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ECON 501B

ECON 501B: Problem Set 3

Due: Thursday, September 13, 2018

Instructions: Answers should be complete proofs of a claim. For True/False questions, either provide a proof that the statement is true or provide a counterexample showing that it is false.

Question 1: *True of False.* Fix an environment $\mathcal{E} = (T, B; (\succsim)_{i \in T \cup B})$ with strict preferences. If $\mu_{TD} = \mu_{BD}$, then there is a unique stable match.

Question 2: There are a set of project managers $\mathcal{M} = \{m : m = 1, \ldots, M\}$ and a set of workers $\mathcal{W} = \{w : w = 1, \ldots, W\}$, where $W \geq M$. If Manager m hires worker w, they contribute output level of $f(m, w)$ to overall firm productivity. Their contribution to productivity is given by a production function $f(\cdot, \cdot) : \mathcal{M} \times \mathcal{W} \to (0, \infty)$. This production function is such that (a) for each $w \in \mathcal{W}$, $f(\cdot, w)$ is strictly increasing in m, and (b) for each $m \in \mathcal{M}$, $f(m, \cdot)$ is strictly increasing in w. Thus, higher (informally "better") managers increase firm productivity and higher (informally "better") workers increase firm productivity. If an agent $i \in \mathcal{M} \cup \mathcal{W}$ is unmatched, her productivity level is 0.

Agents care about the productivity of their match, but not the productivity of other matches. In particular, all agents strictly prefer a more productive match (i.e., a match with a higher level of output) to a less productive match. The Firm, however, cares about the productivity of all matches. In particular, the firm seeks to maximize the total productivity of a match. Thus, if the managers and workers are matched according to the matching $\mu : (\mathcal{M} \cup \mathcal{W}) \to (\mathcal{M} \cup \mathcal{W})$, then the Firm's productivity is

$$\sum_{m \in \mathcal{M} : m \text{ is matched}} f(m, \mu(m)).$$

(Note: You may want to pause and reflect on why this is the "right" equation to maximize.)

2a. Which matchings $\mu : (\mathcal{M} \cup \mathcal{W}) \to (\mathcal{M} \cup \mathcal{W})$ are stable?

For the remainder of the problem, assume that $M = W$.

2b. Suppose the production function is $f(m, w) = m \times w$. Does the stable match maximize the Firm's productivity? Either show that it does or provide an example of a match that yields higher Firm productivity.

2c. Suppose the production function is $f(m, w) = (m \times w)^{.5}$. Does the stable match maximize the Firm's productivity? Either show that it does or provide an example of a match that yields higher Firm productivity.

Hint: First think of the case where $M = W = 2$. Then, $M = W = 3$.