

Exercise #3

$$R_{\text{school}} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$$

$$R_{\text{stay}} = \begin{bmatrix} -5 \\ 1 \end{bmatrix}$$

Step 1:

For Sunny:

$$r_{\pi} = 0.5(5) + 0.5(-5) = 2.5 + (-2.5) = 0$$

$$r_{\pi} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

For Cloudy:

$$r_{\pi} = 0.5(3) + 0.5(1) = 1.5 + 0.5 = 2$$

Step 2:

Row 1 (Sunny):

$$\bullet P_{\pi}(1,1) = 0.5(0.8) + 0.5(0.9) = 0.4 + 0.45 = 0.85$$

$$\bullet P_{\pi}(1,2) = 0.5(0.2) + 0.5(0.1) = 0.1 + 0.05 = 0.15$$

Row 2 (Cloudy):

$$\bullet P_{\pi}(2,1) = 0.5(0.4) + 0.5(0.3) = 0.2 + 0.15 = 0.35$$

$$\bullet P_{\pi}(2,2) = 0.5(0.6) + 0.5(0.7) = 0.3 + 0.35 = 0.65$$

$$P_{\pi} = \begin{bmatrix} 0.85 & 0.15 \\ 0.35 & 0.65 \end{bmatrix}$$

Step 3: (Sunny)

$$v_1 = 0 + 0.9(0.85v_1 + 0.15v_2)$$

$$v_1 = 0 + 0.765v_1 + 0.135v_2$$

$$v_1 = 0.765v_1 + 0.135v_2 = 0$$

$$= 0.235v_1 - 0.135v_2 = 0$$

Step 3: (cloudy)

$$v_2 = 2 + 0.9(0.35v_1 + 0.65v_2)$$

$$v_2 = 2 + 0.315v_1 + 0.585v_2$$

$$v_2 = -0.315 - 0.585 = 2$$

$$= -0.315v_1 + 0.415v_2 = 2$$

Step 4:

Cloudy

$$0.235v_1 - 0.135 = 0$$

$$0.235v_1 = 0 + 0.135v_2$$

$$0.235v_1 = 0 + 0.135v_2$$

$$\frac{0.235}{0.235} \quad \frac{0.235}{0.235}$$

$$v_1 = \frac{0 + 0.135v_2}{0.235}$$

$$-0.315v_1 + 0.415v_2 = 2$$

$$-0.315 \left(\frac{0 + 0.135v_2}{0.235} \right) + 0.415v_2 = 2$$

$$\left(-0.135 \frac{0}{0.235} \right) \left(-0.135 \times \frac{0.135}{0.235v_2} \right) + 0.415 = 2$$

$$(-0.135 \times 0) (-0.135 \times 0.574v_2) + 0.415v_2 = 2$$

$$0 - 0.181v_2 + 0.415v_2 = 2$$

$$0 + 0.234v_2 = 2$$

$$0.234v_2 = 2 + 0$$

$$v_{\pi}(\text{Cloudy}) = \frac{2+0}{0.234} = \frac{2}{0.234} = 8.547$$

Sunny

$$v_1 = \frac{0 + 0.135v_2}{0.235}$$

$$v_1 = \frac{0 + 0.135(8.547)}{0.235} = \frac{0 + 1.154}{0.235} = \frac{1.154}{0.235}$$

$$v_{\pi}(\text{Sunny}) = 4.911$$

Step 5:

For Sunny (v_1) using school:

$$v_1(\text{sunny}) = 5 + 0.9(0.8v_1 + 0.2v_2)$$

$$v_1 = 5 + 0.72v_1 + 0.18v_2$$

$$v_1 = 0.72v_1 + 0.18v_2 = 5$$

$$\boxed{0.28v_1 = 0.18v_2 = 5}$$

For Cloudy (v_2) using School:

$$v_2(\text{cloudy}) = 3 + 0.9(0.4v_1 + 0.6v_2)$$

$$v_2 = 3 + 0.36v_1 + 0.54v_2$$

$$v_2 = 0.36v_1 + 0.54v_2 = 3$$

$$\boxed{-0.36v_1 + 0.46v_2 = 3}$$

Step 6:

$$\text{From Sunny: } 0.28v_1 = 5 + 0.18v_2$$

$$\text{From Cloudy: } -0.36v_1 + 0.46v_2 = 3$$

$$v_1 = \frac{5 + 0.18v_2}{0.28}$$

$$-0.36 \left(\frac{5 + 0.18v_2}{0.28} \right) + 0.46v_2 = 3$$

Step 6 (cloudy)

$$-0.36 \left(\frac{5 + 0.18v_2}{0.28} \right) + 0.46v_2 = 3$$

$$\left(-0.36 \times \frac{5}{0.28} \right) + \left(-0.36 \times \frac{0.18v_2}{0.28} \right) + 0.46v_2 = 3$$

$$-6.429 - 0.231v_1 + 0.46v_2 = 3$$

$$-6.429 + 0.229v_2 = 3$$

$$0.229v_2 = 3 + 6.429$$

$$0.229v_2 = 9.429$$

$$\boxed{v_2(\text{cloudy}) = \frac{9.429}{0.229} = 41.175}$$

Sunny:

$$v_1 = \frac{5 + 0.18v_2}{0.28}$$

$$v_1 = \frac{5 + 0.18(41.175)}{0.28}$$

$$v_1 = \frac{5 + 7.412}{0.28} = \frac{12.412}{0.28}$$

$$\boxed{v_1(\text{sunny}) = 44.329}$$

Step 7:

$$q(1, \text{School}) = 5 + 0.9(0.8v_1 + 0.2v_2) = 44.33$$

$$q(1, \text{Home}) = -5 + 0.9(0.9v_1 + 0.1v_2) = 34.61$$

$$q(2, \text{School}) = 3 + 0.9(0.4v_1 + 0.6v_2) = 41.18$$

$$q(2, \text{Home}) = 1 + 0.9(0.3v_1 + 0.7v_2) = 38.91$$