501A 11/5

* Chapter 16. EMP. (continuing...)

by the definition.

- $H(p, \bar{v})$ is the set of cheapest bundles (at p), which achieves at least util. $\bar{v}$
- Given a continuous and LNS util. fcn. $H(p, \bar{v})$ is equal to the set of optimal bundles (in the UMP) given prices P and wealth just enough to achieve $\bar{v}$.

same

- ★★★ The law of Hicks Compensated Demand.

Given a utility target $\bar{v} \in [u(0), \sup_x u(x))$ and two price vectors p^0, p^1.

If $h^0 \in H(p^0, \bar{v})$ and $h^1 \in H(p^1, \bar{v})$, then $\underbrace{(p^1 - p^0)}_{\text{vector}} \cdot \underbrace{(h^1 - h^0)}_{\text{vector}} \leqq \underbrace{0}_{\#}$.

That $h^0 \in H(p^0, \bar{v})$, then $u(h^0) \geqq \bar{v}$ and $p^0 h^0 \leqq p^0 x$ for all x where $u(x) \geqq \bar{v}$.

$h^1 \in H(p^1, \bar{v})$, then $u(h^1) \geqq \bar{v}$ and $p^1 h^1 \leqq p^1 x$ for 〃.

$\Rightarrow p^0 h^0 \leqq p^0 h^1$ ($\because h^1$ is optimal)

$\Rightarrow p^1 h^1 \leqq p^1 h^0$ ($\because h^0$ is optimal)

) induced from the def. of $H(p, \bar{v})$

$\Downarrow$

Therefore, $p^0 h^0 + p^1 h^1 \leqq p^0 h^1 + p^1 h^0$.

i.e., $p^1 h^1 - p^1 h^0 + p^0 h^0 - p^0 h^1 \leqq 0$

$p^1 (h^1 - h^0) - p^0 (h^1 - h^0) \leqq 0$

$\downarrow$

$(p^1 - p^0) \cdot (h^1 - h^0) \leqq 0$

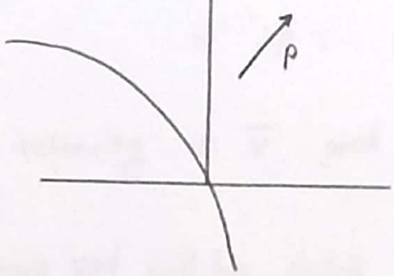
→ Think of $e(p, \bar{v})$, the expenditure fcn.

$$e(p, \bar{v}) = \min_{x;\, u(x) \geqq \bar{v}} px$$

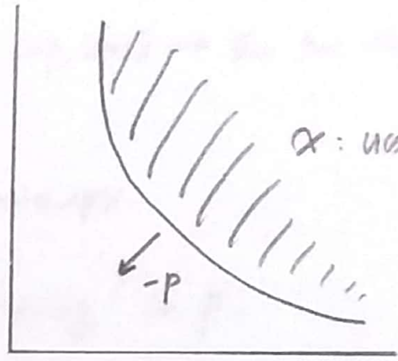
prop 16.3

Remark the producer's problem

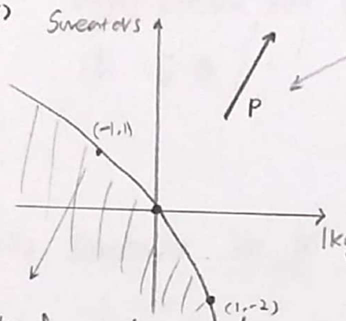
$$\max_{y \in T} py$$



$$e(p, \bar{v}) = \min_{x: u(x) \geq \bar{v}} px$$



ex)



Let $p := (1, 2)$, where $(1,2) = (p_{yarn}, p_{sweater})$

Consider three choice : $(-1,1)$, $(0,0)$, $(1,-2)$

$$\max_{y \in T} py$$

As 1kg of yarn is consumed, 1 of Sweater is produced. ⇒ choose $(-1,1)$ b/c of p. vector.

- Lower contour convex set with p $\Longleftrightarrow$ Upper contour convex set with $-p$.

$\Uparrow$ Mathematically Same!

$y^*(p) = \arg\max_{y \in T} py$ (usc)	$H(p, \bar{v}) = \arg\min_{x: u(x) \geq \bar{v}} px$ (usc)
$\Pi(p) = \max_{y \in T} py$	$e(p, \bar{v}) = \min_{x: u(x) \geq \bar{v}} px$

Back to EMP.

prop 16.3 Suppose u is continuous and LNS.

The expenditure function $e : \mathbb{R}^k_{++} \times [u(0), \sup_x u(x)) \to \mathbb{R}_+$ has the following properties

(a) HD1 in p : $e = \min px$

if λp, $e' = \lambda e = \min \lambda p x$

(b) Strictly increasing in $\bar{v}$ and nondecreasing in p:

$\Downarrow$

uppercontour set will be shrink if $\bar{v} \Uparrow$.

$\Downarrow$

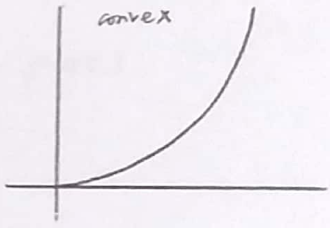
Constraint $u(x) \geqq \bar{v}$ is changing and px, objective fcn is increasing if p is changing.

(c) Concave in p

(d) Continuous in p and $\bar{v}$

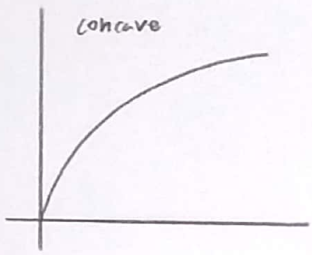
proof of (c)): < Concavity of e in p > $p^t := tp' + (1-t)p^0$, $t \in [0,1]$

if $e(p^0, \bar{v}) = e(p^1, \bar{v})$, then $e(p^t, \bar{v})$



WTS : $\underline{e(p^t, \bar{v}) \geqq t\, e(p^1, \bar{v}) + (1-t)\, e(p^0, \bar{v})}$

Suppose $h^t \in H(p^t, \bar{v})$

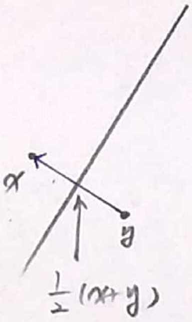


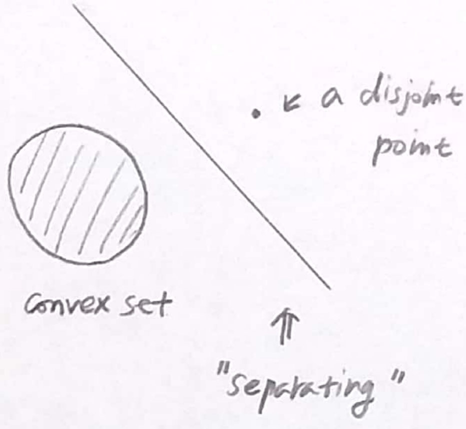
$$
\begin{aligned}
e(p^t, \bar{v}) &= p^t h^t = \{tp' + (1-t)p^0\} \cdot h^t \\
&= t p^1 h^t + (1-t) p^0 h^t \\
&\geqq t\, e(p^1, \bar{v}) + (1-t)\, e(p^0, \bar{v}) . \quad //
\end{aligned}
$$

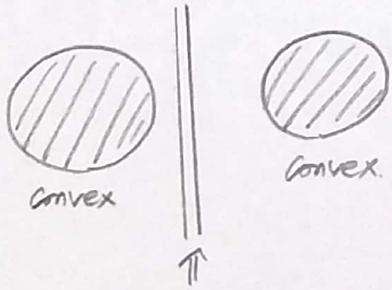
$\uparrow$

$\begin{cases} p^1 h^t \geqq e(p^1, \bar{v}) \\ p^0 h^t \geqq e(p^0, \bar{v}) \end{cases}$: h^t is feasible for $\bar{v}$, but h^t is not the solution of $e(p^1, \bar{v})$ and $e(p^0, \bar{v})$

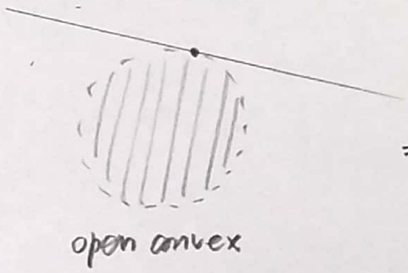
* Chapter 17. Separating Hyperplanes







$\Rightarrow$ Two disjoint convex sets $\Rightarrow$ "Separated" $\exists$ positive gap



$\Rightarrow$ strict separated

(NO positive gap b/w the set and the point)