#1 By Second Welfare Theorem, (silly ver)

(1) Each Y_j convex

(2) each $\succsim$ LNS, convex. then Since (x^*, y^*) is Pareto Optimal,

$\exists\ p^* \neq 0$ s.t. (x^*, y^*, p^*) is a quasi-price equilibrium with transfers.

Since each $\succsim_i$ is strongly increasing,

then $p_\ell^* > 0$ for all ℓ, otherwise $\exists$ no maxima for consumers.

$y^* = 0$ because $p^* \cdot y < 0$ for any $y \in \mathbb{R}^L_-$ and $y \neq 0$

WTS: quasi-price equilibrium with transfers $\Rightarrow$ price equilibrium with transfers.

(Exclude the case that $\exists$ no transfers)

Let $E = \{i \in I \mid W_i = 0\}$ Check (x^*, y^*, p^*) is a quasi-price equilibrium with transfers on $I \setminus E$

(1) production $p^* \cdot y^* = 0 \geq p^* \cdot y$ for any $y \in \mathbb{R}^L_-$

(2) consumption for any $i \in I \setminus E$

$x_i^* \succsim_i x_i, \ \forall x_i \in B_i(W_i)$ from quasi-price equilibrium on I

$B_i(W_i) = \{x_i \in X_i \mid p^* x_i \leq W_i\}$.

(3) $\sum_{i \in I\setminus E} x_i^* = \sum_{i \in I \setminus E} W_i \Leftarrow \sum_{i \in I} x_i^* = \sum_{i \in I} \omega_i$

(persons with $W_i = 0$ and $p_\ell^* > 0, \forall \ell$)

(4) $\sum_{i \in I\setminus E} W_i = \sum_{i \in I\setminus E} p^* \cdot \omega_i \Leftarrow \sum_{i \in I} W_i = \sum_{i \in I} p^* \cdot \omega_i$

That means (x^*, y^*, p^*) is a quasi-price equilibrium with strict transfers on $I \setminus E$. Use the corollary, the conditions are satisfied

(a) X_i is convex and contains 0

(b) $\succsim_i$ is continuous

So (x^*, y^*, p^*) is a price equilibrium with transfers on $I \setminus E$.

Add E to this economy. ___ which is still a price equilibrium. ||

#Q2. a. False

Two goods: x, y one type: $\succsim_i$ $u(x,y) = xy$ (T=1, type of 1??)

$w = (2, 0)$

One firm can convert one x to one y, but has capacity of 1.

So for N=1, (1,1) is P.O.

But for N=2, you can at most get an allocation (3,1),

which cannot reach a pair of payoffs (1,1), (1,1)

b. False.

Give all endowments to one consumer,

Let him optimize through production and take all the goods.

Because of strongly monotonicity,

there is no waste, the outcome is P.O.

#2. (c) True

Strictly convexity ⇒ unique maximal consumption.

Suppose there are two distinct optimal points $x, x' \in B_i(w_i, p)$,

Then, for $\lambda \in (0,1)$, $\lambda x + (1-\lambda)x' \in B_i(w_i, p)$

and $\lambda x + (1-\lambda)x' \succ x, x'$, contradiction.

In the competitive equilibrium (x^{*M}, y^*, p^*),

(t,i) and (t,i') have the same endowments,

the same budget set, the same strictly convex preference,

and the same consumption bundle.

(d) Yes.

#3. (a) No.

① $p_2 >> 0$, otherwise No maximum in the consumption of good 2.

② $p_2 > 0$, $\pi = p_2 y_1^2 - p_1 y_1 \rightarrow \infty$ $(y_1 \rightarrow \infty)$

(b) CE

$p^* = (1, 0)$
$\quad\;\; p_1 \; p_2$

$x^* = (10, 0)$

$y^* = (0, 0)$ (No incentive to produce)

#Q4.

Interior solution

$$P_1^* = \frac{1}{3} \quad P_2^* = \frac{1}{4} \quad P_3^* = 1$$

$\boxed{\text{zero profits}}$

$W_1 = W_2 = 5.$

Consumer's optimization : $\left.\begin{array}{ll} \dfrac{2}{5x_{11}} = P_1^* & \dfrac{1}{2x_{12}} = P_1^* \\ \dfrac{2}{5x_{21}} = P_2^* & \dfrac{1}{2x_{22}} = P_2^* \end{array}\right) \Rightarrow \begin{array}{l} x_{11} = \dfrac{6}{5} \; x_{21} = \dfrac{12}{5} \\ x_{12} = \dfrac{3}{2} \; x_{22} = 2 \end{array}$

and $x_{31} = 4$, $x_{32} = 4$

$$x_1 = \frac{27}{10} \qquad x_2 = \frac{22}{5} \qquad \Rightarrow \qquad y_1^* = \left(\frac{27}{10}, 0, -\frac{9}{10}\right)$$

$$\uparrow \qquad\qquad \uparrow \qquad\qquad y_2^* = \left(0, \frac{22}{5}, -\frac{11}{10}\right)$$

$$\frac{9}{10} W_3 \qquad \frac{11}{10} W_3$$

$\Downarrow$

Not P.O.

First Welfare theorem failed

(∵ Not strongly monotone for good 3)