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**ECON 501B**

## ECON 501B: Problem Set 7

Due: Thursday, November 1, 2018

**Instructions:** Answers should be complete proofs of a claim.

**Question 1:** Consider a matching market where there are $n$ Minions $(M_1, \ldots, M_n)$ and $n$ Grus $(G_1, \ldots, G_n)$. (You may assume $n$ is either even or odd, if that makes your life easier.) They look to match with one another in Minion and Gru pairs; all matches are acceptable. If Minion $M_i$ is matched to Gru $G_j$ they each get a utility of $u(M_i, G_j) = (i + j)^\alpha$, where $\alpha \in (0, 1)$.

1. If utility is non-transferable, what is the resulting stable match $\mu^{NTU}$?
2. If utility is transferable, what is the resulting stable match $\mu^{TU}$?
3. What is the welfare associated with $\mu^{NTU}$? What is the welfare associated with $\mu^{TU}$?
4. Consider the ratio of the welfare associated with the match $\mu^{NTU}$ to the welfare associated with the match $\mu^{TU}$, i.e.,
$$\frac{W(\mu^{NTU})}{W(\mu^{TU})}.$$
What happens to this ratio as $n \to \infty$. Interpret this finding.

**Question 2:** Consider the partial equlibrium economy described in class. Let Problem (*) represent the following constrained optimization problem:

$$\max_{(x,q)} [\overline{\omega}_m + S(x, q)] \qquad \text{subject to} \qquad \sum_{i=1}^{I} x_i = \sum_{j=1}^{J} q_j.$$

Show the following.

1. If $(x^*, q^*)$ solves Problem (*), then there exists a Pareto optimal allocation $(x^*, m^*, q^*, z^*)$.
2. If $(x^*, m^*, q^*, z^*)$ is Pareto optimal, then $(x^*, q^*)$ solves Problem (*).

**Question 3:** MWG Question 10.C.2