- Commodities : $\ell = 1, \dots, L$
- Consumers : $i = 1, \dots, I$
  - * consumption set $X_i \subseteq \mathbb{R}^L$
  - * preference $\succsim_i$
  - * endowment : $w_i = (w_{1i}, \dots w_{Li}) \in X_i$

    $\Rightarrow \bar{w}_\ell = \sum_{i=1}^{I} w_{\ell i}$

    $\bar{w} = (\bar{w}_1, \dots, \bar{w}_L)$
  - * shares of firms : θ_{ij}
- firms : $j = 1, \dots, J$

  production set $Y_j \subseteq \mathbb{R}^L$
- allocation : $(x, y) = (x_1, \dots, x_I, y_1, \dots y_J) \in \mathbb{R}^{L \times (I+J)}$

Def An allocation (x, y) is feedback given $\bar{w}$ if, for each $\ell = 1, \dots, L$,

$$\sum_{i=1}^{I} x_{\ell i} = \bar{w}_\ell + \sum_{j=1}^{J} y_{\ell j}$$

Def A feasible allocation (x, y) is pareto optimal if there is no other feasible allocation (x', y') s.t (1) $x_i' \succsim_i x_i \ \forall i$

(2) $\exists \ x_i' \succ_i x_i$ for some i

Def A competitive equilibrium is an allocation (x^*, y^*) and a price vector p^* s.t.

① For each j, $p^* y_j^* \geqq p^* \cdot y_j$ for all $y_j \in Y_j$

② For each i

(2a) $x_i^* \in B_i^* \equiv \{ x_i \in X_i \mid p^* x_i \leqq p^* \cdot w_i + \sum_{j=1}^{J} \theta_{ij} (p^* y_j^*) \}$

(2b) $x_i^* \succsim_i x_i$ for all $x_i \in B_i^*$

③ $\sum_{i=1}^{I} x_i^* = \sum_{i=1}^{I} w_i + \sum_{j=1}^{J} y_j^*$

Aggregate demand | Endowment | Aggregate supply by firms

⊛ Partial Equilibrium (w/ Strong assumptions)

- Marshal
- first cut way to put firms & consumers together.

Focus : market with only one "real" commodity

"potentially problematic"

① Ignoring the effects of substitutes & complements

- extreme : perfect substitute

focus on commodity ℓ only : If $P_\ell \Downarrow$, demand for other goods should $\to 0$

② Ignore wealth effects : Suppose ℓ has large wealth effects

If $P_\ell \Uparrow$, then it would significantly impact budget for all other commodities, but ignored.

⇒ When we think of partial equilibrium, we really think of the commodity ℓ as being a small component of consumption at all (relevant) price levels.

↳ Vives (1993)

- Two commodities

앞의 일반 환경에서는 원래는 $x_{\ell i}$로 표시

- ℓ : x_i for i's consumption of good ℓ (상품 ℓ을 i가 소비하는 양을 표시)
- numeraire : m_i for i's consumption of the numeraire.

- $X_i = \mathbb{R} \times \mathbb{R}_+ \ni (m_i, x_i)$

- preferences have a quasi-linear utility representation

$$u_i(m_i, x_i) = m_i + \phi_i(x_i)$$

where ϕ_i is ① twice differentiable & $\phi'(\cdot) > 0$ $\phi''(\cdot) < 0$

② $\phi_i(0) = 0$ and ϕ_i bounded above

- Firm j
  - production of good ℓ : q_j (전에는 y_j로 나타냈는데, 여기서는 q_j로 표시)
  - cost : $C_j(q_j)$ units of the numeraire to produce q_j units of good ℓ

    $C_j : \mathbb{R}_+ \to \mathbb{R}$, twice differentiable with $\underset{\text{marginal cost}}{C'(\cdot)} > 0$ and $C''(\cdot) \geq 0$

- $Y_j := \{ (\underset{\text{생산시 포기해야되는 numeraire(MWG chap 5)}}{-z_j}, \underset{\text{생산}}{q_j}) : q_j \geqq 0 \text{ and } z_j \geqq C_j(q_j) \}$

  $\mathbb{R}^2 \supseteq Y_j$

- Allocation : $(x, m, q, z) = ((x_i, m_i)_{i=1,\dots,I}, (q_j, z_j)_{j=1,\dots,J})$

- Endowments : $\omega_i = (\underset{\text{i가 가진 endowment}}{\omega_{mi}}, \underset{\text{good } \ell.}{0})$

- price : price vector $(1, p)$ $\begin{cases} 1 : \text{the price of the numeraire} \\ p : \quad \text{"} \quad \text{good } \ell \end{cases}$

$\Downarrow$

이런 환경을 바탕으로 Competitive eq.를 살펴보자.

Competitive Equilibrium (x^*, m^*, q^*, z^*) and $(1, p^*)$ s.t.

(1) Firm j solves

$$\max_{q_j \geqq 0} [p^* q_j - C_j(q_j)] \Rightarrow p^* \leqq C'(q_j^*) \text{ w/ equality if } q_j^* > 0 \quad (1_j)$$

(interior solution)

supply curve of firm j condition

Also, $\max_{z_j \geqq C_j(q_j^*)} [-1 z_j] \rightarrow z_j^* = C_j(q_j^*) \quad (2_j)$

(2) Consumer i solves

$$\max_{(m_i, x_i)} [m_i + \phi_i(x_i)] \quad \text{s.t.} \quad m_i + p^* x_i \leq \omega_{mi} + \sum_{j=1}^{J} \theta_{ij} \left(p^* q_j^* - c_j(q_j^*)\right)$$

일반 환경 　　 기술한계.

$$\sum_{j=1}^{J} \theta_{ij} \cdot \underbrace{(p^* \cdot y^*)}_{\Downarrow} \text{ 에서}$$

$$\sum_{\ell=1}^{L} p_\ell^* \cdot y_\ell^* = p^* \cdot q_j^* - 1 z_j^*$$

budget constraint binds

$$\Rightarrow \max_{x_i} \left[\omega_{mi} + \sum_{j=1}^{J} \theta_{ij} [p^* q_j^* - c_j(q_j^*)] - p^* x_i + \phi_i(x_i) \right]$$

(3i)
demand of consumer i

$\Rightarrow \phi_i'(x_i^*) \leq p^*$ w/ equality when ~~if~~ $x_i^* > 0$.

$$m_i^* + p^* x_i^* = \omega_{mi} + \sum_{j=1}^{J} \theta_{ij} \left(p^* q_j^* - c_j(q_j^*)\right) \qquad (4i)$$

budget binding.

(3)

$$\sum_{i=1}^{I} x_i^* = \sum_{j=1}^{J} q_j^* \qquad (5a)$$

Aggregate demand = Aggregate production.
(총수요 = 총생산)

$$\sum_{i=1}^{I} m_i^* = \bar{\omega}_m - \sum_{j=1}^{J} z_j^* \qquad (5b)$$

$\Rightarrow$ solve $(x_1^*, \ldots, x_I^*, m_1^*, \ldots, m_I^*, q_1^*, \ldots, q_J^*)$ & p^*

to satisfy (1_j), (2_j), (3_i), (4_i), (5).

Remark 1) Distribution of $\omega_m = (\omega_{m1}, \ldots, \omega_{mI})$ and the firm shares don't impact the consumption or the production of good ℓ

Welfare

Fix a feasible allocation (x, m, q, z)

- each firm j uses $c_j(q_j)$ units to produce q_j
- gets of z_j units of numeraire : $z_j \geq c_j(q_j)$

$$\Rightarrow \sum_{i=1}^{I} m_i = \bar{w}_m - \sum_{j=1}^{J} z_j \leq \bar{w}_m - \sum_{j=1}^{J} c_j(q_j) \quad \text{(Endowment 조건)}$$

(*)

$$\Rightarrow \underbrace{\sum_{i=1}^{I} u_i(m_i, x_i)}_{\text{전체 Utility 합.}} = \sum_{i=1}^{I} m_i + \sum_{i=1}^{I} \phi_i(x_i) \underset{\text{Endowment 조건 추가.}}{\leq} \bar{w}_m + \underbrace{\Big[\underbrace{\sum_{i=1}^{I} \phi_i(x_i)}_{\text{l을 통해 얻은 Utility}} - \underbrace{\sum_{j=1}^{J} c_j(q_j)}_{\text{l을 생산하는데 들어간 cost.}} \Big]}_{\text{Marshallian Aggregate Surplus for commodity } l \,\equiv\, S(x, q)}$$

Define $U(x, q) = \left\{ v \in \mathbb{R}^I : \sum_{i=1}^{I} v_i \leq \bar{w}_m + S(x, q) \right\}$: (x, q) and total wealth in the economy

Def Say (x, m, q, z) induces $v \in \mathbb{R}^I$ if, for each $i = 1, \dots, I$,

$$v_i = m_i + \phi_i(x_i)$$

(*)에서 생각해봄

Lemma 1 (1) If (x, m, q, z) is feasible, then it produces some $v \in U(x, q)$

(2) If $v \in U(x, q)$ and $\sum_{i=1}^{I} x_i = \sum_{j=1}^{J} q_j$,

then there exists (m, z) s.t. (x, m, q, z) is feasible & induces v.

(2)는?

$$z_j = c_j(q_j)$$

$$m_i = \frac{1}{I}\left(\bar{w}_m - \sum_{j=1}^{J} z_j\right)$$

$\Rightarrow$ Feasiblity :
- supply = demand : $\sum_i x_i = \sum_j q_j$ (by assumption)
- Endowment : $\sum_i m_i = \bar{w}_m - \sum_j c_j q_j$ (by construction)

$u(x, q)$를 보면,

$$\sum v_i \leq \overline{\omega}_m + \sum_{i=1}^{I} \phi_i(x_i) - \sum_{j=1}^{J} c_j(q_j)$$

$$= \cancel{\overline{\omega}_m} + \sum_i \phi_i(x_i) + \left(\sum_i m_i - \cancel{\overline{\omega}_m}\right) = \sum u_i(m_i, x_i)$$ //