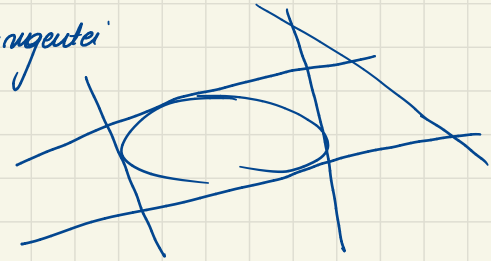
Modele parțial 2

Model 1

1. $\mathcal{E}: \frac{x^2}{30} + \frac{y^2}{24} = 1$

$tg: y = k \cdot x \pm \sqrt{a^2k^2 + b^2}$, k = panta tangentei



$a^2 = 30$, $b^2 = 24$

$d: x + 3y + 3 = 0 \Rightarrow k_d = -\frac{1}{3}$

$tg(\widehat{tg, d}) = \left|\frac{k - k_d}{1 + k \cdot k_d}\right| \Leftrightarrow 1 = \frac{|k + \frac{1}{3}|}{|1 - \frac{k}{3}|} \Leftrightarrow k + \frac{1}{3} = \pm\left(1 - \frac{k}{3}\right)$

$\underline{I}$ $k + \frac{1}{3} = 1 - \frac{k}{3} \Big/ \cdot 3 \Leftrightarrow 3k + 1 = 3 - k \Leftrightarrow 4k = 2 \Rightarrow \underline{k = \frac{1}{2}}$

$\underline{II}$ $k + \frac{1}{3} = -1 + \frac{k}{3} \Big/ \cdot 3 \Leftrightarrow 3k + 1 = -3 + k \Leftrightarrow 2k = -4 \Leftrightarrow \underline{k = -2}$

$k = \frac{1}{2} \Rightarrow tg: y = \frac{1}{2}x \pm \sqrt{30 \cdot \frac{1}{4} + 24} \Leftrightarrow tg: y = \frac{1}{2}x \pm \sqrt{\frac{15}{2} + 24} \Leftrightarrow tg: y = \frac{1}{2}x \pm \sqrt{\frac{63}{2}}$

$k = -2 \Rightarrow tg: y = -2x \pm \sqrt{30 \cdot 4 + 24} \Leftrightarrow tg: y = -2x \pm 12$

2. Stabiliți ecuația hiperbolei care are ca asimptote dreptele $\pm y + \sqrt{3}x = 0$ și este tangentă dreptei $2x - y - 3 = 0$.

Ecuația hiperbolei: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Asimptotele hiperbolei: $y = \pm\frac{b}{a}x$

$y = \pm\sqrt{3}x \Rightarrow \frac{b}{a} = \sqrt{3} \Rightarrow b = a\sqrt{3} \Rightarrow b^2 = 3a^2$

$tg: 2x - y - 3 = 0 \iff y = 2x - 3$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \iff b^2x^2 - a^2y^2 = a^2b^2 \iff$$

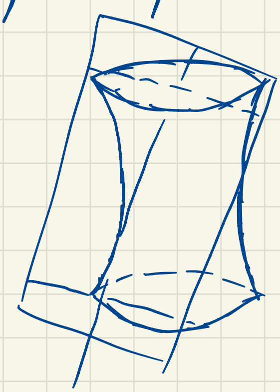
$$\iff b^2x^2 - a^2y^2 - a^2b^2 = 0$$
$$\iff b^2x^2 - a^2(2x-3)^2 - a^2b^2 = 0$$
$$\iff b^2x^2 - a^2(4x^2 - 12x + 9) - a^2b^2 = 0$$
$$\iff b^2x^2 - 4a^2x^2 + 12a^2x - 9a^2 - a^2b^2 = 0$$
$$\iff (b^2 - 4a^2)x^2 + 12a^2x - (9a^2 + a^2b^2) = 0$$

$$\left.\begin{array}{l}\Delta = (12a^2)^2 + 4(b^2-4a^2)(9a^2+a^2b^2) \\ \text{cond de compatibilitate: } \Delta = 0\end{array}\right\} \Rightarrow$$

$$\Rightarrow 144a^4 + (4b^2 - 16a^2)(9a^2 + a^2b^2) = 0$$
$$\iff 144a^4 + (12a^2 - 16a^2)(-9a^2 + 3a^4) = 0$$
$$\iff 144a^4 - 4a^2(-9a^2 + 3a^4) = 0$$
$$\iff 144a^4 + 36a^4 - 12a^6 = 0$$
$$\iff 144a^4 - 12a^6 = 0 \iff 12a^4(15 - a^2) = 0$$
$$\iff a^2 = 0 \text{ sau } a^2 = 15$$
$$\Rightarrow b^2 = 0 \text{ sau } b^2 = 45$$

$$\Rightarrow \mathcal{H}: \frac{x^2}{15} - \frac{y^2}{45} = 1$$

3. Stabiliți ec. planului care intersectează hiperboloidul $x^2 + 4y^2 - 9z^2 = 36$ după 2 drepte care se intersectează în $M(6, -3, 2)$



$$x^2 + 4y^2 - 9z^2 = 36$$
$$\iff x^2 - 9z^2 = 36 - 4y^2$$
$$\iff (x - 3z)(x + 3z) = (6 - 2y)(6 + 2y)$$

$$I \begin{cases} (x - 3z)\lambda = (6 - 2y)\mu \\ (x + 3z)\mu = (6 + 2y)\lambda \end{cases}$$

$M \in \overline{\underline{I}} \Rightarrow \begin{cases} (6-3\cdot 2)\lambda = (6-2\cdot 2)\mu \\ (6+3\cdot 2)\mu = (6+2(-3))\cdot\lambda \end{cases} \rightarrow \mu = 0$ Alegem $\lambda = 1$ $\rightarrow$

$$\rightarrow \Delta_1 : \begin{cases} x - 3z = 0 \\ 2y + 6 = 0 \end{cases}$$

$$\overline{\underline{II}} \begin{cases} (x-3z)\alpha = (6+2y)\beta \\ (x+3z)\beta = (6-2y)\alpha \end{cases} \xrightarrow{M \in \overline{\underline{II}}} \begin{cases} (6-3\cdot 2)\alpha = (6+2\cdot(-3))\beta \\ (6+3\cdot 2)\beta = (6-2\cdot(-3))\alpha \end{cases}$$

$$\rightarrow \alpha = \beta = 1$$

$$\rightarrow \Delta_2 : \begin{cases} x - 2y - 3z - 6 = 0 \\ x + 2y + 3z - 6 = 0 \end{cases}$$

$$\Delta_1 : \begin{cases} x - 3z = 0 & \rightarrow \vec{u_{11}}(1,0,-3) \\ 2y + 6 = 0 & \rightarrow \vec{u_{12}}(0,2,6) \end{cases} \rightarrow \vec{d_1} = \vec{u_{11}} \times \vec{u_{12}} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & -3 \\ 0 & 2 & 0 \end{vmatrix} =$$

$$= 6\vec{i} - 0\vec{j} + 2\vec{k} \rightarrow \vec{d_1}(3,0,1)$$

$$\Delta_2 : \begin{cases} x - 2y - 3z - 6 = 0 \Rightarrow \vec{u_{21}}(1,-2,-3) \\ x + 2y + 3z - 6 = 0 \Rightarrow \vec{u_{22}}(1,2,3) \end{cases} \rightarrow \vec{d_2} = \vec{u_{21}} \times \vec{u_{22}} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -2 & -3 \\ 1 & 2 & 3 \end{vmatrix}$$

$$= 0\cdot\vec{i} - 6\vec{j} + 4\vec{k} \Rightarrow \vec{d_2}(0,-3,2)$$

$$\begin{matrix} \vec{d_1}(3,0,1) \\ \vec{d_2}(0,-3,2) \end{matrix} \rightarrow \pi : \begin{vmatrix} x-6 & y+3 & z-2 \\ 3 & 0 & 1 \\ 0 & 3 & -2 \end{vmatrix} = 0 \Leftrightarrow \pi : x - 2y - 3z - 6 = 0$$

4. Se dă paraboloidul $x^2 - y^2 = 2z$ și planul $\pi : x + y + z - 1 = 0$. Det. ec. unui plan paralel cu π care taie paraboloidul după două drepte. Det. ec. acestor drepte și unghiul dintre ele.

R:

$$(x-y)(x+y) = 2z$$

$$\mathrm{I}\begin{cases}(x-y)\alpha = 2\beta z\\(x+y)\beta = \alpha\end{cases} \qquad \mathrm{II}\begin{cases}(x+y)\lambda = 2\mu z\\(x-y)\mu = \lambda\end{cases}$$

$$\mathrm{I}\begin{cases}\alpha x-\alpha y-2\beta z=0 & \to \vec{n_{11}}(\alpha,-\alpha,-2\beta) \to\\ \beta x+\beta y-\alpha=0 & \to \vec{n_{12}}(\beta,\beta,0)\end{cases}$$

$$\to \begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ \alpha & -\alpha & -2\beta\\ \beta & \beta & 0\end{vmatrix} = (2\beta^2, -2\beta^2, 2\alpha\beta) \;/ : \beta$$

$$\Leftrightarrow \vec{d_1}(2\beta, -2\beta, 2\alpha)$$

Fie $\vec{n}(1,1,1)$ vector normal la π

$$\to \vec{n} \perp \vec{d_1} \Rightarrow n \cdot d_1 = 0 \Leftrightarrow 2\beta - 2\beta + 2\alpha = 0 \to \alpha = 0 \Rightarrow$$

Alegem $\beta = 1$

$$\to \vec{d_1}(2,-2,0) \Rightarrow \vec{d_1}(1,-1,0)$$

$$\mathrm{II}\begin{cases}\lambda x+\lambda y-2\mu z=0 & \to \vec{n_{21}}(\lambda,\lambda,-2\mu)\\ \mu x-\mu y-\lambda=0 & \to \vec{n_{22}}(\mu,-\mu,0)\end{cases}$$

$$\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ \lambda & \lambda & -2\mu\\ \mu & -\mu & 0\end{vmatrix} = (-2\mu^2, -2\mu^2, -2\lambda\mu) \;/ : \mu$$

$$\vec{d_2}(-2\mu, -2\mu, -2\lambda)$$

$$\to -2\mu - 2\mu - 2\lambda = 0 \Leftrightarrow -4\mu = 2\lambda \Rightarrow \lambda = -2\mu$$

Alegem $\lambda = \frac{1}{2} \Rightarrow \mu = -1$

$$\to \vec{d_2}(2,2,-1)$$

$\vec{d_1}\ (1,-1,0)$

$\vec{d_2}\ (2,2,-1)$

$\Delta_1: \begin{cases} -2z = 0 \\ x+y = 0 \end{cases}$ $\Delta_2: \begin{cases} \frac{1}{2}x + \frac{1}{2}y + 2z = 0 \\ -x+y-\frac{1}{2} = 0 \end{cases}$

$z=0$ $-x+y-\frac{1}{2}=0 \Leftrightarrow -x+y=\frac{1}{2}$

$x+y=0 \quad (+)$

$2y = \frac{1}{2} \Rightarrow y=\frac{1}{4} \Rightarrow x=-\frac{1}{4}$

$V: \frac{1}{2}\cdot\left(-\frac{1}{4}\right)+\frac{1}{2}\cdot\frac{1}{4}+2\cdot 0 = 0 \ \checkmark$

$\Rightarrow M\left(-\frac{1}{4}, \frac{1}{4}, 0\right) = \Delta_1 \cap \Delta_2 \in$ planului

$\Rightarrow$ plan: $\begin{vmatrix} x+\frac{1}{4} & y-\frac{1}{4} & z \\ 1 & -1 & 0 \\ 2 & 2 & -1 \end{vmatrix} = 0 \Leftrightarrow x+y+8=0$

$\cos\theta = \frac{\vec{d_1}\cdot\vec{d_2}}{\|\vec{d_1}\|\cdot\|\vec{d_2}\|} = 0 \Rightarrow \theta = \frac{\pi}{2}$

5. Scrieți ecuația conului de rotație în jurul axei $\Delta: \begin{cases} x = 2+pz-p^2 \\ y = 1 \end{cases}$, știind că acest con are ca generatoare dreapta $d: \begin{cases} y=1 \\ z=p \end{cases}$. Determinați p a.î. conul să treacă prin origine.

R: Se intersectează Δ și d?

$\begin{cases} y=1 \\ x=2+pz-p^2 \\ z=p \end{cases} \Rightarrow x=2 \Rightarrow$ se intersectează

$\Delta: \begin{cases} x = 2+pz-p^2 \\ y=1 \end{cases} \rightarrow \vec{d}: \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & -p \\ 0 & 1 & 0 \end{vmatrix} = p\vec{i}+0\vec{j}+\vec{k}$

$\Rightarrow \vec{d}(p,0,1)$

$\Rightarrow \Delta: \frac{x-2}{p} = \frac{y-1}{0} = \frac{z-p}{1}$

$$\begin{cases} px+z=\mu \\ (x-2)^2+(y-1)^2+(z-p)^2=\lambda^2 \\ y=1 \\ z=p \end{cases}$$

$$\begin{cases} p\lambda+z=\mu \\ z=p \end{cases} \Rightarrow p\cdot x+p=\mu \Rightarrow x=\frac{\mu-p}{p} \Rightarrow \left(\frac{\mu-p}{p}\right)^2=\lambda^2 \Leftrightarrow$$

$$\Leftrightarrow (\mu-p)^2=p^2\lambda^2 \Leftrightarrow (px+z-p)^2=p^2((x-2)^2+(y-1)^2+(z-p)^2)$$

$O \in$ conului $\Rightarrow p^2=p^2(4+1+p^2) \Leftrightarrow p^2=p^2(p^2+5) \Leftrightarrow$

$\Leftrightarrow p^2(p^2+5)-p^2=0 \Leftrightarrow p^2(p^2+4)=0 \Leftrightarrow p^2=0 \Rightarrow p=0$

sau $p^2=-4$ nu convine

$\Rightarrow \underline{p=0}$

6. $ABC\Delta$: $A(0,0)$, $B(2,0)$, $C(2,2)$, $\Delta(0,2)$

$A'B'C'\Delta'$: $A'\left(3-\frac{\sqrt{2}}{2}, -1-\frac{3\sqrt{2}}{2}\right)$

$B'\left(3+\frac{3\sqrt{2}}{2}, -1+\frac{\sqrt{2}}{2}\right)$

$C'\left(3+\frac{\sqrt{2}}{2}, -1+\frac{3\sqrt{2}}{2}\right)$

$\Delta'\left(3-\frac{3\sqrt{2}}{2}, -1-\frac{\sqrt{2}}{2}\right)$

Dem că $A'B'C'\Delta'$ este un dreptunghi și det. o nec. de transformări ce transformă pătratul în dreptunghi.

$\left.\begin{array}{l} A'B'(2\sqrt{2}, 2\sqrt{2}) \\ \Delta'C'(2\sqrt{2}, 2\sqrt{2}) \end{array}\right\} \Rightarrow A'B'C'\Delta'$ paralelogram $\Big\}$ $\Rightarrow A'B'C'\Delta'$ dreptunghi

$B'C'(-\sqrt{2}, \sqrt{2}) \Rightarrow B'C' \perp \Delta'C'$

N-am de gând să pricep mai departe

Model 2

1. Det. ec. suprafeţei cilindrice a cărei curbă directoare este:

$$(c): \begin{cases} x^3 + y^3 - 3xy = 0 \\ z = 0. \end{cases}$$

iar generatoarele au vectorul director $v(2,-2,1)$.

R: $\Delta: \frac{x}{2} = \frac{y}{-2} = \frac{z}{1}$

$$\Rightarrow \begin{cases} 2x + 2y = 0 \\ y + 2z = 0 \end{cases} \Leftrightarrow \Delta: \begin{cases} x + y = 0 \\ y + 2z = 0 \end{cases}$$

$$\begin{cases} x + y = \lambda \\ y + 2z = \mu \\ x^3 + y^3 - 3xy = 0 \\ z = 0 \end{cases}$$

$$\left.\begin{array}{l} x + y = \lambda \\ \left.\begin{array}{l} y + 2z = \mu \\ z = 0 \end{array}\right\} \Rightarrow y = \mu \end{array}\right\} \Rightarrow x = \lambda - \mu$$

$$(\lambda - \mu)^3 + \mu^3 - 3(\lambda - \mu)\cdot\mu = 0$$

$$(x - 2z)^3 + (y + 2z)^3 - 3(x - 2z)(y + 2z) = 0$$

2. $x^2 + y^2 - z^2 = 1 \Leftrightarrow x^2 - z^2 = 1 - y^2 \Leftrightarrow (x-z)(x+z) = (1-y)(1+y)$

$$\text{I} \begin{cases} (x-z)\lambda = (1-y)\mu \\ (x+z)\mu = (1+y)\lambda \end{cases} \qquad \text{II} \begin{cases} (x-z)\alpha = (1+y)\beta \\ (x+z)\beta = (1-y)\alpha \end{cases}$$

$M(1,1,1)$

$$\text{I}\begin{cases} 0\cdot\lambda = 0\cdot\mu \\ 2\mu = 2\lambda \end{cases} \Rightarrow \lambda=\mu$$

Alegem $\lambda=\mu=1 \Rightarrow \Delta_1\begin{cases} x+y-z-1=0 \\ x-y+z-1=0 \end{cases}$

$$\text{II}\begin{cases} 0\cdot\alpha = 2\beta \\ 2\cdot\beta = 0\cdot\alpha \end{cases} \Rightarrow \beta=0, \text{ Alegem } \alpha=1 \Rightarrow \Delta_2\begin{cases} x-z=0 \\ y-1=0 \end{cases}$$

3. $\Delta ABC \quad A(1,1),\ B(4,1),\ C(2,3)$

$$\text{Trans}(\vec{v}) = \begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

$\Delta: x+2y-3=0$

$Q(3,0)\in\Delta$

$\vec{d}(-2,1) \Rightarrow \vec{w} = \dfrac{\vec{d}}{\|\vec{d}\|} = \dfrac{\vec{d}}{\sqrt5} = \left(-\dfrac{2\sqrt5}{5}, -\dfrac{\sqrt5}{5}\right) \Rightarrow \vec{w}^{\perp} = \left(\dfrac{\sqrt5}{5}, -\dfrac{2\sqrt5}{5}\right)$

$$\vec{w}^{\perp}\otimes\vec{w}^{\perp} = \begin{pmatrix} \frac{\sqrt5}{5} \\ -\frac{2\sqrt5}{5} \end{pmatrix}\cdot\begin{pmatrix} \frac{\sqrt5}{5} & -\frac{2\sqrt5}{5} \end{pmatrix} = \begin{pmatrix} \frac{1}{5} & -\frac{2}{5} \\ -\frac{2}{5} & \frac{4}{5} \end{pmatrix} = \frac{1}{5}\begin{pmatrix} 1 & -2 \\ -2 & 4 \end{pmatrix}$$

$$I_2 - 2(\vec{w}^{\perp}\otimes\vec{w}^{\perp}) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \frac{2}{5}\begin{pmatrix} 1 & -2 \\ -2 & 4 \end{pmatrix} = \begin{pmatrix} \frac{3}{5} & \frac{4}{5} \\ \frac{4}{5} & \frac{-3}{5} \end{pmatrix}$$

$$2(\vec{w}^{\perp}\otimes\vec{w}^{\perp})\cdot Q = \frac{2}{5}\begin{pmatrix} 1 & -2 \\ -2 & 4 \end{pmatrix}\cdot\begin{pmatrix} 3 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{6}{5} \\ -6 \end{pmatrix}$$

$$\Rightarrow \text{Mirror}(Q,w) = \begin{pmatrix} \frac{3}{5} & \frac{4}{5} & \frac{-6}{5} \\ \frac{4}{5} & -\frac{3}{5} & -6 \\ 0 & 0 & 1 \end{pmatrix}$$

$$M_t = \text{Mirror} \cdot \text{Trans} = \frac{1}{5}\begin{pmatrix} 3 & 4 & 6 \\ 4 & -3 & -30 \\ 0 & 0 & 5 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} =$$

$$= \frac{1}{5}\begin{pmatrix} 3 & 4 & -6-4+6 \\ 4 & -3 & -8+3-30 \\ 0 & 0 & 5 \end{pmatrix} = \frac{1}{5}\begin{pmatrix} 3 & 4 & -4 \\ 4 & -3 & -35 \\ 0 & 0 & 5 \end{pmatrix}$$

$$(A' \; B' \; C') = M_t \cdot (A \; B \; C) = \frac{1}{5}\begin{pmatrix} 3 & 4 & -4 \\ 4 & -3 & -35 \\ 0 & 0 & 5 \end{pmatrix} \cdot \begin{pmatrix} 1 & 4 & 2 \\ 1 & 1 & 3 \\ 1 & 1 & 1 \end{pmatrix} =$$

$$= \frac{1}{5}\begin{pmatrix} 3 & 12 & 14 \\ -34 & -22 & -36 \\ 5 & 5 & 5 \end{pmatrix} \Rightarrow \begin{array}{l} A'\left(\frac{3}{5}, \frac{-34}{5}\right) \\ B'\left(\frac{12}{5}, -\frac{22}{5}\right) \\ C'\left(\frac{14}{5}, -\frac{36}{5}\right) \end{array}$$

4. $\mathcal{H}: 7x^2 - 2y^2 = 14 \; | :14 \Leftrightarrow \mathcal{H}: \frac{x^2}{2} - \frac{y^2}{7} = 1$

$tg: y = kx \pm \sqrt{a^2k^2 - b^2}$

$$\left.\begin{array}{l} \left.\begin{array}{l} tg \perp d \Rightarrow k_{tg} \cdot k_d = -1 \\ \qquad\qquad k_d = -\frac{1}{2} \end{array}\right\} \Rightarrow k_{tg} = 2 \end{array}\right\} \Rightarrow tg: y = 2x \pm \sqrt{2 \cdot 4 - 7}$$

$\Rightarrow tg: y = 2x - 1$

$\Leftrightarrow tg: 2x - y - 1 = 0$

$\Rightarrow$ a), c)

5. $\mathcal{E}: \frac{x^2}{9} + \frac{y^2}{4} + \frac{z^2}{8} = 1$

$$\left.\begin{array}{l} \pi: x\frac{x_0}{9} + y\frac{y_0}{4} + z\frac{z_0}{8} = 1 \Rightarrow \vec{n}\left(\frac{x_0}{9}, \frac{y_0}{4}, \frac{z_0}{8}\right) \\ \pi': 3x - 2y + 5z = 0 \Rightarrow \vec{n}'(3, -2, 5) \end{array}\right\} \Rightarrow$$

$$\Rightarrow \frac{x_0}{27} = \frac{y_0}{-8} = \frac{z_0}{40} \Leftrightarrow \begin{cases} -8x_0 = 27y_0 \\ 40y_0 = -8z_0 \end{cases}$$

$$\begin{cases} -8x_0 = 27y_0 \Rightarrow x_0 = \dfrac{27y_0}{-8} \\ 40y_0 = -8z_0 \rightarrow z_0 = -5y_0 \\ \dfrac{x_0^2}{9} + \dfrac{y_0^2}{4} + \dfrac{z_0^2}{8} = 1 \;/\cdot 72 \end{cases} \qquad \begin{cases} -8x = 27y \\ 40y = -8z \\ \dfrac{x^2}{9} + \dfrac{y^2}{4} + \dfrac{z^2}{8} = 1 \end{cases}$$

$$\Leftrightarrow 8x_0^2 + 18y_0^2 + 9z_0^2 = 72$$

$$\Leftrightarrow \cancel{8} \cdot \frac{27^2}{64_{\cancel{8}}} \cdot y_0^2 + 18y_0^2 + 9\cdot 25 y_0^2 = 72 \;/\cdot 8$$

$$\Leftrightarrow (27^2 + 8\cdot 18 + 8\cdot 9\cdot 25) y_0^2 = 72\cdot 8$$

$$\Rightarrow y_0 = \frac{72\cdot 8}{27^2 + 8\cdot 18 + 8\cdot 9\cdot 25} = \frac{64}{27\cdot 3 + 8\cdot 2 + 8\cdot 25} = \frac{64}{27\cdot 11} \rightarrow$$

$$\rightarrow y_0 = \frac{8}{3\sqrt{33}} = \pm \frac{8\sqrt{33}}{99} \longrightarrow x_0 = \pm \frac{27}{-\cancel{8}} \cdot \frac{\cancel{8}\sqrt{33}}{99} = \pm - \frac{27\sqrt{33}}{99} = \mp \frac{3\sqrt{33}}{11}$$

$$z_0 = \mp \frac{40\sqrt{33}}{99}$$

$$\Rightarrow \pi: -\frac{\cancel{3}\sqrt{33}}{11} \cdot \frac{x}{\cancel{9}3} + \frac{\overset{2}{\cancel{8}}\sqrt{33}}{99} \cdot \frac{y}{\cancel{4}} - \frac{\overset{5}{\cancel{40}}\sqrt{33}}{99} \cdot \frac{z}{\cancel{8}} = 1 \;/\cdot 99$$

$$\Leftrightarrow -3\sqrt{33}\,x + 2\sqrt{33}\,y - 5\sqrt{33}\,z = 99 \;/:\sqrt{33}$$

$$\Leftrightarrow -3x + 2y - 5z = \frac{99}{\sqrt{33}} = \frac{\overset{3}{\cancel{99}}\sqrt{33}}{\cancel{33}}$$

$$\Leftrightarrow -3x + 2y - 5z - 3\sqrt{33} = 0 \;/\cdot(-1)$$

$$\Leftrightarrow 3x - 2y + 5z + 3\sqrt{33} = 0$$

nou $\pi: \dfrac{3\sqrt{33}}{11} \cdot \dfrac{x}{9} - \dfrac{8\sqrt{33}}{99} \cdot \dfrac{y}{4} + \dfrac{40\sqrt{33}}{99} \cdot \dfrac{z}{8} = 1 \;/\cdot 99$

$\ldots \quad 3x - 2y + 5z - 3\sqrt{33} = 0 \quad \Rightarrow$ a), c)

6. $\dfrac{x^2}{2} - \dfrac{y^2}{2} - z^2 = 1$ hiperboloid cu două pânze

$\pi: y + z = 0$

$$\begin{cases} \frac{x^2}{2} - \frac{y^2}{2} - z^2 = 1 \\ y + 2 = 0 \to y = -2 \end{cases}$$

$$\Rightarrow \frac{x^2}{2} - \frac{4}{2} - z^2 = 1 \iff$$

$$\iff \frac{x^2}{2} - z^2 - 2 = 1$$

$$\iff \frac{x^2}{2} - z^2 = 3 \,/ :3$$

$$\iff \frac{x^2}{6} - \frac{z^2}{3} = 1 \to \text{hiperbolă}$$

$$\Rightarrow b)$$

7. Mirror $(0, i)$

$$\vec{i} \otimes \vec{i} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$I_3 - 2(\vec{u} \otimes \vec{u}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - 2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\Rightarrow \text{Mirror}(0, i) = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\text{Mirror}(0, j) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\text{Mirror}(0, k) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$\Rightarrow$ c)

+ de verificat celelalte

Model 3

1. $$\mathrm{Scale}(Q, 2) = \begin{pmatrix} 2 & 0 & 0 & -2x \\ 0 & 2 & 0 & -2y \\ 0 & 0 & 2 & -2z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

2. $$x\frac{x_0}{a^2} + y\frac{y_0}{b^2} + z\frac{z_0}{c^2} - 1 = 0$$

3. $\Delta: \frac{x}{2} = \frac{y}{3} = \frac{z}{1}$

$P_h: \frac{x^2}{4} - \frac{y^2}{9} = 2z$

$\Delta: \frac{x}{2} = \frac{y}{3} = \frac{z}{1} \rightarrow \begin{cases} x = 2t \\ y = 3t \\ z = t \end{cases}$

$\frac{4t^2}{4} - \frac{9t^2}{4} = 2t \Rightarrow t^2 - t^2 = 2t \Rightarrow \underline{t=0} \rightarrow x = y = z = 0$

$\Rightarrow$ a)

4. Triunghi congruent cu cel dat

5. Asimptotele hiperbolei: $\frac{x}{2} = \pm\frac{y}{3}$

$A_1: \frac{x}{2} = \frac{y}{3} \,/\cdot 6 \Leftrightarrow 3x = 2y \Leftrightarrow 3x - 2y = 0$

$A_2: \frac{x}{2} = -\frac{y}{3} \,/\, 6 \Leftrightarrow 3x = -2y \Leftrightarrow 3x + 2y = 0$

$\Delta: 2x+3y-7=0$

$$\begin{cases} 3x-2y=0 \\ 3x+2y=0 \\ 2x+3y-7=0 \end{cases}$$

$$\left.\begin{aligned} 3x-2y&=0 \\ 3x+2y&=0 \end{aligned}\right\} \overset{+}{\Rightarrow} 6x=0 \Leftrightarrow x=0 \Rightarrow y=0$$

$\rightarrow A_1 \cap A_2 = M(0,0)$

$3x-2y=0 \rightarrow y=\frac{3x}{2}$

$2x+3y-7=0$

$\Leftrightarrow 2x+\frac{9x}{2}-7=0 \,/\cdot 2 \Leftrightarrow 4x+9x-14=0 \Leftrightarrow 13x=14 \Leftrightarrow x=\frac{14}{13} \Rightarrow y=\frac{3\cdot\frac{14}{13}}{2}=$

$=\frac{42}{26}=\frac{21}{13}$

$3x+2y=0 \Rightarrow y=-\frac{3x}{2}$

$2x+3y-7=0 \Rightarrow 2x-\frac{9x}{2}-7=0 \,/\cdot 2 \Leftrightarrow 4x-9x-14=0 \Leftrightarrow -5x=14 \Rightarrow x=-\frac{14}{5}$

$\Rightarrow y=-\frac{3\cdot\frac{-14}{5}}{2}=\frac{42}{10}=\frac{21}{5}$

$$A_\Delta=\frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ \frac{14}{13} & \frac{21}{13} & 1 \\ -\frac{14}{5} & \frac{21}{5} & 1 \end{vmatrix} = \frac{1}{2}\left(\frac{14}{13}\cdot\frac{21}{5}+\frac{14}{5}\cdot\frac{21}{13}\right)=\frac{14\cdot 21}{13\cdot 5}=\frac{294}{65} \Rightarrow c)$$

$$\begin{array}{r} 14\cdot \\ 21 \\ \hline 14 \\ 28 \\ \hline 294 \end{array} \qquad \begin{array}{r} 13\cdot \\ 5 \\ \hline 65 \end{array}$$

6. $P: y^2=4x \Rightarrow p$

7. $\Delta: \frac{x}{1}=\frac{y}{1}=\frac{z}{1}$ dreaptă cu vector director $u(1,1,1)$

$$\begin{cases} x=y \\ y=z \end{cases} \Leftrightarrow \begin{cases} x-y=0 \\ y-z=0 \end{cases}$$

$$\begin{cases} x-y=\lambda \\ y-z=\mu \\ y^2+2x^2=4 \\ z=2 \end{cases}$$

$$z=2 \Rightarrow y=\mu+2 \Rightarrow x=\lambda+\mu+2$$

$$\Rightarrow (\mu+2)^2+2(\lambda+\mu+2)^2=4$$

$$\Leftrightarrow (y-z+2)^2+2(x-y+y-z+2)^2=4$$

$$\Leftrightarrow (y-z+2)^2+2(x-z+2)^2=4$$

$$\Leftrightarrow y^2+z^2+4-2yz-4z+4y+2(x^2+z^2+4-2xz-4z+4x)=4$$

$$\Leftrightarrow 2x^2+y^2+3z^2-4xz-2yz+8x+4y-12z+12=4$$

$$\Leftrightarrow 2x^2+y^2+3z^2-4xz-2yz+8\lambda+4y-12z+8=0$$

8. $AB\Gamma\Delta$: $A(0,0)$, $B(2,0)$, $\Gamma(2,2)$, $\Delta(0,2)$

$$Trans(w)=\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$Rot(1,1,30^\circ)=\begin{pmatrix} \frac{\sqrt3}{2} & -\frac12 & 1-\frac{\sqrt3}{2}+\frac12 \\ \frac12 & \frac{\sqrt3}{2} & -\frac12+1-\frac{\sqrt3}{2} \\ 0 & 0 & 1 \end{pmatrix}=\begin{pmatrix} \frac{\sqrt3}{2} & -\frac12 & \frac{3-\sqrt3}{2} \\ \frac12 & \frac{\sqrt3}{2} & \frac{1-\sqrt3}{2} \\ 0 & 0 & 1 \end{pmatrix}$$

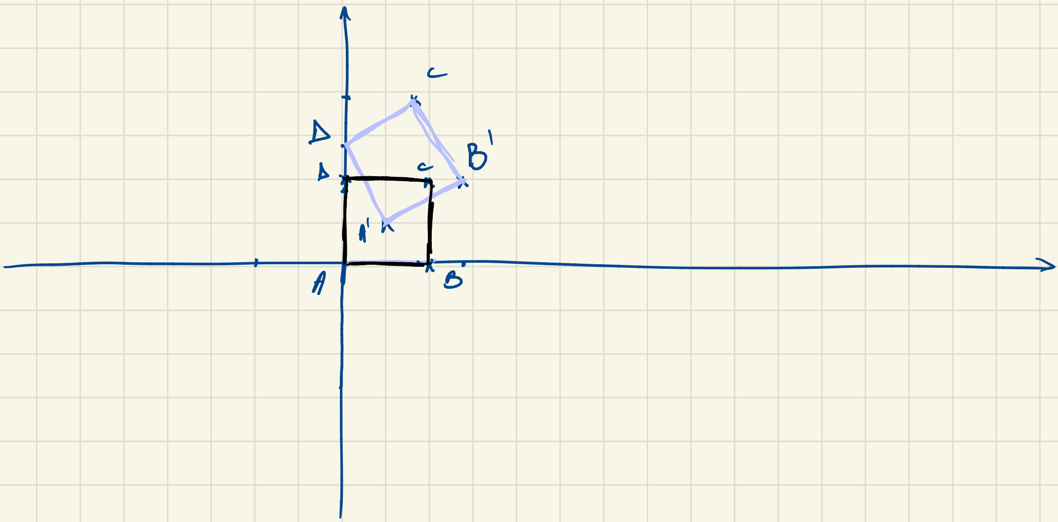
$$M_t=Rot\cdot Trans=\begin{pmatrix} \frac{\sqrt3}{2} & -\frac12 & \frac{3-\sqrt3}{2} \\ \frac12 & \frac{\sqrt3}{2} & \frac{1-\sqrt3}{2} \\ 0 & 0 & 1 \end{pmatrix}\cdot\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}=$$

$$=\begin{pmatrix} \frac{\sqrt3}{2} & -\frac12 & \frac{\sqrt3}{2}-\frac12+\frac{3-\sqrt3}{2} \\ \frac12 & \frac{\sqrt3}{2} & \frac12+\frac{\sqrt3}{2}+\frac{1-\sqrt3}{2} \\ 0 & 0 & 1 \end{pmatrix}=\begin{pmatrix} \frac{\sqrt3}{2} & -\frac12 & \frac22 \\ \frac12 & \frac{\sqrt3}{2} & \frac22 \\ 0 & 0 & 1 \end{pmatrix}=$$

$$= \begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & 1 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$(A' \ B' \ C' \ \Delta') = M_4 (A \ B \ C \ \Delta) = \begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & 1 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 1 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 & 2 & 2 & 0 \\ 0 & 0 & 2 & 2 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & \sqrt{3}+1 & \sqrt{3}-1+1 & -1+1 \\ 1 & 1+1 & 1+\sqrt{3}+1 & \sqrt{3}+1 \\ 1 & 1 & 1 & 1 \end{pmatrix} \Rightarrow \begin{matrix} A(1, 1) \\ B(\sqrt{3}+1, 2) \\ C(\sqrt{3}, \sqrt{3}+2) \\ \Delta(0, \sqrt{3}+1) \end{matrix} \quad \checkmark$$



Model 4

1. $E: 4x^2 + 5y^2 = 120 \ \Big| :120 \Rightarrow E = \frac{x^2}{30} + \frac{y^2}{24} = 1$

$tg: y = kx \pm \sqrt{a^2k^2 + b^2}$ $\qquad d: 4x - 2y + 15 = 0$

$tg \parallel d \Rightarrow k = k_d$
$k_d = -\frac{4}{-2} = 2$ $\Big\} \Rightarrow k = 2 \rightarrow tg: y = 2x \pm \sqrt{30 \cdot 4 + 24} \Leftrightarrow$

$\Leftrightarrow tg: y = 2x \pm \sqrt{144}$

$\Leftrightarrow tg: y = 2x \pm 12$

$tg_1: 2x - y + 12 = 0$
$tg_2: 2x - y - 12 = 0$ $\rightarrow$ distanța $= \frac{|12 - (-12)|}{\sqrt{2^2 + (-1)^2}} = \frac{24}{\sqrt{5}} = \frac{24\sqrt{5}}{5}$

2. $2x^2 - y^2 = 36z \,/ :18 \Leftrightarrow \frac{x^2}{9} - \frac{y^2}{18} = 2z \rightarrow$ Paraboloid hiperbolic

$\Leftrightarrow \left(\frac{x}{3} - \frac{y}{3\sqrt{2}}\right)\left(\frac{x}{3} + \frac{y}{3\sqrt{2}}\right) = 2z \cdot 1$

$$\Rightarrow \text{I} \begin{cases} \left(\frac{x}{3} - \frac{y}{3\sqrt{2}}\right)\alpha = 2\beta z \\ \left(\frac{x}{3} + \frac{y}{3\sqrt{2}}\right)\beta = \alpha \end{cases} \qquad \text{II} \begin{cases} \left(\frac{x}{3} + \frac{y}{3\sqrt{2}}\right)\lambda = 2\mu z \\ \left(\frac{x}{3} - \frac{y}{3\sqrt{2}}\right)\mu = \lambda \end{cases}$$

$$M \in \text{I} \Rightarrow \begin{cases} \left(\frac{36}{3} - \frac{36}{3\sqrt{2}}\right)\alpha = 2 \cdot 36\beta \\ \left(\frac{36}{3} + \frac{36}{3\sqrt{2}}\right)\beta = \alpha \end{cases} \Leftrightarrow \begin{cases} \frac{36\sqrt{2} - 36}{3\sqrt{2}} \cdot \alpha = 2 \cdot 36\beta \\ \frac{36\sqrt{2} + 36}{3\sqrt{2}}\beta = \alpha \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} \frac{12\sqrt{2} - 12}{\sqrt{2}}\alpha = 2 \cdot 36\beta \\ \frac{12\sqrt{2} + 12}{\sqrt{2}}\beta = \alpha \end{cases}$$

Alegem $\beta = \sqrt{2} \Rightarrow \alpha = 12\sqrt{2} + 12 \Rightarrow \Delta_1: \begin{cases} \left(\frac{x}{3} - \frac{y}{3\sqrt{2}}\right)12(\sqrt{2}+1) = 2\sqrt{2}z \\ \left(\frac{x}{3} + \frac{y}{3\sqrt{2}}\right)\sqrt{2} = 12(\sqrt{2}+1) \end{cases}$

$$\text{II} \begin{cases} \left(\frac{x}{3} + \frac{y}{3\sqrt{2}}\right)\lambda = 2\mu z \\ \left(\frac{x}{3} - \frac{y}{3\sqrt{2}}\right)\mu = \lambda \end{cases}$$

$M(36,36,36) \in \pi \Rightarrow \begin{cases} \left(12 + \frac{12}{\sqrt{2}}\right)\lambda = 2 \cdot 36\mu \\ \left(12 - \frac{12}{\sqrt{2}}\right)\mu = \lambda \end{cases}$

Alegem $\mu = \sqrt{2} \Rightarrow \lambda = 12\sqrt{2} - 12 \Rightarrow \Delta_2 \begin{cases} \left(\frac{x}{3} + \frac{y}{3\sqrt{2}}\right) 12(\sqrt{2}-1) = 2\sqrt{2}\,z \\ \left(\frac{x}{3} - \frac{y}{3\sqrt{2}}\right)\sqrt{2} = 12(\sqrt{2}-1) \end{cases}$

3. $\pi: z = 0$

$\begin{cases} x^2 + y^2 + z^2 - 1 = 0 \\ 2x + 2y - 1 = 0 \end{cases}$

$\begin{cases} x = \lambda y \\ z = \mu \\ x^2 + y^2 + z^2 - 1 = 0 \\ 2x + 2y - 1 = 0 \end{cases}$

$\Delta: \begin{cases} x = 0 \\ y = 0 \end{cases}$

$x = \lambda y$

$z = \mu$

$2x + 2y - 1 = 0 \Rightarrow 2\lambda y + 2y - 1 = 0 \Rightarrow y = \frac{1}{2\lambda + 2} \Rightarrow x = \frac{\lambda}{2\lambda + 2}$

$\Rightarrow \left(\frac{\lambda}{2\lambda+2}\right)^2 + \left(\frac{1}{2\lambda+2}\right)^2 + \mu^2 - 1 = 0$

$\Leftrightarrow \left(\frac{x}{y} \cdot \frac{y}{2x+2y}\right)^2 + \left(\frac{y}{2x+2y}\right)^2 + z^2 - 1 = 0$

$\Leftrightarrow \left(\frac{x}{2x+2y}\right)^2 + \left(\frac{y}{2x+2y}\right)^2 + z^2 - 1 = 0$

4. $ABCD: A(0,0), B(3,0), C(3,0), D(0,3)$

$\Delta: 2x + y - 3 = 0 \Rightarrow \vec{d}(-1,2) \Rightarrow \vec{w} = \frac{\vec{d}}{\|\vec{d}\|} = \frac{\vec{d}}{\sqrt{5}} \Rightarrow \vec{w}\left(-\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right)$

$\Rightarrow \vec{w}\left(-\frac{\sqrt{5}}{5}, \frac{2\sqrt{5}}{5}\right)$

$Q(0,3) \in \Delta$

<u>$\text{Mirror}\left(0,3,-\frac{\sqrt{5}}{5}, \frac{2\sqrt{5}}{5}\right)$:</u>

$$w^{\perp} = \left(-\frac{2\sqrt{5}}{5}, -\frac{\sqrt{5}}{5}\right)$$

$$w^{\perp} \otimes w^{\perp} = \begin{pmatrix} \frac{-2\sqrt{5}}{5} \\ \frac{-\sqrt{5}}{5} \end{pmatrix} \begin{pmatrix} \frac{-2\sqrt{5}}{5} & \frac{-\sqrt{5}}{5} \end{pmatrix} = \begin{pmatrix} \frac{4}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{1}{5} \end{pmatrix}$$

$$i_2 - 2(w^{\perp} \otimes w^{\perp}) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} \frac{8}{5} & \frac{4}{5} \\ \frac{4}{5} & \frac{2}{5} \end{pmatrix} = \begin{pmatrix} -\frac{3}{5} & -\frac{4}{5} \\ -\frac{4}{5} & \frac{3}{5} \end{pmatrix}$$

$$2 \cdot (w^{\perp} \otimes w^{\perp}) \cdot Q = \begin{pmatrix} \frac{8}{5} & \frac{4}{5} \\ \frac{4}{5} & \frac{2}{5} \end{pmatrix} \begin{pmatrix} 0 \\ 3 \end{pmatrix} = \begin{pmatrix} \frac{12}{5} \\ \frac{6}{5} \end{pmatrix}$$

$$\Rightarrow \text{Mirror}\left(0,3,-\frac{\sqrt{5}}{5}, \frac{-2\sqrt{5}}{5}\right) = \begin{pmatrix} -\frac{3}{5} & -\frac{4}{5} & \frac{12}{5} \\ -\frac{4}{5} & \frac{3}{5} & \frac{6}{5} \\ 0 & 0 & 1 \end{pmatrix}$$

<u>$\text{Rot}(2,-1,-45°)$:</u> $\quad \sin(-45°) = -\frac{\sqrt{2}}{2} \quad \cos(-45°) = \frac{\sqrt{2}}{2}$

$$\frac{2}{\sqrt{2}} - \frac{2}{1} + \frac{\sqrt{2}}{2} = \frac{3\sqrt{2}-2}{2}$$

$$\text{Rot}(2,-1,-45°) = \begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 2\left(1-\frac{\sqrt{2}}{2}\right) + \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & -2\left(-\frac{\sqrt{2}}{2}\right) - 1 + \frac{\sqrt{2}}{2} \\ 0 & 0 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & \frac{4-\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & \frac{3\sqrt{2}-2}{2} \\ 0 & 0 & 1 \end{pmatrix}$$

$$M_t = \text{Rot.}\ \text{Mirror} = \begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & \frac{4-\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & \frac{3\sqrt{2}-2}{2} \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} -\frac{3}{5} & -\frac{4}{5} & \frac{12}{5} \\ -\frac{4}{5} & \frac{3}{5} & \frac{6}{5} \\ 0 & 0 & 1 \end{pmatrix} =$$

$$= \frac{1}{2} \begin{pmatrix} \sqrt{2} & \sqrt{2} & 4-\sqrt{2} \\ -\sqrt{2} & \sqrt{2} & 3\sqrt{2}-1 \\ 0 & 0 & 2 \end{pmatrix} \cdot \frac{1}{5} \begin{pmatrix} -3 & -4 & 12 \\ -4 & 3 & 6 \\ 0 & 0 & 5 \end{pmatrix}$$

$$= \frac{1}{10} \begin{pmatrix} -7\sqrt{2} & -\sqrt{2} & 13\sqrt{2}+20 \\ -\sqrt{2} & 7\sqrt{2} & 9\sqrt{2}-5 \\ 0 & 0 & 10 \end{pmatrix}$$

$$(A'\,B'\,C'\,D') = M_t \cdot (A\ B\ C\ D) =$$

$$= \frac{1}{10} \begin{pmatrix} -7\sqrt{2} & -\sqrt{2} & 13\sqrt{2}+20 \\ -\sqrt{2} & 7\sqrt{2} & 9\sqrt{2}-5 \\ 0 & 0 & 10 \end{pmatrix} \cdot \begin{pmatrix} 0 & 3 & 3 & 0 \\ 0 & 0 & 3 & 3 \\ 1 & 1 & 1 & 1 \end{pmatrix} =$$

$$= \frac{1}{10} \cdot \begin{pmatrix} 13\sqrt{2}+20 & 8\sqrt{2}+20 & -11\sqrt{2}+20 & 10\sqrt{2}+20 \\ 9\sqrt{2}-5 & 6\sqrt{2}-5 & 27\sqrt{2}-5 & 30\sqrt{2}-15 \\ 10 & 10 & 10 & 10 \end{pmatrix}$$

...

Model 5

$$\Delta : \begin{cases} x = 2y \\ z = 2y \end{cases} \Rightarrow \begin{cases} x - 2y = 0 \\ z - 2y = 0 \end{cases}$$

$$\begin{cases} x - 2y = \lambda & \Rightarrow x = \lambda + 2y \\ z - 2y = \mu & \Rightarrow z = \mu + 2y \\ 2x - 3y + z = 0 \\ x^2 + y^2 + z^2 - 1 = 0 \end{cases}$$

$$\Rightarrow 2\lambda + 4y - 3y + \mu + 2y = 0$$

$$\Leftrightarrow 3y + 2\lambda + \mu = 0 \Rightarrow y = -\frac{2\lambda + \mu}{3} \Rightarrow x = \lambda - \frac{4\lambda + 2\mu}{3}$$

$$z = \mu - \frac{4\lambda + 2\mu}{3}$$

$$\Rightarrow \left(\lambda - \frac{4\lambda + 2\mu}{3}\right)^2 + \left(\frac{2\lambda + \mu}{3}\right)^2 + \left(\mu - \frac{4\lambda + 2\mu}{3}\right)^2 - 1 = 0$$

$$\Leftrightarrow \left(x - 2y - \frac{4(x-2y) + 2(z-2y)}{3}\right)^2 + \left(\frac{2(x-2y) + (z-2y)}{3}\right)^2 + \left(z - 2y - \frac{4(x-2y) + 2(z-2y)}{3}\right)^2 - 1 = 0.$$

2. $P : y^2 = 10x$

$\Delta : y = 4x - 5$

$$\begin{cases} y^2 = 10x \\ y = 4x - 5 \end{cases} \rightarrow (4x-5)^2 = 10x \Leftrightarrow 16x^2 - 40x + 25 = 10x \Leftrightarrow$$

$$\Leftrightarrow 16x^2 - 50x + 25 = 0$$

$$\Delta = 50^2 - 4 \cdot 16 \cdot 25 = 2500 - 1600 = 900$$

$$\Rightarrow x_1 = \frac{50 - 30}{32} = \frac{20}{32} = \frac{5}{8} \rightarrow y_1 = \cancel{4} \cdot \frac{5}{\cancel{8}_2} - 5 = -\frac{5}{2}$$

$$x_2 = \frac{50 + 30}{32} = \frac{80}{32} = \frac{10}{4} = \frac{5}{2} \Rightarrow y_1 = \cancel{4}^2 \cdot \frac{5}{\cancel{2}} - 5 = 5$$

$\Rightarrow$ Pct. de intersecție sunt $M_1\left(\frac{5}{8}, -\frac{5}{2}\right)$
$M_2\left(\frac{5}{4}, 5\right)$

Ec. tang. în (x_0, y_0): $yy_0 = 5(x+x_0)$

$M_1\left(\frac{5}{8}, -\frac{5}{2}\right)$: $y \cdot \frac{-5}{2} = 5\left(x+\frac{5}{8}\right) \Leftrightarrow \frac{-5y}{2} = 5x + \frac{25}{8} \Big/ \cdot 8 \Leftrightarrow$

$\Leftrightarrow -20y = 40x + 25 \Leftrightarrow 40x + 20y + 25 = 0 \Leftrightarrow 8x + 4y + 5 = 0$

$M_2\left(\frac{5}{4}, 5\right)$: $y \cdot 5 = 5\left(x + \frac{5}{4}\right) \Leftrightarrow 5y = 5x + \frac{25}{4} \Big/ \cdot 4 \Leftrightarrow 20y = 20x + 25 \Leftrightarrow$

$\Leftrightarrow 20x - 20y + 25 = 0 \Big/ :5 \Leftrightarrow 4x - 4y + 5 = 0$

4. $\frac{x^2}{30} + \frac{y^2}{16} = 1$

tg: $y = kx \pm \sqrt{a^2k^2 + b^2}$

Δ: $x - 2y + 5 = 0 \Rightarrow k_\Delta = -\frac{1}{-2} = \frac{1}{2}$

$tg\, 45° = \left| \frac{k - k_\Delta}{1 + k \cdot k_\Delta} \right| \Leftrightarrow 1 = \left| \frac{k - \frac{1}{2}}{1 + \frac{k}{2}} \right| \Leftrightarrow \frac{k - \frac{1}{2}}{1 - \frac{k}{2}} = \pm 1$

1) $\frac{k - \frac{1}{2}}{1 + \frac{k}{2}} = 1 \Leftrightarrow k - \frac{1}{2} = 1 + \frac{k}{2} \Big/ \cdot 2 \Leftrightarrow 2k - 1 = 2 + k \Leftrightarrow k = 3 \Rightarrow$

$\Rightarrow$ tg: $y = 3x \pm \sqrt{30 \cdot 9 + 16} \Leftrightarrow y = 3x \pm \sqrt{286}$

$\Rightarrow$ tg: $3x - y \pm \sqrt{286} = 0$

$16 \cdot 9 = 144$, $144 + 30 = 174$

2) $\frac{k - \frac{1}{2}}{1 + \frac{k}{2}} = -1 \Leftrightarrow k - \frac{1}{2} = -1 - \frac{k}{2} \Big/ \cdot 2 \Leftrightarrow 2k - 1 = -2 - k \Leftrightarrow 3k = -1 \Rightarrow$
$\Rightarrow k = -\frac{1}{3}$

$\Rightarrow$ tg: $y = -\frac{1}{3}x \pm \sqrt{30 \cdot \frac{1}{9} + 16} =$

$\Leftrightarrow$ tg: $y = -\frac{1}{3}x \pm \frac{\sqrt{30 + 16 \cdot 9}}{3}$

$\Leftrightarrow$ tg: $y = -\frac{1}{3}x \pm \frac{\sqrt{174}}{3} \Big/ \cdot 3$

$\to tg:\ x + 3y \pm \sqrt{174}$ $\Rightarrow$ a) c) d)

5. $3x^2 - 12y^2 + z^2 - 3 = 0$

$\Pi:\ 3x_0 \cdot x - 12y_0 y + z_0 \cdot z - 3 = 0 \to \vec{n}\,(2x_0, -12y_0, z_0)$

$\Pi':\ 2x + 3y - z + 11 = 0 \Rightarrow \vec{n'}\,(2, 3, -1)$

$\Pi \parallel \Pi'$ $\Rightarrow$

$52 : 2 = 26 : 2 = 13$

$\to x_0 = -4y_0 = -z_0 \to \begin{cases} x_0 = -4y_0 \\ z_0 = 4y_0 \end{cases}$

$$\begin{cases} x_0 = -4y_0 \\ z_0 = 4y_0 \\ 3x_0^2 - 12y_0^2 + z_0^2 - 3 = 0 \end{cases} \Rightarrow 3\cdot 16y_0^2 - 12y_0^2 + 16y_0^2 - 3 = 0$$

$\Rightarrow 52y_0^2 - 3 = 0 \Rightarrow y_0^2 = \frac{3}{52} \Rightarrow y_0 = \pm\frac{\sqrt{3}}{2\sqrt{13}} = \pm\frac{\sqrt{39}}{26}$

$\to x_0 = \mp\frac{2\sqrt{39}}{13} \Rightarrow z_0 = \pm\frac{2\sqrt{39}}{13}$

$\Rightarrow -3\frac{2\sqrt{39}}{13}x - 12\frac{\sqrt{39}}{26}y + \frac{2\sqrt{39}}{13}\cdot z - 3 = 0 \;\Big/ \cdot 26$

$\Leftrightarrow -12\sqrt{39}\,x - 12\sqrt{39}\,y - 4\sqrt{39}\,z - 3\cdot 26 = 0 \;/ : \sqrt{39}$

c) $-12x - 12y$???,??!!

6. $\frac{x^2}{4} - \frac{y^2}{2} + \frac{z^2}{2} = 1$

$x = 2 \Rightarrow 1 - \frac{y^2}{2} + \frac{z^2}{2} = 1 \Leftrightarrow y^2 = z^2 \to |y| = |z| \to y = -z \land y = z$

$\to$ a)

$y = 1 \Rightarrow \frac{x^2}{4} - \frac{1}{2} + \frac{z^2}{2} = 1$

$\Leftrightarrow \frac{x^2}{4}+\frac{z^2}{2}=\frac{3}{2} \,\Big|\cdot\frac{2}{3}$

$\Leftrightarrow \frac{x^2}{6}+\frac{z^2}{3}=1 \Rightarrow$ paraboloid

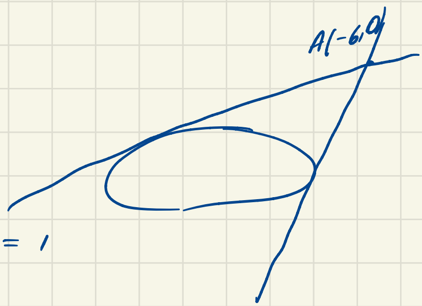
$x=-2 \Rightarrow$ c)

a), c)

7. Se calculează

Model 6

1. $\frac{x^2}{18}+\frac{y^2}{8}=1$



$tg: x\frac{x_0}{18}+y\frac{y_0}{8}=1$

$A(-6,0)\in tg$

$\Rightarrow \cancel{-6}\cdot\frac{x_0}{\cancel{18}}=1$

$\Leftrightarrow \frac{x_0}{-3}=1 \Rightarrow x_0=-3$

$\frac{(-3)^2}{18}+\frac{y_0^2}{8}=1 \Rightarrow \frac{1}{2}+\frac{y_0^2}{8}=1 \Leftrightarrow \frac{y_0^2}{8}=\frac{1}{2} \Leftrightarrow y_0^2=4 \Rightarrow$

$\Rightarrow y_0=\pm 2$

$\Rightarrow tg: -\frac{1}{6}x \pm \frac{1}{4}y=1 \,\Big|\cdot(-12) \Leftrightarrow tg: 2x \pm 3y+12=0$

$\Rightarrow$ b) c)

2. $\frac{x^2}{9}-\frac{y^2}{4}+\frac{z^2}{3}-1=0$

$\Rightarrow \pi: x\frac{x_0}{9}-y\frac{y_0}{4}+z\frac{z_0}{3}-1=0$

$\Rightarrow \vec{n}\left(\frac{x_0}{9}, -\frac{y_0}{4}, \frac{z_0}{3}\right)$

$\pi': 2x+3y-z+11=0 \Rightarrow \vec{n}(2,3,-1)$

$\rightarrow \frac{x_0}{18} = -\frac{y_0}{12} = -\frac{z_0}{3} \Rightarrow \begin{cases} \frac{x_0}{18} = -\frac{z_0}{3} \Rightarrow x_0 = -6z_0 \\ -\frac{y_0}{12} = -\frac{z_0}{3} \Rightarrow y_0 = 4z_0 \end{cases}$

$\frac{(-6z_0)^2}{9} - \frac{(4z_0)^2}{4} + \frac{z_0^2}{3} - 1 = 0$

$\Leftrightarrow \frac{36z_0^2}{9} - \frac{16z_0^2}{4} + \frac{z_0^2}{3} - 1 = 0 \,/\cdot 3$

$\Leftrightarrow 12z_0^2 - 12z_0^2 + z_0^2 - 3 = 0 \Leftrightarrow z_0 = \pm\sqrt{3}$

$\Rightarrow y_0 = \pm 4\sqrt{3}$
$x_0 = \mp 6\sqrt{3}$

$\Rightarrow -\frac{\cancel{6}^{2}\sqrt{3}}{\cancel{9}_{3}} x - \frac{\cancel{4}\sqrt{3}}{\cancel{4}} y + \frac{\sqrt{3}}{3} z - 1 = 0 \,/\cdot 3$

$\Leftrightarrow -2\sqrt{3}x - 3\sqrt{3}y + \sqrt{3}z - 1 = 0 \,/ :-\sqrt{3}$

$\Leftrightarrow 2x + 3y - z + \sqrt{3} = 0$

$2x + 3y - z - \sqrt{3} = 0 \Rightarrow$ a), c)

3. $3x^2 - 4y^2 + 5z = 0$

$\pi: y = 0 \Rightarrow 3x^2 + 5z = 0 \Rightarrow x^2 = -\frac{5}{2} z \Leftrightarrow x^2 = 2\left(-\frac{5}{4}\right) z \Rightarrow$

$\Rightarrow$ parabola $\rightarrow$ a)

4. Calcul

5. $\begin{matrix} x = y \\ y = z \end{matrix} \Leftrightarrow \begin{cases} x - y = 0 \\ y - z = 0 \end{cases}$

$\begin{cases} x - y = \lambda \Rightarrow x = \lambda + \mu \\ y - z = \mu \Rightarrow y = \mu \end{cases}$

$$\begin{cases} x-y^2=0 \\ z=0 \end{cases}$$

$$\lambda+\mu-\mu^2=0 \Leftrightarrow x-y+y-z-(y-z)^2=0 \Leftrightarrow (x-z)-(y-z)^2=0.$$

6. $\frac{x^2}{9}+\frac{y^2}{4}-\frac{z^2}{16}=1$

$$\Rightarrow \left(\frac{x}{3}-\frac{z}{4}\right)\left(\frac{x}{3}+\frac{z}{4}\right)=\left(1-\frac{y}{2}\right)\left(1+\frac{y}{2}\right)$$

$$I \begin{cases} \left(\frac{x}{3}-\frac{z}{4}\right)\lambda=\left(1-\frac{y}{2}\right)\mu \\ \left(\frac{x}{3}+\frac{z}{4}\right)\mu=\left(1+\frac{y}{2}\right)\lambda \end{cases}$$

$$M(6,2,8) \Rightarrow \begin{cases} (2-2)\lambda=(1-1)\mu \\ (2+2)\mu=(1+1)\lambda \end{cases} \Leftrightarrow \lambda=2\mu$$

Alegem $\lambda=1$ și $\mu=2$

$$\Rightarrow I \begin{cases} \frac{x}{3}-\frac{z}{4}=2-y \\ \frac{2x}{3}+\frac{z}{2}=1+\frac{y}{2} \end{cases}$$

$II \{$ Analog

7. <u>Calcul.</u>

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3. $x^2-4y^2+5z^2=0$

$z+\frac{1}{\sqrt{5}}=0 \Rightarrow z=-\frac{\sqrt{5}}{5}$

$$\Rightarrow x^2-4y^2+5\cdot\frac{5}{25}=0$$

$$\Rightarrow x^2-4y^2+1=0$$

$\Leftrightarrow -x^2 + 4y^2 = 1 \rightarrow$ hiperbola

6. $V: \begin{cases} x = 0 \\ y = 0 \\ z = 0 \end{cases}$ $(C): \begin{cases} x = y^2 \\ x - y - z + 1 = 0 \end{cases}$

$\begin{cases} x = \lambda y \\ z = \mu y \\ x - y^2 = 0 \\ x - y - z + 1 = 0 \end{cases}$

$\Rightarrow \lambda y - y - \mu y + 1 = 0 \rightarrow y(\lambda - \mu - 1) = -1 \Rightarrow y = -\frac{1}{\lambda - \mu - 1} \Leftrightarrow y = \frac{1}{1 - \lambda + \mu}$

$\Rightarrow x = \frac{\lambda}{1 - \lambda + \mu}$

$y = \frac{\mu}{1 - \lambda + \mu}$

$\rightarrow \frac{\mu}{1 - \lambda + \mu} - \frac{\lambda^2}{(1 - \lambda + \mu)^2} = 0$

$\Leftrightarrow \frac{\mu(1 - \lambda + \mu) - \lambda^2}{(1 - \lambda + \mu)^2} = 0$

$\Leftrightarrow \mu - \mu\lambda + \mu^2 - \lambda^2 = 0$

$\Leftrightarrow \mu^2 - \lambda^2 - \mu\lambda + \mu = 0$

$\Leftrightarrow \left(\frac{z}{y}\right)^2 - \left(\frac{x}{y}\right)^2 - \frac{z}{y} \cdot \frac{x}{y} + \frac{z}{y} = 0 \;/\cdot y^2$

$\Leftrightarrow z^2 - x^2 - xyz + yz = 0$

4. $\frac{x^2}{45} - \frac{y^2}{5} = 1$

asimptotele: $x = \pm \frac{45}{5} y \Leftrightarrow x = \pm 9y \Rightarrow A_1: x - 9y = 0$

$A_2: x + 9y = 0$

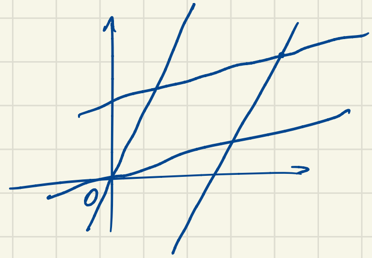
$d_1: y - y_0 = k_1(x - x_0) \Leftrightarrow y - \sqrt{6} = \frac{1}{9}(x - 3) \;/\cdot 9 \Leftrightarrow$

$\Leftrightarrow 9y - \sqrt{6} = x - 3 \Leftrightarrow$ $\left.\begin{array}{l} x - 9y - 3 + \sqrt{6} = 0 \\ x + 9y = 0 \end{array}\right\} \Rightarrow -18y - 3 + \sqrt{6} = 0$

$\Rightarrow y = \frac{3 - \sqrt{6}}{18}$

$\Rightarrow M_1\left(\frac{\sqrt{6}-3}{2}, \frac{3-\sqrt{6}}{18}\right)$

$\Rightarrow x = \frac{\sqrt{6}-3}{2}$



$d_2: \; y - y_0 = k_2(x - x_0) \Leftrightarrow y - \sqrt{6} = -\frac{1}{9}(x-3) \,/\cdot 9$

$\Leftrightarrow 9y - \sqrt{6} = -x + 3 \Leftrightarrow \left.\begin{matrix} x + 9y - \sqrt{6} - 3 = 0 \\ x - 9y = 0 \end{matrix}\right\} \Rightarrow$

$\Rightarrow 18y = \sqrt{6} + 3 \Leftrightarrow y = \frac{\sqrt{6}+3}{18} \Rightarrow x = \frac{\sqrt{6}+3}{2}$

$\Rightarrow M_2\left(\frac{3+\sqrt{6}}{2}, \frac{\sqrt{6}+3}{18}\right) \Rightarrow \vec{OM_1} = \left(\frac{\sqrt{6}-3}{2}, \frac{3-\sqrt{6}}{9}\right)$

$\vec{OM_2} = \left(\frac{3+\sqrt{6}}{2}, \frac{\sqrt{6}+3}{18}\right)$

$\Rightarrow A = \|\vec{OM_1} \times \vec{OM_2}\|$

$\Delta: \begin{cases} x - 2 = 0 \\ y = 0 \end{cases}$

$\pi: \; z = 0$

$(C): \begin{cases} \frac{x^2}{4} + \frac{z^2}{9} - 1 = 0 \\ y - 2 = 0 \end{cases}$