

## \* Welfare MWG 3.I.

Consider a particular consumer facing a change from  $p^0$  to  $p^1$ .

How good or bad would this change be for her?

\* Today's point.

• Consumer has a utility function  $u$ , and wealth  $W$ .

$$(1) \Delta V = \frac{V(p^1, W) - V(p^0, W)}{\text{How good or bad.}}$$

(ex if  $V=2u$ ,  
 $\Delta V$  will be doubled)

(2) Hicks Compensation (Compensating Variation)  $\rightarrow$

$$V(p^0, w^0) = V(p^1, w^1)$$

Define  $CV \in \mathbb{R}$  (CV: Compensating Variation, real number)

$$V(p^0, w^0) = V(p^1, w^0 - CV) \Rightarrow \text{So, } w^1 = w^0 - CV : CV = w^0 - w^1$$

$\begin{cases} CV : + & \text{goodness } (p^0 > p^1) \\ CV : - & \text{badness} \end{cases}$

$$\text{Let } \bar{v}^0 = V(p^0, w^0)$$

$\uparrow$   
a level of utility

$$w^0 = e(p^0, \bar{v}^0) \quad (e \text{ is inverse of } \bar{v}^0 \text{ with } p^0)$$

(Throughout this section, assume  $u$  is continuous, LNS and  $g, h$  are single-valued)

$$w^1 = e(p^1, \bar{v}^0)$$

$$CV = w^0 - w^1 = e(p^0, \bar{v}^0) - e(p^1, \bar{v}^0)$$

$$\left( \begin{array}{c} CV(p^0, p^1, w^0) \\ \uparrow \quad \uparrow \quad \uparrow \\ \text{initial price} \quad \text{new price} \quad \text{initial wealth} \end{array} \right)$$

(NOTE)  $f: \mathbb{R} \rightarrow \mathbb{R}$

$a, b \in \mathbb{R}$

$f(b) - f(a)$

### (3) Equivalent variation

$$V(p^0, w^1) = \underbrace{V(p^1, w^0)}_{:= \bar{v}^1}$$

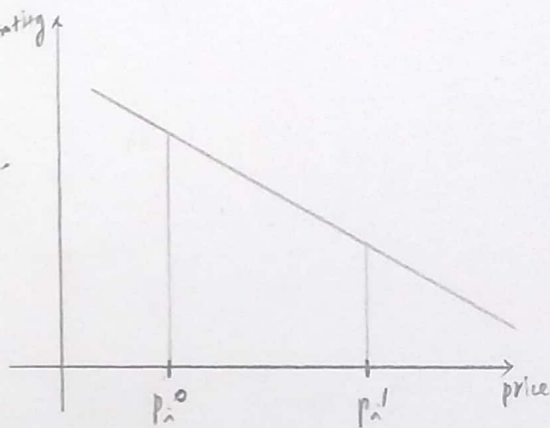
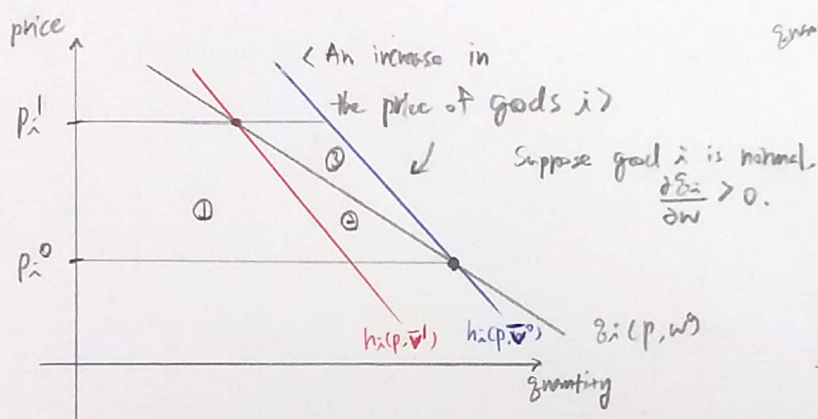
$$V(p^0, w^0 + EV) = V(p^1, w^0) \quad \text{Then, } w^0 + EV = e(p^0, \bar{v}^1), \quad w^0 = e(p^1, \bar{v}^1)$$

Define  $EV \in \mathbb{R}$ .  $w^1 = w^0 + EV$  :  $EV = e(p^0, \bar{v}^1) - e(p^1, \bar{v}^1)$  (New level of utility)

$$\begin{cases} p^0 > p^1, & EV : + \\ p^0 < p^1, & EV : - \end{cases}$$

### (4) Area variation (From MWG)

Now let us suppose that the price of only one good  $i$  changes  $p_j^1 = p_j^0, \forall j \neq i$



$$\Rightarrow AV = \int_{p_i^1}^{p_i^0} g_i(p, w^0) dp_i$$

$$\begin{cases} AV : \textcircled{1} + \textcircled{2} \\ CV : \textcircled{1} + \textcircled{2} + \textcircled{3} \\ EV : \textcircled{1} \end{cases}$$

when  $p^0 > p^1$   
 $EV \in AV \in CV$

Note:  $f(b) - f(a) = \int_a^b f(x) dx$

$e$  is a single-valued fn. So,  $CV = \int_{p_i^1}^{p_i^0} \frac{\partial e(p_i, p_{-i}^0, \bar{v}^0)}{\partial p_i} dp_i$

(Only  $p_i$  is changed, NOT  $p_{-i}$ )

$$= \int_{p_i^1}^{p_i^0} h_i(p, \bar{v}^0) dp_i$$

$$EV = \int_{p_i^1}^{p_i^0} h_i(p, \bar{v}^1) dp_i$$

$\Rightarrow$  Generally,  $AV, CV, EV$  are different b/c wealth effect.

Note:  $p_{-1} = (p_2, \dots, p_n)$