501B PS 2 Answer

Round 1

$A^1(t_i) = \{b_1, \dots, b_i\}\ \forall i$

$\hat{p}^1(t_i) = b_i \quad \forall i$

$p^1(b_i) = t_i \quad \forall i$

$\hat{\mu}^1(b_i) = \begin{cases} t_i & i=1 \\ \emptyset & o/w \end{cases}$

Round 2

$A^2(t_i) = \begin{cases} \{t_1\} & i=1 \\ \{b_1, \dots b_{i-1}\} & i \neq 1 \end{cases}$

$\hat{p}^2(t_i) = \begin{cases} b_1 & i=1 \\ b_{i-1} & i \neq 1 \end{cases}$

$p^2(b_i) = \begin{cases} \{t_1, t_2\} & i=1 \\ t_{i+1} & i<M \\ \emptyset & i=M \end{cases}$

$\hat{\mu}^2(b_i) = \begin{cases} t_2 & i=1 \\ \emptyset & ow \end{cases}$

Round k

$A^k(t_i) = \begin{cases} \emptyset & k > i+1 \\ \{b_1\} & k = i+1 \\ \{b_1, b_2, \dots b_{i-k+1}\} & k \le i \end{cases}$

$\hat{p}^k(t_i) = \begin{cases} b_{i-k+1} & i \ge k \\ \emptyset & ow \end{cases}$

$p^k(b_i) = \begin{cases} \{t_k, t_{k-1}\} & i=1 \\ \{t_{i+k-1}\} & 1 \le i \le M-k+1 \\ \emptyset & ow \end{cases}$

$\hat{\mu}^k(b_i) = \begin{cases} t_k & i=1 \\ \emptyset, & ow \end{cases}$

$\hat{\mu}^k(b_1) = t_k$ for any $k \le M$

$\hat{\mu}^k(b_1) = t_M$ for any $k > M$

$\hat{\mu}^k(b_i) = \emptyset$ for $i \neq 1$, for any k

If $M = \infty$, DA Algorithm does not terminate.

$\hat{\mu}^k$ does not weakly converge

$\hat{\mu}^k(b_1) \to t_k,\ k \to \infty$

Case 1. $M = \infty$

Round 1:

$A^1(t) = B$, for all t.

$\hat{p}^1(t) = b_2$ for all t.

$$P^1(b) = \begin{cases} T, & b = b_2 \\ \emptyset, & ow \end{cases}$$

$$\hat{\mu}^1(b) = \begin{cases} t_1 & b = b_2 \\ \emptyset & ow \end{cases}$$

Round 2:

$$A^2(t) = \begin{cases} B, & \text{if } t = t_1 \\ B \setminus \{b_2\} & ow \end{cases}$$

$$\hat{p}^2(t) = \begin{cases} b_2, & \text{if } t = t_1 \\ b_4, & ow \end{cases}$$

$$P^2(b) = \begin{cases} \{t_1\} & b = b_2 \\ T \setminus \{t_1\} & b = b_4 \\ \emptyset & ow \end{cases}$$

$$\hat{\mu}^2(b) = \begin{cases} t_1 & b = b_2 \\ t_2 & b = b_4 \\ \emptyset & ow \end{cases}$$

Inductively

Round k+1:

$$A^{k+1}(t) = \begin{cases} A^k(t) & \text{if } t \in \{t_1, \dots t_k\} \\ A^k(t) \setminus \{b_{2k}\} & \text{otherwise} \end{cases}$$

$$\hat{p}^{k+1}(t) = \begin{cases} \hat{p}^k(t) & \text{if } t \in \{t_1 \dots t_k\} \\ b_{2(k+1)} & ow \end{cases}$$

$$P^{k+1}(b) = \begin{cases} t_j & \text{if } b = b_{2j},\ j = 1, 2, \dots K \\ T \setminus \{t_1, \dots, t_k\} & b = b_{2(k+1)} \end{cases}$$

$$\hat{\mu}^{k+1}(b) = \begin{cases} t_j & \text{if } b = b_{2j} \text{ for } j = 1, 2, \dots, k+1 \\ \emptyset & \text{otherwise} \end{cases}$$

$\hat{\mu}^{\infty}$ is stable

Suppose (t_m, b_n) blocks $\hat{\mu}^{\infty}$

It must be that n is even

$m < n/2$ and $n < 2m$

and this implies contradiction

$\hat{\mu}^k(b_{2k}) = t_k$, for any k

$\hat{\mu}^{\infty}(b_{2j}) = t_j$

$\hat{\mu}^{\infty}(b_{2j-1}) = \emptyset$ and thus it terminates weakly (converges pt. wise)

Finite Market $M=2N$, Round k, $k \leq N$ (repeat) ③

Round $N+1$

$$A^{N+1}(t) = \begin{cases} A^N(t) & \text{if } t \in \{t_1, \ldots, t_N\} \\ A^N(t) \setminus \{b_{2N}\} & \text{otherwise} \end{cases}$$

$$= \begin{cases} B \setminus \{b_2, \ldots, b_{2(i-1)}\} & \text{if } t = t_i, i \leq N \\ \{b_{2i-1}\}_{i=1}^N & \text{ow} \end{cases}$$

$$\hat{p}^{N+1}(t) = \begin{cases} b_{2i} & \text{if } t = t_i \ \ i \leq N \\ b_1 & \text{ow} \end{cases}$$

$$P^{N+1}(b) = \begin{cases} \{t_i\} & i \leq N \\ T \setminus \{t_i\}_{i=1}^N & b = b_1 \\ \phi & \text{ow} \end{cases}$$

$$\hat{\mu}^{N+1}(b) = \begin{cases} \{t_i\} & b = b_{2i} \ i \in N \\ t_{N+1} & b = b_1 \\ \emptyset & \text{ow} \end{cases}$$

Inductively

$N+k+1$

$$A^{N+k+1}(t) = \begin{cases} A^{N+k}(t) & \text{if } t \in \{1, \ldots, t_{N+k}\} \\ A^{N+k}(t) \setminus \{b_{2k-1}\} & \text{ow} \end{cases}$$

$$\hat{p}^{N+k+1}(t) = \begin{cases} \hat{p}^{N+k}(t) & \text{if } t \in \{t_1, \ldots, t_{N+k}\} \\ b_{2k+1} & \text{ow} \end{cases}$$

$$P^{N+k+1}(b) = \begin{cases} t_j & \text{if } b = b_{2j} \\ t_{N+j} & \text{if } b = b_{2j-1} \ \ j = 1, \ldots, k \\ T \setminus \{t_1, \ldots, t_{N+k}\} & b = b_{2k+1} \\ \emptyset & \text{ow} \end{cases}$$

$$\hat{\mu}^{N+k+1}(b) = \begin{cases} t_j & b = b_{2j}, \ j \leq N \\ t_{N+j} & b = b_{2j-1} \ \ j = 1, \ldots, k+1 \\ \phi & \text{ow} \end{cases}$$

DA Algo. terminates after M periods.

matching:

$(t_1, b_2)\ (t_2, b_4) \ldots (t_N, b_m)$

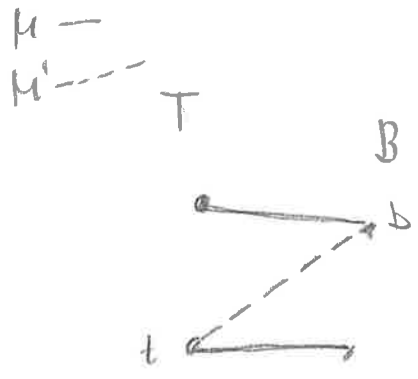
$(t_{N+1}, b_1), (t_{N+2}, b_3), \ldots (t_n, b_{m-1})$

Question 2

(1.) Under strict preferences, stable matches $\Rightarrow$ Pareto Efficient

Contrapositive: If not PE, then there exists a stable matching μ' and $\exists t \in T$:

$$\mu'(t) \succsim_t \mu(t)$$



To show $\mu'(t) \neq t$; because μ is IR and $\mu'(t) \succsim_t \mu(t) \succsim_t t$

Let $b = \mu'(t)$

By assumption: $\mu'(b) \succsim_b \mu(b)$
then (t, b) form blocking pair for μ

Problem 2.2

Ex. 1
$t_1 = b_1 \; b_2 \; t_1$
$t_2 = b_2 \; b_1 \; t_2$
$b_1 = t_1 \sim t_2 \; b_1$
$b_2 : t_1 \sim t_2 \; b_2$

$\mu(t_1) = b_2 \quad \mu(t_2) = b_1$

2.3

$t_1 : b_1 > b_2 > t_1$

$t_2 : b_1 > b_2 > t_2$

$b_1 : t_1 > t_2 > b_1$

$b_2 : t_1 > t_2 > b_2$

$\mu(t_1) = b_2 \quad \mu(t_2) = b_1$

Pareto Efficient but not stable.

(t_1, b_1) block

Question 3

$t_i : b_1 \; b_2 \; t_i \quad i = 1,2$

$b_i : t_1 \sim t_2 \; b_i \quad i = 1,2$

$\begin{cases} \mu(t_1) = b_1 \\ \mu(t_2) = b_2 \end{cases} \qquad \begin{cases} \mu'(t_1) = b_2 \\ \mu'(t_2) = b_1 \end{cases}$

both μ and μ' stable

$\succsim_T$ is not complete

Note: Answer to both (1) and (2)

(3) Suppose $\mu \succsim_T \mu'$ and $\mu' \succsim_T \mu''$

Than for any $t \in T$, $\mu(t) \succsim_t \mu'(t)$

$\mu'(t) \succsim_t \mu''(t)$

thus $\mu \succsim_T \mu''$