Transferable Utility Part 2 10/16

Defn: Say μ is stable relative to $\mathcal{E}^{TU}$ if there exists $(\bar{w}, \underline{w})$ s.t.

① matching μ is stable relative to $(\mathcal{E}^{TU}, \bar{w}, \underline{w})$

② There are $\bar{w}', \underline{w}'$ and (t, b) such that:

$$\bar{w}'(t,b) > \bar{w}(t, \mu_t(t)) \text{ and}$$
$$\underline{w}'(t,b) > \underline{w}(\mu_t(b), b)$$

Lemma μ is stable relative to transferable utility environment iff there are transfer functions $(\bar{w}, \underline{w})$ such that:

(1) μ is stable relative to $(\mathcal{E}^{TU}, \bar{w}, \underline{w})$

② There is no (t, b) such that

$$u(t,b) + v(t,b) > \bar{w}(t, \mu_t(t)) + \underline{w}(b, \mu(b))$$

Thm 1 Fix a transferable utility environment and an associated match. If μ is stable in that environment, then μ maximizes welfare in that environment.

Thm 2 Shapley-Shubik 1971

For each TU environment, there is a matching μ that is stable for that environment

Proof of Theorem 1

Fix some μ that is stable for this environment $\mathcal{E}^{TU}$. There exists some transfer functions such that:

① μ is IR in $(\mathcal{E}^{TU}; \bar{\omega}, \underline{\omega})$

$\forall t \quad \bar{\omega}(t, \mu_t(t)) \geq u(t, \emptyset)$

$\forall b \quad \underline{\omega}(\mu_t(b), b) \geq v(\emptyset, b)$

② For all t and b:

$$u(t,b) + v(t,b) \leq \bar{\omega}(t, \mu_t(t)) + \underline{\omega}(\mu(b), b)$$

Fix some other match $\tilde{\mu}$: will show $W(\tilde{\mu}) \leq W(\mu)$

$$W(\tilde{\mu}) = \underbrace{\sum}_{t \in M(\tilde{\mu}) \cap T} [u(t, \tilde{\mu}(t)) + v(t, \tilde{\mu}(t))] + \underbrace{\sum u(t, \emptyset) + \sum v(\emptyset, b)}_{t \in T \setminus M(\tilde{\mu})}$$

apply (2) apply 1

$$\leq \sum [\bar{\omega}(t, \mu_t(t)) + \underline{\omega}(\mu(\tilde{\mu}(t)), \tilde{\mu}(t)) + \sum \bar{\omega}(t, \mu_t(t)) + \sum \underline{\omega}(\mu_t(b), b)$$

$$= W(\mu)$$

<u>Notice</u>: $b \in B \cap M(\tilde{\mu})$ iff $\exists t \in T \cap M(\tilde{\mu})$

So, in $\mathcal{E}^{TU}$, stability will maximize welfare for us.

To sum up: The inability to transfer wealth is a market friction that appears as a cost to the market.

What is cost of this friction? Is there always a cost?

Example No Costs

Let preferences be assortative. T's and B's serve as compliments to total utility.

$T = \{t_1, \dots, t_n\}$ $B = \{b_1, \dots, b_N\}$

for each t_i : $b_j \succ_{t_i} b_{1+j}$ $j = 1, \dots, n-1$

$b_n \succ t_i$

for each b_j : $t_i \succ_{b_j} t_{i+1}$ $i = 1, \dots, N-1$

$t_n \succ b_j$

NTU environment with Market frictions, we have seen unique stable match

Will match each $\mu(t_i) = b_i$

(Has positive assortative match)

Ex $n=2$

(i)

	b_1	b_2
t_1	10,10	2,2
t_2	2,2	0,0

(ii)

	b_1	b_2
t_1	10,10	8,8
t_2	8,8	0,0

PAM: $10+10+0+0 = 20$ PAM $10+10+0+0 = 20$

NAM: $2+2+2+2 = 8$ $8+8+8+8 = 32$

(i) In (i) PAM makes more utility

(ii) Here, negative assortative matching wins

That is: if agents are compliments they will give higher utility with PAM

If they are substitutes, NAM gives higher utility

Defn:

$$w(t_i, b_j) - w(t_i, b_{j+1}) \geq w(t_{i+1}, b_j) - w(t_{i+1}, b_{j+1})$$

If true, then increasing differences
$\Rightarrow$ compliments $\Rightarrow$ USE PAM

If $\leq$ $\Rightarrow$ decreasing differences $\Rightarrow$ Substitutes
$\Rightarrow$ use NAM

If differences are increasing (compliments) PAM maximizes welfare

If differences are decreasing (sub) NAM will max welfare.