501B 11/6

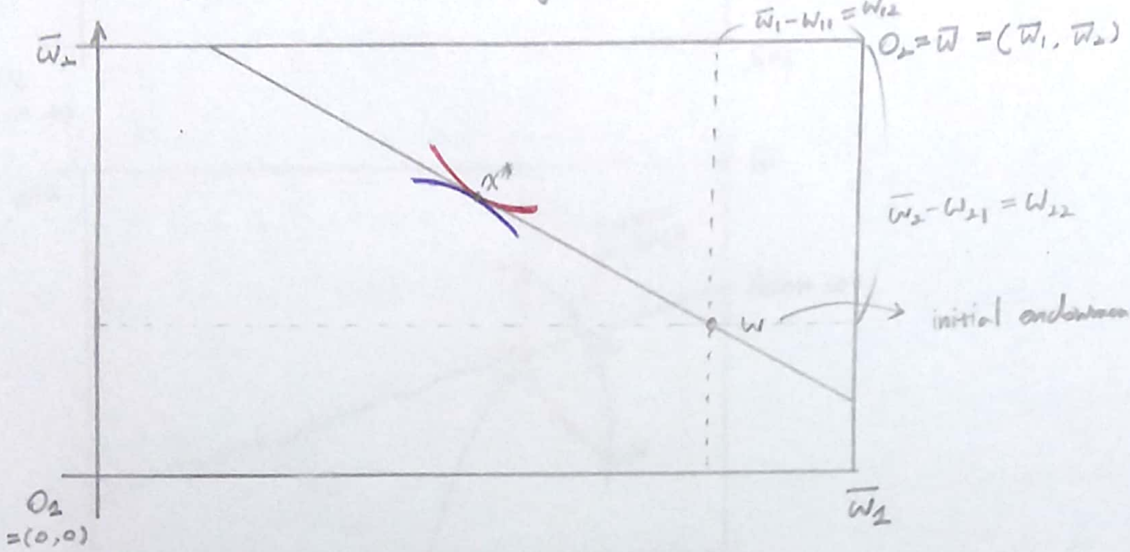
Edgeworth Box Pure Exchange Economy

$l = 1, 2$ and $i = 1, 2$

$X_i = \mathbb{R}^2_+$

$\succsim_i$: Strongly monotone, strictly convex, continuous

$W_{\square\triangle}$ — $\square$ ⇓ goods, $\triangle$ ⇓ agent i

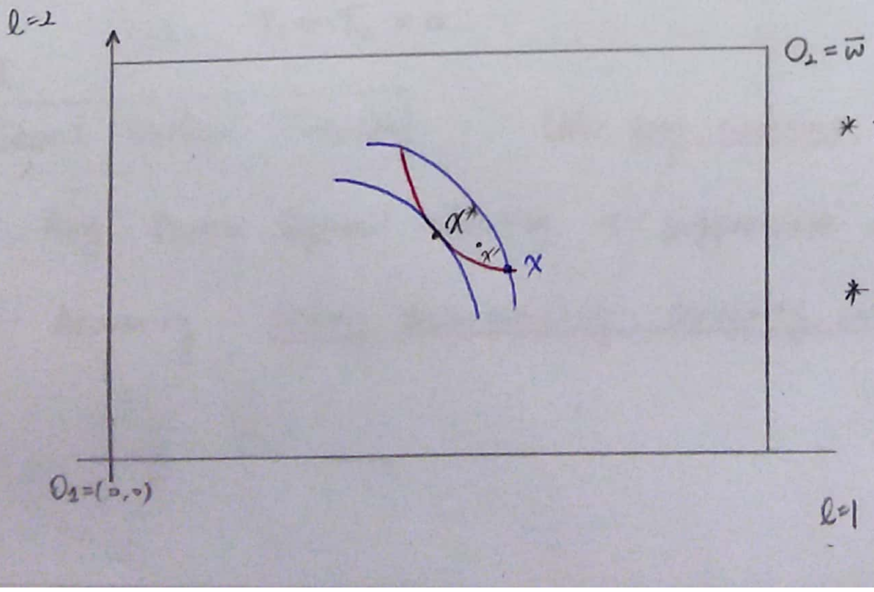


$$\begin{cases} x_1^* = x^* \\ x_2^* = (\bar{w} - x^*) \end{cases}$$

⊙ Today : Connection b/w competitive eq. and pareto optimality.

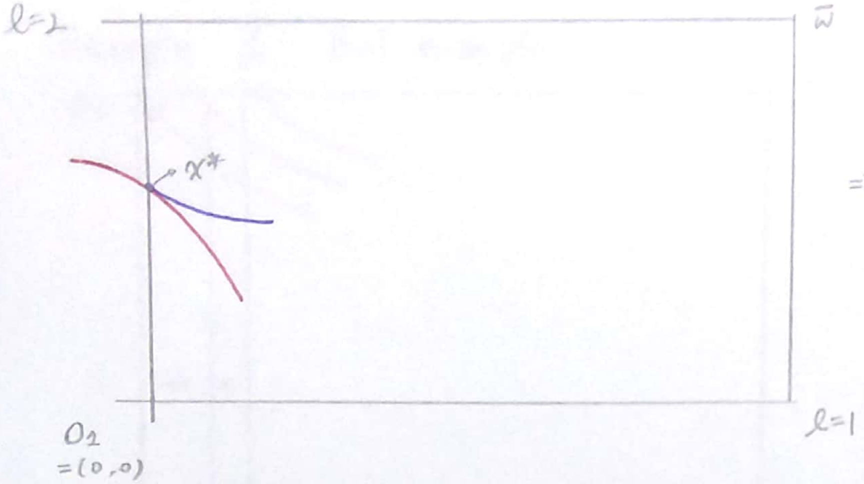
$\boxed{\text{First welfare theorem}}$; Competitive Equilibrium ⇒ Pareto optimal.

- Immediate
- Any competitive equilibrium allocation is Pareto optimal

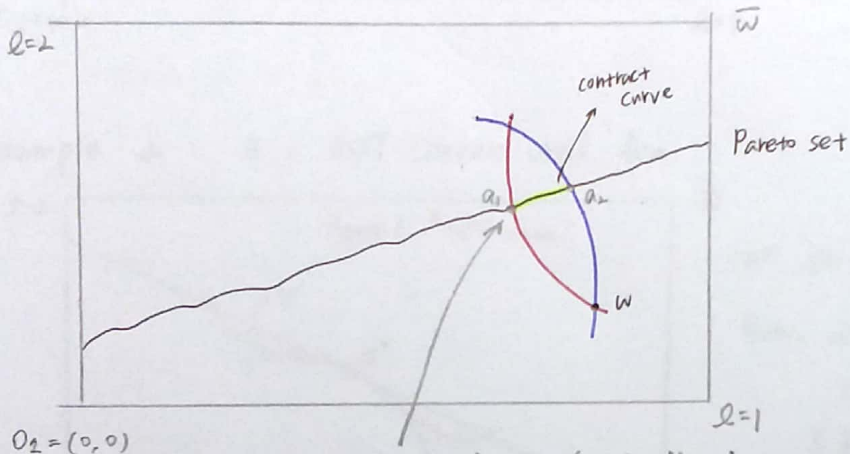


* Indifference curves are tangent at x^* ⇒ then, x^* is Pareto Optimal (PO).

* If the indifference curves are not tangent at x^* and x^* is in the interior of Edgeworth Box, then x^* is not PO



=> NOT PO.



{ · as good as at least other bundles for agent 1
· strictly preferred to a_2 for agent 2.

(Def) An allocation $(x_1^*, x_2^*) = (x_1^*, \bar{w} - x_1^*)$ is supportable as an equilibrium with transfers if $\exists$ $p^* = (p_1^*, p_2^*)$ and transfers T_1, T_2 s.t for each $i = 1, 2$

(1) $p^* \cdot x_i^* \leq p^* \cdot w_i + T_i$

(2) $x_i^* \succsim x_i$ for each $x_i \in X_i = \mathbb{R}_+^2$ with $p^* \cdot x_i \leq p^* \cdot w_i + T_i$

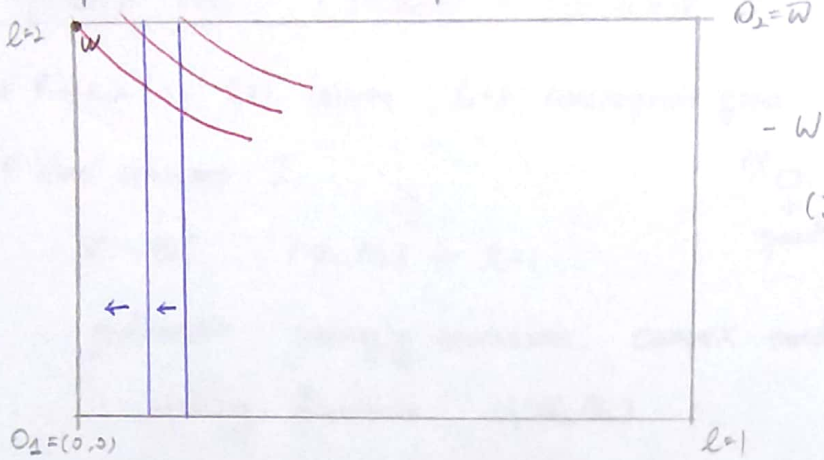
(3) $T_1 + T_2 = 0$

☆☆☆

$\boxed{\text{Second Welfare Theorem}}$ Under some conditions, PO $\Rightarrow$ Competitive Equilibrium.

- Any Pareto Optimal allocation is supportable as an equilibrium with transfers
- Assuming : Strong monotonicity, convexity and continuity

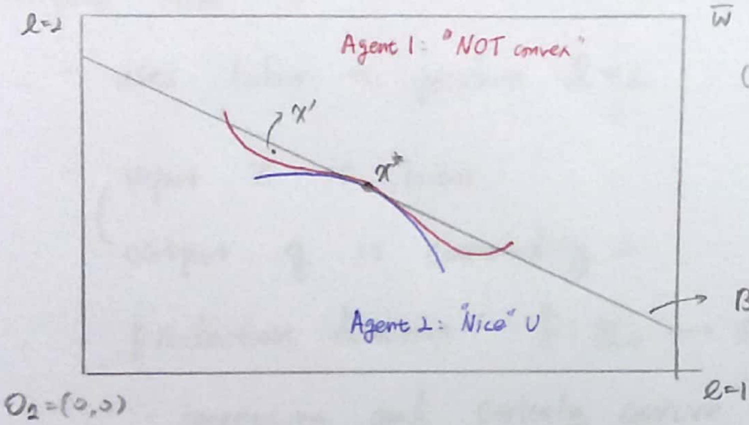
Example 1: Evil example



- W is a Pareto optimal allocation

(In this case, W is a PO, but not Comp. eq.)

Example 2: ∃ a NOT convex util. fcn.



$(x^*, \bar{w} - x^*)$ is Pareto optimal allocation

But, $i=1$: could deviate and choose some $x_i \in B_i(p)$ with $x_i \succ_i x_i^*$

(In this picture, $x' \succ x^*$ for agent 1 but $px' < px^*$)

Remark In a pure exchange economy, we can instead alter endowments, instead of giving transfers

Edgeworth Box : 1 Producer + 1 consumer

* $\ell = 1, 2$: $\ell = 1$ leisure $\ell = 2$ consumption good

* One consumer : I $\qquad x_{\square}$ ($\because \exists$ only one consumer)
  $\uparrow$ goods
  - $X = \mathbb{R}_+^2$ $\qquad (x_1, x_2) \leftarrow \ell = 1$
  - preference : strongly monotonic, convex and continuous.
    utility function : $u(x_1, x_2)$
  - endowment : $(\bar{L}, 0)$

* one firm : J
  - uses labor to produce $\ell = 2$
  - input z is labor
    output q is commodity 2
  - production function : $f : \mathbb{R}_+ \longrightarrow \mathbb{R}_+$
    - increasing and strictly concave
    - $f(z)$: z units of labor gives $f(z)$ units of q_z
  - production set of J

    $$Y = \{(-z, q) \in \mathbb{R}_- \times \mathbb{R}_+ : q \leq f(z)\}$$

  - Consumer owns firm : $\theta_{IJ} = 1$ (consumer 1, firm 1)

* price vector : (w, p)
  w ↑ wage of $\ell = 1$; p ↑ price of $\ell = 2$

$\boxed{\text{Competitive Equilibrium}}$; (x_1^*, x_2^*, z^*, q^*) and (w^*, p^*)

$\boxed{\text{Competitive Equilibrium}}$; (x_1^*, x_2^*, z^*, q^*) and (w^*, p^*)

① Firm J max profit at (w^*, p^*)

* (z^*, q^*) solve $\underline{\max_{z \geq 0} \left[p^* f(z^*) - w^* z^* \right] \text{ s.t. } f(z^*) = q^*}$

$\pi(w^*, p^*) = p^* q^* - w^* z^*$; optimal profit at (w^*, p^*)

($\pi(w, p)$: optimal profit given (w, p))

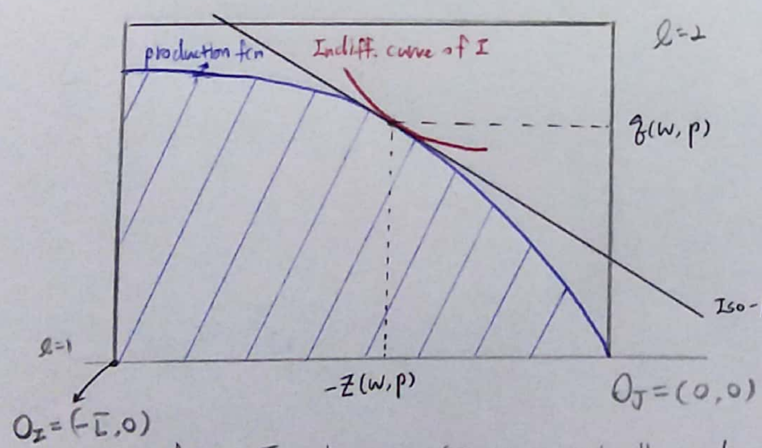
② Consumer I max utility at (x_1^*, x_2^*)

$$\underline{\max_{(x_1, x_2) \in \mathbb{R}^2_+} u(x_1, x_2) \text{ s.t. } p^* x_2 \leq \underbrace{w^*(\bar{L} - x_1)}_{\text{wage}} + \underbrace{\pi(w^*, p^*)}_{\text{profit he gains}}}$$

③ Market clearing Conditions : $\bar{L} - x_1^* = z_1^*$

$x_2^* = q^*$

$\boxed{\text{X. Inducing Competitive Equilibrium}}$



*key : Iso-profit line is also consumer I's budget line

$B_I(w,p) = \{(x_1, x_2) ; w(\bar{L} - x_1) + \underbrace{\pi(w,p)}_{=K} \geq p x_2\}$

Iso-profit line : $\{(-z, q) : K = pq - wz \text{ for some } k\}$

- When firm J chooses (z, q) optimally chooses it so that an isoprofit line is tangent to production function at (z, q)
- If Consumer I chooses x_1, input $= \bar{L} - x_1$

$$\boxed{\begin{aligned} x_2 &= f(\bar{L} - x_1) \\ z &= \bar{L} - x_1 \\ q &= x_2 \end{aligned}}$$

* • (x_1^*, x_2^*, z^*, q^*) are a competitive equilibrium iff it solves $\max_{(x_1, x_2) \in X} u(x_1, x_2)$ s.t.