Welfare and Missing Markets

- Externality
- Public goods

Example : Externalities

* Action that impacts negatively the utility of other agents.

* Consumers : $I = 2$

- $i=1$: has an action that impacts utility of $i=2$
- Denote that action : $h \in \mathbb{R}$
- each agent $i=1,2$ has preferences over $\underset{\text{consumption}}{X_i} \times \underset{\text{action}}{\mathbb{R}}$ If $i=2$, then i does not pick this.
- $u_i : X_i \times \mathbb{R} \to \mathbb{R}$ represents $\succsim_i$

  - $\boxed{u_i(x_{1i}, \ldots, x_{Li}, h) = \underset{\text{numeraire}}{x_{1i}} + \underset{\text{increasing strictly}}{\psi_i(x_{2i}, \ldots, x_{Li})} + \phi_i(h)}$ ⇐ h influences the level of utility but not how i substitutes between two commodities.
  - $\phi_i(\cdot)$ is twice differentiable and concave
  - $\phi_2'(\cdot) < 0$ negative externality
  - no assumptions on $\phi_1'(\cdot)$

- "Choice of h" is costless to 1
- One firm with $Y_J = \{(0, \ldots, 0)\}$
- Change $i=1$'s maximization problem is

$$\boxed{\max_{(x_{11}, \ldots, x_{L1}, h)} \left[x_{11} + \psi_1(x_{21}, \ldots, x_{L1}) + \phi_1(h) \right] \text{ s.t } \sum_{\ell=1}^{L} p_\ell^* \cdot x_{\ell 1} \leq \sum_{\ell=1}^{L} p_\ell^* \cdot \omega_{\ell 1}}$$

- 1's optimal choice of h is h^* with $\phi_1'(h^*) = 0$
- utilitarian efficient choice of h solves $\max [\phi_1(h) + \phi_2(h)]$

or h^e solves $\phi_1'(h^e) + \phi_2'(h^e) = 0$.

Since $\phi_2'(\cdot) < 0$ for all h, $\Rightarrow$ $\phi_1'(h^e) > 0 = \phi_1'(h^*)$

Since ϕ_1 is concave $\Rightarrow$ $h^* > h^e$

$\Rightarrow$ Thus, 1 is consuming more than is utilitarian efficient.

NOTE

Potential Fix: Property Rights

- $i=2$ has the right to an "externality free environment"
- $i=1$ must buy sufficient rights to be able to consume h.
- Market clearing: agent 2 has to want to sell the right to agent 1.

* Endowment of h

- $W_{h1} = 0$ and $W_{h2} = \overline{W}_h$
- P_h : price of h
- amount of right consumed by i is h_i
- $h_1 + h_2 = \overline{W}_h$ by market clearing
- define $\hat{\phi}_2(\cdot)$ function of h_2

$$\hat{\phi}_2(h_2) = \phi_2(\overline{W}_h - h_2) = \phi_2(h_1)$$

Now, $i = 1$'s maximization

$$\max_{(x_{11},\dots,x_{L1},h_1)} \left[x_{11} + \varphi_1(x_{21},\dots,x_{L1}) + \phi_1(h_1) \right] \text{ s.t.}$$

$$\sum_{l=1}^{L} p_l^* \cdot x_{l1} + p_h^* h_1 \leq \sum_{l=1}^{L} p_l^* \cdot w_{l1}$$

Now, $i = 2$'s maximization

$$\max_{(x_{12},\dots,x_{L2},h_2)} \left[x_{12} + \varphi_2(x_{22},\dots,x_{L2}) + \hat{\phi}_2(h_2) \right] \text{ s.t.}$$

$$\sum_{l=1}^{L} p_l^* \cdot x_{l2} + p_h^* h_2 \leq \sum_{l=1}^{L} p_l^* \cdot w_{l2} + p_h^* \cdot w_{h2}$$

Interior Solution $\left.\begin{array}{l} \phi_1'(h_1^*) = p_h^* \\ \hat{\phi}_2'(h_2^*) = p_h^* \end{array}\right\} \Rightarrow \hat{\phi}_2'(h_2^*) = -\phi_2'(w_2 - h_2^*) = \phi_1'(h_1^*)$

$$\Rightarrow \phi_1'(h_1^*) = -\phi_2'(h_1^*)$$

$$\Rightarrow \phi_1'(h_1^*) + \phi_2'(h_1^*) = 0$$ [max utilitarian welfare]

Remark

- can think of the inefficiency as related to a missing market.
- property rights allowed to trade on h, then we return to efficiency.

  (In a missing market, trading on h makes the market efficient

  i.e., eliminates negative externality by trading on h)

- Endowment of h was irrelavant : $\overline{w}_h$ could have taken any value

  & $\overline{w}_h$ could have been distributed any way between $i=1,2$

Example) Take away

An implicit assumption in the First Welfare Theorem is "universal price quoting of the commodities" or "complete markets"

Example: Public goods

* good that you cannot prevent other agents from having access to it.
* jargon : non-rivalrous & non-excludable
* Under provision of public good relative to efficient.

- Commodities : $\ell = 1, 2$ ($\ell = 1$: numeraire, $\ell = 2$: public good)
- Firm $J = 1$:
  - turn numeraire into public good
  - $f : \mathbb{R}_+ \to \mathbb{R}_+$ Strictly increasing, concave, differentiable, invertible inverse differentiable, $f(0) = 0$.
  - $Y_J = \{(-z, q) : z, q \geq 0 \text{ and } f(z) \geq q\}$ $f(z) = q$ in equilibrium.

    $z = f^{-1}(q) := c(q)$ cost function.

    $\Downarrow$

    differentiable $\Rightarrow$ so $c(q)$ is differentiable

- Consumers $i = 1, \dots, I$:
  - Consumer i : (x_{1i}, x_{2i})

    $\Rightarrow$ buys x_{2i}

    profile of consumption bundles is $(x_1, \dots, x_I)$

    $\Rightarrow$ consumes : $x_{2i} + \sum_{k \neq i} x_{2k}$
  - $w_{2i} = 0$, for all i.

- $\succsim_i$ represents by u_i

$$u_i(x_{1i}, \dots, x_{2I}) = x_{1i} + \phi_i(s(x_{21}, \dots, x_{2I}))$$

where $\phi_i : \mathbb{R} \to \mathbb{R}$ is strictly increasing, strictly concave and twice differentiable

& $s(x_{21}, \dots, x_{2I}) = \sum_{k=1}^{I} x_{2k}$

($\rightarrow$ depend on how much the opposite guys buy)

- Amend the definition of a competitive equilibrium to have all consumers having correct beliefs about what others consume :

$((x_1^*, \dots, x_I^*, -z^*, q^*), (p_1^*, p_2^*))$ solves

$$\max_{(x_{1i}^*, x_{2i}^*)} \left[x_{1i} + \phi_i\left(x_{2i} + \sum_{k \neq i} x_{2k}\right) \right] \quad \text{s.t.} \quad p_1^* x_{1i}^* + p_2^* x_{2i}^* \leq p_1^* \omega_{1i}$$

- Normalize $p_1^* = 1$

* Conditions for competitive equilibrium

① Firm max profit

$$q^* = f(z^*) \quad \text{and} \quad \max_z \left[p_2^* f(z) - z \right]$$

$\Rightarrow$ F.O.C $p_2^* f'(z) \leq 1$ with $= 1$ if $z^* > 0$

$\Rightarrow$ $z^* = f^{-1}(q^*) = c(q^*)$. Thus $p_2^* \leq c'(q)$

② Consumer max :

$$\phi_i'\left(x_{2i}^* + \sum_{k \neq i} x_{2k}^*\right) \leq p_2^* \quad \text{with} = \text{if } x_{2i}^* > 0.$$

③ Market clearing

$$x_{2i}^* + \sum_{k \neq i} x_{2k}^* = q^* \rightarrow \text{for each}:$$

$$\phi_i'(q^*) \leq p_2^* \leq c'(q^*)$$

from ② from ①

$$\text{if } x_{2i}^* > 0 \Rightarrow \phi_i'(q^*) = c'(q^*)$$

* Implication for provision of public goods.

- Order $i = 1, \ldots, I$ s.t at q^*

$$\phi_1'(q^*) \leq \phi_2'(q^*) \leq \ldots \leq \phi_I'(q^*)$$ (⇐ For convinience, we can order $\phi_i'(q)$, $\forall i$)

case 1) : $q^* = 0$. No public good.

case 2) : $q^* > 0$ $\quad p_2^* = c'(q^*)$ and

$\exists$ some M s.t. $\phi_i'(q^*) = p_2^*$ for $i = M, \ldots, I$

$\phi_i'(q^*) < p_2^*$ for $i < M$

⇓

$$\sum_{i=M}^{I} \phi_i'(q^*) = (I - M + 1) \cdot c'(q^*). \qquad (A)$$

- efficient q: $\max_q \sum_{i=1}^{I} \phi_i(q) - c(q)$ (B)

*Check ⇒ $q^e > q^*$ ← compare (A) to solution (B)

(Remember assumption of ϕ_i, $s(x_{21}, \ldots, x_{2I}) = \sum_{k=1}^{I} x_{2k}$)

⇒ Complete the market : (firm choose consumer specific q_i
consumer specific prices