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ECON 501B

ECON 501B: Problem Set 1

Due: Thursday, August 30, 2018

Instructions: For True/False questions, either provide a proof that the statement is true or provide a counterexample showing that it is false.

Question 1: *True or False.* Fix an environment $(T, B; (\succsim)_{t\in T}, (\succsim)_{b\in B})$ so that the following holds: there exists $t_* \in T$ and $b_* \in B$, with

1. $b_* \succ_{t_*} b$, for each $b \in B\backslash\{b_*\}$, and
2. $t_* \succ_{b_*} t$, for each $t \in T\backslash\{t_*\}$.

Then, in any stable match $\mu : (T \cup B) \to (T \cup B)$, $\mu(t_*) = b_*$.

Question 2: *True or False.* Fix an environment $(T, B; (\succsim)_{t\in T}, (\succsim)_{b\in B})$, so that

$$|\{t \in T : A(t) = B\}| = |\{b \in B : A(b) = T\}|.$$

If all agents have strict preferences and $A(t) = B$, then there is some stable match in which t is matched.

Question 3: In class we said that a pair (t, b) **blocks** a matching $\mu : (T \cup B) \to (T \cup B)$ if (i) $b \succ_t \mu(t)$, and (ii) $t \succ_b \mu(b)$. Consider instead the following definition: A pair (t, b) **blocks*** a matching $\mu : (T \cup B) \to (T \cup B)$ if either (i) $b \succsim_t \mu(t)$ and $t \succ_b \mu(b)$, or (ii) $b \succ_t \mu(t)$ and $t \succsim_b \mu(b)$. Say a matching $\mu : (T \cup B) \to (T \cup B)$ is **stable*** if it is individually rational and there is no block* pair (t, b).

1. If a matching $\mu : (T \cup B) \to (T \cup B)$ is stable* is it stable? Either provide a proof that it is or a counter example.
2. If a matching $\mu : (T \cup B) \to (T \cup B)$ is stable is it stable*? Either provide a proof that it is or a counter example.
3. Is the notion of blocks* stronger or weaker than the notion of blocks? Is the notion of stable* stronger or weaker than the notion of stable?

Question 4: There are three agents on each side of the market: $T = \{t_1, t_2, t_3\}$ and $B = \{b_1, b_2, b_3\}$. Matched agents can share a pie; unmatched agents get no pie. The following describes the fraction of the pie that a t_i agent would get when matched with (b_1, b_2, b_3):

- t_1's Fraction: $(\frac{1}{4}, \frac{2}{4}, \frac{3}{4})$;
- t_2's Fraction: $(\frac{3}{4}, \frac{1}{4}, \frac{2}{4})$;
- t_3's Fraction: $(\frac{2}{4}, \frac{3}{4}, \frac{1}{4})$.

(So, if (t_1, b_1) are matched, t_1 gets $\frac{1}{4}$ of the pie and b_1 gets $\frac{3}{4}$ of the pie.) All agents strictly prefer larger fractions of pie to smaller fractions of pie.

Which matches are (resp. are not) stable? (That is, provide a compelte argument for why each matching is or is not stable.)