* Until now... and next steps.

I. Abstract Choice ($\succsim$, u and C(·))

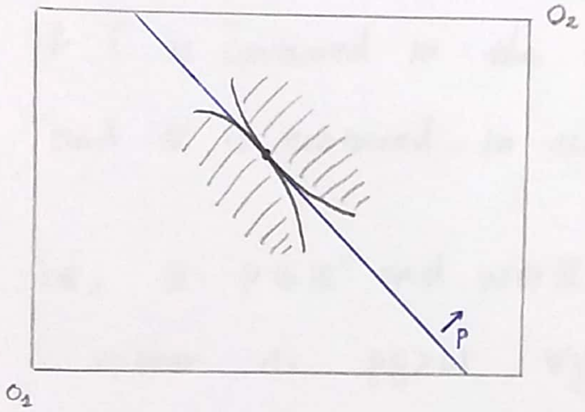
II. Consumer Demand

III. Choice under uncertainty → In the future...

IV. Producer theory → "

) Main topics.

Today: Chap. 17. Separating hyperplanes (continue)



→ p separates two hyperplanes

⇑ convex set

weak separation

$\boxed{\text{Separating hyperplane thm (I)}}$

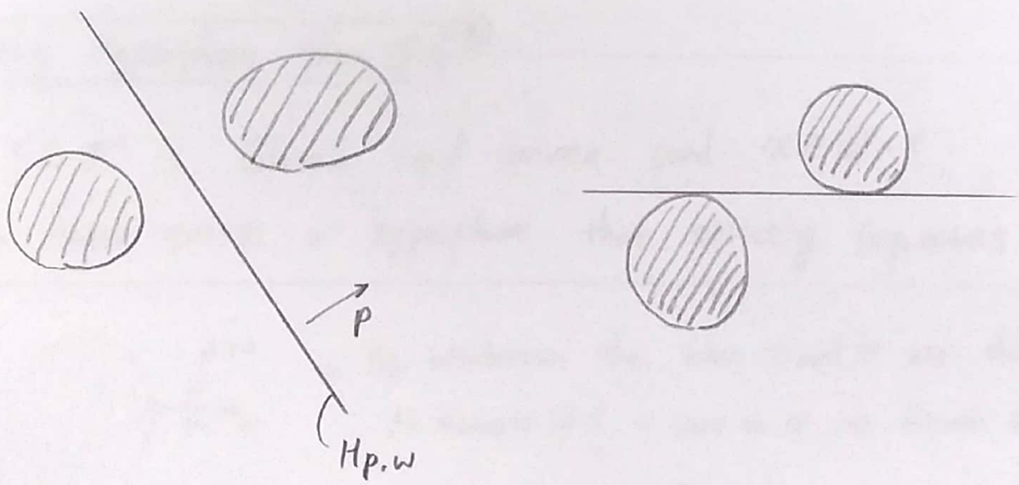
If we have two disjoint convex sets $X, Y \subset \mathbb{R}^k$,

then $\exists$ a hyperplane that "separates" X and Y,

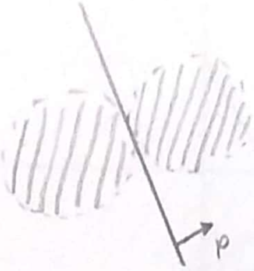
i.e., a vector $p \in \mathbb{R}^k$ and a real number w s.t. $\begin{cases} px \leq w, & \forall x \in X \\ py \geq w, & \forall y \in Y \end{cases}$

⇓

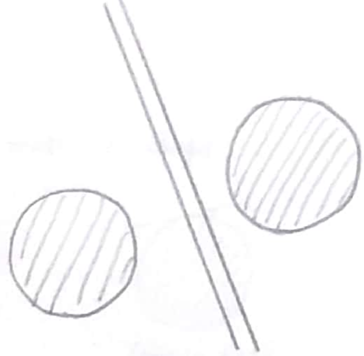
If $\exists$ a ε gap, this will be a strict separation.



· Think of "open convex sets"



No intersection.



Strictly separated.
but NO gap b/w them.

Strictly separated
$\exists$ a gap b/w them.

(Def) A hyperplane H in $\mathbb{R}^k$ strictly separates a set $Y \subset \mathbb{R}^k$ from a point $x \in \mathbb{R}^k$ if Y is contained in the open half space on one side of H and x is contained in the open half space on the other side of H.

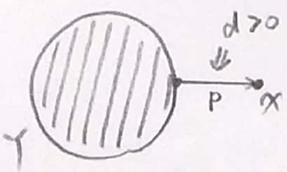
i.e., $\exists$ $p \in \mathbb{R}^k$ and $w \in \mathbb{R}$ s.t

either (1) $py > w$, $\forall y \in Y$ and $px < w$

or (2) $py < w$, $\forall y \in Y$ and $px > w$

Separating Hyperplane thm (I) ⊛

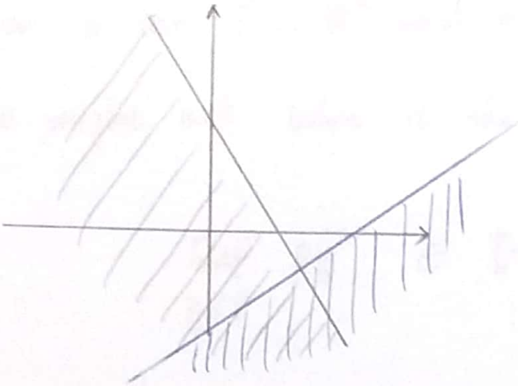
If $Y \subset \mathbb{R}^k$ is Closed and convex and $x \in \mathbb{R}^k \setminus Y$, then there exists a hyperplane that strictly separates Y and x.



$\Rightarrow$ By Weierstrass thm, when Y and x are disjoint ($\exists$ a gap)

All elements in Y is close to x, $\Rightarrow$ distance b/w Y and x is positive.
(compact, convex.)

In $\mathbb{R}^2$,

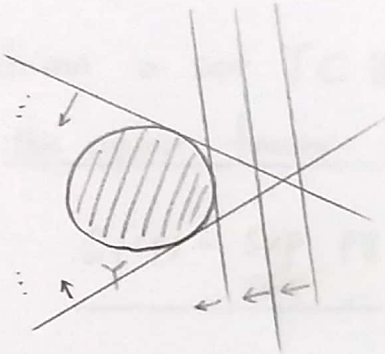


→ Cut!

We can make a shape.

→ But, we cannot make a pack-man  by cut

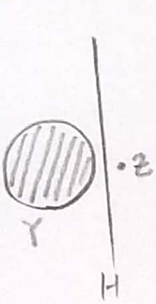
propln.1 Every closed, convex $\Upsilon \subsetneq \mathbb{R}^K$ consists of the intersection of all of the closed half-spaces containing Υ.



proof) $\exists$ $x \in \mathbb{R}^K \setminus \Upsilon$. So $\exists$ a closed half-spaces containing Υ by separating hyperplane theorem. Of course the intersection of all of the closed half-spaces containing Υ itself contains Υ.

WTS: this intersection does not contain any point z outside Υ.

Consider $z \in \mathbb{R}^K \setminus \Upsilon$. Our separating hyperplane theorem implies there exists H that strictly separate Υ and z. Thus $\exists$ a closed half-space that contains Υ but not z. Therefore, z is not in the intersection

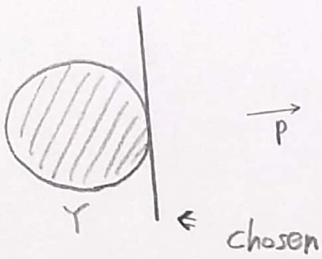


$\blacksquare$

- Given a set $Y \subset \mathbb{R}^K$ and a "price vector" $p \in \mathbb{R}^K$,

one might ask what is the smallest w such that everything in Y is affordable.

$$\sup_{y \in Y} py \in [-\infty, +\infty]$$



← chosen! → support function of Y ; $S_Y(p) = \sup_{y \in Y} py$

Def Given a set $Y \subset \mathbb{R}^K$,
the support function of Y, $S_Y : \mathbb{R}^K \to [-\infty, \infty]$ is defined as

$$S_Y(p) = \sup_{Y \in Y} py$$

ex) EMP → almost support fcn. ← $p \in \mathbb{R}^K$
profit fcn. ↗ ← $p \in \mathbb{R}^K$
production possibility set

we can know her utility fcn from EX fcn.
↓
For next class. We will cover it.

☆☆ **Corollary** (a) If $Y \subset \mathbb{R}^k$ is closed and convex,
then $Y = \bigcap_{p \in \mathbb{R}^K} \{x \in \mathbb{R} : px < S_Y(p)\}$

(b) If X and Y are both closed, convex subsets of $\mathbb{R}^K$,
then $S_X = S_Y \iff X = Y$.

//.