# 16-350
# Planning Techniques for Robotics

## Search Algorithms:
## Multi-goal A*, IDA*

*Maxim Likhachev*

*Robotics Institute*

*Carnegie Mellon University*

# Agenda

- A* with multiple goals
- Iterative Deepening A* (IDA*)

# Support for Multiple Goal Candidates

- How to compute a least-cost path to any one of the possible goals?
  - Example 1: Computing a least-cost path to a parking spot given multiple parking spaces (some are better, some are worse, some are closer, some are further)
  - Example 2: Catching a moving target whose future trajectory is known (i.e., multiple potential intercept points)
  - Example 3: Mapping/exploration (next class)

# A* Search

**Main function**

$g(s_{start}) = 0$; all other $g$-values are infinite; $OPEN = \{s_{start}\}$;

ComputePath();

publish solution;

**ComputePath function**

while($s_{goal}$ is not expanded and $OPEN \neq 0$)

  remove $s$ with the smallest $[f(s) = g(s)+h(s)]$ from $OPEN$;

  insert $s$ into $CLOSED$;

  for every successor $s'$ of $s$ such that $s'$ not in $CLOSED$

    if $g(s') > g(s) + c(s,s')$

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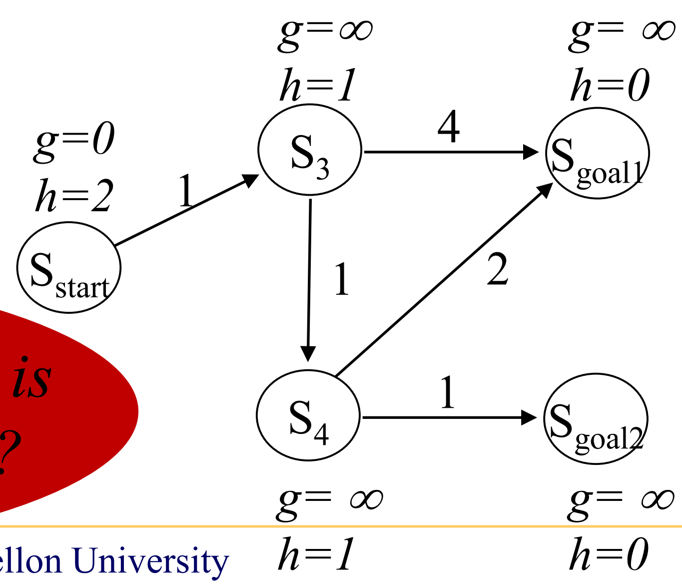
      insert $s'$ into $OPEN$;




*How to find a least-cost path that is lowest across all possible goals?*

# Introducing “**imaginary**” goal

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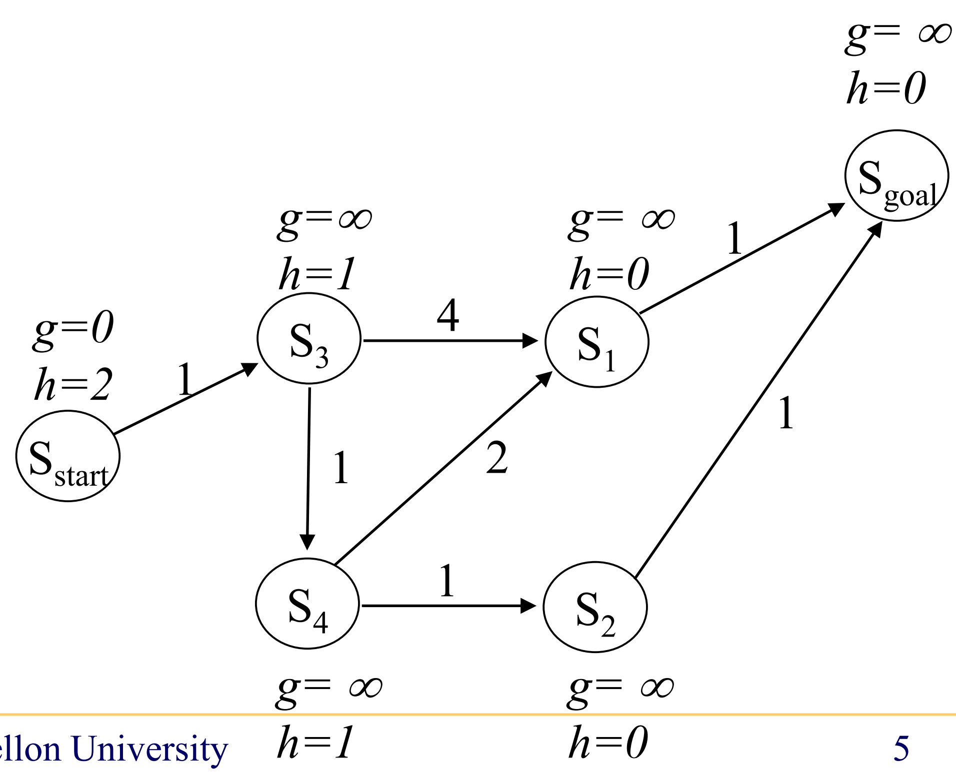
*Equivalent problem but with a single goal!*



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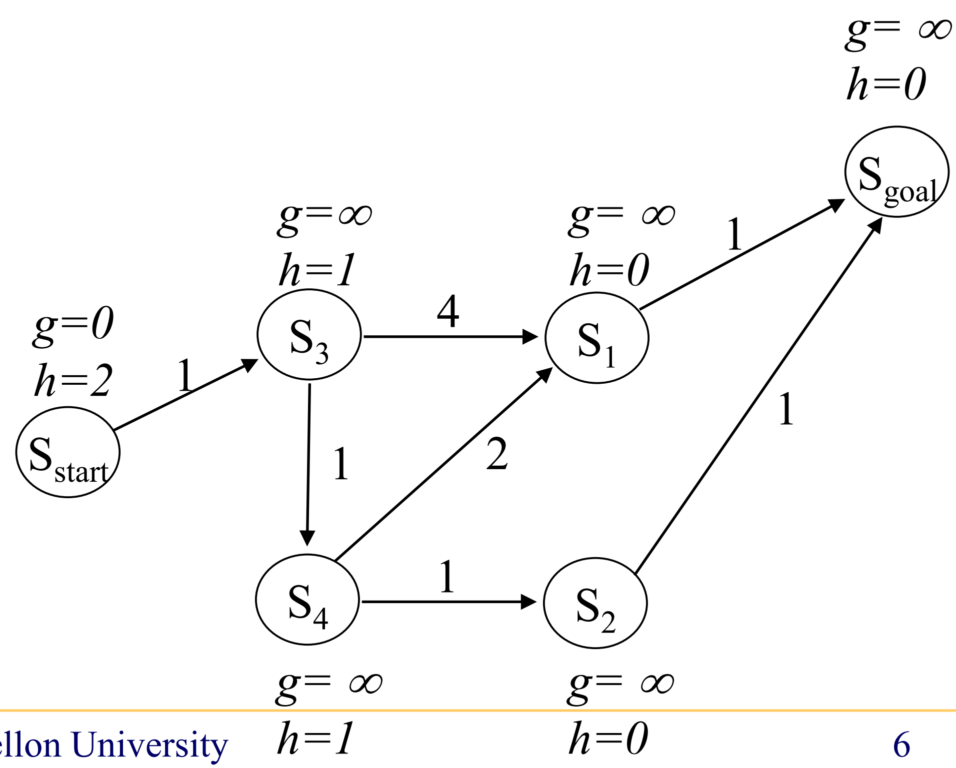
*How to prove it?*



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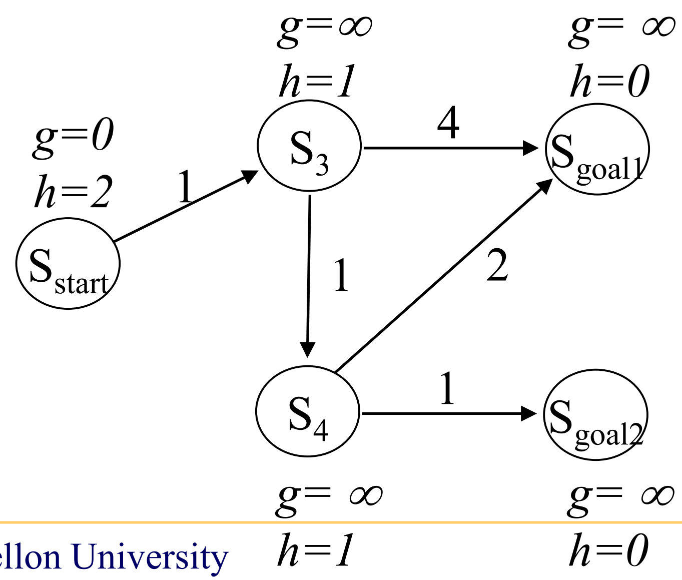

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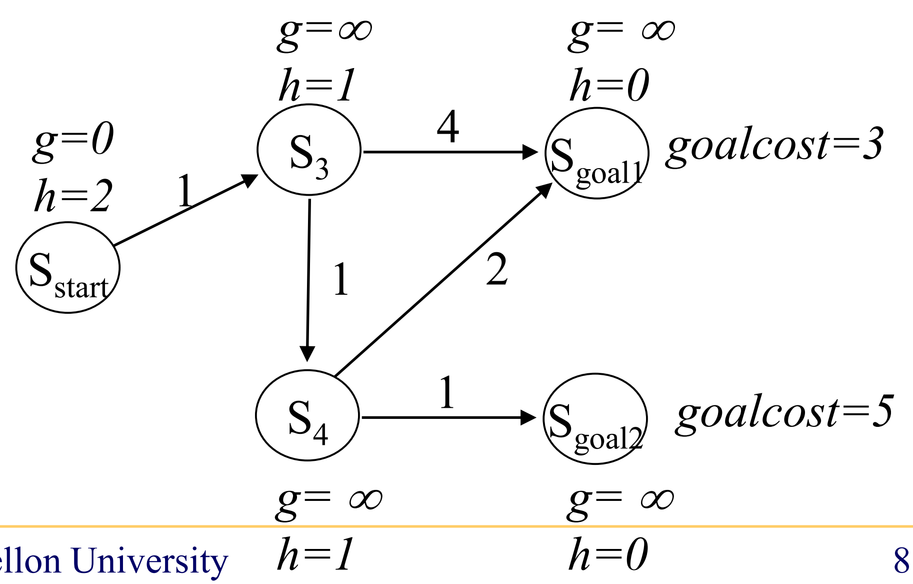

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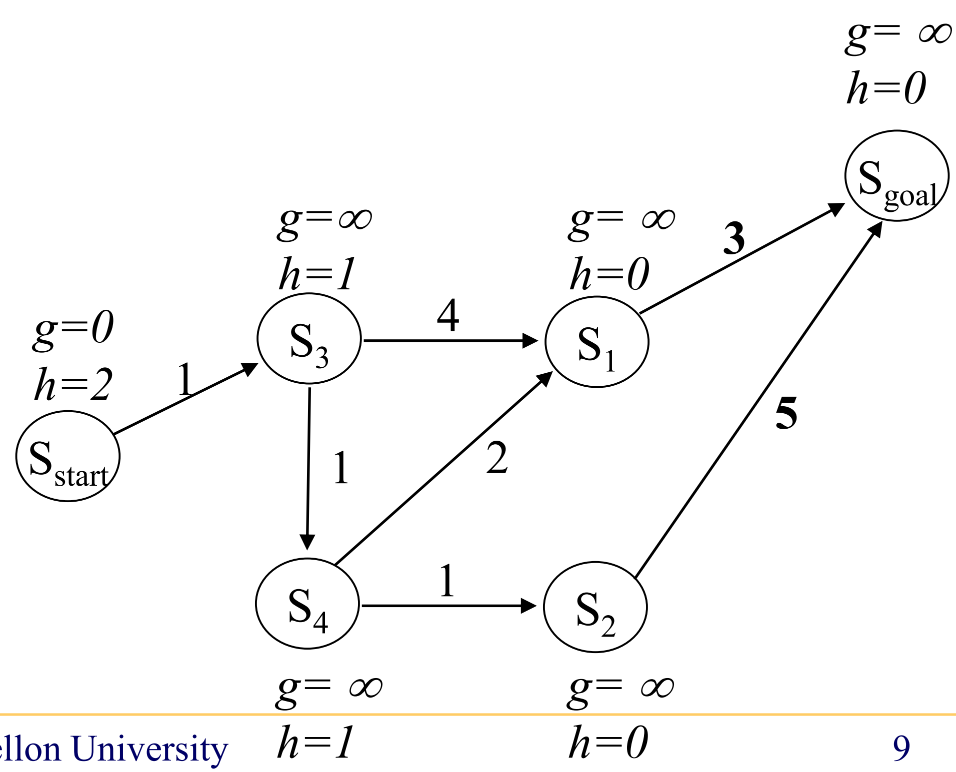

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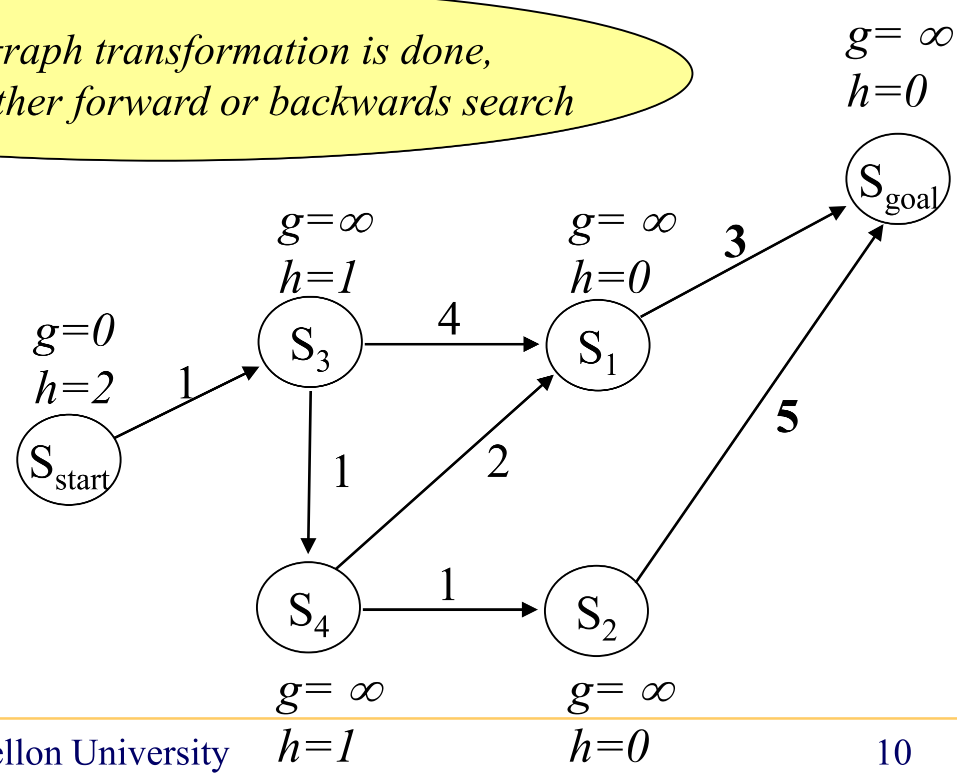

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*Any impact on how heuristics is computed?*

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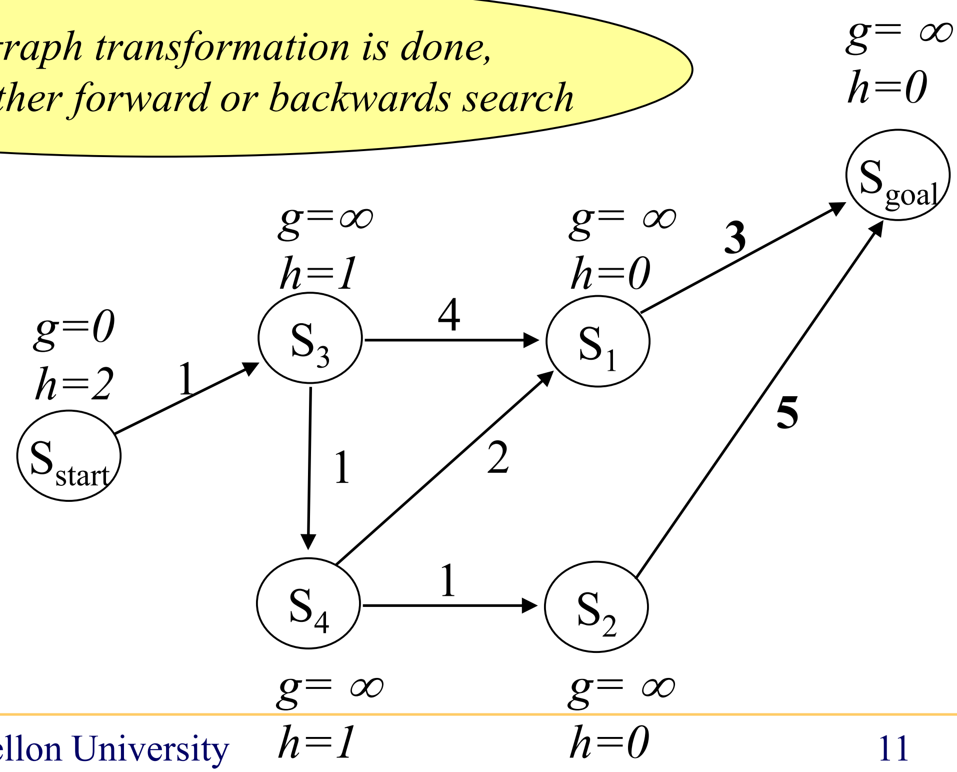

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# Memory Issues

- A* does provably minimum number of expansions (O(n)) for finding a provably optimal solution

- Memory requirements of weighted A* are often but not always better

# Search with Linear Memory Requirement

- Depth-First Search (w/o coloring all expanded states):
  - explore each every possible path one at a time avoiding looping and keeping in the memory only the best path discovered so far

**DFS function**

*LIFO list = {$s_{start}$}; //stack*

*bestpathsofar = NONE;*

*While (list != 0)*

    *s = list.pop();*

    *if (s = $s_{goal}$)*

      *if (cost of the found path from $s_{start}$ to s < cost of bestpathsofar)*

        *set bestpathsofar to the current path from $s_{start}$ to s*

    *else*

      *for every successor s' of s*

        *list.push(s');*

*return bestpathsofar;*

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    *for every successor s' of s*
      *list.push(s');*
*return bestpathsofar;*

*What is memory complexity?*

*What are its disadvantages?*

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  - Complete and optimal (assuming finite state-spaces)
  - Memory: $O(bm)$, where $b$ – max. branching factor, $m$ – max. pathlength in graph
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  - Example:
    - graph: a 4-connected grid of 40 by 40 cells, start: center of the grid
    - A* expands up to 800 states, DFS may expand way over $4^{20} > 10^{12}$ states

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*What if goal is few steps away in a huge state-space?*

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# Search with Linear Memory Requirement

- IDA* (Iterative Deepening A*) [Korf, ‘85]:
  1. *set* $f_{max} = 1$ *(or some other small value)*
  2. *execute (previously explained) DFS that does not expand states with* $f > f_{max}$
  3. *If DFS returns a path to the goal, return it*
  4. *Otherwise* $f_{max} = f_{max} + 1$ *(or larger increment) and go to step 2*

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  3. *If DFS returns a path to the goal, return it*
  4. *Otherwise* $f_{max} = f_{max} + 1$ *(or larger increment) and go to step 2*

  - Complete and optimal in any state-space (with positive costs)

  - Memory: $O(bl)$, where $b$ – max. branching factor, $l$ – length of optimal path

  - Complexity: $O(kb^l)$, where $k$ is the number of times DFS is called

# Summary

- Support for multiple potential goals is a common problem in robotics and can often be easily tacked by the graph transformation (introducing "imaginary" goal)

- In the worst case, memory requirements of A* are the full size of the graph

- Iterative Deepening A* (IDA*) – simple alternative with memory requirements linear in the length of the optimal path to the goal.
  - It can perform substantially more work than A* though