flows to debt and equity, $D(s)$ and $E(s)$, are given by [7]

$D(s)$	$E(s)$	State outcome
$X(s)$	0	for $s \in [0, s^1)$
B	$X(s) - B$	for $s \in [s^1, s^2]$
B	$X(s) - B - \tau_c(X(s) - \Delta - B) + \theta\tau_c(X(s) - \Delta - B)$	for $s \in [s^2, s^3]$
B	$X(s) - B - \tau_c(X(s) - \Delta - B) + \Gamma$	for $s \in [s^3, \bar{s}]$

Here, s^1 denotes the state in which earnings just cover debt charges For earnings realizations in the state interval $s \in [0, s^1)$, the firm is in default, all of the firm's earnings are paid to debtholders, the corporate tax bill is zero, and all corporate tax deductions in excess of earnings are unutilized as are all tax credits

For $s \in [s^1, \bar{s}]$, no default occurs, the residual component of the firm's earnings is paid to equityholders, and the corporate tax bill can be zero or positive The extent to which corporate taxes are paid over $[s^1, \bar{s}]$ is state-contingent and depends on earnings $X(s)$, the face value of debt B, and tax shield substitutes for debt (Δ and Γ) Accordingly, s^2 denotes the state in which the corporate tax bill is just driven to zero It follows that, for $s \in [s^1, s^2]$, the corporate tax bill is zero because corporate tax deductions exceed earnings with a consequent loss of excess deductions and all tax credits Similarly, s^3 denotes the state in which all deductions and credits are just fully utilized For $s \in (s^2, s^3)$, the corporate tax bill is positive and deductions are fully utilized but credits are only partially utilized due to the statutory ceiling limiting usable credits to a fraction, θ, of the gross tax liability $(= \tau_c(X(s) - B - \Delta))$ For $s \in [s^3, \bar{s}]$, the corporate tax bill is positive and all deductions and credits are fully utilized Most importantly for our model, notice that for $s \in [s^1, s^3)$, corporate tax shields are lost to the firm even though no default occurs

The current market value of the firm is $V = D + E$ where D and E are the current market valuations (at prices $\{P_D(s), P_E(s)\}$) of the vectors of state-

contingent *before*-personal tax cash flows to debt $\{D(s)\}$ and to equity $\{E(s)\}$

$$D = \int_0^{\bar{s}} D(s) P_D(s)\,ds = \int_{s^1}^{\bar{s}} B P_D(s)\,ds + \int_0^{s^1} X(s) P_D(s)\,ds,$$

$$E = \int_0^{\bar{s}} E(s) P_E(s)\,ds = \int_{s^3}^{\bar{s}} \{X(s) - B - \tau_c(X(s) - \varDelta - B) + \varGamma\} P_E(s)\,ds,$$

$$+ \int_{s^2}^{s^3} \{X(s) - B - (1-\theta)\tau_c(X(s) - \varDelta - B)\} P_E(s)\,ds$$

$$+ \int_{s^1}^{s^2} \{X(s) - B\} P_E(s)\,ds$$

The firm's optimal leverage decision maximizes the current market value of the firm $V = D + E$[8] To see how alternative leverage decisions affect firm value, calculate the marginal value of debt financing $\partial V/\partial B$ (noting that terms involving the limits of integration vanish),

$$\partial V/\partial B = \int_{s^3}^{\bar{s}} \{P_D(s) - P_E(s)(1-\tau_c)\}\,ds$$

$$+ \int_{s^2}^{s^3} \{P_D(s) - P_E(s)(1-\tau_c(1-\theta))\}\,ds$$

$$+ \int_{s^1}^{s^2} \{P_D(s) - P_E(s)\}\,ds \tag{1}$$

Inspecting (1), we see immediately that the presence of corporate tax shield substitutes for debt ($\varDelta, \varGamma > 0$) implies that the leverage decision is necessarily relevant to the firm The leverage decision is irrelevant if and only if $\partial V/\partial B = 0$ for all feasible decisions B But with $\varDelta > 0$ and/or $\varGamma > 0$, it is impossible for $\partial V/\partial B$ to vanish identically for all B so that at least some leverage decisions are strictly preferred to others

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[8]Schneller (forthcoming) asserts that, in a world of differential personal taxation, owners will generally disagree on optimal firm decisions and therefore value maximization is not the proper corporate goal Contrary to Schneller's claim, the logic of the Fisher Separation Theorem [see DeAngelo (1980)] continues to apply under differential personal taxation To see why, note first that in the competitive economy formalized above, a given firm cannot affect the economy's risk-sharing capabilities Both debt and equity markets are complete so that investors' diversification and personal tax attribute demands are fully satisfied Moreover, market prices $\{P_D(s), P_E(s)\}$ are perceived as independent of the decisions of a given firm It follows that a given firm's financing and investment decisions impact on pre-exchange owners' consumption opportunities only through their effect on personal wealth In the absence of technological externalities, the maximization of firm value simultaneously maximizes the wealth and therefore the consumption opportunities and utility of every pre-exchange owner

To better understand the marginal value of debt expression (1), again consider the special case in which all investors are risk-neutral with homogeneous beliefs As shown in section 2 1, risk-neutral valuation implies $P_D(s)=\bar{P}_D\pi(s)$ and $P_E(s)=\bar{P}_E\pi(s)$ for all s where $\bar{P}_D$ and $\bar{P}_E$ are the market prices of before-personal tax *expected* cash flow to debt and equity The marginal value of debt then reduces to the easily interpreted expression

$$\partial V/\partial B = \{\bar{P}_D - \bar{P}_E(1-\tau_c)\} \int_{s^3}^{\bar{s}} \pi(s)\,ds$$

$$+ \{\bar{P}_D - \bar{P}_E(1-\tau_c(1-\theta))\} \int_{s^2}^{s^3} \pi(s)\,ds$$

$$+ \{\bar{P}_D - \bar{P}_E\} \int_{s^1}^{s^2} \pi(s)\,ds \qquad (2)$$

In (2), $\partial V/\partial B$ is the present value of the expected marginal after-corporate (but before personal) tax cash flow resulting from a substitution of one more promised dollar of debt for equity This marginal present value can be decomposed into three components which depend on the extent to which the corporate tax deduction from the marginal unit of debt is utilized The first component, $\bar{P}_D - \bar{P}_E(1-\tau_c)$, is the present value of the debt for equity substitution given full utilization of the corporate tax deduction associated with the marginal unit of debt The first integral in (2) is the probability of full utilization of the marginal debt deduction The second component, $\bar{P}_D - \bar{P}_E(1-\tau_c(1-\theta))$, represents a lower present value due to partial loss of the corporate tax shield caused by the statutory θ-ceiling on usable tax credits The second integral is the probability of partial loss of the marginal corporate tax shield due to the θ-ceiling Similarly, the third component, $\bar{P}_D - \bar{P}_E$, is the present value of the debt for equity substitution given total loss of marginal corporate tax shield because available deductions already shelter all earnings The third integral is the probability of total loss of the corporate tax deduction on the marginal unit of debt Eq (2) does not include a default probability term because additional promised payments to debt have no cash flow (or corporate tax) impact in default states, over the default range, all earnings are already paid to debtholders and no further debt for equity cash flow substitution can occur

3. Miller's leverage irrelevancy theorem

To highlight the difference between Miller's model and ours, we first characterize market equilibrium for a world analytically similar to Miller's and derive his leverage irrelevancy result Given no corporate tax shield

substitutes for debt,[9] partial or total loss of the marginal corporate tax shield benefits of debt never occur Technically, with $\Delta = \Gamma = 0$, we have $s^1 = s^2 = s^3$ and $\partial V / \partial B$ reduces to the first term in (2),

$$\partial V / \partial B = \{\bar{P}_D - \bar{P}_E (1 - \tau_c)\} \int_{s^1}^{\bar{s}} \pi(s) \, ds \tag{3}$$

From (3), we can derive the aggregate debt supply curve. If relative prices satisfy $\bar{P}_D < \bar{P}_E (1 - \tau_c)$, then $\partial V / \partial B < 0$ for all feasible leverage decisions and the firm selects an all equity capital structure If $\bar{P}_D > \bar{P}_E (1 - \tau_c)$, then $\partial V / \partial B > 0$ for all B and an all debt capital structure is uniquely optimal If $\bar{P}_D = \bar{P}_E (1 - \tau_c)$, the firm is indifferent among all feasible debt–equity packages of earnings (i e , riskless and risky debt) so that the supply curve is perfectly elastic over the entire feasible leverage range Since the above analysis applies to all firms, the aggregate debt supply curve (the sum of all firms' supply curves) is also perfectly elastic at relative prices $\bar{P}_D = \bar{P}_E (1 - \tau_c)$ as shown in fig 1
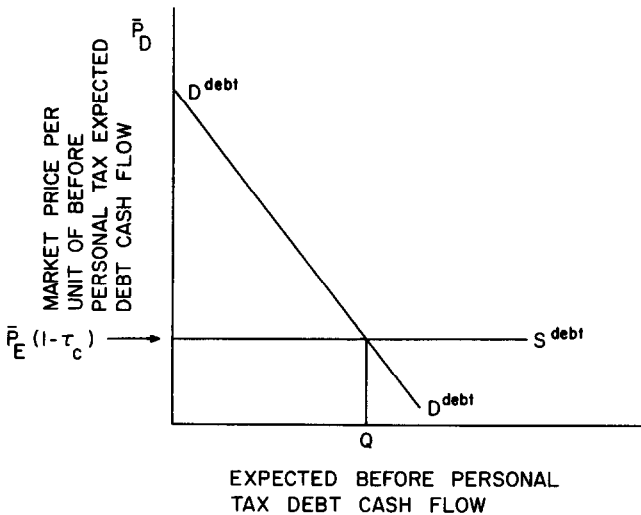
Fig 1 Market equilibrium in a 'debt and taxes' world, D^{debt} = aggregate demand curve, S^{debt} = aggregate supply curve, Q = equilibrium aggregate quantity of debt, $\bar{P}_D = \bar{P}_E (1 - \tau_c)$ = equilibrium debt price

[9] We continue to assume that bankruptcy costs are absent This assumption is critical for Miller's irrelevancy result (see section 5) Also, see DeAngelo–Masulis (1980) for a more complete discussion of the conditions which lead to the irrelevancy theorem

Given the heterogeneous personal tax code of section 2 1, the downward-sloping aggregate debt demand curve must intersect the aggregate supply curve in the perfectly elastic range at relative prices $\bar{P}_D = \bar{P}_E(1-\tau_c)$ Thus, the leverage decision is irrelevant to the individual firm facing market equilibrium prices

On the demand side, in market equilibrium, the marginal investors must be in bracket B 2 As explained in section 2 1, marginal investors are those for whom after-personal tax expected yields on debt and equity are equated at current prices $(1-\tau_{PD}^\mu)/\bar{P}_D = (1-\tau_{PE}^\mu)/\bar{P}_E$ At market equilibrium prices $\bar{P}_D = \bar{P}_E(1-\tau_c)$, marginal investors are in bracket B 2 with tax rates satisfying $(1-\tau_{PD}^\mu) = (1-\tau_{PE}^\mu)(1-\tau_c)$

The duality between equilibrium relative market prices and equilibrium relative marginal personal tax rates yields an intuitive interpretation of market equilibrium Formally, substitute the marginal investors' tax rate condition $\bar{P}_E = \bar{P}_D(1-\tau_{PE}^\mu)/(1-\tau_{PD}^\mu)$ into the marginal value of debt expression (3),[10]

$$\partial V/\partial B = \frac{\bar{P}_D}{1-\tau_{PD}^\mu}\left[\{(1-\tau_{PD}^\mu)-(1-\tau_{PE}^\mu)(1-\tau_c)\} \int_{s^1}^{\bar{s}} \pi(s)\,ds \right] \qquad (4)$$

In equilibrium, the bracketed term is zero as noted above Intuitively, the market endogenously determines the relative marginal personal tax rates on debt and equity so that the personal tax advantage of equity exactly offsets the corporate tax advantage of debt over the entire feasible leverage range In Miller's special case in which equity income is not taxed ($\tau_{PE}^\mu = \tau_{PE}^r = 0$ for all i), we have $\tau_c = \tau_{PD}^\mu$ which says that the marginal corporate tax advantage exactly equals the personal tax disadvantage of debt

4. Tax shield substitutes for debt and interior optimum leverage

With positive corporate tax shield substitutes for debt ($\Delta, \Gamma > 0$), Miller's firm level leverage irrelevancy conclusion no longer holds Instead, relative

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[10]Miller's (1977, p 267) gains from leverage expression can be derived from (4) quite easily For riskless debt, the probability of default is zero ($s^1 = 0$) and the integral in (4) equals one Also, for riskless debt, the present market value of debt $D = \bar{P}_D B$ Combining these two facts, (4) may be rewritten as Miller's expression,

$$\partial V/\partial D = [1 - ((1-\tau_{PE}^\mu)(1-\tau_c))/(1-\tau_{PD}^\mu)]$$

It is worth noting that if firms are constrained to issue only riskless debt, then $\bar{P}_D = \bar{P}_E(1-\tau_c)$ need not hold in equilibrium so that leverage can be relevant to individual firms See DeAngelo-Masulis (1980) for an explanation of how constraints on firms' supply adjustment capabilities can overturn the leverage irrelevancy result It is also worth noting that cross-sectional variation in corporate tax rates [as in Black (1971)] implies leverage relevancy to individual firms and corporate capital structure clienteles (firms with relatively high corporate tax rates will be highly levered, etc)

market prices will adjust until in market equilibrium, each firm has a unique interior optimum leverage decision This unique interior optimum exists because there is a *constant* expected marginal *personal* tax disadvantage to debt while positive tax shield substitutes imply that the expected marginal *corporate* tax benefit *declines* as leverage is added to the capital structure [11] At the unique optimum, the expected marginal corporate tax benefit just equals the expected marginal personal tax disadvantage of debt

We begin, as with Miller's irrelevancy theorem by deriving the aggregate supply curve for before-personal tax expected debt cash flow With corporate tax shield substitutes for debt, each firm's debt supply curve and therefore the aggregate debt supply curve will have a brief perfectly elastic section and then be smoothly upward sloping as shown in fig 2 The perfectly elastic
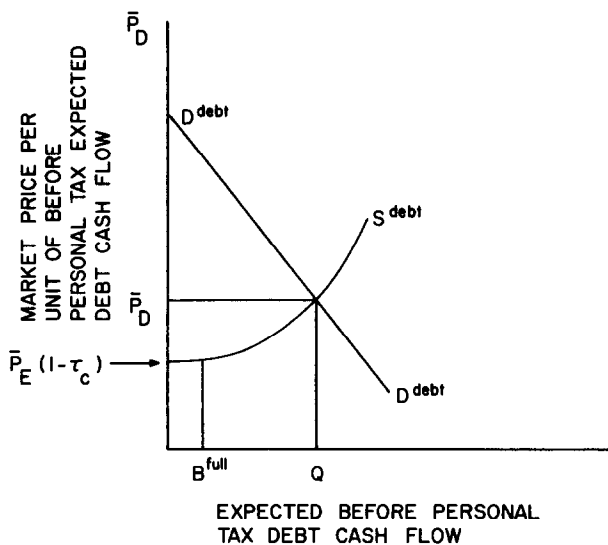
Fig 2 Market equilibrium with tax shield substitutes for debt, D^{debt} = aggregate demand curve, S^{debt} = aggregate supply curve, Q = equilibrium aggregate quantity of debt, $\bar{P}_D > \bar{P}_E(1 - \tau_c)$ is the equilibrium debt price, B^{full} = aggregate quantity of debt supplied when all firms are at the maximum debt level allowing full utilization of all corporate tax shields

[11]Strictly speaking, the marginal personal tax disadvantage is constant, independent of B, only for risk-free debt The unique interior optimum still obtains with risky debt, but now the intuitive explanation is that the expected marginal corporate tax benefit declines more rapidly than the expected marginal personal tax disadvantage declines See footnotes 13 and 17 Our earlier working paper established an unique interior optimum leverage without the assumption of risk neutrality, homogeneous beliefs, and conventional debt contracts

section occurs at relative prices $\bar{P}_D = \bar{P}_E(1 - \tau_c)$ and extends only over those low levels of leverage which allow all corporate tax shields (Δ, Γ, and B) to be fully utilized in every state of nature Beyond this full utilization level, the supply curve is upward sloping because firms are willing to supply more debt to the market only if they are compensated [with higher unit debt prices $\bar{P}_D > \bar{P}_E(1 - \tau_c)$] for the increased probability of partial/total loss of the corporate tax shield associated with additional debt For any given set of relative prices on this upward sloping section of the supply curve, each firm has a unique interior optimum leverage decision And under weak assumptions about the personal tax code, the aggregate demand curve intersects the aggregate supply curve in the upward sloping section at relative prices $\bar{P}_E > \bar{P}_D > \bar{P}_E(1 - \tau_c)$ which dictates the unique interior optimum for each firm

To derive the supply curve formally, notice first that $\Delta, \Gamma > 0$ imply $s^3 > s^2 > s^1 \geq 0$ so that the marginal value of debt is once again given by the general expression (2) If relative prices satisfy $\bar{P}_D < \bar{P}_E(1 - \tau_c)$, then as in Miller's world, $\partial V / \partial B < 0$ everywhere and no firm will supply any debt At relative prices $\bar{P}_D = \bar{P}_E(1 - \tau_c)$, there is a brief perfectly elastic section of the supply curve which extends only to the point where the selected debt level just results in full utilization of all corporate tax shields (Δ, Γ, and the selected B) in every state of nature Full utilization of the tax credit Γ in every state requires that leverage B be set low enough that θ times the resulting gross tax liability is always greater than or equal to Γ,

$$\theta \tau_c (X(s) - \Delta - B) \geq \theta \tau_c (X(0) - \Delta - B) \geq \Gamma \quad \text{for all} \quad s$$

Since $X(0)$ is the lowest possible earnings, the maximum promised debt level or maximum leverage, B^{full}, which is consistent with full utilization of all corporate tax shields in every state is[12]

$$B^{full} \equiv X(0) - \Delta - \Gamma / \theta \tau_c < X(0)$$

Technically, for leverage B in the range $0 \leq B \leq B^{full}$, $s^1 = s^2 = s^3 = 0$, because default risk is zero and all corporate tax shields (Δ, Γ, and B) are fully utilized in every state of nature Over this range (2) reduces to

$$\partial V / \partial B = \bar{P}_D - \bar{P}_E(1 - \tau_c) \quad \text{for} \quad 0 \leq B \leq B^{full} < X(0)$$

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[12]We assume $X(0) - \Delta - \Gamma / \theta \tau_c > 0$ for at least some firms If the reverse inequality holds, firms cannot issue any debt without risking loss of corporate tax shield In this case the aggregate debt supply curve would be upward sloping everywhere [there would not be a perfectly elastic section at $\bar{P}_D = \bar{P}_E(1 - \tau_c)$]

Thus, when relative prices satisfy $\bar{P}_D = \bar{P}_E(1 - \tau_c)$, the firm is indifferent among all leverage decisions which allow full utilization of all corporate tax sheild and its supply curve is perfectly elastic over the debt range $0 \leq B \leq B^{\text{full}}$ Correspondingly, at $\bar{P}_D = \bar{P}_E(1 - \tau_c)$, the aggregate supply curve is perfectly elastic until the quantity at which all firms have reached their full utilization debt levels

For higher debt levels B in the range $B^{\text{full}} < B \leq X(0)$, debt is still riskless but some corporate tax shield is lost in low earnings states Over this leverage range, $s^1 = 0$ but $s^3 > s^2 \geq 0$ so that, at relative prices $\bar{P}_D = \bar{P}_E(1 - \tau_c)$, the first term in (2) vanishes and the sum of the second and third terms is strictly negative In other words, $\partial V/\partial B < 0$ and no debt will be supplied beyond B^{full} at relative prices $\bar{P}_D = \bar{P}_E(1 - \tau_c)$

Therefore, to induce a supply of debt greater than B^{full}, a higher unit price $\bar{P}_D > \bar{P}_E(1 - \tau_c)$ must be paid This higher debt price is required to compensate for the loss of corporate tax shield which will occur on marginal units of debt in states of the world $[s^1, s^3]$ For any given set of prices $\bar{P}_E > \bar{P}_D > \bar{P}_E(1 - \tau_c)$, each firm has a unique interior optimum leverage decision B^* which solves the first-order condition $\partial V/\partial B = 0$ [13] As $\bar{P}_D$ is raised above $\bar{P}_E(1 - \tau_c)$ over the range $\bar{P}_E > \bar{P}_D > \bar{P}_E(1 - \tau_c)$, each firm's optimum capital structure involves more and more debt so that the aggregate supply curve is indeed smoothly upward sloping over this price range [14]

Under reasonable conditions, market equilibrium will occur along the upward sloping portion of the debt supply curve The perfectly elastic section of the supply curve is relatively short because, at $\bar{P}_D = \bar{P}_E(1 - \tau_c)$, firms are willing to supply only the 'safest' of riskless debt they will issue debt only up

[13] With $\bar{P}_E > \bar{P}_D > \bar{P}_E(1 - \tau_c)$, it is straightforward to show that the right-hand derivative $(\partial V/\partial B)[B = 0] > 0$ and the left-hand derivative $(\partial V/\partial B)[B = X(\bar{s})] < 0$ so that interior leverage decisions strictly dominate corner solutions A unique interior optimum exists if V is convex in B, $(\partial^2 V/\partial B^2 < 0)$ Differentiating (2) yields

$$\partial^2 V/\partial B^2 = -\theta\tau_c\bar{P}_E(\partial s^3/\partial B)\pi(s^3) - (1 - \theta)\tau_c\bar{P}_E(\partial s^2/\partial B)\pi(s^2) - (\bar{P}_D - \bar{P}_E)(\partial s^1/\partial B)\pi(s^1),$$

where s^1, s^2, and s^3 are defined in footnote 7 The first two terms are strictly negative For riskless debt, $\partial s^1/\partial B = 0$ so that the third term vanishes and $\partial^2 V/\partial B^2 < 0$ necessarily follows For risky debt, $\partial s^1/\partial B > 0$ so that the third term is positive In this case, we assume the first two terms dominate the third [which they will, e g, if $\pi(s^3) = \pi(s^2) = \pi(s^1)$ and $\partial s^1/\partial B = \partial s^2/\partial B = \partial s^3/\partial B$]

Our argument for a unique interior optimum does not require that firms earnings be risky Under certainty (one state at $t = 1$ with earnings X), the firm's unique interior optimum leverage sets $B = X - \Delta - \Gamma/\theta\tau_c$ so that all corporate credits and deductions are fully utilized when $\bar{P}_D - \bar{P}_E(1 - \tau_c) > 0$ and $\bar{P}_D - \bar{P}_E(1 - \tau_c(1 - \theta)) < 0$

[14] Footnote 13 established the unique interior leverage optimum B^* which solves $(\partial V/\partial B)[B^*] = 0$ at a given set of relative prices $\bar{P}_E > \bar{P}_D > \bar{P}_E(1 - \tau_c)$ We wish to show that as $\bar{P}_D$ is raised (relative to fixed $\bar{P}_E$) over this range, larger quantities of debt will be supplied Define ECFD $\equiv$ expected before-personal tax debt cash flow supplied as debt Technically, the supply curve is upward sloping if

$$\partial(ECFD)/\partial\bar{P}_D = (\partial(ECFD)/\partial B)(\partial B^*/\partial\bar{P}_D) > 0$$

to the point where there is not only zero probability of default but also zero probability of losing any corporate tax shield More precisely, firms are willing to issue debt only over the relatively brief range $0 \leq B \leq B^{full}$ This requires not only riskless debt but also the stronger condition that earnings never fall so low as to cause any part of available corporate tax shields to be lost in any state of nature

Recalling the aggregate demand discussion of section 2 1, we know that investors in bracket B 1 will demand positive quantities of corporate debt at relative prices $\bar{P}_D = \bar{P}_E(1 - \tau_c)$ The greater the number of investors (and the amount of current wealth) in B 1, the larger the quantity of debt demanded at these prices We assume that investors in B 1 are sufficiently important in the market to demand a larger aggregate quantity of debt than B^{full} at $\bar{P}_D = \bar{P}_E(1 - \tau_c)$[15] In this case, aggregate quantity supplied will fall short of demand and relative prices must adjust to provide $\bar{P}_D > \bar{P}_E(1 - \tau_c)$ to equilibrate supply and demand In market equilibrium, these relative prices imply that each firm will have a unique interior optimum leverage decision and marginal investors will be in bracket B 1 with tax rates satisfying $(1 - \tau_{PD}^\mu) > (1 - \tau_{PE}^\mu)(1 - \tau_c)$

We can provide an intuitive interpretation of the trade-off which determines the firm's unique interior optimum leverage decision by invoking the duality relationship between relative market prices and relative marginal personal tax rates Since the analytical expressions are complicated in the general case, we will examine Miller's special case in which equity income is not taxed ($\tau_{PE}^\mu = \tau_{PE}^i = 0$ for all i) In this case, equilibrium prices $\bar{P}_D > \bar{P}_E(1 - \tau_c)$ imply $\tau_{PD}^\mu < \tau_c$ for the marginal investors Also for simplicity of interpretation, we assume that the firm's interior optimum leverage B^* occurs at a level implying risk-free debt ($B^{full} < B^* \leq X(0)$ so that $s^1 = 0$)

But this follows immediately because

$$\partial(ECFD)/\partial B = \int_{s^1}^{\bar{s}} \pi(s)\,ds > 0,$$

$$\partial B^*/\partial \bar{P}_D = (\partial^2 V/\partial \bar{P}_D \partial B)/(-\partial^2 V/\partial B^2) = \left(\int_{s^1}^{\bar{s}} \pi(s)\,ds\right) \bigg/ \left(-\partial^2 V/\partial B^2\right) > 0$$

Here, $\partial B^*/\partial \bar{P}_D$ is derived by setting (2) equal to zero and totally differentiating while allowing variation in B^* and $\bar{P}_D$

[15] If investors in B 1 are relatively unimportant in the market, then in the aggregate, quantity supplied = quantity demanded over the perfectly elastic range of the supply curve Each firm will be indifferent to leverage satisfying $0 \leq B \leq B^{full}$ and these capital structures will strictly dominate leverage satisfying $B > B^{full}$ This is an intuitively unappealing picture of equilibrium as it implies (i) no firm ever defaults on debt and (ii) no firm ever loses any corporate tax shield [Equilibrium at $\bar{P}_D > \bar{P}_E(1 - \tau_c)$ does not require (i) and (ii)] It seems reasonable that B 1 investors will be important in the market For example, in Miller's special case in which equity income is not taxed, B 1 investors have $\tau_{PD}^i < \tau_c$ In the U S, $\tau_c = 0\,48$ and a large number of investors have marginal tax rates on ordinary income $< 0\,48$

Substituting the simplified marginal personal tax rate condition $\bar{P}_E = \bar{P}_D/(1 - \tau_{PD}^\mu)$ into (2) and gathering terms yields the leverage optimality condition

$$\frac{\partial V}{\partial B}[B^*] = \frac{\bar{P}_D}{(1 - \tau_{PD}^\mu)}\left[\tau_c\left\{\int_{s^3}^{\bar{s}}\pi(s)\,ds + (1 - \theta)\int_{s^2}^{s^3}\pi(s)\,ds\right\} - \tau_{PD}^\mu\right] = 0,$$

(5)

At B^*, the term in square brackets equals zero which says that the expected marginal corporate tax benefit is equated to the expected marginal personal tax disadvantage of debt at the optimum The second term in square brackets, $-\tau_{PD}^\mu$, is the expected marginal *personal* tax disadvantage of debt since, for riskless debt, increasing B by one unit increases the personal tax liability by τ_{PD}^μ in every state of nature The first term in brackets is the expected marginal *corporate* tax saving from debt which reflects the fact that the corporate shield on the marginal unit of risk-free debt is fully lost in states $[0, s^2)$, partially lost in states $[s^2, s^3)$, and fully realized in states $[s^3, \bar{s}]$

Examining (5) more closely, we see that the presence of corporate tax shield substitutes for debt affects the extent to which the corporate tax shield from the marginal unit of debt is lost to the firm This loss of corporate tax shield results from properties of the tax code which rule out negative taxes or subsidies and thereby set a ceiling on the total use of tax shields (Δ, Γ, and B) which are potentially available to the firm [16] These ceilings ensure that the expected marginal *corporate* tax saving from additional debt declines as debt is added to the capital structure [17] But for each additional dollar of debt substituted for equity the same higher marginal *personal* tax on debt, rather than on equity income, must be paid Thus, for relatively low levels of leverage (less than B^*), the marginal value of debt is positive because there is a relatively high probability that

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[16]The loss of corporate tax shield which is relevant for our model is not triggered by default and, in fact, has nothing to do with risk per se our arguments go through even under certainty as noted in footnote 13 Brennan–Schwartz (1978) and Kim (1978) consider the default-related loss of corporate tax shield in the context of corporate tax–bankruptcy cost models (without differential personal taxes) See Miller (1977), Chen–Kim (1979), and DeAngelo–Masulis (1980) for a discussion of this phenomenon with differential personal taxes DeAngelo–Masulis demonstrate that the default-related loss of corporate tax shield has no effect on Miller's irrelevancy theorem (Also, see section 3 of this paper in which Miller's theorem was shown to hold even though corporate debt tax shelter is lost in default)

[17]Notice that the expected marginal corporate tax benefit from debt is strictly less than the statutory corporate tax rate τ_c To see that the expected marginal corporate tax benefit declines with higher leverage, differentiate the first term within the brackets in expression (5) with respect to B and note that

$$\tau_c\{-\theta\pi(s^3)(\partial s^3/\partial B) - (1 - \theta)\pi(s^2)(\partial s^2/\partial B)\} < 0$$

Notice that this equation is a positive scalar multiple of the second-order condition presented in footnote 13 because, for risk-free debt, $\partial s^1/\partial B = 0$

additional debt can be fully utilized to reduce the firm's tax liabilities and this corporate tax reduction outweighs the higher personal taxes paid on additional debt For relatively high levels of leverage (greater than B^*), the marginal value of debt is negative because the tax shield substitutes imply a relatively high probability that the potential corporate shield from additional debt will be partially or totally lost, while an additional personal tax liability for holding debt is incurred At the unique interior optimum B^*, the expected marginal corporate tax saving just balances the marginal personal tax disadvantage of additional debt

By using a one-period model, we have implicitly assumed away tax loss carrybacks and carryforwards (CB–CF) which could be introduced in a multi-period formulation From our one-period model, we can infer the likely effects CB–CF provisions would have on our predictions CB–CF provisions should reduce the impact of tax shield substitutes on the leverage decision More specifically, CB provisions reduce the probability that a corporate tax shield will be lost by allowing current tax losses to be applied immediately against several previous years' unsheltered taxable income Current tax losses can be large relative to previously unsheltered income so that CB provisions will not always result in full utilization of the current period's potential corporate tax shield [18] Similarly, CF provisions reduce the probability that corporate tax shield will be lost by allowing excess shields to be applied against future taxable income for several years However, this just shifts the problem forward since future leverage decisions will be affected by the existence of now greater future tax shield substitutes for debt Furthermore, the CF deferral inherently involves a time-value loss of corporate tax shield In sum, CB–CF provisions would reduce, but not eliminate, the expected value of the corporate tax shield loss on the marginal unit of debt

Thus, with CB–CF provisions, we expect that the expected marginal corporate tax savings of debt will still decline as leverage is added to the capital structure (but not as rapidly as in our one-period formulation) As a result, the debt supply curve will still be upward sloping beyond the debt level allowing full utilization of tax shield The supply curve will be more elastic than in our formulation (but not infinitely elastic) since firms will require smaller price compensation to increase their debt supply because CB–CF provisions reduce the expected loss of corporate tax shield Given a sufficiently large number of investors in bracket B 1, equilibrium should still obtain in the upward sloping region of the debt supply curve and each firm will have a unique interior optimum leverage decision (but with quantitatively higher leverage than our one-period formulation predicts) Most importantly, we expect that a multi-period formulation encompassing

[18]Moreover, utilizing tax loss carrybacks implies a previous loss of corporate tax shield because taxes were already paid in earlier periods Although a rebate is obtained in the current period, it does not include interest the government earned on the earlier tax payment

CB–CF provisions would leave our qualitative predictions unchanged (see section 6) [19]

5. Bankruptcy costs and horse-and-rabbit stew

In section 4, each firm's unique interior optimum leverage decision resulted solely from the interactions of the personal and corporate tax treatment of income Default-related costs were completely absent in that discussion [20] With positive default costs (and with or without tax shield substitutes for debt), each firm will still have a unique interior optimum leverage decision in market equilibrium Moreover, Miller's (1977, pp 262–264) horse-and-rabbit stew criticism of traditional corporate tax-default cost models is not applicable in this case [21] Miller faults the traditional models for requiring unrealistically large expected marginal bankruptcy costs to offset the expected marginal corporate tax savings of debt at observed debt–equity ratios In our model, regardless of whether default costs are large or small, the market's relative prices of debt and equity will adjust so that the net (corporate and marginal personal) tax advantage of debt is of the same order of magnitude as expected marginal default costs The relative prices must equilibrate in this way to induce firms to supply the proper quantities of debt and equity to satisfy the demands of investors

More concretely, assume that all tax shield substitutes are absent ($\Delta = \Gamma = 0$) and let $C[B, X(s)]$ denote state-contingent default costs where $C[B, X(s)] \equiv 0$ for no-default states $s \in [s^1, \bar{s}]$ and $C[B, X(s)] > 0$, $\partial C/\partial B > 0$ for $s \in [0, s^1)$ Then the marginal value of debt is easily shown to be

$$\partial V/\partial B = \{\bar{P}_D - \bar{P}_E(1 - \tau_c)\} \int_{s^1}^{\bar{s}} \pi(s)\,ds - \bar{P}_D \int_0^{s^1} \frac{\partial C}{\partial B} \pi(s)\,ds$$

$$= \frac{\bar{P}_D}{(1 - \tau_{PD}^\mu)} \{(1 - \tau_{PD}^\mu) - (1 - \tau_{PE}^\mu)(1 - \tau_c)\} \int_{s^1}^{\bar{s}} \pi(s)\,ds$$

$$- \bar{P}_D \int_0^{s^1} \frac{\partial C}{\partial B} \pi(s)\,ds$$

[19]We could also modify our model to introduce markets for transferring corporate tax shields Markets for leasing and acquisitions provide two obvious avenues for transferring non-debt tax shields from one firm to another Since firms have incentives to avoid excess corporate tax shields, they will be motivated to alter their level of leverage and/or their level of non-debt tax shields Assuming that markets do not allow costless transfer of tax losses, we would still expect a negative relationship between debt and non-debt tax shield (holding earnings constant)

[20]More generally, agency costs or other leverage-related costs were also ignored [see Jensen–Meckling (1976), Galai–Masulis (1976), and Myers (1977)] Formally introducing these costs into our framework would have the same effect as introducing default costs in equilibrium, each firm would have an interior optimum leverage decision and the endogenously determined net tax benefit of debt would be of the same order of magnitude as marginal agency or leverage-related costs

[21]Some indication that Miller was moving toward this conclusion can be found in 'Debt and Taxes' on p 271

In market equilibrium, relative prices will again satisfy $\bar{P}_E > \bar{P}_D > \bar{P}_E(1-\tau_c)$ In this case, the higher debt price $\bar{P}_D > \bar{P}_E(1-\tau_c)$ is required in market equilibrium as compensation for the expected marginal costs of default Equivalently stated, in market equilibrium there will be a net corporate — marginal personal tax advantage to debt financing $\{(1-\tau_{PD}^{\mu}) > 1(1-\tau_{PE}^{\mu})(1-\tau_c)\}$ which will compensate firms for the expected marginal default costs and thereby. induce them to supply risk debt In market equilibrium, each firm will have a unique interior optimum leverage decision which equates the present value of the expected marginal net tax advantage of debt to the present value of expected marginal default costs

Thus, even if expected marginal default costs are small relative to the *corporate* tax advantage of debt [such as the 5% average ex-post cost found by Warner (1977) for court costs and lawyers' fees associated with the bankruptcy proceedings of eleven railroads] they can still be significant relative to the net corporate–marginal personal tax advantage of debt $\{(1-\tau_{PD}^{\mu})-(1-\tau_{PE}^{\mu})(1-\tau_c)\} > 0$ Moreover, the market-determined marginal personal tax rates will adjust to increases in the supply of debt *so as to decrease the expected net tax advantage of debt to firms* In equilibrium, expected default costs equal the expected net tax advantage of debt In other words, expected marginal default costs are of significant magnitude in our model because the expected net (corporate and marginal personal) tax benefit of debt is *endogenously determined* by the interaction of supply and demand to be of the same order of magnitude as marginal expected default costs [22]

6. Testable hypotheses

We have demonstrated that each firm has a unique interior optimum capital structure in market equilibrium in a world characterized by (i) the equity-biased personal tax code of section 2 1 and (ii) corporate tax shield substitutes for debt and/or positive default costs

From this expanded model, we can derive the following cross-sectional and time-series predictions [23]

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[22]When both bankruptcy costs and corporate tax shield losses induced by tax shield substitutes are present, we cannot say that bankruptcy costs alone are significant measured relative to net tax savings We can only say that bankruptcy costs and tax losses taken together are significant

[23]H 1 and H 2 were derived earlier in the text Here we sketch the derivation of H 3 (similar calculations lead to H 4 and H 5) Let B^* denote the firm's unique interior optimum leverage decision for which $\partial V/\partial B = 0$ Totally differentiating this first-order condition, letting α denote a dummy parameter $(\Delta, \Gamma, \tau_c,$ marginal default-costs), and rearranging terms yields $\partial B^*/\partial \alpha = (\partial^2 V/\partial B \partial \alpha)/(-\partial^2 V/\partial B^2)$ Assuming the second-order condition $\partial^2 V/\partial B^2 < 0$ is satisfied, it follows that $\text{sign}(\partial B^*/\partial \alpha) = \text{sign}(\partial^2 V/\partial \alpha \partial B)$ To derive H 3, note that differentiating the general

H 1 The leverage decision is relevant to the individual firm in the sense that a pure change in debt (holding investment constant) will have a valuation impact

H 2 In equilibrium, relative market prices will imply a net (corporate and personal) tax advantage to corporate debt financing — i e, the implied marginal personal tax rates will satisfy $(1-\tau_{PD}^\mu)>(1-\tau_{PE}^\mu)(1-\tau_c)$ or, equivalently, $\tau_{PD}^\mu<\tau_c+\tau_{PE}^\mu(1-\tau_c)$

H 3 *Ceteris paribus*, decreases in allowable investment related tax shields (e g, depreciation deductions or investment tax credits) due to changes in the corporate tax code or due to changes in inflation which reduce the real value of tax shields will increase the amount of debt that firms employ In cross-sectional analysis, firms with lower investment related tax shields (holding before-tax earnings constant) will employ greater debt in their capital structures

H 4 *Ceteris paribus*, decreases in firms' marginal bankruptcy costs will increase the use of debt financing Cross-sectionally, firms subject to greater marginal bankruptcy costs will employ less debt

H 5 *Ceteris paribus*, as the corporate tax rate is raised, firms will substitute debt for equity financing Cross-sectionally, firms subject to lower corporate tax rates will employ less debt in their capital structures (holding earnings constant)

Hypotheses H 1 and H 2 are statements about the capital market pricing implications of our tax shield substitutes model Both are in direct conflict with the predictions of Miller's 'Debt and Taxes' model Obviously, H 1 predicts that a leverage change will affect the market value of the firm while Miller's model predicts the absence of any valuation impact H 2 predicts

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(i e, not necessarily risk-neutral) valuation expression (1) yields

$$V_{B\Delta} = -\theta\tau_c P_E(s^3)(\partial s^3/\partial\Delta) - \tau_c(1-\theta)P_E(s^2)(\partial s^2/\partial\Delta) < 0 \quad \text{implies} \quad \partial B^*/\partial\Delta < 0,$$

$$V_{B\Gamma} = -\theta\tau_c P_E(s^3)(\partial s^3/\partial\Gamma) < 0 \qquad\qquad\qquad \text{implies} \quad \partial B^*/\partial\Gamma < 0$$

Notice that H 3 was derived from our general valuation expression (1) while assuming that a unique interior optimum leverage exists Notice also that we hold investment fixed while allowing debt to vary in response to a parameter shift While this assumption is consistent with previous analyses of the capital structure decision [e g, see Scott (1976)], one should nevertheless recognize that potential interactions of investment and financing decisions can technically overturn H 3–H 5 when investment is not held fixed However, H 3–H 5 can be derived technically when allowing investment to change if we impose sign and/or magnitude assumptions on interaction effects We have also assumed that market prices are fixed which means that our time series (but not our cross-sectional) predictions must be viewed as partial equilibrium results Finally, notice that our formulation assumes away progressivity in the corporate tax code Taking this progressivity into account should, *ceteris paribus*, make leverage more attractive to larger firms

that relative market prices for debt and equity will imply a net (corporate and personal) tax advantage to debt — i e, the after-personal tax cash flow to marginal debtholders from investing one more dollar as debt, $(1 - \tau_{PD}^{\mu})$, exceeds the after-personal tax cash flow sacrificed by marginal equity holders' $(1 - \tau_{PE}^{\mu})(1 - \tau_c)$, due to the debt-for-equity substitution In contrast, Miller's model predicts that the endogenously determined net tax effect is zero — i e, $(1 - \tau_{PD}^{\mu}) = (1 - \tau_{PE}^{\mu})(1 - \tau_c)$

The intuition supporting H 3 is quite appealing Debt is desirable to firms because it provides a tax shield for corporate income Depreciation deductions, investment tax credits, and many other features of the corporate tax code provide firms with substitutes for the corporate tax shield attributes of debt H 3 quite reasonably predicts that the use of leverage will be negatively related to the magnitude of available investment related corporate tax shield substitutes for debt To our knowledge, H 3 has not been previously derived However, H 4 and H 5 have been derived by Scott (1976) in a corporate tax–bankruptcy cost model

7. Empirical evidence

This section reviews existing evidence regarding the empirical relevance of investment tax shields and hypotheses H 1–H 5 While this evidence provides preliminary support of our model, we recognize that a more careful empirical analysis is clearly in order

In our model, we assumed that investment-related tax shields are economically significant relative to debt tax shields Two pieces of evidence support this assumption First, direct measurement by the I R S suggests that both shields are of the same order of magnitude, although debt deductions are larger ($64 3 billion to $49 5 billion for U S corporations in 1975) Second, over the period 1964–1973, 27% of all U S corporations filing tax returns in a given year paid no taxes at all [24] Given the relatively low bankruptcy rate for this period,[25] the evidence suggests that investment-

[24]The 27% figure was derived from *Statistics of Income* by calculating the ratio of the number of corporations filing income tax returns with net 'taxable' income to the total number of corporations filing returns The yearly ratio was very stable it never deviated more than 2% from the ten-year average of 27% However, over time there is likely to be significant positive serial correlation of firms paying or not paying taxes

These estimates may not be representative of large firms However, Vanik (1978) offers some estimates of taxes paid in recent years for some 168 of the largest corporations in the U S In 1975 approximately 20% of these firms paid no federal income tax while in 1976 this fell to 10% of these firms This evidence suggests that even the largest firms experience excess tax shields The total corporate tax liabilities avoided by investment tax shields is estimated by Muskie (1976)

[25]The average annual failure (bankruptcy) rate of commercial and industrial firms for the period 1965–1974 is 43 per 10,000 concerns based on annual figures of Dun and Bradstreet (1977) Of these failures, approximately 2 5% had liabilities of $1 million or greater

related tax shields were important in reducing corporate tax bills to zero (and perhaps resulted in excess tax shields) — i e, not all the low tax bills were a result of financial distress

7 1   Evidence on the leverage irrelevancy theorem

The fundamental hypothesis of our expanded model, H 1, predicts that firm market value is a function of its capital structure Early evidence in support of H 1 can be found in Miller–Modigliani's (1966) study of 63 electric utility firms in which they found a significant positive relationship between market values of the firm and the debt tax shield (shown in their table 4) While supportive of H 1, this evidence may justifiably be criticized because of limitations in the statistical methodology as well as the inherent difficulties involved in controlling for cross-sectional differences in firms' underlying asset values and biases associated with studying a regulated industry

In a more recent study which avoided these difficulties, Masulis (1980) developed a model to estimate the average effect across firms of a change in the debt tax shield on the market value of a given firm To do this he studied 117 intrafirm exchange offers and recapitalizations which approximated pure capital structure changes, most of which involved a change in the firm's debt tax shield The basic approach was to utilize the ex post capital structure change to explain the magnitude of the common stock rate of return on the announcement date of the exchange offer Modelling the hypothesis that a change in the firm's debt tax shield causes a change in the same direction in firm value, Masulis obtained an estimated *average* change in firm value between 10% to 20% of the change in debt market value Masulis obtained similar estimates under the assumption that only positive changes in debt affected firm value Together, the Miller–Modigliani and Masulis studies represent direct evidence of the empirical importance of the leverage decision to firm value

7 2   Industry cross-sectional predictions

Hypotheses H 3 and H 4 predict that differential investment tax shields and/or differential marginal costs of leverage should induce differential optimal leverage ratios for firms While little empirical evidence of differential leverage-related costs is available, there is considerable evidence of significant variations in investment tax shields across industries as documented in Vanik (1978), Muskie (1976), Siegfried (1974), and Rosenberg (1969) Given these differences across industries, our model predicts that (1) firm leverage (debt–asset ratio) should also differ across industries with differing non-debt tax

shields relative to EBIT (earnings before interest and taxes) while exhibiting much greater homogeneity within these individual industry classifications, and (2) as the ratio of non-debt tax shield to EBIT rises, leverage should fall

Taking a sample of firms from selected industry groups Scott–Martin (1975) and Scott (1972) studied the leverage ratios of twelve major industries and Schwartz–Aronson (1967) studied the leverage ratios of four major industrial groups They found statistically significant differences in firms' leverage ratios (defined as shareholder equity divided by firm book value) across industries while within industries firm leverage ratios were relatively homogeneous and stable over time These results were based on a one-way analysis of variance, and are consistent with the predictions of our model Unfortunately, these studies analyzed only a small number of industries (the industries analyzed in the first two studies being almost identical), which severely limits our ability to deduce evidence supporting or refuting prediction (2) of this section

One piece of casual evidence in support of H 3 is that Drugs, Mining and Oil Industries had the lowest leverage ratio in the Scott–Martin study and these industries all benefit from special tax code features which increase their investment-related tax shields considerably, e g, expensing of R&D costs, mineral and oil depletion allowances

7 3   Aggregate time-series predictions due to changes in the corporate tax code and rate of inflation

Our model predicts changes over time in firms' capital structures due to changes in the cost of leverage (bankruptcy and reorganization or other agency cost of debt) (H 4), changes in the corporate tax rate (H 5), or changes in the firm's investment tax shield (H 3) Over the last fifty years, there have been a number of significant changes in the federal income tax code including major increases in the corporate tax rate as well as increases in the size of corporate investment tax deductions and credits [see Fromm (1971) and Oakland (1972)] There have also been changes in the personal tax code Given these non-simultaneous changes in tax rates and investment tax shields, we would expect to see significant *aggregate* changes in firms' leverage decisions over time

Since a large number of corporate tax deductions are based on historical costs, such as depreciation, depletion allowances, and cost of goods sold, *ceteris paribus*, increases in inflation which increase nominal revenues, will decrease the real value of investment tax shields inducing firms to replace this tax shield loss by increasing their use of debt [26] Given the significant

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[26]However, there could be additional effects due to inflation-induced changes in the relative marginal personal tax rates on debt and equity See Aaron (1976)

increases in the rate of inflation over the period 1965–1974, our model predicts related increases in the level of firm leverage [27]

The empirical evidence of aggregate firm leverage behavior uncovered in Corcoran (1977) and Zwick (1977) clearly shows that in the period 1948–1975 the behavior of firm leverage has been far from stable Corcoran observes that the average debt to firm value ratio in market value terms for non-financial corporations 'rose from 22% in 1965 to 42% in 1974, a movement which paralleled the acceleration in the domestic inflation rate' A similar pattern was found by Zwick using a different measure of leverage (face value of debt to firm book value) The inflation impact was compounded by a corporate tax surcharge over the period 1968–mid 1970, the termination of the investment tax credit between April 1969 and the end of 1970, and a reduction in depletion allowances as of 1969 Together these two studies support the predictions of H 3 and H 5

In a more comprehensive study, Holland–Myers (1977) measured the year by year aggregate debt and equity market values for all non-financial corporations in the U S for the years 1929–1975 The resulting ratios of market values of debt to firm assets was, as expected, highly variable with significant increases in leverage (1) in the 1940–1942 period when corporate taxes were substantially increased and (2) again in the 1967–1975 period of high inflation (early in this period corporate tax rates were also increased) This evidence is consistent with hypotheses H 3 and H 5 [28]

7 4 Equilibrium marginal tax rates

Hypothesis H 2 of our expanded model predicts the market determined marginal tax rate relationship

$$\tau_{PD}^{\mu} < \tau_c + \tau_{PE}^{\mu}(1 - \tau_c)$$

[27]See Nichols (1968), Bradford (1974), and Lintner (1975) for a theoretical discussion Hong (1977) presents evidence that inflation adversely and differentially affects firms' profitability by causing depreciation and cost of inventory withdrawal expenses to be understated This evidence is corroborated by the studies of Fama–Schwert (1977), Jaffe–Mandelker (1976), Bodie (1976), and Nelson (1976) which also find a significant negative relationship between common stock rates of return and the rate of inflation

[28]One piece of apparently non-supportive evidence is Miller's (1963) study for the Commission on Money and Credit Miller measured the ratio of face value of long-term debt to book value of assets for five-year intervals over the period 1926–1956 for both all non-financial corporations and all manufacturing corporations He found that this ratio was highly stable for all non-financial corporations but increased significantly for manufacturing corporations However, by ignoring short-term debt, he admitted that he had downward biased the change in the leverage ratio over the sample period Moreover, since corporations are likely to make initial adjustments in leverage by altering their short-term debt, Miller's methodology is likely to be dampening the instability in the time series of his measured leverage ratios

If we assume as Miller does that $\tau_{PE}^\mu = 0$, then the above condition simplifies to $\tau_c > \tau_{PD}^\mu$ Notice that $\tau_{PE}^\mu = 0$ implies that corporate stocks and non-taxable municipal bonds are perfect substitutes in terms of after-personal tax cash flow, so that in equilibrium the before-personal tax yields must be equal (ignoring differences in risk) Given this relationship, one can estimate the marginal personal tax rate on debt by comparing the ratio of the municipal bond yield to the corporate bond yield for bonds of equivalent maturity and risk,[29] since the before-personal tax yield on municipals must be sufficiently below that of the corporate bonds so that there is a marginal investor who is indifferent to holding the two assets Hence

$$1/P_M = (1/P_D)(1 - \tau_{PD}^\mu)$$

or

$$\tau_{PD}^\mu = 1 - (1/P_M)/(1/P_D)$$

where $1/P_M$ = yield on non-taxable municipal bonds and $1/P_D$ = yield on taxable corporate bonds [30]

Following this approach, Sharpe (1978) compared the yields on municipal bonds with yields on long-term Aa public utility bonds For the years 1950–1975 the average ratio of yields was approximately 0 70 which implies a 30% marginal personal tax rate on debt Given a corporate tax rate of 48%, this finding supports our expanded model's predicted relationship between the equilibrium marginal tax rates and is inconsistent with Miller's prediction

Moreover, Sharpe's evidence is also inconsistent with the leverage irrelevancy theorem when equity is taxed at the personal level In this case equilibrium requires $P_D = P_E(1 - \tau_c)$ and $P_E \leq P_M$ because municipal bonds offer a personal tax advantage over equity Equivalently, equilibrium requires $(1 - \tau_c) \geq (1/P_M)/(1/P_D)$ which is inconsistent with Sharpe's empirical estimates

8. Summary

In this paper, we generalized Miller's differential personal tax model to include an often overlooked but major feature of the U S tax code the existence of corporate tax shield substitutes for debt such as accounting

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[29]This is assuming bonds are selling at par so there are no capital gains or losses realized The methodology is given in Sharpe (1978, pp 142–143)

[30]The yield on a bond in a one-period model is usually defined as $r_D = (1/P_D) - 1$ Here we are interpreting $1/P_D$ as the yield which is strictly appropriate in the case of a consol bond (or perpetuity) in a multi-period context This is consistent with Sharpe's estimation of bond yields from multiperiod cash flows of long term bonds This is justified because we have formulated our one-period model to approximate the results of a multi-period model where the capital structure decision is permanent

depreciation deductions and investment tax credits Introduction of these realistic corporate tax code features leads to a market equilibrium in which each firm has a unique interior optimum leverage decision due solely to the interaction of personal and corporate tax treatment of debt and equity Our model also allows for positive default costs In particular, the presence of tax shield substitutes for debt and/or default costs implies a unique interior optimum leverage decision in market equilibrium Moreover, Miller's horse-and-rabbit stew criticism of corporate tax saving–default cost models is inapplicable to our model because the net corporate–marginal personal tax saving is endogenously determined to be of the same order of magnitude as expected marginal default costs Our model yields a number of testable hypotheses regarding both the cross-sectional and time-series properties of firms' leverage decisions as well as the marginal personal tax rates implicit in relative market prices We argued that existing evidence provides indirect support for these hypotheses However, we believe that a more thorough empirical analysis is clearly in order

References

Aaron, H , 1976, Inflation and the income tax, American Economic Review 66, 193–199

Baxter, N , 1967, Leverage, risk of ruin and the cost of capital, Journal of Finance 22, 395–403

Black, F , 1971, Taxes and capital market equilibrium, Working paper no 21A, mimeo (Associates in Finance, Belmont, MA)

Black, F and M Scholes, 1974, The effects of dividend yield and dividend policy on common stock prices and returns, Journal of Financial Economics 1, 1–22

Bodie, Z , 1976, Common stocks as a hedge against inflation, Journal of Finance 31, 459–470

Bradford, W , 1974, Inflation and the value of the firm Monetary and depreciation effects, Southern Economic Journal 41, 414–427

Brennan, M and E Schwartz, 1978, Corporate income taxes, valuation, and the problem of optimal capital structure, Journal of Business 51, 103–114

Chen, A and H Kim, 1979, Theories of corporate debt policy A synthesis, Journal of Finance 34, 371–384

Corcoran, P , 1977, Inflation, taxes, and corporate investment incentives, Federal Reserve Bank of New York Quarterly Review 2, 1–9

DeAngelo, H , 1979, Competition and unanimity Working paper (University of Washington, Seattle, WA)

DeAngelo, H and R Masulis, 1980, Leverage and dividend irrelevancy under corporate and personal taxation, Journal of Finance, forthcoming

Dun & Bradstreet, 1977, The business failure record 1976 (New York)

Fama, E and M Miller, 1972, The theory of finance (Holt, Rinehart and Winston, New York)

Fama, E and G W Schwert, 1977, Asset returns and inflation, Journal of Financial Economics, 115–146

Fromm, G , 1971, Tax incentives and capital spending, Studies of Government Finance (The Brookings Institution, Washington, DC)

Galai, D and R Masulis, 1976, The option pricing model and the risk factor of stock, Journal of Financial Economics 3, 115–146

Holland, D and S Myers, 1977, Trends in corporate profitability and capital cost, Working paper (MIT, Cambridge, MA)

Hong, H , 1977, Inflation and the market value of the firm Theory and tests, Journal of Finance 32, 1031–1048

Jaffe, J and G Mandelkar, 1976, The 'Fisher effect' for risky assets An empirical investigation Journal of Finance 31, 447–458

Jensen, M and W Meckling, 1976, Theory of the firm Managerial behavior agency costs and ownership structure, Journal of Financial Economics 3, 305–360

Kim, H , 1978, A mean-variance theory of optimal capital structure and corporate debt capacity, Journal of Finance 33, 45–64

Kim, H , W Lewellen and J McConnell, 1978, Financial leverage clienteles Theory and evidence, Journal of Financial Economics 7, 83–109

Kraus, A and R Litzenberger, 1973, A state preference model of optimal financial leverage, Journal of Finance 28, 911–921

Lintner, J , 1975, Inflation and security return, Presidential address, Journal of Finance 30, 259–280

Masulis, R , 1978, The effects of capital structure change on security prices, Unpublished Ph D dissertation (University of Chicago, Chicago, IL)

Miller, M , 1963, The corporation income tax and corporate financial policies, in Stabilization policies, Commission on Money and Credit (Prentice-Hall, Englewood Cliffs, NJ)

Miller, M , 1977, Debt and taxes, Journal of Finance 32, 261–275

Miller, M and F Modigliani, 1961, Dividend policy, growth and the valuation of shares, Journal of Business 34, 411–433

Miller, M and F Modigliani, 1966, Some estimates of the cost of capital to the electric utility industry, 1954–57, American Economic Review 56, 333–391

Modigliani, F and M Miller, 1958, The cost of capital, corporation finance and the theory of investment, American Economic Review 48, 261–297

Modigliani, F and M Miller, 1963, Corporation income taxes and the cost of capital A correction, American Economic Review 53, 433–443

Muskie, E , 1976, Tax expenditures Compendium of background material on industrial provisions, Committee on the Budget, U S Senate 94th Congress, 2nd session

Myers, S , 1977, Determinants of corporate borrowing, Journal of Financial Economics 4, 147–176

Nelson, C , 1976, Inflation and rates of return on common stocks, Journal of Finance 31, 471–483

Nichols, D , 1968, A note on inflation and common stock values, Journal of Finance 23, 655–657

Oakland, W , 1973, Corporate earnings and tax shifting in U S manufacturing, 1930–1968, Review of Economics and Statistics 54, 235–244

Robichek, A and S Myers, 1966, Problems in the theory of optimal capital structure, Journal of Financial and Quantitative Analysis 1, 1–35

Rosenberg, L , 1969, Taxation of income from capital by industry group, in Harberger and Bailey, eds , Taxation of income from capital (Brookings Institution, Washington, DC)

Schneller, M , 1980, Taxes and the optimal capital structure of the firm, Journal of Finance 35, 119–127

Schwartz, E and J Aronson, 1967, Some surrogate evidence in support of the concept of optimal financial structure, Journal of Finance 22, 10–19

Scott, D , 1973, Evidence on the importance of financial structure, Financial Management, 45–50

Scott, D and D Martin, 1975, Industry influence on financial structure, Financial Management, Spring, 67–73

Scott, J , 1976, A theory of optimal capital structure, Bell Journal of Economics and Management Science 7, 33–54

Sharpe, W , 1978, Investments (Prentice-Hall, Englewood Cliffs, NJ)

Siegfried, J , 1974, Effective average U S corporation income tax rates, National Tax Journal 27, 245–259

U S Treasury, Internal Revenue Department, Statistics of income, 1964–1973 (U S Government Printing Office, Washington, DC)

Vanık, Hon Charles, 1978, Annual corporate tax study, Tax year 1976, Congressional Record, 95th Congress, 2nd Session, E168-E176

Warner, J , 1977, Bankruptcy costs Some evidence, Journal of Finance 32, 337–347

Zwıck, B , 1977, The market for corporate bonds, Federal Reserve Bank of New York Quarterly Review 2, 27–36