Amanda Friedenberg
ECON 501B

ECON 501B: Problem Set 4

Due: Thursday, September 27, 2018

Instructions: Answers should be complete proofs of a claim. For True/False questions, either provide a proof that the statement is true or provide a counterexample showing that it is false. (You may reference results from class, should that be pertinent.)

Question 1: Fix a one-to-one environment $\mathcal{E} = (T, B; (\succsim_t : t \in T), (\succsim_b : b \in B))$. We said that a coalition C blocks $\mu : (T \cup B) \to (T \cup B)$ if there exists some matching $\mu' : (T \cup B) \to (T \cup B)$ so that, for each $i \in C$, (a) $\mu'(i) \in C$, and (b) $\mu'(i) \succ_i \mu(i)$. This presumes that coalition members can deviate, so long as they are deviating "to each other."

Suppose instead that non-coalition members impose constraints on the block. So, agents can deviate from their own match and join a coalition. But they cannot change the match for non-coalition members that they are not matched with. With this in mind, consider the following definition: a coalition C blocks* $\mu : (T \cup B) \to (T \cup B)$ if there exists some matching $\mu' : (T \cup B) \to (T \cup B)$ so that (a) for each $i \in C$, $\mu'(i) \in C$, (b) for each $i \notin C$, $\mu'(i) = \mu(i)$, and (c) for each $i \in C$, $\mu'(i) \succ_i \mu(i)$. Provide an example that illustrates the "problem" with this definition. Then, provide a "fix" of the definition. Given the fixed definition of block*, is it the case that a coalition blocks* μ if and only if a coalition blocks μ? Either provide a proof or a counterexample.

Question 2: Consider an environment where we are matching firms and workers. Let $F = \{f_1, f_2\}$ be the set of firms and $W = \{w_1, w_2, w_3\}$ be the set of workers. Each firm has a quota of 2. Workers w_1 and w_2 are a couple.

Preferences are assortative: All else equal, each firm strictly prefers w_1 to w_2 to w_3 to being unmatched. All else equal, each worker prefers f_1 to both f_2 and being unmatched. (They differ based on whether f_2 is better or worse than being unmatched.) However, the fact that w_1 and w_2 are a couple causes two issues. First, f_1 dislikes hiring couples: He would prefer a couple to being unmatched, but would prefer a singleton to hiring a couple. Otherwise, his preferences are responsive. Second, if $w_i \in \{w_1, w_2\}$ is matched with f_1, then i's partner $w_j \in \{w_1, w_2\} \backslash \{w_i\}$ prefers to be unmatched versus being matched with f_2. On the other hand, if $w_i \in \{w_1, w_2\}$ is matched with f_2, then i's partner $w_j \in \{w_1, w_2\} \backslash \{w_i\}$ prefers to be matched with f_2 versus being unmatched. In addition: f_2 has responsive preferences and w_3 prefers being matched with f_2 to being unmatched.

1. Use the information above, to formally describe the preference relation of each of the firms.
2. Explain why the example does not fit the framework we studied in class.
3. What are the set of stable matches? (Note, your argument should also contain a proof that captures why other matches are not stable.)
4. Does there exist a worker optimal stable match?
5. Suppose f_1 no longer minds hiring couples. So, his preferences are responsive. However, he now prefers w_1 to w_3 and w_3 to w_2.

- Use this information to formally describe the preference relation of f_1.
- What are the set of stable matches? (Note, your argument should also contain a proof that captures why other matches are not stable.)
- Does there exist a worker optimal stable match?

Question 3: *True or False.* Fix a many to one environment with responsive preferences. Let $B_1, B_2 \subseteq \mathcal{B}\backslash\{\emptyset\}$ be such that

1. $B_1 \backslash B_2 \neq \emptyset$;
2. for each $\hat{B} \subseteq B_2$, $B_2 \succsim_t \hat{B}$; and
3. $B_1 \succ_t B_2$.

Then there exists $b_1 \in B_1 \backslash B_2$ and $b_2 \in B_2 \backslash B_1$ such that $\{b_1\} \succ_t \{b_2\}$.

Question 4: *True or False.* Fix a many to one environment with responsive preferences. Let $B_1, B_2 \subseteq \mathcal{B}\backslash\{\emptyset\}$ be such that

1. $|B_2| \geq |B_1|$;
2. for each $\hat{B} \subseteq B_2$, $B_2 \succsim_t \hat{B}$; and
3. $B_1 \succ_t B_2$.

Then there exists $b_1 \in B_1 \backslash B_2$ and $b_2 \in B_2 \backslash B_1$ such that $\{b_1\} \succ_t \{b_2\}$.

Question 5: *True or False.* Fix a many to one environment with responsive preferences. Let $B_1, B_2 \subseteq \mathcal{B}\backslash\{\emptyset\}$ be such that

1. $B_1 \backslash B_2 \neq \emptyset$;
2. for each $\hat{B} \subseteq B_2$, $B_2 \succsim \hat{B}$; and
3. $B_1 \succ_t t$.

Then there exists $b_1 \in B_1 \backslash B_2$ such that $\{b_1\} \succ_t t$.