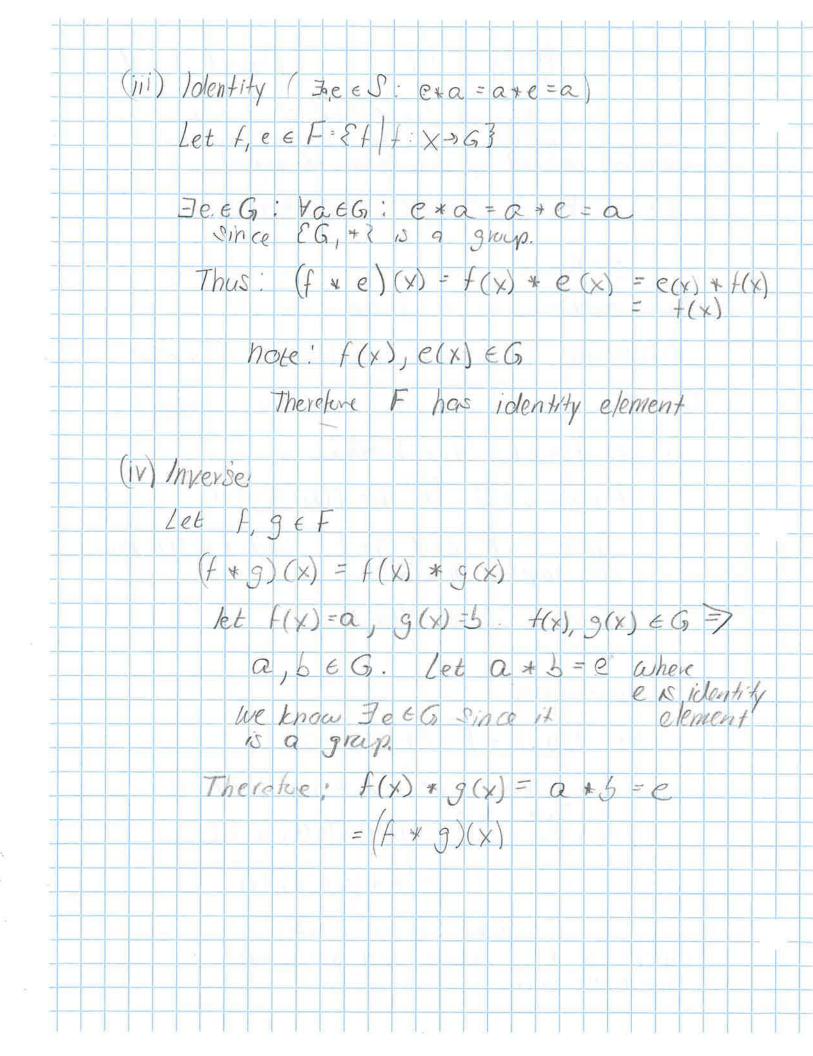
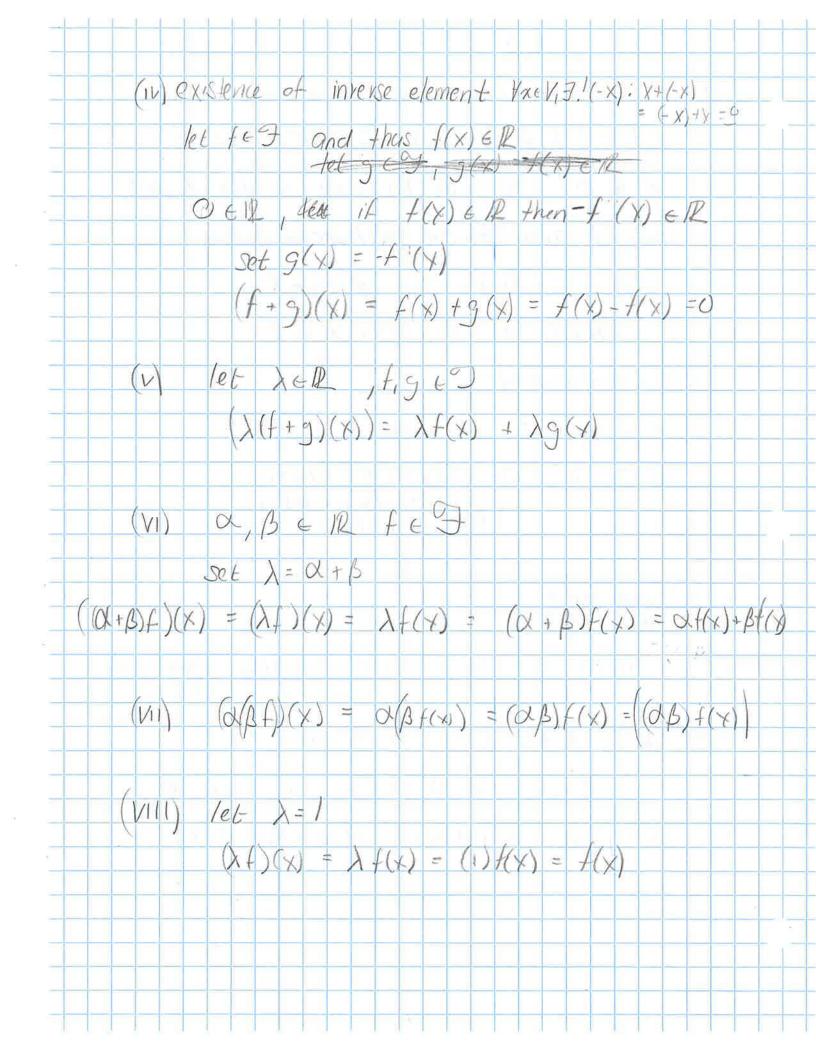
Math 519 David Zynda 1) Let X be a set and &G, +3 be a group.

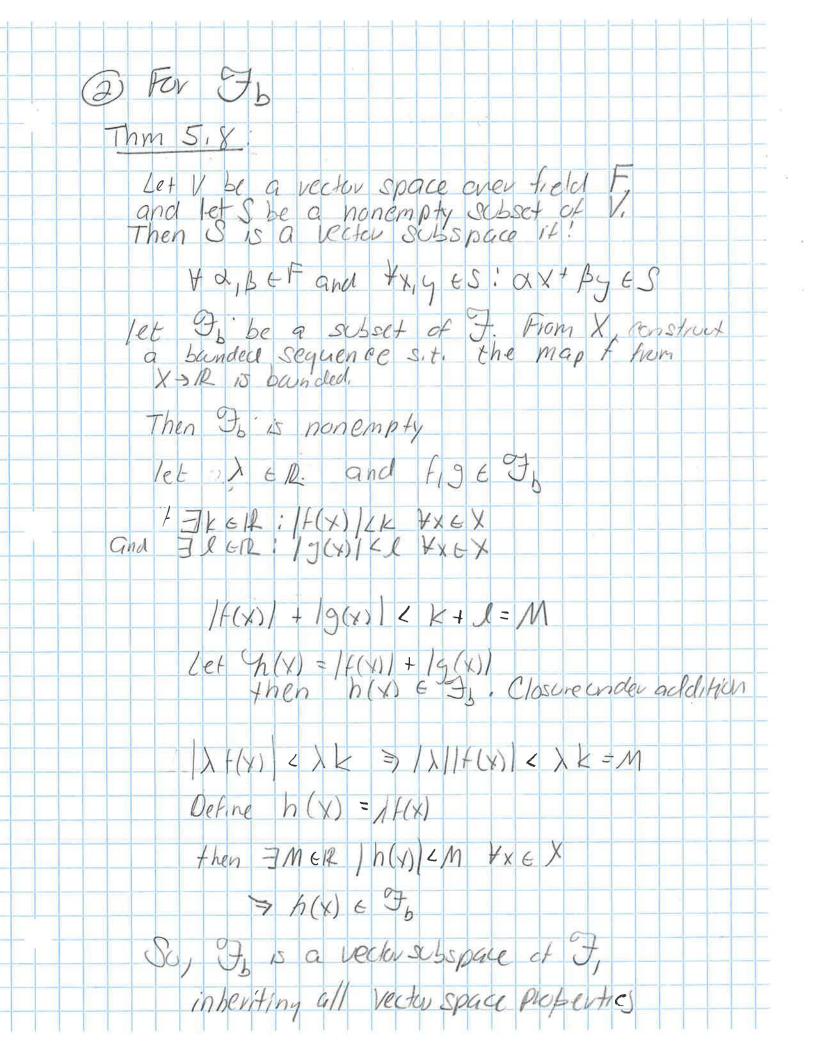
The set of functions of X into G, endowed with operation defined by composition of images, i.e. (1)  $\forall x \in X, (f * 9)(x) = f(x) * 9(x)$ 15 9 group. Four conditions of a group, (i) closure (ii) associativity (iii) Identity (v) inverse Note: X is a set. Gis a set. EG, \*? is a group (1) let f,g & F = Ef/f: X >G3 (f + g)(x) = h = h + g(x) + g(x)Since feF and geF, and F=Sff:x=s6} this implies h ∈ G Therefore, F is closed. (ii) Associativity Let f, 9, b & F = Ef | f'x > G? = ) f + (9 \* h))(x) Since EG, \*7 is group and associates.

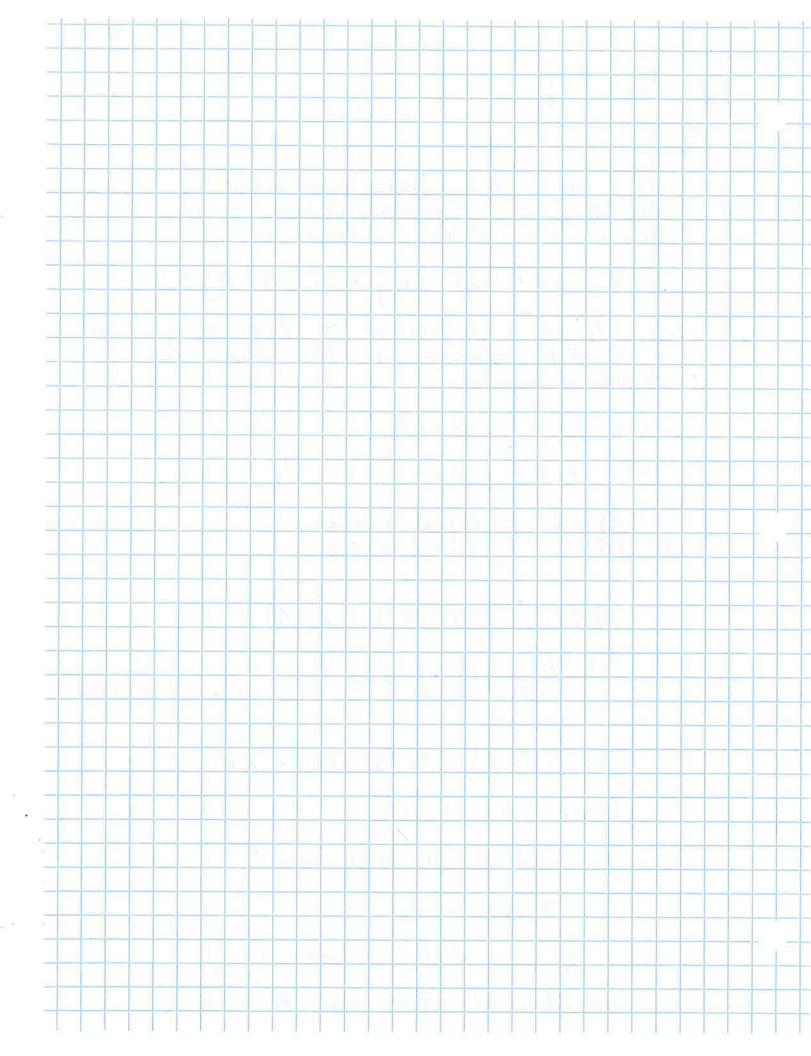
fig. h are in G



2) Let X be a non-empty set. Let: F = E f / f : X > 12 3 and the subset F Consisting of 911 bounded maps from X to 12 1: F = Eft F | 3k & R: | f(x) | ek, tx & X } Show I and I, are real vector spaces (+g)(x) = f(x) + g(x) 4 f, g & F (AF)(X) = (AF)(X) HFEF LER (i) Associative property let f, q, h & J (f+(g+h))(x) = f(x) + (g(x) + h(x))= (f(x) + g(x)) + h(x)= (f+g)+h)(x)(ii) Commutative Jet f, g & 9 (f+g)(x) = A(x) + g(x) = g(x)+f(x) = (g+f)(x) 11) Additive Identity 7! Q = V: x + Q = Q + x = X eeR let f, g & F let g(x) = e (f + g)(x) = f(x) + g(x) = f(x) + e = f(x)







3) Let S ⊆ R. A real-valued function f: S → 12 is strictly inc. if y = 7 = 1 = 1 (2) 7 f(2) Let U be the set of all real-valued tenotions on 9 set X. U:X >/R unu' => 3 a strictly inc function f: u(X) > 12: u'= f(u(X)) Equivalence requires transitivity, reflexivity, and symmetricity Reflexive. unu = Ja strictly increasing function f: u(x) > R: W= f(u(x)) let f(x) = x, then f(u(x)) = a(x)Symmetric: 21, ~ 42 => 3 str. inc f:4,(x) > 12 ' 42 = f(4,(x)) U2 ~ U, => 7 Str. inc f'U2(x) => 12: U, = f(U2(x)) the inverse of a strictly increasing function is a strictly in creasing knother  $X = Y \iff f(X) \in f(Y) \implies f'(X) \in f'(Y) \iff f'(X) \in f'(X) \iff f'(X)$ So it is true that this relation is symmetric. For any Str. increasing function mapping u > 12, its inverse is also strictly increasing

Transitine and Una then una  $U_1 N U_2 = \int_{\mathcal{C}} \left( U_1(x) \right)$ U, ~ U3 =7 U3 = f2 (U2 (X)) <=> U3 = F2 (f, (U(x)) =7 U, NU3 (f, of )(x) is strictly increasing; that is composition of two increasing functions is strictly increasing X>y > f(x)>f(y) let g qto be strictly increasing X7 > f(x) > f(y) let F(X) = a fly) = 5 > 3 > a => g(6) > g(a) => g(f(x)) > g(f(y)) therefore, the relation ~ is transitive. So, uny' is an equivalence relation

EG, \*3 is a non-empty binary relation \*: G × G > G (i) 49 eG: TheG: g+h=h+g=g (ii) 3 h + G: Yg + G: g + h = h + g = g (i) \$\(\frac{1}{2}\) (1) says every g has an h (maybe not the same h) such that g + h = h + g = g (11) says there is (one) h such that every let G, v G, -> G = \*\*

1f g, & G = (g, , g;) = 1 G×G > (92,9) & and (9, 92) otherwise 16 g, = 2 Gx6 > (g, g,) and (g, g) ow let G have 4 element (1, 1)  $h = (1,1),(1,2),(2,L)(2,1) \Rightarrow 9*h=9$ (1,2) h=(1,1),(1,2) = 9 \* h = 9(2,2) h= (2,1)(2,2)(1,1)(1,2)=> 9 x h= 9 5-(211) h=(2,1)(2,2) = 9 x h=9 hence to eGithebigship his are not all the botingthe GitseGisthing) since his are not all the

(b) It EG, + s is a group, then (i) holds since by definition a group must have an identity element playing the we of g (ii) a so holds, again, let h=e, +4 identity element. Then, since 56, + is a grup, it must be the case! and also since (11) >(1) IF (ii) is twe, and it is, (i) is also the

