501A TA 11/17

#16.2 $f^*(t) = \min_{x\in X} f(x,t)$, ($f(x,t)$ is continuous & X is compact, f^* is well defined
f is non decreasing in t

WTS: $f^*(t)$ is nondecreasing in t.

proof) Let $t_2 \geq t_1$

Since f^* is well defined, let ($x' \in \arg\min_{x\in X} f(x,t_1)$
$x'' \in \arg\min_{x\in X} f(x,t_2)$

$$f^*(t_1) = f(x',t_1) \leq f(x'',t_1) \leq f(x'',t_2) = f^*(t_2).$$

Fix u, $e(p,u) = \min_{x:u(x)\geq u} px$

$u \in [u(0), \sup_{x\in X} u(x)] \Rightarrow e(p,u)$ is well-defined.

Since px is nondecreasing in p, then $e(p,u)$ is nondecreasing in p

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#16.3 $u(x_1,x_2) = x_1^\alpha x_2^{1-\alpha} \Rightarrow e(p,u) = \alpha^{-\alpha}(1-\alpha)^{\alpha-1} p_1^\alpha p_2^{1-\alpha} u$.

i) HD1 in p $(t>0)$

$$e(tp,u) = \alpha^{-\alpha}(\alpha-1)^{\alpha-1}(tp_1)^\alpha(tp_2)^{1-\alpha}u = te(p,u)$$

ii) Strictly increasing in u and non-decreasing in p.

$$\frac{\partial e}{\partial u} = \alpha^{-\alpha}(1-\alpha)^{\alpha-1}p_1^\alpha p_2^{1-\alpha} > 0 \quad \text{when } p \gg 0,\ 1>\alpha>0$$

$$\frac{\partial e}{\partial p_1} = \alpha^{1-\alpha}(1-\alpha)^{\alpha-1}p_1^{\alpha-1}p_2^{1-\alpha}u > 0 \quad \text{when } p\gg 0,\ 1>\alpha>0,\ u>0$$

iii) Concave in p. Check Hessian Matrix.

$$H = \begin{pmatrix} \alpha^{1-\alpha}(1-\alpha)^{\alpha-1}(\alpha-1)p_1^{\alpha-2}p_2^{1-\alpha} & , -\alpha^{2-\alpha}(1-\alpha)^{\alpha-1}p_1^\alpha p_2^{-\alpha-1} \\ -\alpha^{2-\alpha}(1-\alpha)^{\alpha-1}p_1^\alpha p_2^{-\alpha-1} & , -\alpha^{1-\alpha}(1-\alpha)^{\alpha}p_1^\alpha p_2^{-\alpha-1} \end{pmatrix}$$

$H_{11} \leq 0$ and $|H| > 0 \Rightarrow H$ is negative semi-definite.

$\Rightarrow e(p,u)$ concave in p

iv) Continuous in p, u.

→ Trivial.

$h_1(p,u) = \left(\frac{\alpha p_2}{(1-\alpha)p_1}\right)^{1-\alpha} u \qquad h_2(p,u) = \left(\frac{\alpha p_2}{(1-\alpha)p_1}\right)^{-\alpha} u$

i) HD0 in p $t>0$. $h_1(tp,u) = \left(\frac{\alpha t p_2}{(1-\alpha)t p_1}\right)^{1-\alpha} u = h(p,u)$

ii) No excess utility

$u(h_1, h_2) = h_1^{\alpha} h_2^{1-\alpha} = u$

iii) $\succsim$ convex $\Rightarrow$ $\underline{h(p,u) : \text{quasi-convex}}$

trivial

17.1 X, Y closed, convex, $S_Y = S_X \Rightarrow X = Y$.

Suppose $X \neq Y$. b/c Y is closed and convex.

WLOG, $\exists\ x \in X \setminus Y$. By theorem 17.1,

$\exists$ hyperplane that strictly separates y and x.

i.e., $\exists\ p \in \mathbb{R}^k$, $p \neq 0$, $w \in \mathbb{R}$ s.t. for any $y \in Y$,

either $py < w < px$ or $px < w < py$.

WLOG, $py < w < px$ $\qquad S_Y(p) = \sup_{y \in Y} py \leq w < px \leq \sup_{x \in X} S_X(p)$

i.e., $S_Y(p) < S_X(p) \Rightarrow S_Y(p) \neq S_X(p)$. //

#14.2 (c) $B = [0,1]$, $B' = [0,1)$

Define $\succsim$ on $\mathbb{R}$ to be $x \succsim y$ if $x \geqq y$.

$u(x) = x$ represents $\succsim$.

By definition, $V(B) = V(B') = 1$. $\Rightarrow V(B') \geqq V(B) \not\Rightarrow B' \succsim^* B$

because for all $x' \in B$, $\exists\ 1 \in B$ s.t. $1 \succ x'$. //