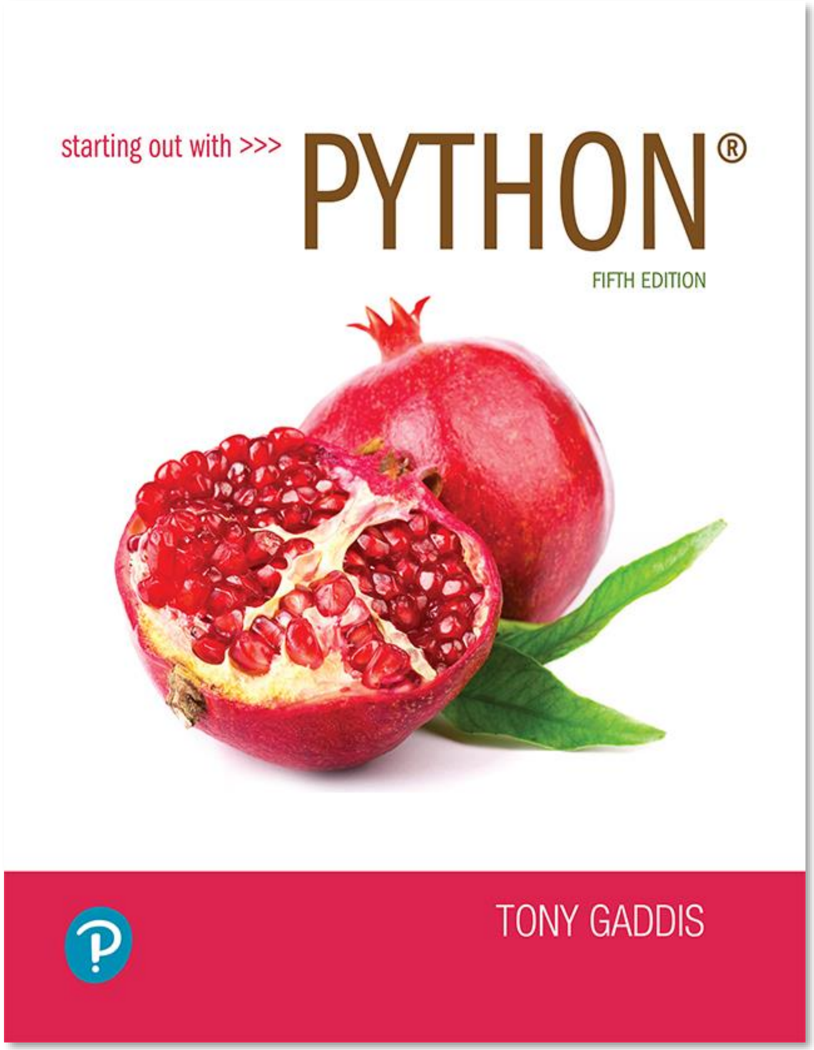
Starting out with Python

Fifth Edition



Chapter 12

Recursion

Topics

- Introduction to Recursion
- Problem Solving with Recursion
- Examples of Recursive Algorithms

Introduction to Recursion (1 of 3)

- Recursive function: a function that calls itself
- Recursive function must have a way to control the number of times it repeats
  - Usually involves an `if-else` statement which defines when the function should return a value and when it should call itself
- Depth of recursion: the number of times a function calls itself

Introduction to Recursion (2 of 3)

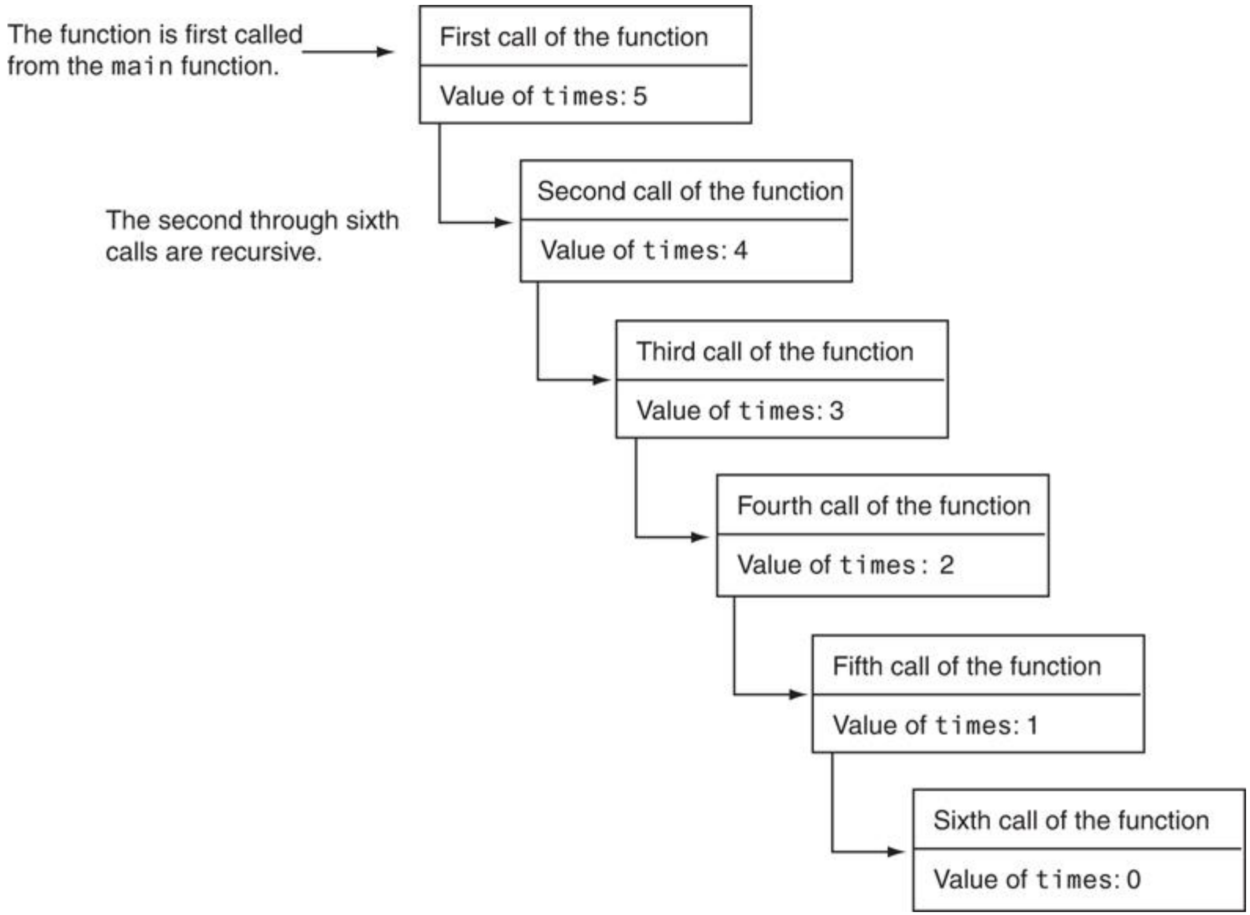


Figure 12-2 Six calls to the message function

Introduction to Recursion (3 of 3)

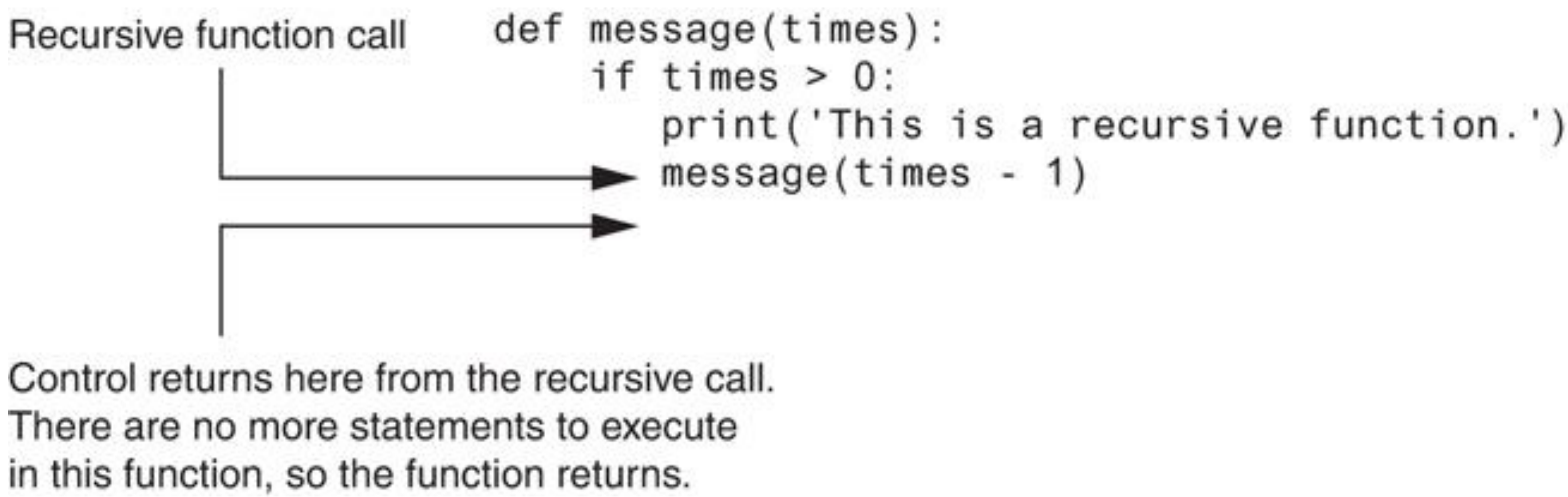


Figure 12-3 Control returns to the point after the recursive function call

Problem Solving with Recursion (1 of 2)

- Recursion is a powerful tool for solving repetitive problems
- Recursion is never required to solve a problem
  - Any problem that can be solved recursively can be solved with a loop
  - Recursive algorithms usually less efficient than iterative ones
    - Due to *overhead* of each function call

Problem Solving with Recursion (2 of 2)

- Some repetitive problems are more easily solved with recursion
- General outline of recursive function:
  - If the problem can be solved now without recursion, solve and return
    - Known as the *base case*
  - Otherwise, reduce problem to smaller problem of the same structure and call the function again to solve the smaller problem
    - Known as the *recursive case*

Using Recursion to Calculate the Factorial of a Number

- In mathematics, the $n!$ notation represents the factorial of a number n
  - For $n = 0$, $n! = 1$
  - For $n > 0$, $n! = 1 \times 2 \times 3 \times \ldots \times n$
- The above definition lends itself to recursive programming
  - $n = 0$ is the base case
  - $n > 0$ is the recursive case
    - factorial(n) = n x factorial(n-1)

Using Recursion (1 of 3)

```
# The factorial function uses recursion to
# calculate the factorial of its argument,
# which is assumed to be nonnegative.
def factorial(num):
      if num == 0:
          return 1
      else:
         return num * factorial(num - 1)
```

Using Recursion (2 of 3)

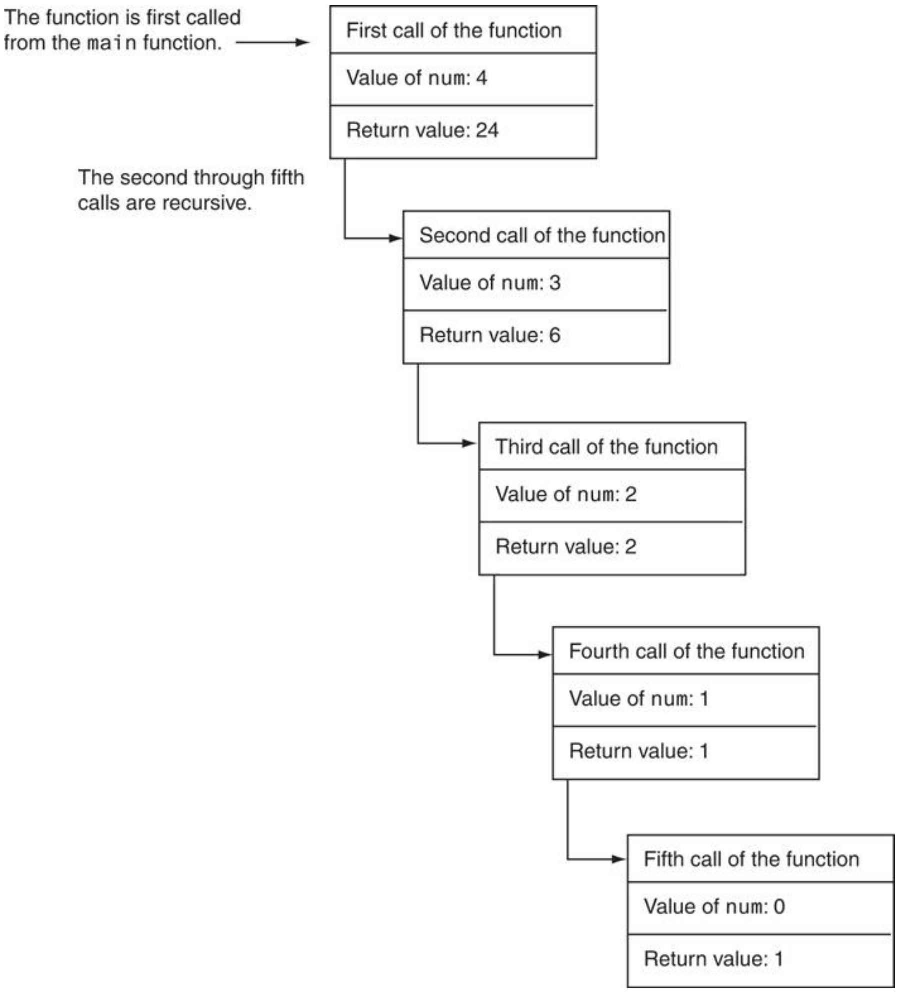


Figure 12-4 The value of num and the return value during each call of the function

Using Recursion (3 of 3)

- Since each call to the recursive function reduces the problem:
  - Eventually, it will get to the base case which does not require recursion, and the recursion will stop
- Usually the problem is reduced by making one or more parameters smaller at each function call

Direct and Indirect Recursion

- Direct recursion: when a function directly calls itself
  - All the examples shown so far were of direct recursion
- Indirect recursion: when function A calls function B, which in turn calls function A

Examples of Recursive Algorithms (1 of 2)

- Summing a range of list elements with recursion
  - Function receives a list containing range of elements to be summed, index of starting item in the range, and index of ending item in the range
  - Base case:
    - `if start index > end index return 0`
  - Recursive case:
    - `return current_number + sum(list, start+1, end)`

Examples of Recursive Algorithms (2 of 2)

```
# The range_sum function returns the sum of a
specified
# range of items in num_list. The start parameter
# specifies the index of the starting item. The end
# parmeter specifies the index of the ending item.
def range_sum(num_list, start, end):
      if start > end:
         return 0
      else:
         return num_list[start] +
         range_sum(num_list,start + 1, end)
```

The Fibonacci Series

- Fibonacci series: has two base cases
  - `if n = 0 then Fib(n) = 0`
  - `if n = 1 then Fib(n) = 1`
  - `if n > 1 then Fib(n) = Fib(n-1) + Fib(n-2)`
- Corresponding function code:

```
# The fid function returns the nth number
# in the Fibonacci series
def fib(n):
     if n == 0:
       return 0
     elif n == 1:
         return 1
     else:
        return fib(n - 1) + fib(n - 2)
```

Finding the Greatest Common Divisor

- Calculation of the greatest common divisor (GCD) of two positive integers
  - If x can be evenly divided by y, then
  - `gcd(x,y) = y`
  - Otherwise, gcd(x,y) = gcd(y, remainder of x/y)
- Corresponding function code:

```
# The gcd function returns the greatest common
# divisor of two numbers.
def gcd(x,y):
      if x % y == 0:
          return y
      else:
            return gcd(x, x % y)
```

The Towers of Hanoi (1 of 5)

- Mathematical game commonly used to illustrate the power of recursion
  - Uses three pegs and a set of discs in decreasing sizes
  - Goal of the game: move the discs from leftmost peg to rightmost peg
    - Only one disc can be moved at a time
    - A disc cannot be placed on top of a smaller disc
    - All discs must be on a peg except while being moved

The Towers of Hanoi (2 of 5)

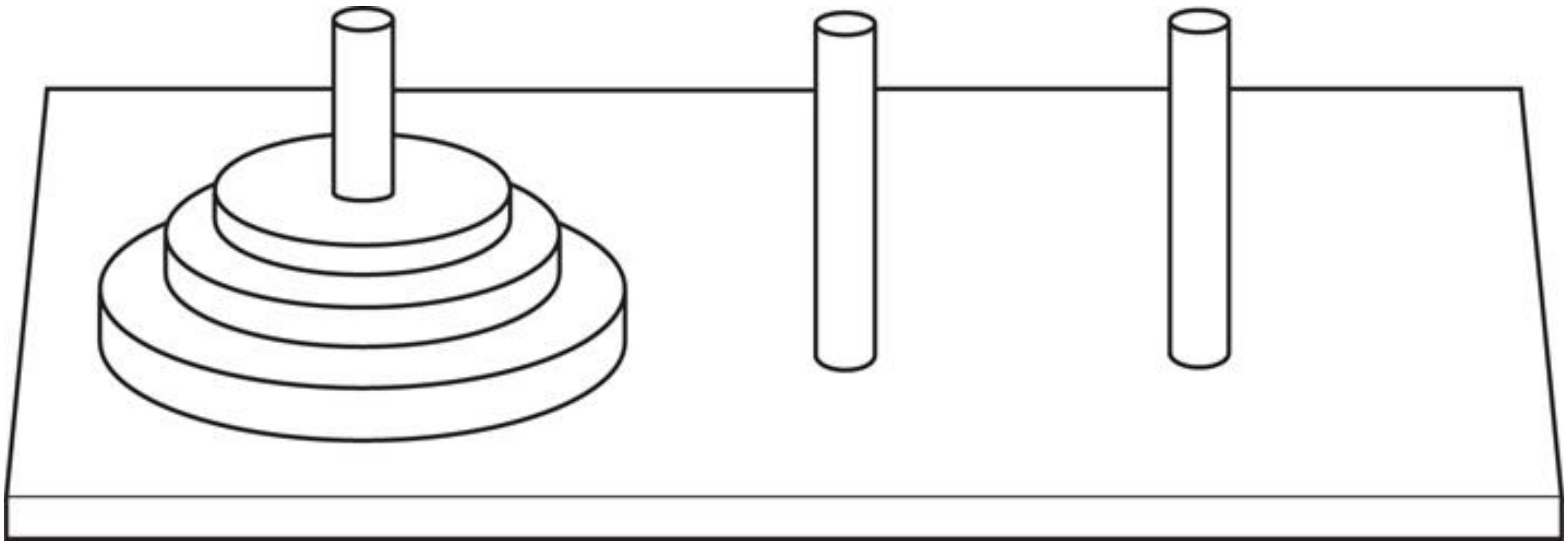


Figure 12-5 The pegs and discs in the Tower of Hanoi game

The Towers of Hanoi (3 of 5)

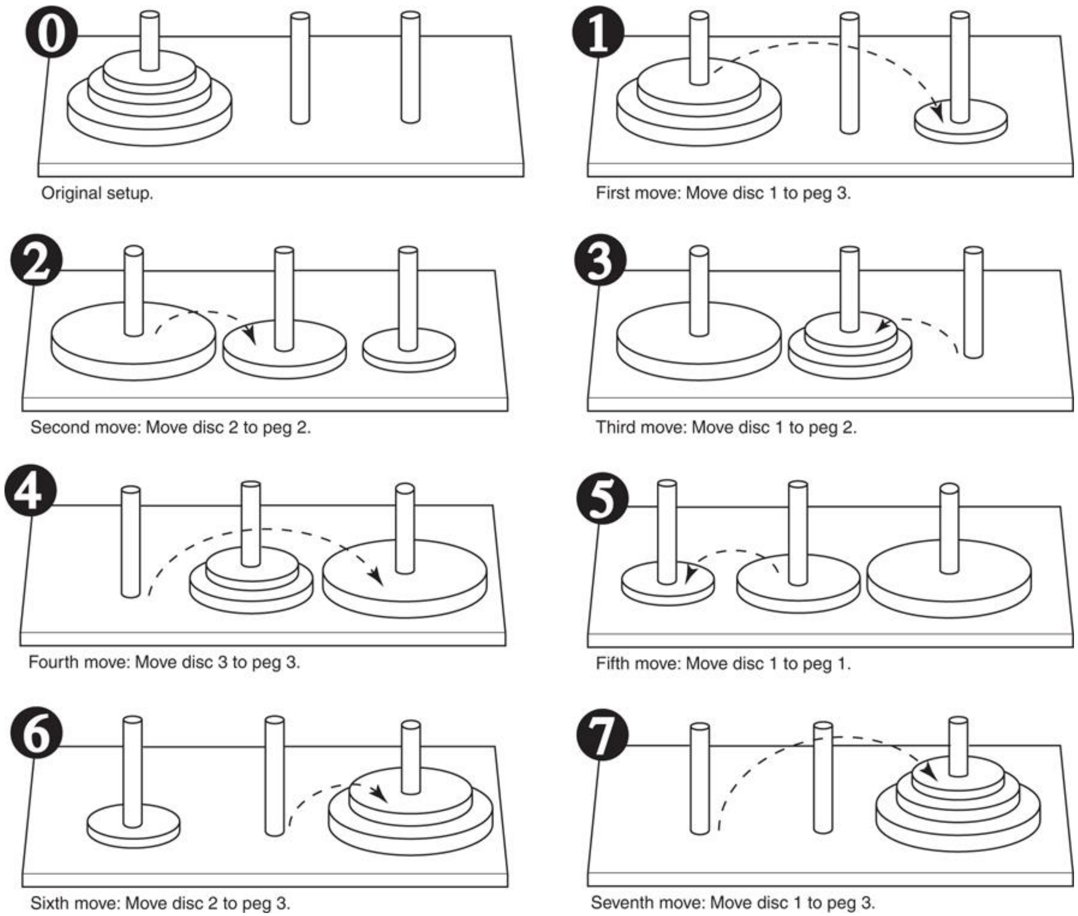


Figure 12-6 Steps for moving three pegs

The Towers of Hanoi (4 of 5)

- Problem statement: move n discs from peg 1 to peg 3 using peg 2 as a temporary peg
- Recursive solution:
  - If n == 1: Move disc from peg 1 to peg 3
  - Otherwise:
    - Move n-1 discs from peg 1 to peg 2, using peg 3
    - Move remaining disc from peg 1 to peg 3
    - Move n-1 discs from peg 2 to peg 3, using peg 1

The Towers of Hanoi (5 of 5)

```
# The moveDiscs function displays a disc move in
# the powers of Hanoi game.
# The parameters are:
#     num:       The number of discs to move.
#     from_peg: The peg to move from.
#     to_peg:   The peg to move to.
#     temp_peg: The temporary peg.
def move_disces(num, from_peg, to_peg, temp_peg):
     if num > 0:
       move_disces(num - 1, from_peg, temp_peg, to_peg)
       print('Move a disc from peg', from_peg, 'to peg',
       to_peg)
       move_disces(num - 1, temp_peg, to_peg, from_peg)
```

Recursion versus Looping

- Reasons not to use recursion:
  - Less efficient: entails function calling overhead that is not necessary with a loop
  - Usually a solution using a loop is more evident than a recursive solution
- Some problems are more easily solved with recursion than with a loop
  - Example: Fibonacci, where the mathematical definition lends itself to recursion

Summary

- This chapter covered:
  - Definition of recursion
  - The importance of the base case
  - The recursive case as reducing the problem size
  - Direct and indirect recursion
  - Examples of recursive algorithms
  - Recursion versus looping