Econ 501A Solutions HW2

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Problem 1: Prove that for finite X, a complete and transitive $\succsim$ has a utility representation.

Strategy: (1) First construct $u(x)$ (2) Show that $u(x)$ represents $\succsim$ i.e.:

$$x \succsim y \iff u(x) \geq u(y)$$

One way to solve this is:

$$\forall x \in X : u(x) = |\{y : y \in X, x \succsim y\}$$

Another way to solve this is using Rubenstein's way:

Definition: an element $a \in X$ is minimal if $x \succsim a$ for any $x \in X$.

Lemma: If any finiste set $A \subseteq X$ There is a minimal element.

Proof:

By induction on the size of A, A is a singleton. By completeness, the only element is minimal.

Assume $|A| = n$ is true. For $|A| = n + 1$, let $x \in A$ then for the set of $A \setminus x$, there is a minimal element denoted by y. If $x \succsim y$ then y is minimal in A. If $y \succsim x$, the by transitivity then $z \succsim x$ for all $z \in A \setminus x$ and therefore x is minimal.

Problem: Prove that for finite X, a complete and transitive $\succsim$ has a utility representation.

Proof:

Contruct a seqence of sets inductively. Let X be the subsets of elements that are minimal in X. Hence ($X_1 \neq \emptyset$)

Assume that we have constructed $X_1, ..., X_k$. Now if $X = X_1 \cup X_2 \cup ... \cup X_k$, we are done. If not, define X_{k+1}, the set of minimal elements in $X - X_1 - X_2 - X_3 - ... - X_k$. And by lemma $X_{k+1} \neq \emptyset$.

Since X_i is finite, we must be done after at most $|X|$ steps. Define $u(x) = k$ if $x \in X_k$. To verify that u represents $\succsim$ let $a \succsim b$. Then $b \notin X - X_1 - X_2 - ... - X_{u(a)}$.

Thus, $u(a) \geq u(b)$

Problem 2:

(a) if $\succsim$ is complete, B contains two elements $\implies C \succsim (B) \neq \emptyset$ if $\succsim$ is not complete, $B = \{x, y\}$. Since$\succsim$ is complete: x not succsim y and y not succsim x. $X \in C_{\succsim}(B), y \in C_{\succsim}(B)$

If $\succsim$ is not complete, the there exists $x, y \in X$ such that x not succsim y and y not succsim x.

(b) $\succsim$ complete and transitive, $\mathcal{B}, C_{\succsim}(B)$ satisfies finite nonemptiness.

We already proved that for any finite set, there exists maximum element (minimal above in prob 1) in X by induction. Denote this maximal element by X^*. By defition, for any finite B, $\exists x^* \in X$ such that for any $y \in B$ finite, $x^* \succsim y \implies x^* \in C_{\succsim}(B)$

So, $C_{\succsim}(B) \neq \emptyset$.

Problem 3

X is finite. Consider $C(\cdot)$ defined on all nonempty subsets of X. Then: there exists a complete and transitive preference $\succsim$ such that it generates the choice rule $C(\cdot)$ if and only if $C(\cdot)$ satisfies weak axiom and finite nonemptiness (Proposition 3.1b). This implies that $C_{\succsim}(\cdot)$ satisfies finite nonemptiness by proposition 3.2. Then $\succsim t \implies C_{\succsim}(\cdot)$ has weak axiom.

So, $C(\cdot) = C_{\succsim}(\cdot)$.

Essentially, repeat 3.3 in lecture notes.

Problem 4 (3.3)

$X = \{x_1, ..., x_n\}$ is a finite set of prices of wine with lower the price lower the subscript on x.

(a) Ann always chooses the cheapest bottle. Is her choice rule rationalizable?

First answer yes or no. If yes, then define a preference. Then, show that it is complete and transitive. Show that the choice rule generated by this preference is equal to Ann's original choice rule ($C_{\succsim}(\cdot) = C_A(\cdot)$)

The preference Ann has is: $x_i \succsim x_j$ if $i \leq j$. So, it is complete. It is transitive (easy to show). For any B the choice rule $C_{\succsim}(B) = \{x_{i^*}\}$ where $i^* = \min\{i : X_i \in B\}$

Then $C_A(B) = \{x_{i^*}\}$ and $C_A(B) = C_{\succsim}(B)$

(b) Bob's choice rule second cheapest in a set. Show that his preference does not fulfill weakness axiom. This would then imply that his choice is not rationalizable.

Consider $B = \{x_1, x_2, x_3\}$ and $B' = \{x_2, x_3, x_4\}$.

Bob chooses x_2 in B and x_3 in B' So, it violates the weakness axiom.

(c) Also not rationalizable since in one set she could choose an x_i but another set she chooses nothing.

Problem 5

Consider single valued choice rule.

(i) $C(\cdot)$ satisfies WA.

(ii) $x, y \in B, B'$ and $x \neq y$ if $C(B) = x \implies C(B') \neq y$

(iii) $x, y \in B, C(B) = x, c(B') = y \implies x \notin B'$.

(iv) to (ii):

WARP states that if x is revealed at least as good as y, then y cannot be revealed preferred to x. Then $x, y \in B : x \in C(B) \implies x \succsim y$. And if $y \notin C(B) \implies x \succ y$.

So, if $y \in C(B')$ then you cannot have x in this choice set.

(ii) to (iii): Proof by contradiction. Suppose $x \in B, y \in B, C(B) = x$

To be continued in the next recitation