Lecture 1

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One-to-One Matching Markets

"A firm is matched to exactly one worker for the position"

Beginning Notation and Concept

Let:

$$T = \{t_1, ..., t_{|T|}\}$$
$$B = \{b_1, ..., b_{|B|}\}$$

Where is not necessarily $|T| = |B|$ in all cases. Consider an agent $i \in T \cup B$. This agent is in either one side of the market or the other. That is $I \in \{T, B\}$ J will be on the other side of the market. So, $J \in \{T, B\} \setminus \{I\}$

Agents $i \in I$ only care to match with agents inside J.

Preferences of Agents

Preferences are represented as a binary relation $\succsim_i$ on the set $J \cup \{i\}$ (this set says that agent $i \in I$ prefers the J side of the market or to keep to himself).

$j \succsim_i k$ means i weakly prefers j to k where $j, k \in J$.

$j \succsim_i i$ means i weakly prefers j to remaining alone.

Two Fundamental Assumptions:

(1) Preferences are complete

(2) Preferences are transitive

Completeness requires either $j \succsim_i k$ or $k \succsim_i j$

Transitivity requires that if $j \succsim_i k$ and $k \succsim_i l$ then $j \succsim_i l$

In the event that $j \succsim_k$ and $\neg(k \succsim_i j)$, the j is strictly preferred to k.

Some Implicit Assumptions

(1) Agents only care about "name" of agents on other side of the market. We can account for this by re-labeling and thinking of names of agents as being associated with different prices or wages.

This account does rule out that you care about match of other agents in the market (externalities matching). We assume that agents don't care about where other agents end up.

(2) Agents only rank a single agent in other side of market and not groups of agents. This is the case only in 1-to-1 matching. In short, each T is matched to one B and one B is matched to one T.

The Matching Map

A matching is a map (function) $\mu : (T \cup B) \to (T \cup B)$ such that for each:

$$I \in \{T, B\}$$
$$B \in \{T, B\} \setminus \{I\}$$

the following hold:

1. for each $i \in I : \mu(i) \in J \cup \{i\}$
2. if $\mu(i) = j \in J$ then $\mu(j) = i \in I$

A bit less dense, the first point says that for each $t \in T$, $\mu(t) \in B \cup \{t\}$ or for each $b \in B$, $\mu(b) \in T \cup \{b\}$

Likewise, (2) can be expressed $\mu(t) = b$ if and only if $\mu(b) = t$.

We write t is matched if $\mu(t) \neq t$. That is, your are matched if you are not with yourself.

Remark: Matching μ is a one-to-one (injective) function.

Proof: Suppose $\exists i_1, i_2 \in I$ such that $\mu(i_1) = \mu(i_2)$. That is, two agents matched to the same j. Then, $i_1 = i_2$ since this is one-to-one matching.

Lastly, matching is of order 2. That is:

$$\mu^2(i) \equiv \mu(\mu(i)) = i$$

Fix $\mu(i) = j$. Then since if $\mu(i) = j \in J$ then $\mu(j) = i \in I$:

$$\mu(\mu(i)) = \mu(j) = i$$