501B HW9 11/16 TA

#Q1

① For each $l=1,2$, $\succsim_i$ strongly monotone. So, $p_l^* > 0$;
otherwise, each consumer has no maximum on $B_i(p^*)$

"For garbage case"

② Since $p_l^* > 0$, $l=1,2$, profit max. entails that $y_l^* = 0$. Thus, $p_l^* y_l^* = 0$

(1) $w_1 = (10, 2)$ $w_2 = (2, 10)$

utility max: $\max u^1$ s.t $p x_{11} + x_{21} = 10p + 2$

FOC x_{11}: $\alpha_1 x_{11}^{\alpha_1 - 1} x_{21}^{\alpha_1} - \lambda p = 0$ $\rightarrow$ $\frac{x_{21}}{x_{11}} = p$

x_{21}: $\alpha_1 x_{11}^{\alpha_1} x_{21}^{\alpha_1 - 1} - \lambda = 0$

Simarly, $\frac{x_{22}}{x_{12}} = p$

Budget constraint: $\left.\begin{array}{l} 2p x_{11} = 10p + 2 \\ 2p x_{12} = 2p + 10 \end{array}\right) \Rightarrow x_{11} = \frac{5p+1}{p}$ $x_{12} = \frac{p+5}{p}$

Market clearing: $x_{11} + x_{12} = 12$ Thus, $p = 1$.

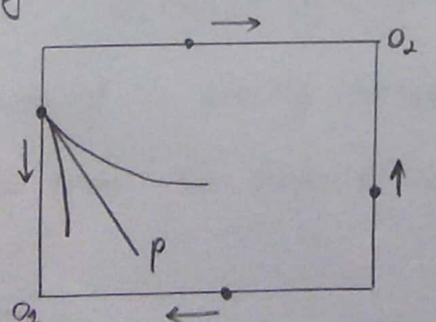
$\therefore$ C.E. $= \{(6,6), (6,6), p=1\}$

(2) · Interior

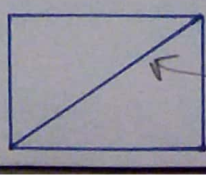
$MRS_1 = MRS_2$: $\frac{x_{21}}{x_{11}} = \frac{x_{22}}{x_{12}} = k$

For Market clearing, since $x_{l1} + x_{l2} = 12$, for $l = 1, 2$, $k = 1$.

· Boundary



$\Rightarrow$ In this case, it could be $MRS_1 \neq MRS_2$.
Thus, along "the side",
p.o bundle can be improved.

Therefore, P.O. set is  P.O. $:= \{(x_{11}, x_{11}), (12 - x_{11}, 12 - x_{11}) \mid x_{11} \in [0, 12]\}$

(3) First Welfare theorem [C.E. $\Rightarrow$ P.O]

For Second Welfare thm, we need transfers to support P.O. as C.E.

(4) Fix a Pareto Optimal allocation (① Tangent point ② find a price vector

$\{(5,5)\ (7,7)\} = (w_1, w_2)$, market price $p=1$.

$\Downarrow$

And then, ③ change endowment by transfers.

Start with $(W_1, W_2) = (12, 12)$ (Given $(10,2)$, $(2,10)$)

Transfer 2 from 1 to 2 : $(W_1', W_2') = (10, 14)$

$\rightarrow$ The meaning of transfers is money.

We can calculate new C.E where another endowment is located.

☆

Q2

- Since $((x_1^*, x_2^*), y^*)$ is Pareto Optimal, (x_1^*, x_2^*) is non-wasteful and $y_\ell^* = 0$, $\ell = 1, 2$

Interior solution $MRS_1 = MRS_2$

$$\phi_1'(x_{21}) = \phi_2'(x_{22}) \quad \cdots \circledast$$

Market clearing : $x_{21} + x_{22} = \overline{w}_2 \Rightarrow x_{22} = \overline{w}_2 - x_{21}$

Thus, x_{22} plug in $\circledast$

$$\phi_1'(x_{21}) - \phi_2'(\overline{w}_2 - x_{21}) = 0.$$

Since ϕ is strictly concave, $\phi_1'(\cdot)$, $\phi_2'(\cdot)$ are decreasing.

That means we have a unique solution for (x_{21}, x_{22}). //

Q3.

① We have shown $p_\ell^* > 0$ for $\ell = 1, 2$, because of strong monotonicity.

② $y_\ell^* = 0$, $\ell = 1, 2$.

Let $w' = x^*$. Check $((x^*, y^*), p^*)$ is a competitive equilibrium w.r.t w'.

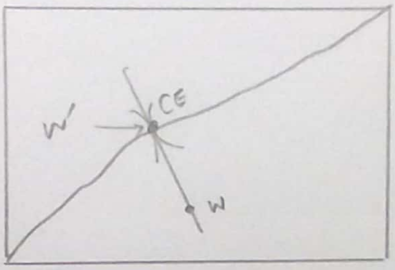
For consumer i, $x_i^* \in \underset{x_i \in \mathbb{R}_+^L}{\arg\max}\ u^i(x)$ s.t $p^* \cdot x_i \leq p^* w_i' = p^* x_i^*$

· Budget constraint is satisfied because in the C.E, $p^* x_i^* = w_i$
(Non-wasteful)

· Feasibility $\sum x_i^* = \bar{w}$, This is from the C.E.

(Given from the Q3)

Concept



Chose $w' =$ C.E. as new endowment.

It can be a C.E.

Q4

1. Tes.

proof) <contradiction>

Suppose a C.E. is not in the core.

A coalition C, for any $i \in C$, and an allocation $\{x_i, i \in C\}$.

s.t. $\sum_{i \in C} x_i \leq \sum_{i \in C} w_i$, then $x_i \succsim_i x_i^*$.

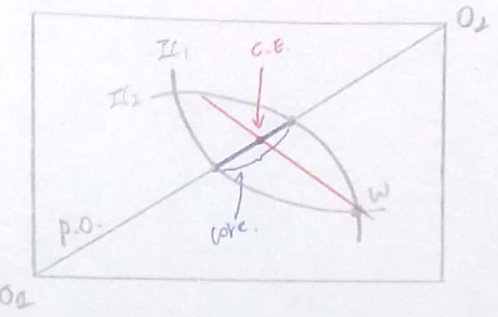
$p^* x_i > p^* x_i^* = p^* w_i$ for any $i \in C$.

$p^* \sum_{i \in C} x_i > p^* \sum_{i \in C} w_i$

$\sum_{\ell=1}^{L} p_\ell \sum_{i \in C} x_{\ell i} > \sum_{\ell=1}^{L} p_\ell \sum_{i \in C} w_{\ell, i}$ Thus, $\exists \ell$, $p_\ell \sum_{i \in C} x_{\ell i} > p_\ell \sum_{i \in C} w_{\ell, i}$

$p_\ell > 0$, $\sum_{i \in C} x_{\ell i} > \sum_{i \in C} w_{\ell i}$, contradiction to feasibility. //

#4. (2) False



"In the Core"

No deviate to motivation

C.E $\underset{\not\Leftarrow}{\Rightarrow}$ In the Core $\underset{\not\Leftarrow}{\Rightarrow}$ P.O