Lecture 11

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Theorem 1:

Suppose preferences are responsive. A matching μ is group stable if and only if it is pairwise stable

It should be clear that if responsive preferences are not group stable, then they cannot be pairwise stable. If they are not group stable, then there is some coalition C that blocks μ. That coalition will have agents who would rather be alone (μ not IR) or rather be with another agent on the other side of the market (blocking pair). Hence, it will not be pairwise stable.

To show: if responsive preferences are pairwise stable, then they are group stable.

Take Contrapositive: If responsive preferences are not group stable, then they are not pairwise stable. Assume μ is not group stable, but μ is IR. But if μ is not IR, then it is not pairwise stable, and we are done.

Given μ is not group stable, there exists by definition some $C \neq \emptyset$ such that C blocks μ via mu'. In other words, μ' dominates μ via C.

Claim: There exists some $t \in C \cap T$ such that $\mu'(t) \cap C \neq \emptyset$.

First notice the following if the above were not true. If $C \subseteq B$: then $\mu'(b) = b$ for each $b \in C$. Then, $b = \mu'(b) \succ_b \mu(b)$ for each $b \in C$. Then, we contradict our assumption of individual rationality made above.

So, we now know that $C \cap T \neq \emptyset$. In fact, for $t \in C \cap T : \mu'(t) \cap C \neq \emptyset$. If this were not so, then there is a $t \in C \cap T$ such that both statements are true:

(1) Either $\mu'(t) = t$ or $\mu'(t) \subseteq \mu(t)$ (t is matched with himself or he is matched with an old matching under μ).
(2) And $\mu'(t) \succ_t \mu(t)$.

Notice, that the first part of the first condition cannot be, since we assumed IR. And, the second part implies that some b is left alone and matches with himself, also breaking the IR assumption. So, the claim stands.

Two Cases:

(1) $|\mu'(t) \leq |\mu(t)|$.
(2) $|\mu'(t)| > |\mu(t)|$.

Case 1:

We have responsive preferences. We also have that $\mu'(t) \setminus \mu(t) \neq \emptyset$. This is because we showed above that $\mu'(t) \cap C \neq \emptyset$.

Note the following:

$$B_1 = \mu'(t)$$
$$B_2 = \mu(t)$$
$$\mu'(t) \setminus \mu(t) = B_1 \setminus B_2 \neq \emptyset$$
$$|B_2| = |\mu(t)| \geq |\mu'(t)| = |B_1|$$

For each $\hat{B} \subseteq B_2$ we have $\mu(t) \succ_t \hat{B}$. We know this since μ is IR. And of course $\mu'(t) \succ_t \mu(t)$.

Now, apply Lemma 1:

Need to finish notes.