Final.

Econ 501A ~~Midterm~~ Exam

December 11, 2017

1. The exam questions are on the back side of this page. Please wait before turning this page over.
2. You may rely without proof on results that I have proven in class in 501A, and results that are proven in the supplementary notes that I have written. You may also rely without proof on mathematical results established apart from 501A.
3. Please do not rely without proof on any result that has been proven on the problem sets in 501A. Relying without proof on such results will yield only partial credit.
4. Don't be stressed. Good luck!

If not

$u(a) > u(8) \Rightarrow a \succ 8$

If not $a \succ 8 \Rightarrow$ not $u(a) > u(8)$, i.e. $a \precsim 8 \Rightarrow u(8) \geq u(a)$

$a \precsim 8 \Rightarrow$ 8 must be in the alphabet before a, or at the same place as a.

$\Rightarrow u(8) \geq u(a)$

1. In the context of part I of this course, suppose the grand set of alternatives X contains only three alternatives: $X = \{a, b, c\}$. Suppose that an individual strictly prefers a to b, and strictly prefers b to c.

(a) Is her preference complete and transitive? If yes, prove it. If no, explain and state additional assumptions that would imply her preference is complete and transitive, and continue to hold those assumptions throughout this problem.

(b) Does her preference have a utility representation? If yes, construct a utility representation for her preference and prove that it is in fact a utility representation. If no, explain.

150 150 DW

AU EU CU, AU.

2. In the context of part II of this course, Ellsworth's utility function is $u(x_1, x_2) = \min\{x_1, x_2\}$, where x_1 is the quantity of good 1, and x_2 the quantity of good 2. Ellsworth has wealth $w = \$150$ and initially faces prices $p_1 = p_2 = 1$. Ellsworth's boss is thinking of sending him to another town where the price of good 1 is still 1 but the price of good 2 is 2. The boss offers no raise in pay. Apart from the difference in the price of good 2, the two towns are equally attractive to Ellsworth.

This move would be bad for Ellsworth, because the prices in the new town are higher than the prices in the old town. A natural question is: *How* bad would this move be for Ellsworth, in some numerical terms? On a previous problem set, you computed two numerical measures of how bad the move would be. In total, I believe that in 501A we have considered five different numerical measures that could be offered as answers to the "how bad" question above. (Although, perhaps one or more of these five measures would not provide a good answer.)

What are these five measures? Briefly explain these five measures conceptually, and briefly explain their relative advantages and disadvantages. Also, compute the five measures for Ellsworth's hypothetical move.

AV
AV
CU
EV

3. In the context of part III of this course, consider an individual who has a preference over lotteries. Is true that if that preference has an expected utility representation, then the preference is complete, transitive, continuous and satisfies the independence axiom? If it is true, prove it. If it is false, give a counter example and prove whichever parts are true.

$U(L) = \sum_{n=1}^{N} p_n u_n$

CU.

$U \sum$

Let

W

$U(\sum p_n u_n) = \sum p_n$

$p^0 \to p^1$

$U(\sum p_n u_n) = \sum p_n U u_n$

$L = (L_1, L_2, L_K)$

$U(\sum_{k=1}^{K} L_k) = \sum_{k=1}^{K} U(L_k)$

$U \sum_{n=1}^{N} \quad - \quad U(\sum_{k=1}^{K} L_k) = \sum_{k=1}^{K} U(L_k)$

$(\sum_{k=1}^{K} \sum_{n=1}^{N} p_n u_n) =$

$U(\sum_{k=1}^{K} L_k) = \sum_{k=1}^{K} U(L_k)$

$\sum_{k=1}^{K} u(L_k) = \sum_{k=1}^{K} \sum_{n=1}^{N} u_n p_n = \sum_{n=1}^{N} u_n \sum_{k=1}^{K} p_n$

$$U\left(\sum_{k=1}^{K} L_k\right) = \sum_{k=1}^{K} U(L_k)$$

$$\sum_{n=1}^{N} u_n \left(\sum_{k=1}^{K} p_k\right) = \sum_{k=1}^{K} \sum_{n=1}^{N} u_n p_n = \sum_{k=1}^{K} U(L_k)$$

Econ 501A Final Exam

December 12, 2016

1. *Please wait before turning this page over.*
2. The exam questions are on the back side of this page.
3. Please do not rely without proof on any result that has been established in this course. That is, *if you rely on any such result, please prove it.* Relying without proof on such results will yield partial credit.
4. Don't be stressed. This matters less than the comprehensive exam. Good luck!

1. Suppose $\succeq$ is a complete and transitive preference over some space of alternatives X. Prove that if X is finite, then $\succeq$ has a utility representation.

2. Suppose $\succeq$ is a complete and transitive preference over the space of consumption bundles $X = \mathbb{R}^k_+$. Let $p \in \mathbb{R}^k_{++}$.

(a) Fill in the blank to make the following statement true, and prove the completed statement:

If the preference is c & t ; [illegible] . LNS and $x^* \succeq x$ for all x such that $px \leq px^*$, then $px^* \leq px'$ for all x' such that $x' \succeq x^*$.

(b) Fill in the blank to make the following statement true, and prove the completed statement:

If the preference is c & t contin and $px^* \leq px'$ for all x' such that $x' \succeq x^*$, then $x^* \succeq x$ for all x such that $px \leq px^*$.

(c) If the blanks are left empty, then the two previous statements need not be true. Provide examples showing that.

3. This question regards preferences for lotteries over a set of $N > 2$ possible outcomes, $\{1, 2, ..., N\}$.

(a) Given two lotteries L and L', Ann weakly prefers L to L' if and only if the greatest possible outcome that might occur under L is greater than or equal to the greatest possible outcome that might occur under L'. (Here outcome N is greater than outcome 2, which is greater than outcome 1, and so on.) Does Ann's preference satisfy the independence axiom? Prove your answer.

(b) Given two lotteries L and L', Bob weakly prefers L to L' if and only if the probability that L yields outcome N is greater than or equal to the probability that L' yields outcome N. Does Bob's preference satisfy the independence axiom? Prove your answer.

L L

3m

2m

1m 3m <

L'

0.05

L >

0.1

$p_1 > p_0$
$y^1 > y_0$

Econ 501A Final Exam

December 15, 2014

Don't be stressed. This matters less than the comprehensive exam. Good luck!

This exam has four questions, which are printed across the front and back of this sheet of paper.

$p^0 \quad y^0 \qquad x_1 > x_0$

1. A consumer has indirect utility function $v(p, y)$ which satisfies the standard properties. In each of the following four cases, (p^0, y^0) and (p^1, y^1) are both strictly positive price/income pairs. Demand at the first pair is x^0 and demand at the second pair is x^1. The Hicksian demand function is $x^h(p, y)$.

You are asked to compare the indirect utility at the two pairs. In each of the following four cases, one of the following is true: (i) $v(p^0, y^0) = v(p^1, y^1)$, (ii) $v(p^0, y^0)$ is not more than $v(p^1, y^1)$ and may be strictly less, (iii) $v(p^0, y^0)$ is not less than $v(p^1, y^1)$ and may be strictly more, or (iv) there is not enough to information to compare $v(p^0, y^0)$ and $v(p^1, y^1)$. In each case, say which of (i), (ii), (iii) or (iv) holds and show that your answer is correct.

budget correspond is HAO $p \cdot x \le y$

(a) $(p^1, y^1) = (tp^0, ty^0)$ for some $t \in R_{++}$ HAO (i)

(b) $(p^1, y^1) \gg (p^0, y^0)$? (iv)

(c) $y^1 \ge p^1 x^0$ (ii) x^0 is aff under (p^1, x^1) $x^0 p^1 \le y^1$

(d) p^1 and p^0 differ only in the price of good 1: $p^1_1 > p^0_1$ while $p^1_i = p^0_i$ for all $i \neq 1$ and

? $y^1 - y^0 = \int_{p^0_1}^{p^1_1} x^h_1(p, v(p^0, y^0))dp_1$ $\boxed{CV}$ + (iv) $p^1 > p^0$ $p_0 < p^1$

2. Suppose that a particular preference over gambles can be represented by a utility function that has the expected utility property. Both of the following are true: (i) The preference satisfies the substitution axiom and (ii) The preference satisfies the independence axiom.

price ↓ EV + $\Delta V = V(p^1, w^0) - V(p^0, w^0) > 0$

Show either (i) or (ii). (You will get full credit for showing just one, you need not show both. You may have seen more than one version of the independence axiom. If you choose (ii), state which version of the independence axiom you are showing.)

$V(p^1, w^0) > V(p^0, w^0)$

3. Eunice is an expected utility maximizer with twice differentiable vNM utility function $u(\cdot)$ where $u' > 0$. With probability $\alpha \in (0, 1)$ she will suffer an accident that results in a loss of $\$L > 0$ if uninsured. She must decide how much insurance to purchase at a price per unit of $\rho > 0$. She can purchase any quantity of insurance $x \geq 0$, in which case her final wealth will be $\$(w - \rho x - L + x)$ if the accident occurs or $\$(w - \rho x)$ if the accident does not occur.

$u'' < 0$

$r_A(x) = \frac{-u''(x)}{u'(x)}$

If $\alpha = \rho$ insure totally Part (B)

(a) Suppose that $u'' < 0$.

posit $\rho > \alpha$ For which values of $\rho > 0$, if any, will Eunice choose $x^* = 0$? None.

$\rho < \alpha$ For which values of $\rho > 0$, if any, will Eunice choose $x^* = L$? If $\alpha = \rho$. L

for concave $\alpha = \rho$ For which values of $\rho > 0$, if any, will Eunice choose $x^* \in (0, L]$? If $\rho > \alpha$

For which values of $\rho > 0$, if any, will Eunice choose $x^* > L$? ~~None~~.

probal loss = cost per unit

For each of these four questions, show that your answers are correct.

$\Rightarrow u''(x) < 0$ $\Rightarrow$ concave risk averse

(b) Repeat part (a) but suppose instead that $u'' = 0$. risk neutral.

4. A firm has production function $f : R^n_+ \to R$.

Let $c(w, y)$ be its cost function and $\pi(p, w)$ its profit function.

Let $p \in R_{++}$ and $w \in R^n_+$.

(a) Let $x^* \in \arg\max_{x \in R^n_+} pf(x) - wx$. Show that $f(x^*) \in \arg\max_{y \in R_+} py - c(w, y)$.

(b) Let $y^* \in \arg\max_{y \in R_+} py - c(w, y)$. Suppose that $\hat{x}$ is such that $w\hat{x} = c(w, y^*)$ and $f(\hat{x}) = y^*$. Show that $\hat{x} \in \arg\max_{x \in R^n_+} pf(x) - wx$.

(c) Given x^* and y^* from the previous parts, show that $pf(x^*) - wx^* = \pi(p, w) = py^* - c(w, y^*)$.