Amanda Friedenberg
ECON 501B

ECON 501B: Problem Set 9

Due: Thursday, November 15, 2018

Instructions: Answers should be complete proofs of a claim.

Question 1: Consider an Edgeworth Box Pure Exchange Economy described as follows: For each $i = 1, 2$, $X_i = \mathbb{R}^2_+$. Endowments are given by $\omega_1 = (10, 2)$ and $\omega_2 = (2, 10)$. Preferences for $i = 1, 2$ are represented by a utility function $u_i(x_{1,i}, x_{2,i}) = x_{1,i}^{\alpha_i} x_{2,i}^{\alpha_i}$ for $\alpha_i \in (0, 1)$.

1. Characterize the competitive equilibrium allocations in this environment. (That is, which allocations are vs. are not competitive equilibrium allocations?)
2. Characterize the Pareto set. (That is, which allocations are vs. are not in the Pareto set?)
3. Compare the two sets above? How do they speak to the First and Second Welfare Theorems?
4. Fix an allocation in the Pareto set that is not a competitive equilibrium allocation. (That is, pick a specific allocation—with numbers—that satisfies these requirements.) Construct transfers so that it can be sustained as an equilibrium with transfers.

Question 2: Consider an Edgeworth Box Pure Exchange Economy, where the first good $\ell = 1$ is a numeraire and the second good $\ell = 2$ is a consumption good. Suppose, for each consumer $i = 1, 2$, i's preferences admit a quasilinear utility representation

$$u_i(x_{1,i}, x_{2,i}) = x_{1,i} + \phi_i(x_{2,i})$$

where ϕ_i is a twice differentiable strictly increasing and strictly concave function. Show the following: If $((x_1^*, x_2^*), y^*)$ and $((x_1^{**}, x_2^{**}), y^{**})$ are Pareto optimal allocations in the interior of the Edgeworth Box, then $x_{2,i}^* = x_{2,i}^{**}$ for each i.

Question 3: Let $E^{PE} = (Y_J, (X_i, \succsim_i, \omega_i, \theta_{i,J} : i = 1, \ldots, I))$ be a pure exchange economy, where

- $Y_J = \mathbb{R}^L_-$ is the production set of a "garbage firm,"
- each $X_i = \mathbb{R}^L_+$,
- each $\succsim_i$ is strongly monotone, and
- for each $\ell = 1, \ldots, L$, $\overline{\omega}_\ell > 0$.

(Recall, $\theta_{i,J}$ is the share of the firm owned by J.) Suppose $((x^*, y^*), p^*)$ is a price equilibrium with transfers, supported by transfers $(W_1, \ldots, W_I)$. Show that there are endowments $\omega' = (\omega'_1, \ldots, \omega'_I)$ with $\overline{\omega}' = \overline{\omega}$ so that $((x^*, y^*), p^*)$ is a competitive equilibrium of the economy $(Y_J, (X_i, \succsim_i, \omega'_i, \theta_{i,J} : i = 1, \ldots, I))$.

Question 4: Let $E^{PE} = (Y_J, (X_i, \succsim_i, \omega_i, \theta_{i,J} : i = 1, \ldots, I))$ be a pure exchange economy as described above. We will think of a coalition of consumers $C \subseteq \{1, \ldots, I\}$. Say that a coalition $C \subseteq \{1, \ldots, I\}$ **blocks** the allocation (x^*, y^*) if there exists a vector of consumption $x = (x_1, \ldots, x_I)$ so that

1. $x_i \succ_i x_i^*$ for each $i \in C$, and
2. $\sum_{i \in C} x_i \in Y_J + \{\sum_{i \in C} \omega_i\}$.

A feasible allocation (x^*, y^*) is in the **core** if no coalition of consumers block (x^*, y^*). (It is worthwhile to reflect on these definitions, in light of our previous discussions of the core.)

1. *True or False*: Any competitive equilibrium allocation is in the core.
2. *True or False*: Any allocation in the core is a competitive equilibrium allocation.
3. How do your conclusions relate to the Welfare Theorems?