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**ECON 501B**

## ECON 501B: Problem Set 6

Due: Thursday, October 11, 2018

**Instructions:** Answers should be complete proofs of a claim. For True/False either show the result is true or provide a counter example.

**Question 1:** Consider the following many to one environment. The agents are $T = \{t_1, t_2\}$ and $B = \{b_1, b_2, b_3\}$. The quotas are $q_1 = 3$ and $q_2 = 1$. The $T$ agents have responsive preferences with

- $\{b_2\} \succ_{t_1} \{b_1\} \succ_{t_1} \{b_3\} \succ_{t_1} t_1$, and
- $\{b_1\} \succ_{t_2} \{b_2\} \succ_{t_2} t_2 \succ_{t_2} \{b_3\}$.

The $B$ agents have preferences

- $t_1 \succ_{b_1} t_2 \succ_{b_1} b_1$
- $t_2 \succ_{b_2} t_1 \succ_{b_2} b_2$, and
- $t_1 \succ_{b_3} b_3 \succ_{b_3} t_2$.

1. What are the set of all stable matches?
2. Does there exist a match that is $T$-optimal amongst all stable matches?
3. Does there exist a match that is $T$-optimal amongst all core matches?
4. Does there exist a match that is $T$-optimal amongst all IR matches?

**Question 2:** Let $\mathcal{E} = (T, B; (\succ_i^*)_{i \in T \cup B})$ be a one-to-one environment, where $(\succ_i)_{i \in T \cup B}$ represents the agents' true preferences. Assume these involve strict preferences. The designer does not know these preferences and so implements the mechanism $m : \prod_{i \in I} \mathcal{P} \to \mathcal{M}$.

The true preference relation for $t$, viz. $\succ_t^*$, induces an ordered list $[b_{t,1}, b_{t,2}, \ldots, b_{t,K}]$ where

- $b_{t,1}$ is maximal according to the preference relation $\succ_t^*$, i.e., $b_{t,1} \succ_t^* b$ for all $b \neq b_{t,1}$,
- for each $k = 1, \ldots, K-1$, $b_{t,k} \succ_t^* b_{t,k+1}$,
- $b_{t,K}$ is the least acceptable alternative according to the preference relation $\succ_t^*$, i.e., (i) $b_{t,K} \succ_i^* t$ and (ii) $t \succ_t^* b$ for all $b$ not in the ordered list.

A **truncation** of the preference relation $\succ_t^*$ is some report $\hat{\succ}_t$ that induces the ordered list $[b_{t,j}, b_{t,j+1}, \ldots, b_{t,J}]$, for some $j, J$ with $K \geq J \geq j \geq 1$. A **strong truncation** of the preference relation $\succ_t^*$ is some report $\hat{\succ}_t$ that induces the ordered list $[b_{t,1}, b_{t,2}, \ldots, b_{t,J}]$ $J$ with $K \geq J \geq 1$. (So a strong truncation is a truncation.)

Say that agent $t$ has an **incentive to misreport** if there exists some $\hat{\succ}_t \in \mathcal{P}_t$ with $\hat{\succ}_t \neq \succ_t$ so that

$$m(\hat{\succ}_t, \succ_{-t}^*)(t) \succ_t^* m(\succ_t^*, \succ_{-t}^*)(t).$$

(Note, this is really saying that $t$ has an incentive to misreport when everyone is reporting truthfully.) Say that agent $t$ has an **incentive to misreport a truncation** (**strong truncation**) if there exists some truncation (resp. strong truncation) $\hat{\succ}_t \in \mathcal{P}_t$ with $\hat{\succ}_t \neq \succ_t$ so that

$$m(\hat{\succ}_t, \succ^*_{-t})(t) \succ^*_t m(\succ^*_t, \succ^*_{-t})(t).$$

Take $m$ to be the B-Proposing DA mechanism, i.e., the mechanism with $m(\succsim)$ being the matching that results from the B-Proposing DA algorithm applied to preferences $\succsim$.

1. *True or False.* If $t$ has an incentive to misreport, then $t$ has an incentive to misreport a truncation.
2. *True or False.* If $t$ has an incentive to misreport, then $t$ has an incentive to misreport a strong truncation.

**Question 3:** *True or False.* If $|T| = 1$, there exist a stable strategy-proof mechanism.

**Question 4:** *True or False.* There always exists a strategy-proof mechanism.