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ECON 501B

ECON 501B: Problem Set 10

Due: Tuesday, November 27, 2018

Instructions: Answers should be complete proofs of a claim. For True/False either show the result is true or provide a counter example.

Question 1: Fix a pure exchange economy $E^{PE} = (Y_J, (X_i, \succsim_i, \omega_i, \theta_{i,J} : i = 1, \ldots, I))$ that satisfies the following:

1. $Y_J = \mathbb{R}^L_-$,
2. each $X_i = \mathbb{R}^L_+$,
3. each $\succsim_i$ is continuous, convex, and strongly monotone, and
4. $\overline{\omega} = (\overline{\omega}_1, \ldots, \overline{\omega}_L) >> 0$.

Show that, for any Pareto optimal allocation (x^*, y^*), there exists some price vector p^* so that $((x^*, y^*), p^*)$ is a price equilibrium with transfers. (Hint: Use the result on quasi-price equilibrium with transfers.)

Question 2: Consider an economy where consumers are characterized by "type." If two consumers are the same type, they have the same consumption sets, preferences, endowments and firm shares. With this in mind, write $t = 1, \ldots, T$ for a type of a consumer. If there are N consumers are each type, there are $I = N \times T$ consumers in the market. When there are N consumers of each type, we call the economy an **N-replica economy**. It is characterized by

$$E^N = ((Y_j : j = 1, \ldots, J), (X_t, \succsim_t, \omega_t, \theta_{t,1}, \ldots, \theta_{t,J} : t = 1, \ldots, T), N).$$

Adopt the following notation for subscripts: If you want to refer to a consumer i who is the n^{th} agent of type t, write (t, n). (That is, first put the type and then the consumer's position.) Throughout the question, take each $\succsim_t$ to be strongly monotone, strictly convex, and continuous.

a. True of False: If an allocation $(x^{*,N}, y^*)$ is Pareto optimal in the N-replica economy, then there exists a Pareto optimal allocation $(x^{*,N+1}, y^*)$ of the $(N+1)$-replica economy so that, each $x^{*,N}_{(t,n)} = x^{*,N+1}_{(t,n)}$.

Fix an allocation (x, y) of the N-replica economy. The allocation satisfies the **equal treatment property** if, for each t and each n, n', $x_{(t,n)} = x_{(t,n')}$.

b. True of False: If an allocation of the N-replica economy is Pareto optimal, then it satisfies the equal treatment property.

c. True of False: A competitive equilibrium allocation (of the N-replica economy) satisfies the equal treatment property.

d. Consider instead an N-replica pure exchange economy with two types $t = 1, 2$ and different numbers of agents of each type. Will a competitive equilibrium allocation satisfy the equal treatment property?

Question 3: Consider a two-good economy with one firm and one consumer. The firm uses good $\ell = 1$ to produce good $\ell = 2$ according to the production function $f(y_1) = y_1^2$. The consumer has consumption set $X = \mathbb{R}_+$, preferences that are monotonically increasing in $\ell = 2$ and an endowment of $\omega = (10, 0)$.

- Does a competitive equilibrium exist? If so, compute that equilibrium. If not, show it does not exist.
- How would your answer change if the consumer's preferences were represented by the utility function $u(x_1, x_2) = x_1 - x_2$.

Question 4: Consider an economy with three commodities, two consumers, and two firms. Consumer $i = 1$ owns firm $j = 1$ and consumer $i = 2$ owns firm $j = 2$. The endowments are $\omega_1 = \omega_2 = (0, 0, 5)$; so, if there are commodities $\ell = 1, 2$ they must be produced by the firms. Firm $j = 1$ has a technology that turns $\ell = 3$ to $\ell = 1$, while firm $j = 2$ has a technology that turns $\ell = 3$ to $\ell = 2$. Specifically, the production function of firm $j = 1$ is $f_1(y_{3,1}) = 3y_{3,1}$ and the production function of firm $j = 2$ is $f_2(y_{3,2} = 4y_{3,2}$. Preferences of consumers are represented by utility functions

- $u_1(x_{1,1}, x_{2,1}, x_{3,1}) = .4 \ln(x_{1,1}) + .6 \ln(x_{2,1})$, and
- $u_2(x_{1,2}, x_{2,2}, x_{3,2}) = .5 \ln(x_{1,2}) + .5 \ln(x_{2,2})$.
- What is the competitive equilibrium allocation of this economy?
- How would your answer change if consumer $i = 1$ owned firm $j = 2$ and consumer $i = 2$ owned firm $j = 1$?