Exercise Set 4

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(1) A set is convex if and only if every convex combination of vectors in S is also in S.

(i) If every combination of vectors in S is also in S, then a set is convex.

This proof is evident from the definition of a convex set, that is:

A set S is convex in vector space V if for any two points x, y in S and λ in the unit interval $(0, 1)$ the point $(1-\lambda)x + \lambda y$ is in S.

Clearly, if every combination of vectors is convex in S, then there will be two that fulfill that definition of convexity.

(ii) A set is convex if every convex combination of vectros in S is also in S.

Let V be a convex vector space such that:

$$(1-\lambda)y + \lambda x = v \subseteq V$$

For $\lambda > 0$. Now, let $(1-\lambda) = \lambda_2$ and denote λ as λ_1. Then,

$$\lambda_2 y + \lambda_1 x = v \subseteq V$$

Then, for all $\{\lambda_n\}$, $\lambda_i \geq 0$ and $\sum_{i=1}^{n} \lambda_i = 1$

Now, for the first step in the induction proof, let a and b be in the convex set V such that

$$\lambda_1 a + \lambda_2 b = v \subseteq V$$

Let $\lambda_2 = \lambda_2^* + \lambda_3 = (1-\lambda_1)$ such that $(\lambda_1 + \lambda_2^* + lambda_3 = 1)$. Multiply b by a factor of 1, but expressed by: $\frac{\lambda_2^*+\lambda_3}{\lambda_2^*+\lambda_3}$

Such that:

$$\lambda_1 a + (\lambda_2^* + \lambda_3)(\frac{\lambda_2^* + \lambda_3}{\lambda_2^* + \lambda_3})b$$

Now, decompose b into two separate vectors following the scalar fraction:

$$\lambda_1 a + (\lambda_2^* + \lambda_3)(\frac{\lambda_2^* b^*}{\lambda_2^* + \lambda_3} + \frac{\lambda_3 c b^*}{\lambda_2^* + \lambda_3})b$$

(2)