◎ First & Second Welfare theorem (General ver.) ← Today's topic

Distinct concepts

* Competitive equilibrium : optimization given prices, market clearing

* Pareto optimality : welfare

Welfare theorems

- give a connection
- imposing strong conditions

Basic Building Block : Equilibrium with transfers

Economy : $E = \left((Y_j)_{j=1}^{J} ; (X_i, \succsim_i, \omega_i, \theta_{i1}, \dots, \theta_{iJ})_{i=1}^{I} \right)$

Def An allocation (x^*, y^*) and a price vector p^* form a competitive equilibrium

if ① for each $j=1,\dots,J$, $p^* \cdot y_j^* \geq p^* \cdot y_j$ for each $y_j \in Y_j$ [profit max]

② for each $i=1,\dots,I$

2a) $x_i^* \in B_i^*(p^*) = \{ x_i \in X_i : p^* \cdot x_i \leq p^* \cdot \underset{\text{endowment}}{\omega_i} + \underbrace{\sum_{j=1}^{J} \theta_{ij} (p^* \cdot y_j^*)}_{\text{profit from a firm}} \}$ [util. max]

2b) $x_i^* \succsim_i x_i$ for each $x_i \in B_i^*(p^*)$

③ $\sum_{i=1}^{I} x_i^* = \bar{\omega} + \sum_{j=1}^{J} y_j^*$ [Market clearing]

Def An allocation (x^*, y^*) and a price vector p^* form a price equilibrium with transfer

if $\exists$ an assignment of wealth $W = (W_1, \dots, W_I)$ s.t.

① for each $j=1,\dots,J$, $p^* \cdot y_j^* \geq p^* \cdot y_j$ for each $y_j \in Y_j$ [profit max]

② for each $i=1,\dots,I$ 2a) $x_i^* \in B_i(p^*, W_i) = \{ x_i \in X_i ; p^* \cdot x_i \leq \underset{\text{Wealth for } i}{W_i} \}$ [util max]

2b) $x_i^* \succsim x_i$ for all $x_i \in B_i^*(p^*, W_i)$

③ $\sum_{i=1}^{I} x_i^* = \bar{w} + \sum_{j=1}^{J} y_j^*$

④ $\sum_{i=1}^{I} W_i = p^* \cdot \bar{w} + \sum_{j=1}^{J} p^* \cdot y_j^*$

[Market clearing]

(Remark) A competitive equilibrium induces a price equilibrium with transfers

$$W_i = p^* \cdot w_i + \sum_{j=1}^{J} \theta_{ij} p^* \cdot y_j^*$$

$\Downarrow$

Check ④ : $\sum_i W_i = p^* \cdot \bar{w} + \sum_{j=1}^{J} p^* \cdot y_j^*$ (Summation of all i's wealth)

$\boxed{\text{First Welfare Theorem}}$ * C.E. $\Rightarrow$ P.O. Under $\succsim_i$ LNS

Idea: Any competitive equilibrium induces a Pareto allocation.

(Def) A preference relation $\succsim_i$ on $X_i \subseteq \mathbb{R}_+^L$ satisfies Locally NON-satiation

if for each $x_i \in X_i$ and each $\varepsilon > 0$ $\exists$ some $x_i' \in X_i$ s.t.

(1) $\| x_i - x_i' \| < \varepsilon$ (2) $x_i' \succ x_i$

c.f) "Rules out" of LNS : fat indifference curve / bliss point preference.

$\boxed{\text{Theorem : First Welfare Theorem}}$ Under $\succsim_i$ LNS, Price Eq w/ transfers $\Rightarrow$ P.O / Competitive eq $\Rightarrow$ P.O

Fix an economy with locally non-satiated preferences.

If $((x^*, y^*), p^*)$ form a price equilibrium with transfers.

then (x^*, y^*) is Pareto Optimal allocation.

↳ Corollary : This holds for a competitive equilibrium allocation.

Lemma 1. Suppose $\succsim_i$ is locally nonsatiated and let $x_i^* \in B_i(p, W_i)$ that $\succsim_i$ - maximal, i.e., $x_i^* \succsim x_i$ for each $x_i \in B_i(p, W_i)$.

Then $x_i \succsim_i x_i^*$ implies $p \cdot x_i \geq W_i$

proof) * If $x_i \succ_i x_i^*$, $\Rightarrow$ $x_i \notin B_i(p_i, W_i) \Rightarrow px_i > px_i^* = W_i$

* If $x_i \sim_i x_i^*$,

LNS: for each $n \geq 1$, there exists some $x_i^n \in X_i$ with $\|x_i^n - x_i\| < \frac{1}{n}$

and $x_i^n \succ_i x_i$

$\Rightarrow$ By transitivity, $x_i^n \succ_i x_i \sim x_i^*$ i.e., $x_i^n \succ_i x_i^* \Rightarrow px_i^n > W_i$

$\lim_{n\to\infty} x_i^n = x_i$, so $\underbrace{\lim_{n\to\infty} p \cdot x_i^n}_{= px_i} \geq W_i$

Thus, $px_i \geq W_i$ //

proof of Theorem)

Let $((x^*, y^*), p^*)$ be a price equilibrium with transfers that is supported by $(W_1, \dots, W_I)$. Suppose, contra hypothesis, that (x^*, y^*) is not Pareto Optimal.

$\Rightarrow$ Then, $\exists$ a feasible allocation (x, y) s.t. (1) $x_i \succsim_i x_i^*$ for each $i = 1, \dots, I$

(2) $x_i \succ_i x_i^*$ for some $i = 1, \dots, I$

WTS: (x, y) cannot be feasible.

For each i, $x_i^* \in B_i(p, W_i)$ and x_i^* is $\succsim_i$-maximal.

- Lemma 1 + (1): $p^* \cdot x_i \geq W_i$ for each $i = 1, \dots, I$

- (2) says there is some i with $p^* \cdot x_i > W_i$

$$\Rightarrow \sum_{i=1}^{I} p^* \cdot x_i > \underbrace{\sum_{i=1}^{I} W_i = p^* \cdot \bar{w}}_{\Uparrow \text{ By Price Eq. Tr.}} + \sum_{j=1}^{J} p^* \cdot y_j^* \underset{\substack{\uparrow \\ \text{By condition (1)} \\ \text{of P.E.T.}}}{\geq} p^* \cdot \bar{w} + \sum_{j=1}^{J} p^* \cdot y_j$$

$$\sum_{i=1}^{I} p^* \cdot x_i > p^* \cdot \bar{\omega} + \sum_{j=1}^{J} p^* \cdot y_j$$

$$\Downarrow$$

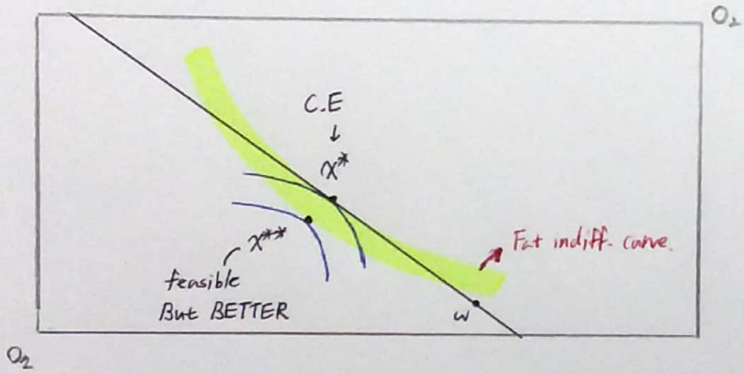
$$\sum_{i=1}^{I} x_i \neq \bar{\omega} + \sum_{j=1}^{J} y_j \quad \leftarrow \text{(x, y) is NOT feasible} \quad ||$$

(*Market Clearing does not hold by the contrahypothesis)

Remark The role of local non-satiation

To give us : "$x_i \succsim_i x_i^*$ and $x_i^* \in B_i(p^*, W_i)$ is $\succsim_i$-maximal" $\Rightarrow$ $p \cdot x_i \geqq W_i$

Example where LNS violated : Fat indifference curves.



Second Welfare Theorem

P.O ⇒ P.E.T / C.E Under "stronger" assumptions

· Idea : If we have a Pareto optimal allocation,

then we can sustain it as an equilibrium – potentially after making transfers.

⇒ This is why we are in the world of a price equilibrium with transfers

※ Will NEED STRONGER Assumptions on preference.

proof strategy : define a "silly equilibrium"

- Show that under [Assumption 1], any Pareto Optimal allocation induces a silly equilibrium
- show that under [Assumption 2], any silly eq. is a price eq. with transfers ↳ "NOT Silly"
- Conclude : Second Welfare Theorem [Assumption 1, 2]

NOT all assumptions are created equal