(Defn) An allocation (x^*, y^*) and price vector $p^* \in \mathbb{R}^L$ constitute a price equilibrium with transfers if there exists $W = (W_1, \ldots, W_I)$ with

(1) For each firm j, $p^* \cdot y_j^* \geq p^* \cdot y_j$, for all $j \in Y_j$

(2) For each consumer i,

2a) $x_i^* \in B_i(p^*, W_i) \equiv \{x_i \in X_i : p^* x_i \leq W_i\}$

2b) $x_i^* \succsim_i x_i$ for all $x_i \in B_i(p^*, W_i)$

(3) $\sum_{i=1}^{I} x_i^* = \sum_{i=1}^{I} \omega_i + \sum_{j=1}^{J} y_j^*$

(4) $\sum_{i=1}^{I} W_i = p^* \cdot \bar{\omega} + \sum_{j=1}^{J} p^* \cdot y_j^*$

Today: Second Welfare theorem

* Approach to Second Welfare theorem
  - Define "silly equilibrium"
  - Show that under [Assumption 1] any Pareto optimal allocation can be sustained as a silly equilibrium
  - Show that under [Assumption 2] any silly equilibrium induces a price equilibrium with transfers

* Motivation

For each $x_i \in X$, $[p^* x_i \leq W_i \Rightarrow x_i^* \succsim_i x_i] \Leftrightarrow [x_i^* \prec_i x_i \Rightarrow p^* x_i > W_i]$

⇓

Instead $[x_i \succ_i x_i^* \Rightarrow p^* \cdot x_i \geq W_i]$ ← can we add this?

⇓

To see how silly, think about contrapositive. $[p^* \cdot x_i < W_i \Rightarrow x_i^* \succsim_i x_i]$

⇓

Could have some x_i s.t. $p^* x_i = W_i$ and $x_i^* \succsim_i x_i^*$

Defn An allocation (x^*, y^*) and a price vector $p^* \in \mathbb{R}^L$ constitute a quasi-price equilibrium with transfers if there exists an assignment of wealth, $W = (W_1, \dots, W_I)$ s.t.

(1) for each j, $p^* \cdot y_j^* \geq p^* \cdot y_j$ for each $y_j \in Y_j$

(2) for each i and each $x_i \in X_i$, $x_i \succsim_i x_i^* \Rightarrow p^* \cdot x_i \geq W_i$

The difference from P.E.T.

(3) $\sum_{i=1}^{I} x_i^* = \sum_{i=1}^{I} W_i + \sum_{j=1}^{J} y_j^*$

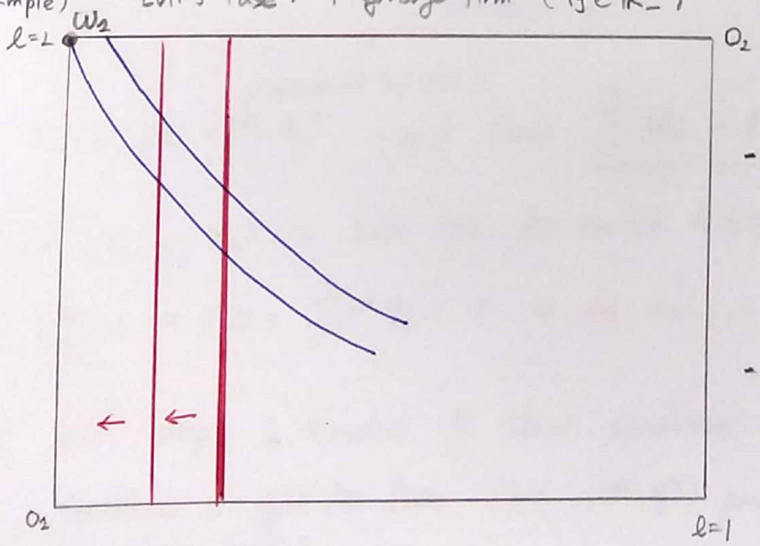
(4) $\sum_{i=1}^{I} W_i = p^* \cdot \bar{\omega} + \sum_{j=1}^{J} p^* \cdot y_j^*$

Theorem Second Welfare Theorem (Silly Ver.)

Fix an economy when (a) each Y_j is convex and (b) each $\succsim_i$ is locally non-satiated and convex.

Then, for every Pareto optimal allocation (x^*, y^*), there exists a price vector $p^* \neq (0, \dots, 0)$ s.t. $((x^*, y^*), p^*)$ constitute a quasi-price equilibrium with transfers.

Example) Evil's case: 1 garbage firm ($Y_J \in \mathbb{R}^2_-$)



$\bar{\omega} = (\bar{\omega}_1, \bar{\omega}_2)$: Pareto optimal allocation.

$((W_1, W_2), y_J^*)$, $y_J^* = (0, 0)$

- If $p_2^* = 0$, 1 can always consume more of good $l=2$ and be better off.

  $\Rightarrow i=1$ does not have an optimal bundle.

- If (p_1^*, p_2^*) s.t. $p_1^* > 0$, $p_2^* = 0$

  $i=1$ can consume $W_1 = (0, \bar{\omega}_2)$

  $\rightarrow \exists\, x_1 = (0, x_{21})$ s.t. $x_{21} > \bar{\omega}_2$

  But, $p^* x_1 = p^* \cdot \omega_1$ does not contradict q-PET ✓

  $W_1 = 0$, $W_2 = p^* \cdot \omega_2 = p_1^* \cdot \bar{\omega}_1 + p_2^* \cdot 0 = p_1^* \cdot \bar{\omega}_1$

  $i=2$ can consume $W_2 = (\bar{\omega}_1, 0)$

Prop 1 Let each $\succsim_i$ be continuous on a convex set X_i.

Let x_i^*, $p \in \mathbb{R}^L$, and W_i be such that for each $x_i \succsim_i x_i^* \Rightarrow p \cdot x_i \geq W_i$

"x_i^* is quasi-optimal given P and W_i"

If $\exists$ a bundle $\tilde{x}_i \in X_i$ s.t. $p \cdot \tilde{x}_i < W_i$,

if $\exists$ a "cheaper consumption"

then for all $x_i \in X_i$, $x_i \succ_i x_i^* \Rightarrow p \cdot x_i > W_i$

x^* is optimal given P and W_i

Corollary Fix an economy. where (a) each $X_i \in \mathbb{R}^L$ is convex and contains $(0, \dots, 0) \in X_i$,

and (b) each $\succsim_i$ is continuous.

If $((x^*, y^*), p^*)$ is a quasi-price equilibrium w/ strictly positive transfers for each i,

then $((x^*, y^*), p^*)$ is a price equilibrium with transfers.

$W = (W_1, \dots, W_I)$
s.t. each $W_i > 0$

Endogenous object.

*Proof of Second Welfare Theorem (Silly ver.)

Fix a pareto optimal allocation (x^*, y^*) and construct sets $V = \{\sum_{i=1}^{I} x_i : \cancel{\text{for}} \text{ each } i,\ x_i \in X_i,\ x_i \succ_i x_i^*\}$ $\subseteq \mathbb{R}^L$

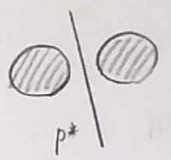
total consumption when each i is better off than at x^*

$Y = \{\sum_{j=1}^{J} y_j : \text{for each } j,\ y_j \in Y_j\} \subseteq \mathbb{R}^L$

$\hookrightarrow$ total possible productions.

Step 1: there exists $p^* \in \mathbb{R}^L \setminus 0$ and $r \in \mathbb{R}$ s.t. (1) $p^* \cdot z \geq r$ for all $z \in V$

(2) $r \geq p^* \cdot z$ for all $z \in \bar{\omega} + Y$



- To show step 1, V is convex, $\bar{\omega} + Y$ is convex, $V \cap (\bar{\omega} + Y) = \emptyset$

$\bar{\omega} + Y = \{z \in \mathbb{R}^L : z = \bar{\omega} + z' \text{ for some } z' \in Y\}$.

(Determined by step 1)

Step 2 take $W_i = p^* \cdot x_i^*$ and show $\sum_{i=1}^{I} W_i = p^* \cdot \bar{\omega} + \sum_{j=1}^{J} p^* \cdot y_j^*$ $\leftarrow$ use step 1 (use feasibility of (x^*, y^*))

Step 3 If $(x_1, \dots, x_I)$ is such that, for each i, $x_i \succsim x_i^*$, then $\sum_{i=1}^{I} p^* \cdot x_i \geq r$. $\leftarrow$ // (Same).

Step 4 $\sum_{i=1}^{I} W_i = p^* \bar{\omega} + \sum_{j=1}^{J} p^* \cdot y_j = r$ $\leftarrow$ use Step 2, 3

Step 5 Use steps 1, 3 and 4 to show conditions (1)-(2) of a quasi equilibrium.

Condition 3) get for free since (x^*, y^*) pareto optimal implies feasible.