Econ 519 Homework 1

David Zynda

August 27, 2018

(1) Prove the following statement:

For $x \neq 1$ and $\forall n \in \mathbb{N}$

$$\sum_{k=0}^{n} x^k = \frac{1-x^{n+1}}{1-x}.$$

Step 1: Let $n = 1$. Then,

$$\sum_{k=0}^{1} x^k = \frac{1-x^{0+1}}{1-x} + \frac{1-x^{1+1}}{1-x} = \frac{1+x^2}{1-x}.$$

$$= \frac{(1+x)(1-x)}{(1-x)} = (1+x) = \frac{1-x^{n+1}}{1-x}, n = 1$$

Step 2: For $n+1$ show that the geometric sum is $\frac{1-x^{n+2}}{1-x}$.

$$\sum_{k=0}^{n+1} x^k = \sum_{k=0}^{n} x^k + x^{k+1} = \frac{1-x^{n+1}}{1-x} + x^{n+1}$$

$$= \frac{1-x^{n+1}+(1-x)(x^{n+1})}{1-x} = \frac{1-x^{n+1}+x^{n+1}-x(x^{n+1})}{1-x}$$

$$= \frac{1-x^{n+1}+x^{n+1}-x^{n+2}}{1-x} = \frac{1-x^{n+2}}{1-x}$$

(2) Prove the following statement.

If A and B are sets, then $A \cap (B-A) = \emptyset$.

$$(B-A) \iff B \setminus A \iff B \cap A^c$$
$$A \cap (B-A) = B \cap A^c \iff A \cap B \cap A^c \iff \emptyset$$

(3) Prove the following statement.

Suppose that $n \in \mathbb{Z}$. If n^2 is odd, then n is odd.

Suppose by contradiction that n^2 is odd but that n is even.

By definition, an even number is $2m$ where $m \in \mathbb{Z}$. Let $b = 2m$ for some $m \in \mathbb{Z}$. Then, $b^2 = (2m)^2 = 4m^2$ So, b^2 is even. But we assumed it was odd. Then, through contradiction, it is clear that if n^2 is odd, then n is odd.

(4) Prove the following statment.

$$\Big(\bigcup_{i\in I} A_i\Big)^c = \bigcap_{i=1} (A_i)^c.$$

$$\Big(\bigcup_{i\in I} A_i\Big)^c = (A_1 \cup A_2 \cup A_3 \cup ... \cup A_I)^c$$
$$= A_1^c \cap A_2^c \cap A_3^c \cap ... \cap A_I^c = \bigcap_{i\in I} (A_i)^c$$

(5.1) Is $\leq$ and equivalence relation? Prove yes or explain why no!

This is not an equivalnce relation. An equivalence relation must satisfy 3 things: + Must be reflexive + Must be symmetric + Must be transitive.

The relation $\leq$ is reflexive and transitive. However, it is not symmetric. In fact, it is anti-symmetric:

$$\forall x, y \in S : (x \leq y) \wedge (y \leq x) \implies x = y$$

(5.2) Give an example for a relation that is reflexive but not symmetric.

The example above suffices, or $\geq$.

$$\forall x \in S : x \geq x$$

$$\forall x, y \in S : (x \geq y) \wedge (y \geq x) \implies x = y$$