Theorem 1 (Cauchy-Schwarz-Inequality)
Let $\alpha, \beta \in R$,
then $(\sum_{i=1}^{n} \alpha_i \beta_i)^2 \leq (\sum_{i=1}^{n} \alpha_i^2)(\sum_{i=1}^{n} \beta_i^2)$
This is equivalent to:
$(\sum \alpha_i \beta_i)^2 \leq \sum \alpha_i^2 \sum \beta_i^2$

Theorem 2
Let $(V, ||\cdot||)$ be a normed vector space.
Let $d : V \times V \to R$ be defined by $d(v, w) = ||v - w||_E$
Then (v, d) is a metric space.

Theorem 3
Let $\{x_n\} \in R$ if $x_n \leq x_{n+1} \forall n \in N$ (increasing sequence) then sup $\{x_1, x_2, ...\} = \lim_{n\to\infty} x_n$
if $x_n \geq x_{n+1} \forall n \in N$ (decreasing sequence) then inf $\{x_1, x_2, ...\} = \lim_{n\to\infty} x_n$

Theorem 4
Let $\{x_n\}$ be a sequence in R
Then $\lim_{n\to\infty} x_n = \gamma \in R \cup \{-\infty, \infty\}$
if and only if
$\lim_{n\to\infty} \inf x_n = \lim_{n\to\infty} \sup x_n = \gamma$

Theorem 5 Rising Sun Lemma
Every Sequence of real numbers contains an increasing or decreasing subsequence or both.

Theorem 6 Bolzano Weierstrass
Every bounded sequence of real numbers contains a convergent subsequence.

Theorem 7
Let (x, d) be a metric space.
Then
i) Ø and X are both open and closed.
ii) the union of an arbitrary collection of empty sets in x is open.
iii) the intersection of a finite collection of open sets in x is open.

Theorem 8
A set A in a metric space (x, d) is closed if and only if for every sequence $\{x_n\}$ in A that converges to $x \in X$, one has $x \in A$.

Theorem 9
Let (x, d) and (y, ρ) be metric spaces and $f : x \to y$. Then f is continuous if and only if the pre-image of every open set in y is open in X.

Theorem 10
Let (x, d) and (y, ρ) be metric spaces. Then $f : x \to y$ is continuous at $x \in X$ if and only if f is sequentially continuous at x.

Theorem 11
Let (x, d) and (y, ρ) be metric spaces with $A \subseteq X$ and $f : A \to Y$. f is continuous at a limit point $x_1 \in A$ if and only if $lim_{x\to x_0} f(x) = f(x_0)$.

Theorem 12 Intermediate Value Theorem
Let $[a, b] \subseteq R, f : [a, b] \to R$ be continuous, and suppose $f(a) < 0$ and $f(b) > 0$, then there exists $C \in (a, b)$ such that $f(x) = 0$.

Theorem 13
Let (x, d) be a metric space. Then $A \subseteq X$ is compact if and only if A is sequentially compact.

Theorem 14
Let (x, d) be a metric space. A subset $A \subseteq X$ is compact if and only if it is complete and bounded.

Theorem 15 Heine-Borel Theorem
Let $A \subseteq R^n$. Then A is compact if and only if it is closed and bounded.

Theorem 16

Let (x, d) and (y, ρ) be metric spaces. If $x \to y$ is continuous, and $C \subset X$ is compact in X, then $f(c)$ is compact in y.

Theorem 17 (Weierstrass Theorem/ Extreme Value Theorem)

Let C be a non-empty and compact set. In a metric space (x, d) and suppose $f : X \to R$ is continuous. Then f attains its minimum and maximum C.

Theorem 18

Let $B = \{v_s \in V | s \in S\}$ be a basis for the vector space V. Then every nonzero vector $x \in V$ has unique representation as a linear combination with coefficients not all zero of a finite number of vectors in B.

Theorem 19

Let X be a finite dimensional vector space, Y be a vector space, and $T : X \to Y$ be a linear transformation. Then dim $X =$ (ker$T$)+ rank$T =$ rank(ker$T$)+ rank(im$T$)

Theorem 20

Let $T : X \to Y$ be an invertible linear mapping. Then $T^{-1} : Y \to X$ is a linear mapping.

Theorem 21

A linear mapping $T : X \to Y$ is injective if and only if $T(x) = 0$ implies $x = 0$. (That is ker $T = \{0\}$)

Theorem 22

Let X and Y be named vector spaces and $T : X \to Y$ be a linear mapping. If T is continuous at some point $x \in X$ that is continuous on x.

Theorem 23

Let X and Y be normed vectors psaces. Al linear mapping $T : X \to Y$ is continuous if and only if it is bounded.

Theorem 24

A linear mapping T from a finite dimensional normed vector space x into a normed vector space Y is continuous.

Theorem 25

Let X and Y be normed vector spaces. Then $B(X, Y)$ endowed with linear operator norm is a normed vector space.

Theorem 26

Let $T \in L(R^m, R^n)$ be a linear mapping with standard matrix representation $A = |ia, t|$ with $i = 1, ..., n, k = 1, ...m,$

Define $\mu = \max\{|a_{in}|, i = 1, ..., n, k = 1, ..., m\}$. Then $\mu \leq ||T|| \leq \mu\sqrt{n \cdot m}$

Theorem 27

Let $R \in L(R^m, R^n)$ and $S \in L(R^n, R^p)$ then $T = S \circ R \in L(R^m, R^p$ and $||T|| \leq ||S \circ R|| \leq ||S|| \cdot ||R||$

Theorem 28

$T \in L(R^n)$ is invertible if and only if ker $T = \{0\}$

Theorem 29

Let S and T be invertible operators in $L(R^n)$: Then $S \circ T \in L(R^n)$ is invertible $(S \circ T)^{-1} = T^{-1} \circ S^{-1}$

Theorem 30

Let T and S be linear operators in R^n. If T is invertible and $||S - T|| \leq \frac{1}{||T^{-1}}$, then S is invertible. This implies that $\Omega(R^*)$ is open in $L(R^k)$. Moreover, $||S^{-1}|| \leq \frac{||T^{-1}||}{1-||T^{-1} \cdot (T-S)||}$

Theorem 31

The function $(\cdot)^{-1} : \Omega(R^n) \to \Omega(R^n)$ that assigns to each invertible operator α inverse is continuous

Theorem 32

Let $f : (a, b) \to R$ if f is differentiable at $x \in (a, b)$, then f is continuous at x.

Theorem 33

Let $f : (a, b) \to R$ be a differentiable function and $x_0 \in (a, b)$ a local maximizer (minimizer) of f. Then $f'(x_0) = 0$.

Theorem 34 (Rolle)

Let $f : [a, b] \to R$ be continuous and differentiable on (a, b). If $f(a) \to f(b) = 0$ then there exists $\theta \in (a, b)$ such that $f'(\theta) = 0$.

Theorem 35 (Mean Value Theorem)

Let $f : R \to R$ be a differentiable function. If $a, b \in R$ satisfies $a < b$, then there exists $\o \in (a, b)$ such that $f'(0 = \frac{f(b)-f(a)}{b-a}$ "slope".

Theorem 36 (Inverse function theorem on R)

Let $f : R \to R$ be continuously differentiable with $f'(a) \neq 0$

The f is indivisible in a neighborhood of a the inverse is continuously differentiable and $(f^{-1}(\cdot))' = \frac{1}{f'(a)}$.

Theorem 39

Let $x \subseteq R^n$ be open and $f : X \to R^m$ be differentiable at $x \in X$. Then f is continuous at x.

Theorem 40

Let $x \subseteq R^n$ be open and $f : X \to R^m$. f is differentiable at $x \in X$ if and only if each of the component functions is differentiable at x. Moreover, if f is differentiable at xm then the partial derivatives of the component functions exists at x, and the derivative of f at x is the matrix:

[insert matrix here]

We call this matrix of partial derivatives of the component functions the "Jacobean" of f at x.

Theorem 41

Let $x \subseteq R^n$ be open and $f : x \to R^m$. If the partial derivatives of the component functions exist and are continuous on x, then f is differentiable on x.