Econ 519 Homework 1

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(1) Prove the following statement:

For $x \neq 1$ and $\forall n \in \mathbb{N}$

$$\sum_{k=0}^{n} x^k = \frac{1 - x^{n+1}}{1 - x}.$$

Step 1: Let $n = 1$. Then,

$$\sum_{k=0}^{1} x^k = \frac{1 - x^{0+1}}{1 - x} + \frac{1 - x^{1+1}}{1 - x} = \frac{1 + x^2}{1 - x}.$$

$$= \frac{(1 + x)(1 - x)}{(1 - x)} = (1 + x) = \frac{1 - x^{n+1}}{1 - x}, n = 1$$

Step 2: For $n + 1$ show that the geometric sum is $\frac{1-x^{n+2}}{1-x}$.

$$\sum_{k=0}^{n+1} x^k = \sum_{k=0}^{n} x^k + x^{k+1} = \frac{1 - x^{n+1}}{1 - x} + x^{n+1}$$

$$= \frac{1 - x^{n+1} + (1 - x)(x^{n+1})}{1 - x} = \frac{1 - x^{n+1} + x^{n+1} - x(x^{n+1})}{1 - x}$$

$$= \frac{1 - x^{n+1} + x^{n+1} - x^{n+2}}{1 - x} = \frac{1 - x^{n+2}}{1 - x}$$

(2) Prove the following statement.

If A and B are sets, then $A \cap (B - A) = \emptyset$.

$$(B - A) \iff B \setminus A \iff B \cap A^c$$
$$A \cap (B - A) = B \cap A^c \iff A \cap B \cap A^c \iff \emptyset$$

(3) Prove the following statement.

Suppose that $n \in \mathbb{Z}$. If n^2 is odd, then n is odd.

Suppose by contradiction that n^2 is odd but that n is even.

By definition, an even number is $2m$ where $m \in \mathbb{Z}$. Let $b = 2m$ for some $m \in \mathbb{Z}$. Then, $b^2 = (2m)^2 = 4m^2$ So, b^2 is even. But we assumed it was odd. Then, through contradiction, it is clear that if n^2 is odd, then n is odd.

(4) Prove the following statment.

$$\Big(\bigcup_{i\in I} A_i\Big)^c = \bigcap_{i=1}(A_i)^c.$$

$$\Big(\bigcup_{i\in I} A_i\Big)^c = (A_1 \cup A_2 \cup A_3 \cup ... \cup A_I)^c$$

$$= A_1^c \cap A_2^c \cap A_3^c \cap ... \cap A_I^c = \bigcap_{i\in I}(A_i)^c$$