520 11/1

◎ General equilibrium

Edgeworth Box Economies

• $\succsim_i$ on $\mathbb{R}^2_+$

* Strongly monotone : if $x \geqq x'$ and $x \neq x'$, then $x \succ x'$

* Strict convexity : if $x' \succsim x$, $x'' \succsim x$, and $x' \neq x''$

then $\alpha x' + (1-\alpha) x'' \succ x$ for each $\alpha \in (0,1)$

* Continuous : if $x^n \to x^\infty$, $z^n \to z^\infty$ and for each n $x^n \succsim z^n$

then $x^\infty \succsim z^\infty$

* two agents & two goods

I. Pure Exchange economies (NO PRODUCTION)

→ No production

→ Firms will not enter explicitly.

* Commodities : $l = 1, 2$

* consumers : $i = 1, 2$

- consumption set of i : $X_i = \mathbb{R}^2_+$

- preference relation : $\succsim_i$ Strongly monotone, Strictly convex & continuous

* allocation : $(x_1, x_2) \in \mathbb{R}^4_+$

↓

$x_i = (x_{1i}, x_{2i})$

x_{li} ↑ goods ↖ consumers

* $w_i = (w_{1i}, w_{2i}) \in \mathbb{R}^2_+$

* allocation $(x_1, x_2) \in \mathbb{R}^4_+$ is feasible if $x_{l1} + x_{l2} \leqq \bar{w}_l$

allocation is non-wasteful if $x_{l1} + x_{l2} = \bar{w}_l$ for each $l = 1, 2$

Remark Special case of an economy with one firm $J=1$

* $\theta_{11} = \theta_{12} = \frac{1}{2}$
* $Y_J = \mathbb{R}^2_-$

- An allocation (x_1, x_2, J) (Only here)
  - standard definition of feasibility : $x_1 + x_2 = \bar{w} + y$
  - Here : (x_1, x_2) satisfies feasibility according to *

    iff $\exists\ y \in Y_J$ s.t. (x_1, x_2, y) satisfies the standard def'n of feasibility.

- If (x_1^*, x_2^*, y^*) and $p^* = (p_1^*, p_2^*)$ is a competitive equilibrium, then $p_\ell^* > 0$ for each $\ell = 1, 2$

  $\Rightarrow$ if $p_\ell^* \leqq 0$: then NO consumer would have an optimal x_i^*

  $\rightarrow$ Just want to consume more & more.

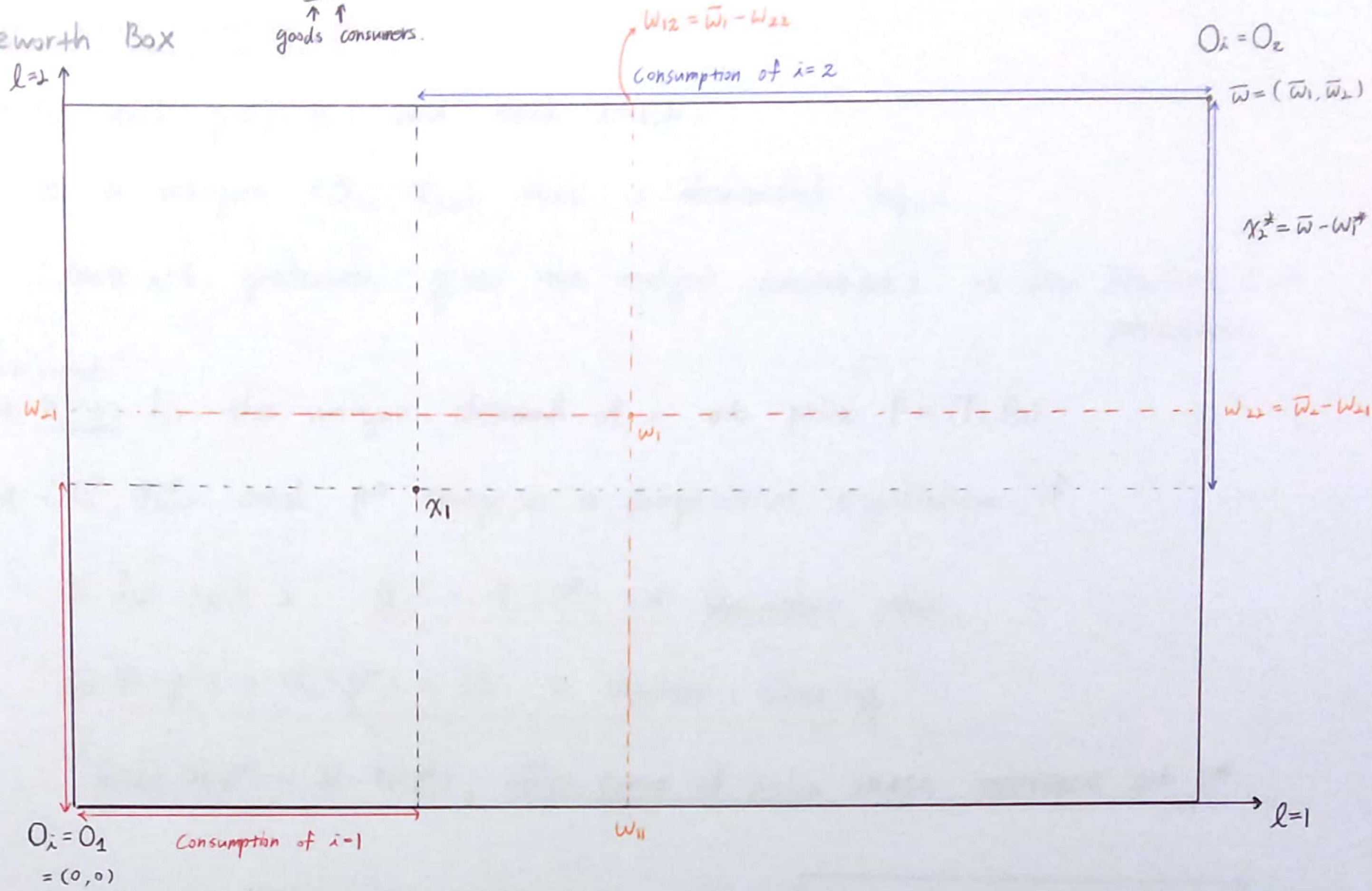
  $\Rightarrow$ if $y^* = (0,0)$,

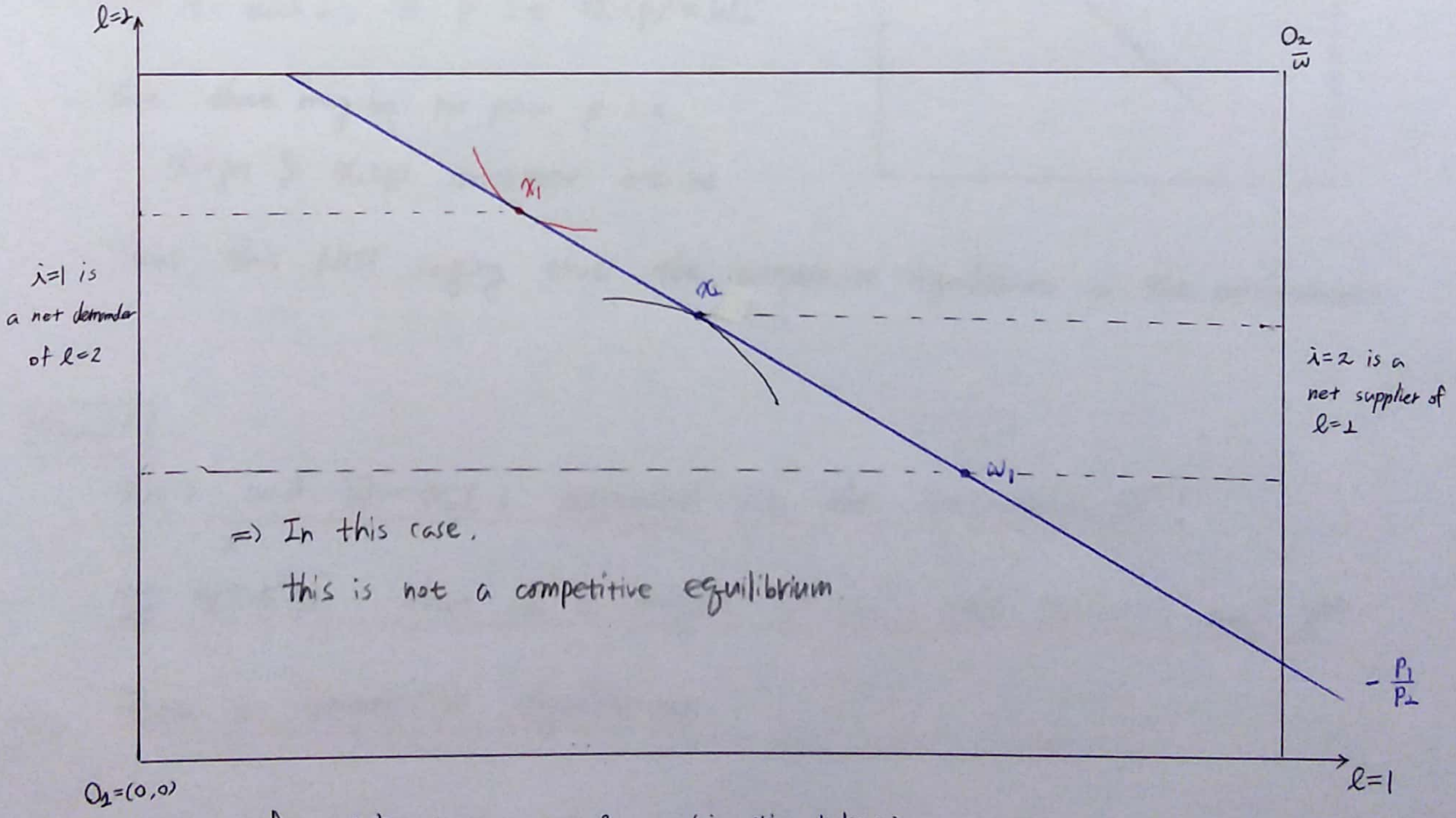
  - firm J chooses y to maximize $\max\limits_{y \in \mathbb{R}^2_-} [p^* \cdot y]$

  $\Rightarrow$ (x_1^*, x_2^*) are non-wasteful.

Edgeworth Box

$x_{\square\triangle}$ ← goods, consumers





$$B_i(p) = \{ x_i \in \mathbb{R}_+^2 : p \cdot x_i \leq p \cdot w_i \} \quad (i: \text{price takers})$$

$$\Downarrow$$

$$p_1(x_{1i} - w_{1i}) + p_2(x_{2i} - w_{2i}) \leq 0$$

Think about varying prices

* for each $p=(p_1, p_2)$ and each $i=1,2$,

  $\exists$ a unique (x_{1i}, x_{2i}) that is demanded by i

  (max i's preference given the budget constraint) $\Rightarrow$ See MWG 3.D.2 proposition.

"Offer curve"

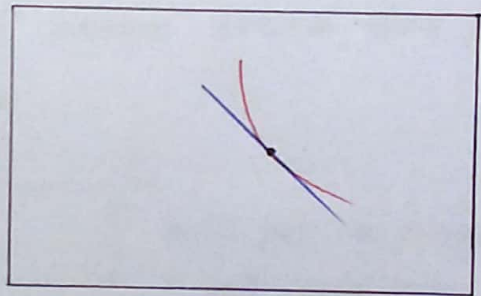
* $x_i(p)$ for the unique demand of i at price $P=(P_1,P_2)$

* (x_1^*, x_2^*) and p^* comprise a competitive equilibrium if

  ① for each i, $x_i^* = x_i(p^*)$ $\leftarrow$ Consumers max.

  ② $x_1(p^*) + x_2(p^*) = \bar{w}$ $\leftarrow$ Market clearing

  Since $x_2(p^*) = \bar{w} - x_1(p^*)$, offer curve of $i=1,2$ must intersect at p^*



$\boxed{\text{Remark}}$ For each i, $\exists$ p s.t $x_i(p) = w_i$

- But there may be no price p s.t

  $x_1(p)$ & $x_2(p)$ intersect at w.

- Thus, this NOT saying that the competitive equilibrium is the endowment.

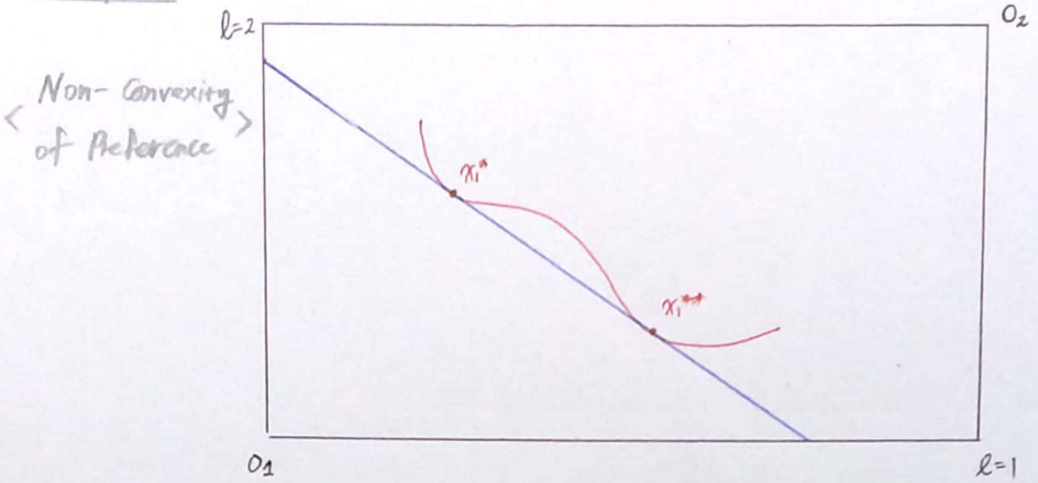
$\boxed{\text{Remark}}$

$x_1(\cdot)$ and $\bar{w} - x_2(\cdot)$ intersect at the consumption x^*:

If $x_1^* \neq w_1$, then $\exists$ a unique p^* s.t. $(x_1^*, \bar{w}-x_1^*)$ and p^* form a competitive equilibrium.

Remark : With slight amendment to our assumptions about preferences, we may not have a competitive equilibrium.

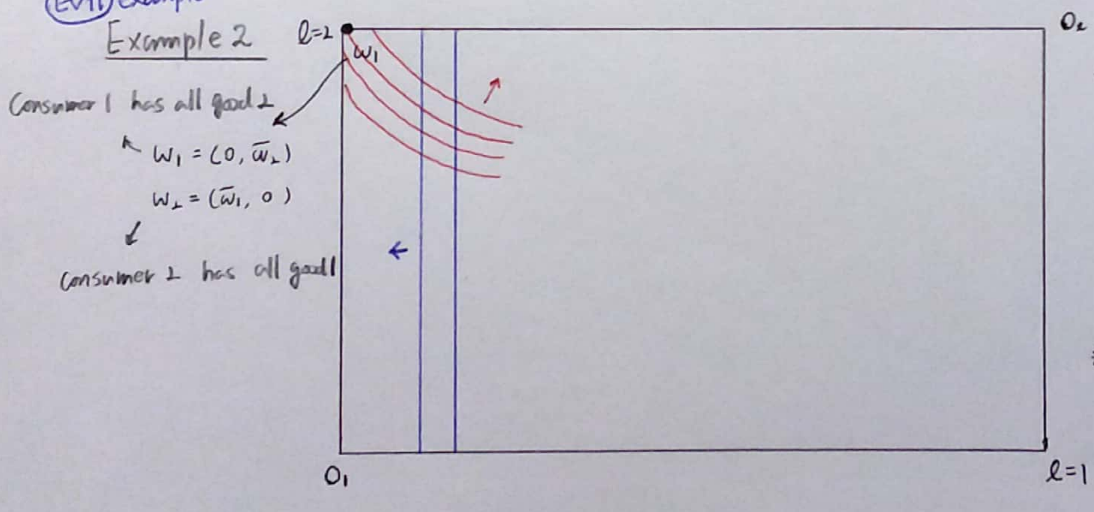
Example 1



- offer curve goes through 2 points these prices.
- induces a jump in the offer curve between these points.
- Consumer 2's offer curve "would" intersect between these points we have a problem for existence.

Evil example.

Example 2



* 1's prefs are strongly monotone and strictly convex
  → the indifference curve $\{x \in \mathbb{R}^+ : x \sim_1 \omega_1\}$ has an infinit slope at ω_1
* 2 only cares about $l=1$
  - prefs are monotone (not strongly)
  - prefs are convex (not strictly)
  - continuous

case 1) $\frac{P_1}{P_2} > 0$

- In this case : 2's best response is to consume $\omega_2 = (\bar{\omega}_1, 0)$
- At the endowment, $\omega_1 = (0, \bar{\omega}_2)$: $MRS_1(\omega_1) >$ relative price

⇒ NO BUNDLE satisfies Market clearing.

Case 2) $P_2 \leqq 0$ and $P_1 > 0$ $\Rightarrow$ and $(0, x_2) \in B_1(p) \Rightarrow 1$ has no best response
so can't have competitive eq

$P_2 > 0$ and $P_1 \leqq 0$

$\Rightarrow$ and $(x_1, 0) \in B_2(p) \Rightarrow 2$ has no best response.