Homework 9 Solutions **Spencer Cooper**

ECON 501B Solutions

Prof. Friedenberg November 2018

Note:

- For each $l = 1, 2$, preference is strong monotone, so $p_l^* > 0$, otherwise, each consumer has no maximum on $B_i(p^*)$
- Since $p_l^* > 0, l = 1, 2$, profit maximization entails $y_l^* = 0$. Thus $p_l^* \cdot y_l^* = 0$

1. (a) FOC: $x_{11} : \alpha_1 x_{11}^{\alpha_1 - 1} x_{21}^{\alpha_1} - \lambda p = 0$
   $x_{21} : \alpha_1 x_{11}^{\alpha_1} x_{21}^{\alpha_1 - 1} - \lambda = 0$

   $\frac{x_{21}}{x_{11}} = p$
   $\frac{x_{22}}{x_{12}} = p$
   Applying budget constraints:
   $2px_{11} = 10p + 2$ and
   $2px_{12} = 2p + 10$
   $\Rightarrow x_{11} = \frac{5p+1}{p}$
   $\Rightarrow x_{12} = \frac{p+5}{p}$
   By market clearing:
   $x_{11} + x_{12} = 12 \Rightarrow p = 1$
   Then CE allocation: $\{(6, 6), (6, 6)\}, p = 1\}$

   (b) Interior:
   P.O. implies $\text{MRS}_1 = \text{MRS}_2$
   $\frac{x_{21}}{x_{11}} = \frac{x_{22}}{x_{12}} = k$
   Because $x_{l1} + x_{l2} = 12$, for $l = 1, 2$, then $k = 1$

   Boundary:
   Must be on origin corners.
   Then P.O.: $\{(x_{11}, x_{11}), (12 - x_{11}, 12 - x_{11}) | x_{11} \in [0, 12]\}$

   (c) FWT gives CE $\Rightarrow$ PO
   For SWT, we need transfers to support P.O. as CE.

   (d) Consider the PO allocation $\{(5, 5), (7, 7)\} = (\omega_1, \omega_2)$ with market price $p = 1$
   Start with $(w_1, w_2) = (12, 12)$
   Transfer 2 from 1 to 2:
   $(w_1', w_2') = (10, 14)$

2. Since $((x_1^*, x_2^*), y^*)$ is P.O., (x_1^*, x_2^*) is non-wasteful and $y^* = 0$

Interior Solution:
$\text{MRS}_1 = \text{MRS}_2$
$\phi_1'(x_{21}) = \phi_2'(x_{22})$
Market clearing gives $x_{22} = \bar{\omega_2} - x_{21}$
Plugging in we get:
$\phi_1'(x_{21}) - \phi_2'(\bar{\omega_2} - x_{22}) = 0$
Becasue of strict concavity, ϕ_1' and ϕ_2' are decreasing. That means we have a unique solution for (x_{21}, x_{22})

3. We can again assume $y^* = 0$ by above argument.
Let $\omega' = x^*$, check $((x^*, y^*), p^*)$ is a C.E. with respect to ω'
For consumer i :
$x_i^* \in argmax(u^i(x_i))$ s.t. $p^* \cdot x_i \leq p^* \omega_i' = p^* x_i^*$
Budget constraint is satisfied because in the C.E. , $p^* x_i^* = W_i$ by non-wasteful.

Now check feasibility:
$\sum x_i^* - \bar{\omega}$
This is from the C.E.

4. (a) Yes - proof by contradiction
Suppose a C.E. is not in the core. Then there is a coalition C such that for any $i \in C$, and an allocation $\{x_i | i \in C\}$ such that $\sum x_i \leq \sum w_i | i \in C$, then $x_i \succ_i x_i^*$
Therefore $p^* x_i > p^* x_i^* = p^* \omega_i$ for any $i \in C$
Summing all $i \in C$:
$p^* \sum x_i > p^* \sum \omega_i$
Then $\sum_{l=1}^{L} p_l \sum_{i \in C} x_{li} > \sum_{l=1}^{L} p_l \sum_{i \in C\ li}$
$\Rightarrow \exists l : p_l \sum x_{li} > p_l \sum x_{li}$
Because $p_l > 0 : \sum x_{li} > \sum \omega_{li}$ which is a contradiction to feasibility.

(b) False