Homework 3 Solutions **Spencer Cooper**

ECON 501B

Prof. Friedenberg September 2018

1. If preferences are strict, then T-Proposing DA gives μ_{TD}, which is T optimal and B pessimal.
   Therefore, for any stable μ: $\forall t \in T : \mu_{TD}(t) \succsim_t \mu(t) \succsim_t \mu_{BD}(t)$ and
   for any stable μ: $\forall b \in B : \mu_{BD}(b) \succsim_b \mu(b) \succsim_b \mu_{TD}(b)$.
   If there exists a stable μ' and some $t \in T$, such that $\mu'(t) \neq \mu_{TD}(t)$ or $\mu'(t) \neq \mu_{BD}(t)$.
   Then $\mu_{TD}(t) \succ_t \mu'(t)$ or $\mu'(t) \succ_t \mu_{BD}(t)$, a contradiction of a unique stable match.

2. The following problem demonstrates assortative preferences, which we define as:
   $w_{i+1} \succ_m w_i$ for all $m \in M$ and
   $m_{i+1} \succ_w m_i$ for all $w \in MW$ and
   For each i agent, all j agents are acceptable.

   (a) μ^* stable?
   WLOG, assume $W \geq M$.
   $\mu^*(m_i) = w_{i+(W-M)}$ for $i = 1, 2, ..., M$
   $\mu^*(w_i) = w_i$ for $i = 1, 2, ..., W - M$
   Call μ^* positive assortative matching (PAM).

   M-Proposing DA:
   In round 1: For each $i, p'(m_i) = w_M$
   $\hat{\mu}'(w_M) = m_M$ and $\hat{\mu}'(w_j) = \emptyset$ for $j \neq M$

   In round (k+1):
   $p^{\hat{k+1}}(m_{M-j}) = w_{M-j}$, for $j = 0, 1, ..., k$
   For $m \in \{m_1, m_2, ..., m_{M-k-1}\}, p^{\hat{k+1}}(m) = w_{W-k-1}$
   For $j = 1, ..., k+1 : \mu^{\hat{k+1}}(w_{m-j}) = m_M - j$
   For $j > k+1 : \mu^{\hat{k+1}}(w_{M-j}) = \emptyset$
   Therefore, $\mu_{MD} = \mu*$ is stable.
   Similar for W-proposing DA.

   (b) The production function satisfies:
   For any $w_i > w_j$, then $f(m, w_i) - f(m, w_j)$ is increasing in m, and
   For $m_i > m_j$, then $f(m_i, w) - f(m_j, w)$ is increasing in w.

   Suppose the production function satisfies increasing differences. Then the match that maximizes the sum of productivity is a positive assortative matching.
   Proof:
   Case where $N = 2$: $f(m_2, w_2) + f(m_1, w_1)$ and $f(m_2, w_1) + f(m_1, w_2)$.
   Because of increasing differences:

$f(m_2, w_2) - f(m_2, w_1) > f(m_1, w_2) - f(m_1, w_1)$

General case where $N = n$: Let $M' \cup W'$ be the set such that $m_i \in M' \Rightarrow \mu(m_i) \neq w_i$ and likewise for w's.
Let k be the highest index of agents in $M' \cup W'$.
Consider an alternate match μ' with $\mu'(m_k) = w_k$ and $\mu'(m_j) = \mu'(w_i)$ while other matchings stay the same.
$f(m_k, w_k) + f(m_j, w_i) \sim f(m_k, w_i) + f(m_j, w_k)$ (where $\sim$ is the relation we are trying to attain).
$f(m_k, w_k) - f(m_j, w_k) > f(m_k, w_i) - f(m_j, w_i)$ from ID.
The set $M' \cup W'$ can always be improved upon, no matter the number of agents in the set. For the most productive set, $M' \cup W' = \emptyset$

(c) For any $w_i > w_j$
$f(m, w_i) - f(m, w_j) = \sqrt{mw_i} - \sqrt{mw_j} = \sqrt{m}(\sqrt{w_i} - \sqrt{w_j}$ is increasing in m.
Also then check for $m_i > m_j$.