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ECON 501B

Final 2018

Instructions: You have two hours to complete the exam. *All* answers should involve complete proofs to back up your assertions. You may reference results from class, provided that your reference is not self-referential or close to self-referential. (E.g., Don't say "amend this proof from class." Instead give the new proof. But feel free to use Lemmata that still hold.) Note: Questions 1 and 4 will require less time than Questions 2 and 3. (Or at least that is the case for me!)

Question 1: There are three workers w_1, w_2, w_3 and three government positions g_1, g_2, g_3. Each position is run by a manager who can hire at most one worker. Every worker is acceptable to every manager and every position is acceptable to every worker. The associated wages are fixed by the government—managers cannot offer transfers to workers and visa versa. Given those wages, we have the following utilities for any given matched pair. (For instance, if g_1 and w_2 are matched together, then g_1 obtains a utility of 9 and w_2 obtains a utility of 10.)

	w_1	w_2	w_3
g_1	10,10	9,10	1,3
g_2	9,9	8,2	1,2
g_3	9,1	8,1	1,1

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Given this situation the stable matching μ results. However, the government is thinking of changing the policy to allow transfers between matched agents. The transfers must not exceed the sum of the utility between a matched pair of agents. But, otherwise, any transfer is allowed. Would this change the resulting stable match? Is this good or bad for the agents?

Question 2: In both the matching and the general equilibrium literatures, you have encountered the definition of a core. The idea is that a matching or an allocation is in the core if no coalition has a strict incentive to deviate. This was formalized in terms of a blocking coalition. *In everything that follows, take the phrase "matching" to mean "one-to-one matching" and take the economy to be a pure exchange economy (with potentially a garbage firm).*

2a. (Matching+GE Question) State the formal definitions of what it means for a coalition to (1) block a matching, and (2) block an allocation. Use your definition to define the core in each of these contexts.

In the context of the matching literature, you also came across a variant of the blocking definition—one that I called block*. This was the idea that every member of the coalition is at least as well off—some are strictly better off.

2b. (Matching+GE Question) State the formal definitions of what it means for a coalition to (1) block* a matching, and (2) block* an allocation. This amended definition defines a new concept called a core*. (Don't waste time writing it out.)

2c. (Matching+GE Question) Show that, in each of these contexts, core* implies Pareto efficiency/optimality. (That is, show that if a matching is in the core* then it is Pareto efficient and if an allocation is in the core* then it is Pareto optimal.)

2d. (GE Question) Suppose that preferences are locally non-satiated. Show the following: If $((x^*, y^*), p^*)$ is a competitive equilibrium allocation, then (x^*, y^*) is in the core*.

2e. (Matching Question) Suppose that preferences are strict. Show that a matching is in the core if and only if it is in the core*.

2f. (GE Question) Suppose that preferences are locally non-satiated. Imagine you wanted to show that (x^*, y^*) is in the core* if and only if it is in the core. What "difficulty" would you run into for such a proof? Does it matter? (Think about why you might care about such a result and tell me whether you need such a result given that "care.")

Question 3: Consider a pure exchange economy $E = (Y_1; X_1, X_2, \succsim_1, \succsim_2, \omega_1, \omega_2, \theta_{1,1}, \theta_{2,1})$ where:

- Firm 1 has a production set $Y_1 = \mathbb{R}^2_-$,
- Consumer i has consumption set $X_i = \mathbb{R}^2_+$ and a "bliss consumption bundle" $\hat{x}_i \in X_i$,
- Consumer i has preferences represented by $u_i(x_i) = -\|x_i - \hat{x}_i\|^2$, and

Note, $\hat{x}_i \in X_i$ reflects consumer i's "bliss consumption bundle;" that is, when i consumes $\hat{x}_i$ her utility is zero, but when she consumes $x_i \neq \hat{x}_i$ her utility is less than zero. Loosely, moving away from the bliss bundle lowers i's utility. If two bundles x_i and x_i' have the same Euclidean distance from $\hat{x}_i$, then they provide i with the same utility.

(Hint: A picture of i's indifference curves may help you see what is going on.)

3a. Consider two scenarios:

1. Scenario A: There exists $\ell = 1, 2$ with $\hat{x}_{\ell,1} + \hat{x}_{\ell,2} > \overline{\omega}_\ell$.
2. Scenario B: For each $\ell = 1, 2$, $\hat{x}_{\ell,1} + \hat{x}_{\ell,2} \leq \overline{\omega}_\ell$.

For each of these scenarios: Suppose $((x^*, y^*), p^*)$ constitutes a price equilibrium with transfers. Can it be the case that both $x_1^* = \hat{x}_1$ and $x_2^* = \hat{x}_2$? (In words: In a price equilibrium with transfers, can both consumers obtain their bliss consumption bundles?) If so, provide a proof that such an equilibrium exists; if not, show that such an equilibrium cannot exist.

3b. For each of the scenarios in 3a, does your answer shed light on whether or not the Second Welfare Theorem holds in this example?

3c. Let $((x^*, y^*), p^*)$ constitute a price equilibrium with transfers (W_1, W_2). Is (x^*, y^*) Pareto optimal?

3d. Suppose that $((x^*, y^*), p^*)$ constitutes a price equilibrium with transfers (W_1, W_2). Show the following:

1. $x_i^* \leq \hat{x}_i$.
2. If $x_i^* \neq \hat{x}_i$, then there exists some ℓ so that $x_{\ell,i}^* < \hat{x}_{\ell,i}$ and $p_\ell^* > 0$.

3e. Suppose that $\hat{x}_{1,1} + \hat{x}_{1,2} > \overline{\omega}_1$ and $\hat{x}_{2,1} + \hat{x}_{2,2} > \overline{\omega}_2$. Let $((x^*, y^*), p^*)$ constitute a price equilibrium with transfers (W_1, W_2), where $x_1^* \neq \hat{x}_1$ and $x_2^* \neq \hat{x}_2$. Draw an Edgeworth Box that depicts the consumption of each consumer, relative to her preferences and budget sets. Be sure to indicate how you know the picture looks as it does. In addition, describe the equilibrium conditions cannot be seen from the picture alone.

Question 4: Consider an economy with one firm and two consumers. Each consumer i has a consumption set $X_i = \mathbb{R}^2_+$. Moreover, consumer i's preferences are represented by a utility function $u_i : X_i \to \mathbb{R}$ that is strictly increasing in the first commodity $\ell = 1$ and strictly decreasing in the second commodity $\ell = 2$. Endowments are given by $\omega_1 = (5, 0)$ and $\omega_2 = (0, 5)$.

4a. First suppose that the production set of the firm is $Y = \mathbb{R}^2_-$. If $((x^*, y^*), p^*)$ is a competitive equilibrium of this economy, what must x^*, y^*, and p^* be?

4b. Now suppose that the production set of the firm is $Y = (0, 0)$. If $((x^*, y^*), p^*)$ is a competitive equilibrium of this economy, what must x^*, y^*, and p^* be?