Midterm Solutions **Spencer Cooper**
ECON 501B
Prof. Friedenberg October 2018

1. v is IR and no blocking pair:
   $IR : \forall i \in T \cup B : v(i) \succ_i \mu(i) \succsim_i i$ because μ is stable.
   Suppose $\exists$ a blocking pair (t, b) for v:
   $b \succ_t v(t) \succsim_t \mu(t)$
   $t \succ_b v(b) \succsim_b \Rightarrow (t, b)$ blocks μ

2. Proof by contrapositive:
   Suppose $\exists \mu'$ Pareto dominates μ
   Let $C = \{i \in T \cup B | \mu'(i) \succ_i \mu(i)\}$
   C is non-empty by the definition of Pareto dominance.
   For any $t \in C, \mu'(t) \subseteq \mu(t) \cup C$
   For any $b \notin \mu(t) \cup C$, by definition of C:
   $\mu'(b) = \mu(b) \neq t$.
   Then, $b \notin \mu'(t)$
   For any $b \in C, \mu'(b) \succ_b \mu(b)$
   Let $t' = \mu'(b)$
   Then $b \in \mu'(t')$, but $b \notin \mu(t')$, so $\mu'(t') \neq \mu(t')$
   Because of strict preferences and Pareto dominance, $\mu'(t') \succ_{t'} \mu(t')$
   $\Rightarrow t' \in C$
   So C forms a block to μ.

3. (a) The unique stable match, μ^* :
   $\mu^*(t_1) = b_k, \mu^*(t_i) = b_{i-1}$ for $i = 2, ..., K$
   To show that μ^* is unique stable, if suffices to show it is the outcome of both the T and B-Proposing DA.

   T-Proposal:
   - Round 1: t_1 matched to b_k and b_1 matched to t_2 and these matches do not change.
   - Round k: b_k matched to t_{k+1}, they won't change later.

   B-Proposal:
   - Round 1: t_1 matched to b_k and b_1 matched to t_2 and these matches do not change.
   - Round k: t_k matched to b_{k-1}, they won't change later.

   Thus we have μ^*

   (b) Notice that t_1 and b_k have no incentive to misreport. If $b_k(k < K)$ misreports, through B-Proposal DA, at round i, such that $i < k$, b_i will be matched to t_{i+1} and they won't change later.
   So they cannot get better matches.

   (c) If t_k misreports $(k > 1)$, through B-proposal DAA, at round $k - 1$, all unmatched b agents will propose to t_k, which does not include $(b_1, ..., b_{k-2})$ who have been matched

and won't change.
So t_k cannot get a better match.

Alternate answers for b and c:
Notice that under truthful reporting, the T-proposal DAA yields the same match as that from the B-proposal. So it is equivalent to treat mechanism $m(\cdot)$ as the B-proposal DAA.
Apply Theorem 3 (shown on October 4th in class) and truthful reporting is dominant for any agent $i \in T \cup B$.

4. By Contrapositive: $\forall t : mu(t) \succ_t \mu_{TD}(t)$
Want to show that all agents are matched under μ and μ_{TD}
For $\mu : \forall t : \mu(t) \succ_t \mu_{TD}(t) \succsim_t t$
All t matched. Because $|T| = |B|$, all b matched.

For μ_{TD} to be stable, $\forall b : \mu_{TD}(b) \succsim_b \mu(b) \succ_b b$
Suppose $\exists b : \mu(b) \succ_b \mu_{TD}(b)$. Let $t' = \mu(b)$
$\mu(t') = b \succ_{t'} \mu_{TD}(t')$
Thus (t', b) blocks μ_{TD}
All b agents are matched. $|T| = |B|$ so all t matched.

Because all agents matched under μ_{TD}, there must be a b agent newly matched in the last round. Denote this agent as b^*.
Let $\mu_{TD}(b^*) \succ_{b^*} \mu(b^*)$
Remark: all $t, \mu_{TD}(t) \neq \mu(t)$ so for all $b, \mu_{TD})b \neq \mu(b)$ and $t^* = \mu(b^*)$.
We know $\mu(t^*) \succ_{T^*} \mu_{TD}(t^*)$.
At the last round of T-proposal DAA, t^* must be matched to $\mu_{TD}(t^*)$, and t^* must have proposed to b^* before. We know t^* is acceptable to $b^* \Rightarrow b^*$ must be matched to someone at the second to the last round. But b^* rejects this t' agent at the last round and switch to $\mu_{TD}(b^*)$
At the last round, t' is unmatched, contradicting "the last round."