Homework Solutions 4 **Spencer Cooper**

ECON 501B

Prof. Friedenberg September 2018

1. To revise the definition of block*:
   A coalition C blocks$^*\mu$ if there exists:
   (a) same
   (b) for each $i \notin C$ with $\mu(i) \notin C, \mu'(i) = \mu(i)$.
   (c) same

   Problem:
   $T = \{t\}, B = \{b_1, b_2\}$
   Preferences:
   $t : b_1 \succ b_2 \succ t$
   $b_i : t \succ b_i$

   Think about μ such that $\mu(t) = b_2, \mu(b_1) = b_1$
   Then $C = \{t, b_1\}$ should form a block, but does not under the definition of block*

   Under the new definition of block*, neither condition is stronger; that is, block is equivalent to block*

   block* $\Rightarrow$ block is obvious.
   block $\Rightarrow$ block*:
   Suppose C blocks μ
   That means $\exists \mu'$ such that:
   $a)$ for each $i \in C : \mu'(i) \in C$
   $b)$ for each $i \in C : \mu'(i) \succ_i \mu(i)$

   Constraint μ'' :
   $a)$ if $i \in C, \mu''(i) = \mu'(i)$
   $b)$ if $i \notin C$ and $\mu(i) \notin C, \mu''(i) = \mu'(i)$
   $c)$ if $i \notin C$ and $\mu(i) \in C, \mu''(i) = i$

   By construction, for each $i \; inC, \mu''(i) = \mu'(i) \succ_t \mu(i)$ and $\mu''(i) = \mu'(i) \in C$

2. (a) $f_1 : W \succ \{w_1, w_3\} \succ \{w_2, w_3\} \succ \{w_1\} \succ \{w_2\} \succ \{w_3\} \succ \{w_1, w_2\} \succ f_1$
   $f_2 : W \succ \{w_1, w_2\} \succ \{w_1, w_3\} \succ \{w_2, w_3\} \succ \{w_1\} \succ \{w_2\} \succ \{w_3\} \succ f_2$

   (b) No, externalities.

(c) If $\mu(f_1) = f_1$, then (f_1, w_i) will block.
If $\mu(f_1) = \{w_i\}$ then there exists a $w_j \neq w_j$ such that f_1 and w_j will form a block.
If $\mu(f_1) = \{w_1, w_2\}$ then f_1 blocks.
If $\mu(f_1) = \{w_2, w_3\}$ then (f_1, w_1) form a block.
$\mu(f_1) = \{w_1, w_3\}\mu(w_2)$ and $\mu(f_2) = f_2$ unique stable match.

(d) Yerp.

(e) $f_1 : W \succ \{w_1, w_3\} \succ \{w_1, w_2\} \succ \{w_2, w_3\} \succ \{w_1\} \succ \{w_2\} \succ \{w_3\} \succ f_1$

Only unique match is again $\mu(f_1) = \{w_1, w_3\}\mu(w_2)$ and $\mu(f_2) = f_2$
(proof omitted cause tired).
Worker preferred same.

3. False:
$T = \{t\}, B = \{b_1, b_2, b_3\}$
Preferences:
$t : \{b_2, b_3\} \succ \{b_1\} \succ \{b_2\} \succ \{b_3\}$
Let $B_1 = \{b_2, b_3\}$ and $B_2 = \{b_1\}$
Then for each $b_i \in B_1, \{b_1\} \succ_t \{b_i\}$

4. True.
Since $B_1 \succ_t B_2$, then it is not the case that $B_1 \subseteq B_2$
Thus $B_1 \backslash B_2 \neq \emptyset$
Then see Lemma 2.

5. False.
$t : \{b_2\} \succ \{b_1, b_2\} \succ t \succ \{b - 1\}$
$B_1 = \{b_1, b_2\}$ and $B_2 = \{b_2\}$
$\{b_1\} = B_1 \backslash B_2$ but $t \succ_t b_1$