ELE725 Lab Report #1

Sampling and Quantization (Audio)

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***Abstract*—This document explains the methodology and purpose of sampling and quantizing audio signals. It explains the effects of sampling in time and frequency domain. It will also examine what parameters are involved in the quantization process and how the quality of the audio changes along with varying these parameters.**

Keywords—sampling, quantization, midrise quantizer, midtread quantizer, mean-squared error, quantization level, companding

I. Introduction

Sampling and quantization is a fundamental practice when it comes to digital multimedia signal processing. The purpose of this lab is to investigate the effects of up sampling, down sampling, uniform quantization and non-uniform quantization using μ -law to an audio signal.

II. Theory and Methodology

A. Sampling

An analog signal can be broken down to discrete values through sampling. Periodical sampling of a continuous time signal $x_a(t)$ can be mathematically described as [1]:

$$x[n] = x_a(t)|_{t=nT_s} = x_a(nTs)$$
$$where: n = \cdots, -2, -1, 0, 1, 2, \ldots$$

Down sampling is a form of subsampling where only selective samples from a sequence is retained. Down sampling removes every i^{th} sample(s) in a signal. This practice is usually done when there is a limited amount of memory in a system. A function labeled as *DownSample (inFile, outFile, N, pf)* is created in MATLAB to simulate this practice. The parameter *inFile* is the audio file to be down sampled, *outFile* is the down sampled file to be saved, *N* is the sampling factor, and *pf* is a Boolean datatype indicating whether a filtering is used on the signal prior to down sampling. In this function, the function *decimate(X,N)* is used to down sample where it resamples the signal at 1/*N* times the original sample rate. Its frequency plot is then graphed shown in Figure 2(a), 3(a), and 4(a), and the down sampled audio is played back.

Up sampling, also known as interpolation, samples a signal at a higher sampling rate, usually by a sampling factor of *N*. This is done by inserting *N* sample(s) equidistantly spaced between two existing samples for all samples. This increases the resolution of the media. The *interp()* function is used in MATLAB to simulate up sampling. The output is then plotted in frequency domain as shown in Figure 2(b), 3(b), 4(b).

B. Quantization

The second part of digitalizing a signal involves processing the values from sampling and evaluating them to values that are equal in memory size. It is the process of mapping input values with various amplitudes to output values from a set of predetermined values specified by a constant number of bits to represent each output value. These predetermined values are called quantization levels which are equally spaced among the amplitude range of the signal, so if there are N bits, then there will be 2^N levels available for mapping.

The two types of quantization are shown in Figure 1. The major difference between these two is the number of levels where midrise quantizers have 2^N levels and midtread quantizers have 2^N-1 levels.

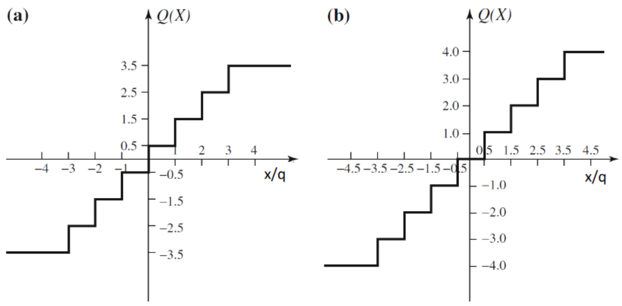


Figure 1. Midrise (a) vs Midtread (b) Quantizers [2]

The following equations represent how to map the input values to output values for midrise and midtread quantizers respectively. The difference being midrise utilizes the floor() function whereas the midtread quantizer applies the round() function.

$$Q(x) = floor\left(\frac{x}{q}\right) + \frac{1}{2}$$
$$Q(x) = round(\frac{x}{q})$$

Where Q(x) represents the quantized output value, x is the input value of the signal, and q is the step size of the quantization levels.

The step size is the difference between consecutive quantization levels or the value from one level to the next. The step size is determined using the following equation for a sampled signal in the range [$-V_{max}$, V_{max}].

$$q = \frac{2 * V_{\max}}{M} = \frac{V_{\max}}{2^{N-1}}$$

Where N is the number of bits used to represent each quantization level and $M = 2^N$ is the number of levels.

C. Mean Squared Error

When quantizing input signals, the output will be an approximate to the closest quantization level. Therefore, there will be some error between the actual input value and output value which can be characterized by the mean square error (MSE). The MSE is the average squared difference between the input values and the estimated approximations which is represented by the following equation:

$$MSE = \frac{1}{N}\sum_{i=0}^{N-1}(x' - x)^2$$

Where x' is the quantized signal and x is the original signal.

D. μ-Law Quantization

The quantization prior to this point has been uniform but now, a companding algorithm is introduced to the process. The purpose of the companding algorithm is to increase the dynamic range of the signal, give more weight to smaller amplitudes of the signal and can reduce quantization error as the human ear is more sensitive to quantization errors at small magnitudes. The μ-law is an example of a companding algorithm where the signal is initially compressed, then quantized by one of the quantization methods introduced earlier and finally expanded. The following equations are used to compress and expand the signals respectively.

$$y = X_{max}\frac{\log\left[1 + \mu\frac{|x|}{X_{max}}\right]}{\log[1+\mu]}sign(x)$$

$$x = \frac{X_{max}}{\mu}\left[10^{\frac{\log(1+\mu)}{X_{max}}|y|} - 1\right]sign(y)$$

III. Results

A. Sampling

Figure 3(a), 4(a), and 5(a), shows the down sampled signal with sampling factor N = 2, 4, 8 respectively. These plots have varying center frequency and signal amplitude seen in Table I that shows a direct relation with the sampling factor where the relationship is described as the following:

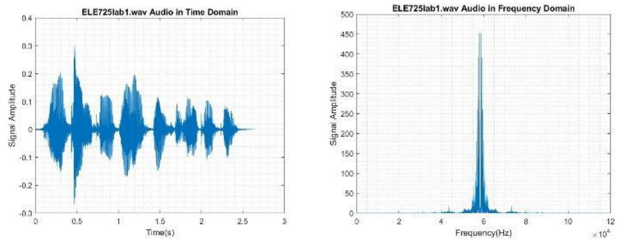
$$center\ frequency\ = \frac{center\ frequency\ of\ orignal\ signal}{sampling\ factor\ (N)}$$

$$amplitude\ = \frac{amplitude\ of\ orignal\ signal}{sampling\ factor\ (N)}$$

Similarly, Figure 3(b), 4(b), and 5(b), shows the up sampled signal with sampling factor N = 2, 4, 8 respectively. Its center frequencies and signal amplitude's relation with the sampling factor can be described as the following:

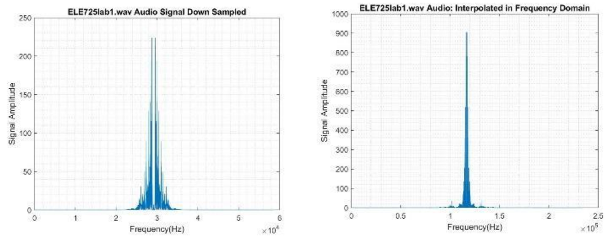
$$centre\ frequency = \ center\ fequency\ of\ orignal\ signal \\ \times sampling\ factor\ (N)$$

$$amplitude\ = amplitude\ of\ orignal\ signal \\ \times sampling\ factor\ (N)$$



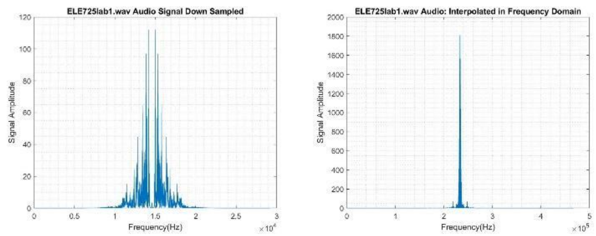
(a) (b)

Figure 2. (a) Original audio signal in time domain. (b) Original audio signal in frequency domain



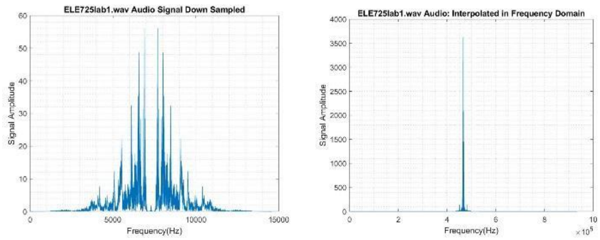
(a) (b)

Figure 3. Audio signal plots for sampling factor of N=2.. (a) Down sampled signal in frequency domain. (b) Up sampled signal in frequency domain.



(a) (b)

Figure 4. Audio signal plots for sampling factor of N=4. (a) Down sampled signal in frequency domain. (b) Up sampled signal in frequency domain.



(a) (a)

Figure 5. Audio signal plots for sampling factor of N=8. (a) Down sampled signal in frequency domain. (b) Up sampled signal in frequency domain.

TABLE I.
APPROXIMATE CENTER FREQUENCIES AND AMPLITUDES OF RESAMPLED SIGNALS

	Sampling Factor (N)	Approximate Center Frequency (Hz)	Approximate Signal Amplitude
	Original	60 000	450
Down sampling	2	30 000	225
	4	15 000	112
	8	7 500	56
Up sampling	2	120 000	900
	4	240 000	1 800
	8	480 000	3 600

The sound quality of the down sampled audio decreases as the sampling factor increases. The audio with N=8 sounded the most muffled. This is to be expected because down sampling removes some sample from the audio thus playback is not as clear. On the other hand, the sound quality of the up sampled audio did not show much significant difference from the original audio. This is an example of Weber's law is due to the fact that no significant additional information is retrieved when up sampling the original audio which was of high quality to begin with.

B. *Quantization*

Applying the UniformQuant(inFile, outFile, N) function where the input is uniformly quantized to the audio signal of 'ELE725_lab1.wav' file gives the results shown in Figure 6 where different values of N were tested.

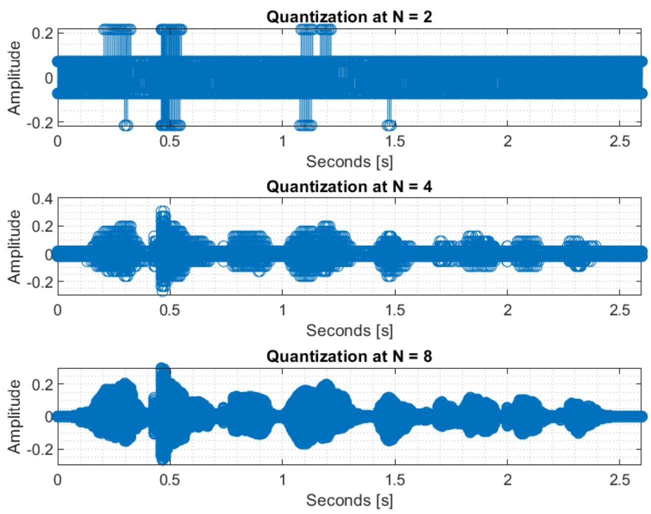


Figure 6. Uniform Quantization with N=2, 4, and 8

It can be seen as the value of N increases, the resulting waveform begins to resemble more closely to the original audio signal previously seen in Figure 2(a). The MSE of the various quantized signals can be seen in Table II.

TABLE II.
UNIFORM QUANTIZATION ERROR WITH DIFFERENT BITS

Number of bits representing each quantization level (N)	Number of quantization levels (M)	MSE
2	4	0.0084
4	16	$4.6900 * 10^{-4}$
8	256	$1.6839 * 10^{-6}$

It can be seen as N and M increase, the MSE steadily decreases resulting in small quantization error values. Listening to the audio files, increasing N drastically increases the quality of the audio. At the lower values of N, the audio sounds extremely static and the voice is interpreted as robotic. However, at higher values of N, the audio becomes less static and more clear.

The Figure 7 below show the results of applying the MulawQuant(inFile, outFile, N, μ) function with μ=100 where the signal will first be compressed using the μ-law algorithm, then the signal will be quantized as previously done, and finally expanded using the μ-law algorithm again.

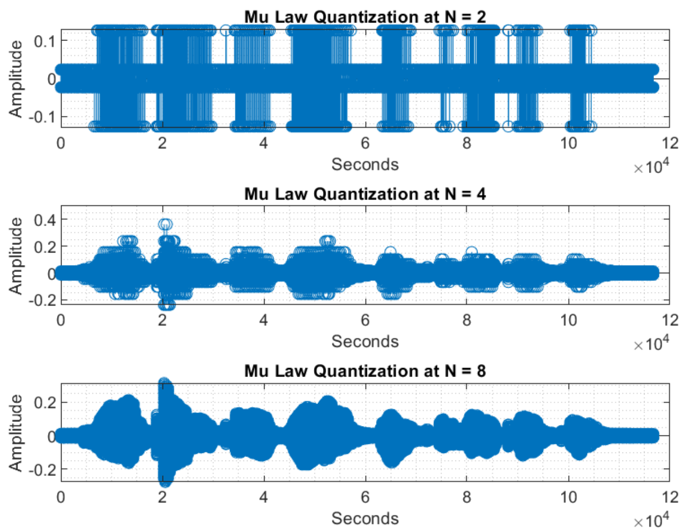


Figure 7. Mu Law Quantization with N=2, 4, and 8

It is evident from looking at the graph above that the quantization levels are not equally spaced where as in uniform quantization, they were. The space between the levels are exponentially increased as we go from lower to higher levels. Similar to uniform quantization, in μ-law quantization, as the number of bits is increased, the signal is resembled more closely to the original audio signal. Another observation is that lower value samples are indeed given more weight during the quantization process, this is visible when comparing the μ-law and uniform quantization graphs with N=2. It is visible more samples are being quantized to the approximate -0.3 quantization level in the μ-law graph than the uniform quantization graph. The MSE comparing the μ-law quantized signals and the original signal can be seen in Table III.

TABLE III.
MU LAW QUANTIZATION ERROR WITH DIFFERENT BITS

Number of bits representing each quantization level (N)	**Number of quantization levels (M)**	**MSE**
2	4	$9.3639 * 10^{-4}$
4	16	$1.4632 * 10^{-4}$
8	256	$9.7599 * 10^{-5}$

Comparing Tables II and III, it can be noted that the MSE at N=2 and N=4 is lower in µ-law quantization than uniform quantization, however, as the number of bits is increased to 8, the MSE in µ-law is higher than uniform quantization. The MSE is significantly lower at N=2 in µ-law, the difference in MSE at N=4 is negligible between the two methods and at N=8, the uniform method gives lower quantization errors.

Figure 8 shows the original, uniform quantized, and µ-law quantized signals with N=2. It can be seen that the µ-law quantized signal resembles the original signal more closely than the uniformly quantized signal. Since the mid rise quantizer cannot contain any zero valued samples, if the sample is slightly above or below zero, it will be quantized to the closest quantization level which can create large quantization errors. Thus, it should be tested to see which method between uniform and µ-law quantization methods are better for small and high valued signals.

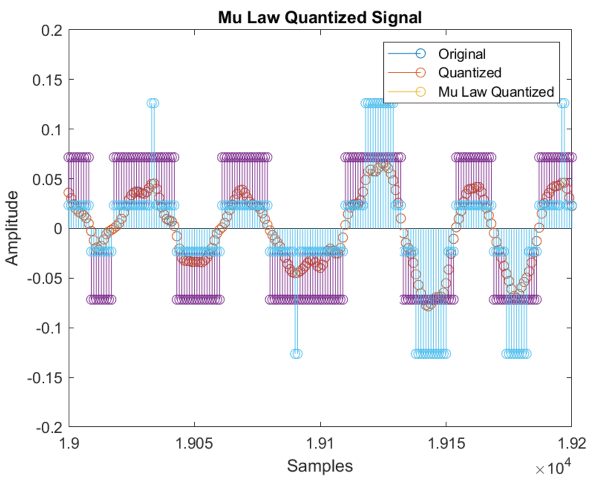


Figure 8. Plotting 200 samples of original, quantized, and µ-law quantized signals

A custom MATLAB function called MeanSquaredError() was created to calculate the MSE of values of a signal over or under a specified amplitude. The function accepts one input to specify an amplitude and another input to specify if values over or under that value should be accounted towards the MSE. For example, the uniform and µ-law quantized signals with N=2,4 and 8 were tested using this function to calculate the MSE for high valued signals over the value of 0.15. Then tested again, this time to find the low valued signals under 0.15. The results can bee seen below in Table IV. It is evident for low valued signals with small N bits, the µ-law quantization provides lower MSE values but as N becomes large, the opposite is true. For high valued signals, the uniform quantization is better suited to provide low MSE values for all values of N.

TABLE IV.
COMPARING MSE OF LOW AND HIGH SIGNAL VALUES WITH DIFFERENT NUMBER OF BITS

	MSE for Low Signal Values		**MSE for High Signal Values**	
N	Uniform	µ -law	Uniform	µ -law
2	0.0030	$8.9222 * 10^{-4}$	$2.2779 * 10^{-7}$	$4.4217 * 10^{-5}$
4	$1.4938 * 10^{-4}$	$1.4073 * 10^{-4}$	$9.5030 * 10^{-7}$	$5.6026 * 10^{-6}$
8	$4.2847 * 10^{-7}$	$9.6737 * 10^{-5}$	$4.3461 * 10^{-9}$	$8.5974 * 10^{-7}$

IV. CONCLUSION

In conclusion, down sampling an audio file decreases the quality of the audio whereas up sampling the same audio file did not differ from the original audio due to no additional information being retrieved as accordance with Weber's law. However, interpolating or up sampling the downsampled audio file produces a near identical original audio before being downsampled. The quality of the audio after down sampling and up sampling remained noticeably unchanged. After investigating the properties of uniform and µ-law quantization, it can be determined that smaller quantization errors are obtained using the µ-law algorithm. From Tables II and III, it was seen that the MSE, a measurement of quantization error was lower in µ-law quantization for smaller sizes of N, however, for large sizes of N, the uniform quantization was more accurate. This process tested all values of the input signal whereas Table IV separated the signal by high and low valued samples, and then the MSE was found. It was evident that for low valued signals, the µ-law quantization method gave lower MSE values for smaller sizes of N but as N was increased, the uniform quantization was more accurate. For high valued signals, the uniform quantization method gave lower MSE values for all sizes of N. This evidence complies with the purpose of the µ-law algorithm to give more weight to low-valued samples, but it was also discovered that the benefits of this method was true until a large enough N was used, where then the uniform quantization would be preferred.

REFERENCES

[1] Z. Li and M. Andrew, *Fundamentals of Multimedia*, 1st ed. New Jersey: Pearson Education, 2004.

[2] N. Khan, "Week 2 – Sampling and Quantizatin", Ryerson University.