16-350
Planning Techniques for Robotics

Planning Representations:
Probabilistic Roadmaps for Continuous Spaces

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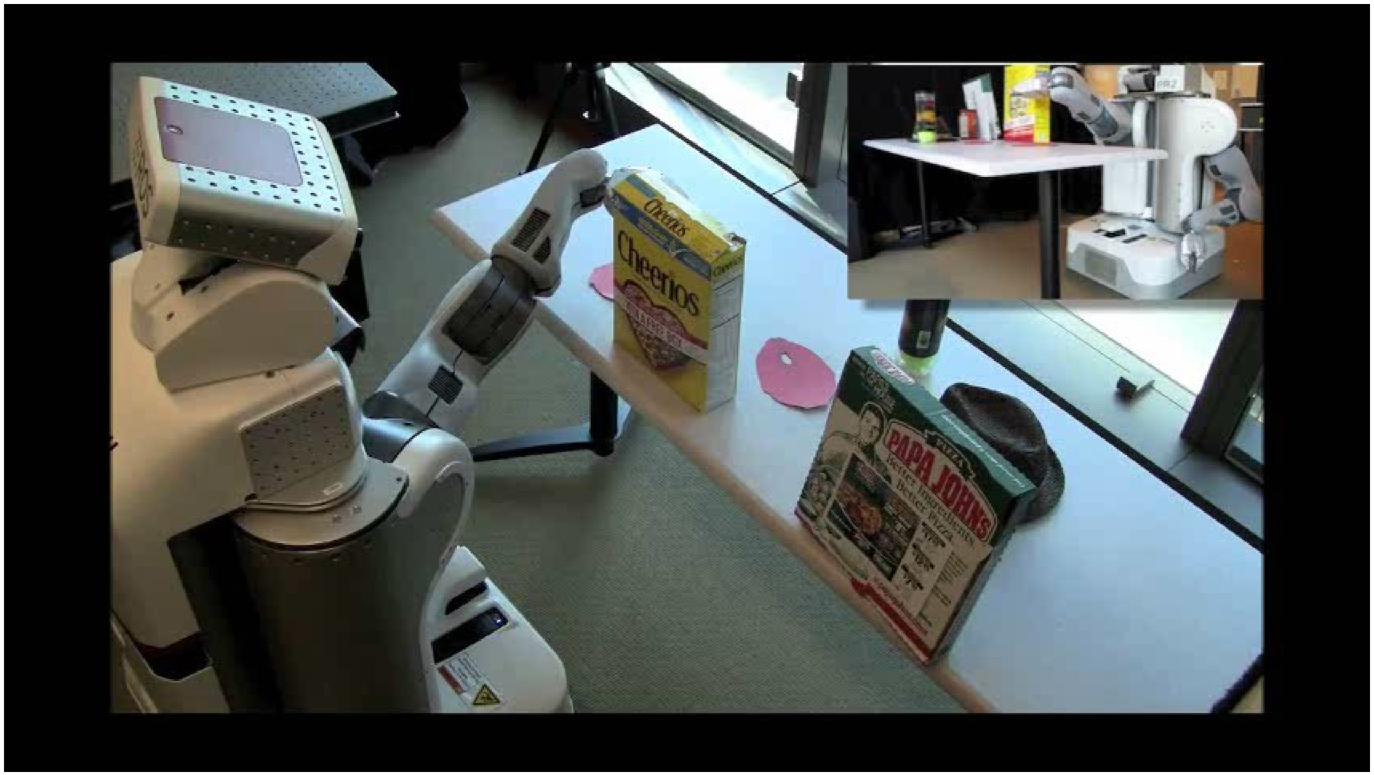
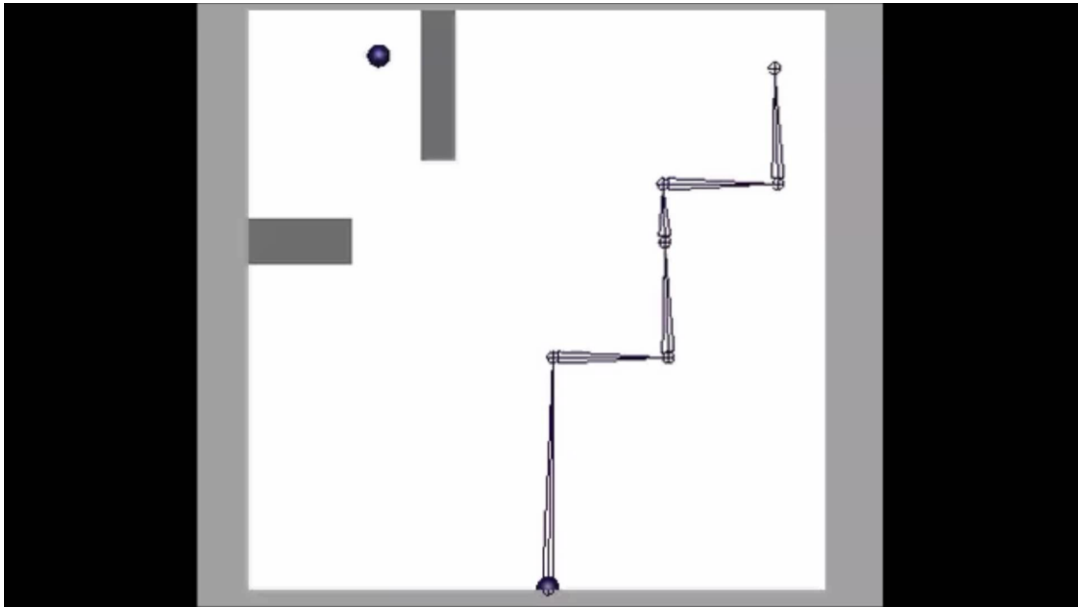
Example

- Planning for manipulation



Example

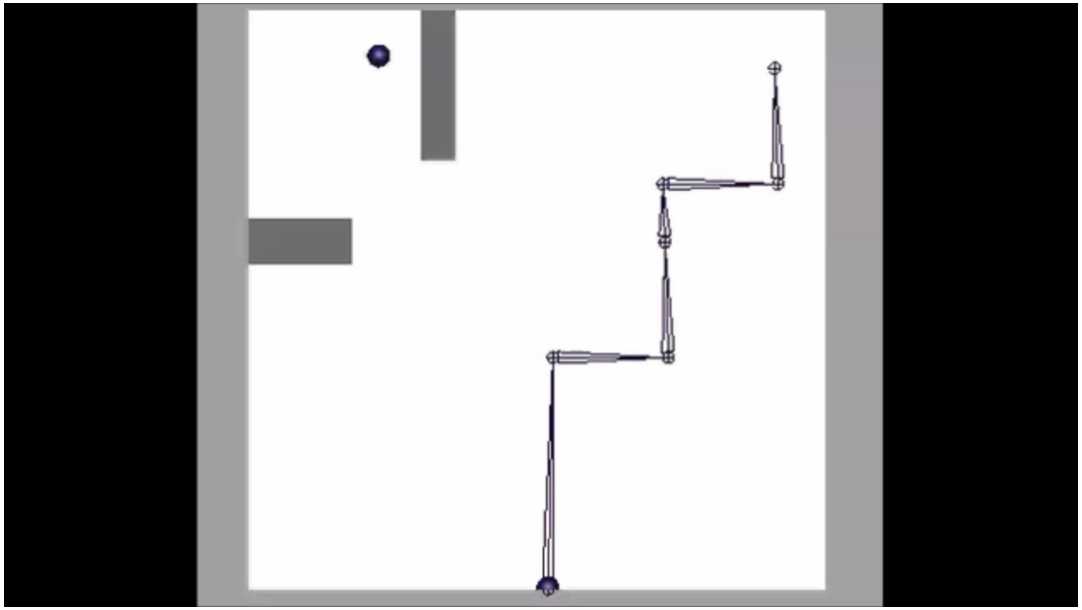
- Planning for manipulation



Example

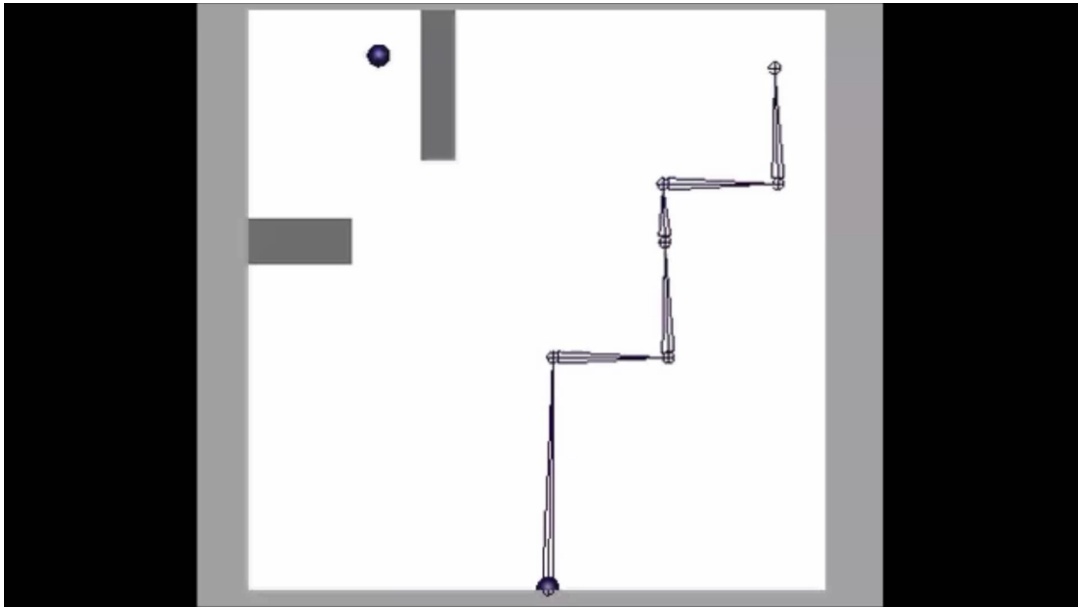
- Planning for manipulation
  - robot state is defined by joint angles $Q = \{q_1,\ldots,q_6\}$
  - need to find a (least-cost) motion that connects Q_{start} to Q_{goal}

Constraints?



Example

- Planning for manipulation
  - robot state is defined by joint angles $Q = \{q_1, \ldots, q_6\}$
  - need to find a (least-cost) motion that connects Q_{start} to Q_{goal}
  - Constraints:
    - All joint angles should be within corresponding joint limits
    - No collisions with obstacles and no self-collisions



Exampl

Can we use a grid-based representation for planning?

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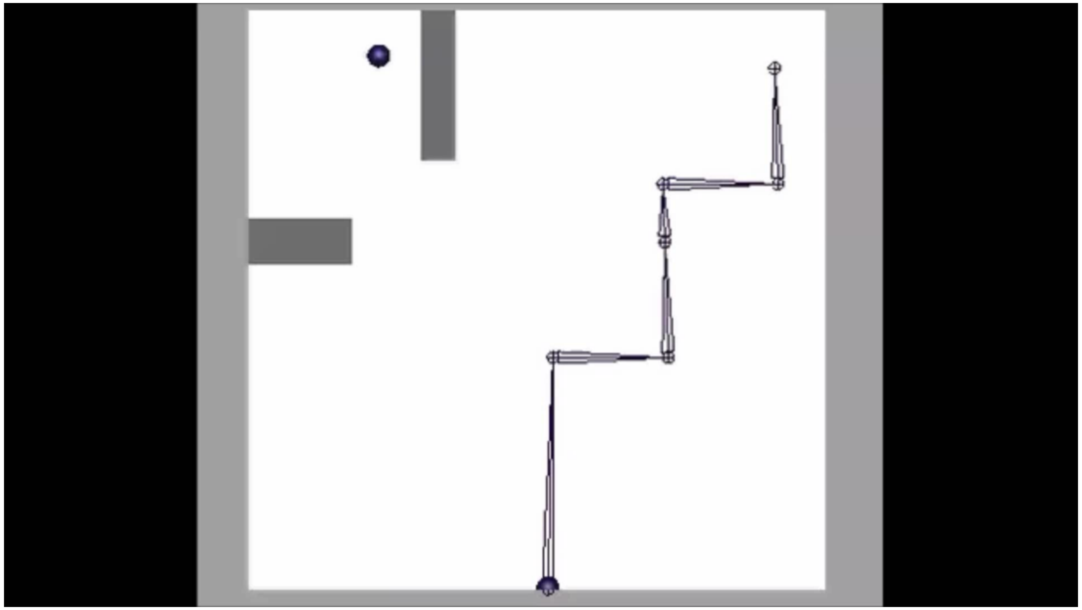
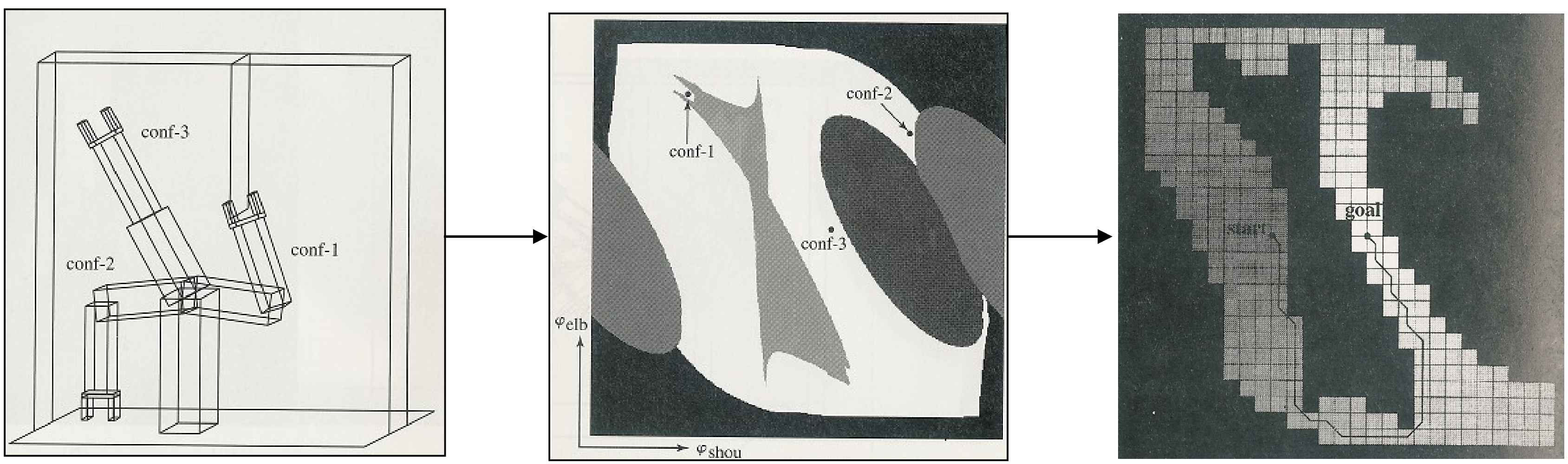


Resolution Complete vs. Sampling-based Planning

- Resolution complete planning (e.g. Grid-based):
  - generate a systematic (uniform) representation (graph) of a free C-space (C_{free})
  - search the generated representation for a solution guaranteeing to find it if one exists (completeness)
  - can interleave the construction of the representation with the search (i.e., construct only what is necessary)

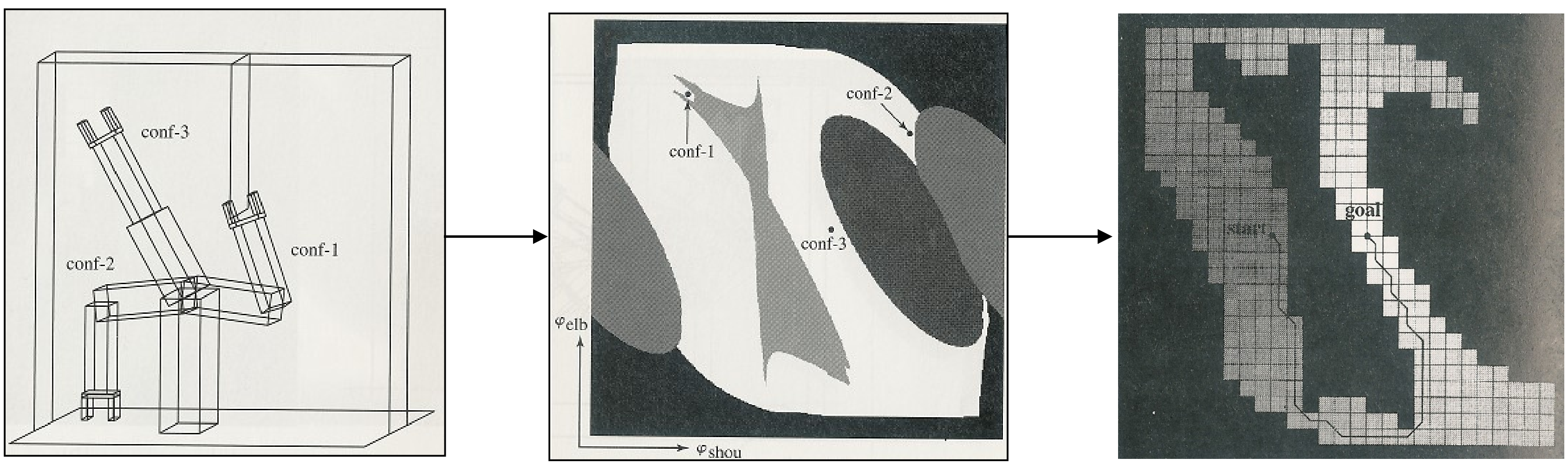


the example above is borrowed from "AI: A Modern Approach" by S. RusselL & P. Norvig

Resolution Complete vs. Sampling-based Planning

- Resolution complete planning (e.g. Grid-based):
  - complete and provide sub-optimality bounds on the solution

Great. Any issues?



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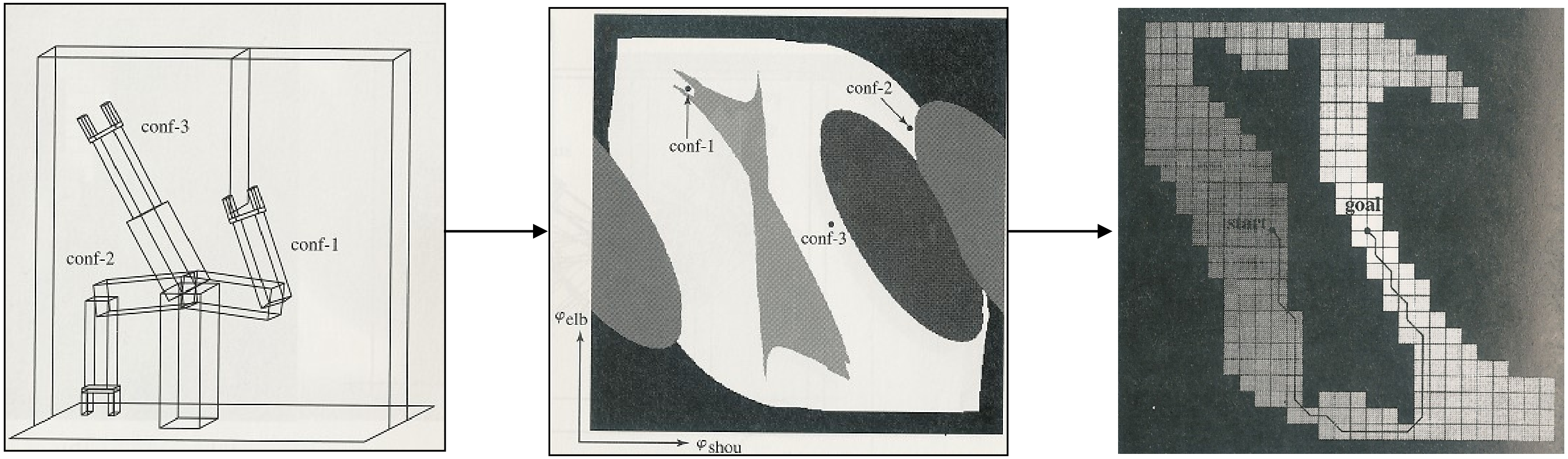
Resolution Complete vs. Sampling-based Planning

- Resolution complete planning (e.g. Grid-based):
  - complete and provide sub-optimality bounds on the solution
  - can get computationally very expensive, especially in high-D

Estimate # of states
for 6DOF arm on the board



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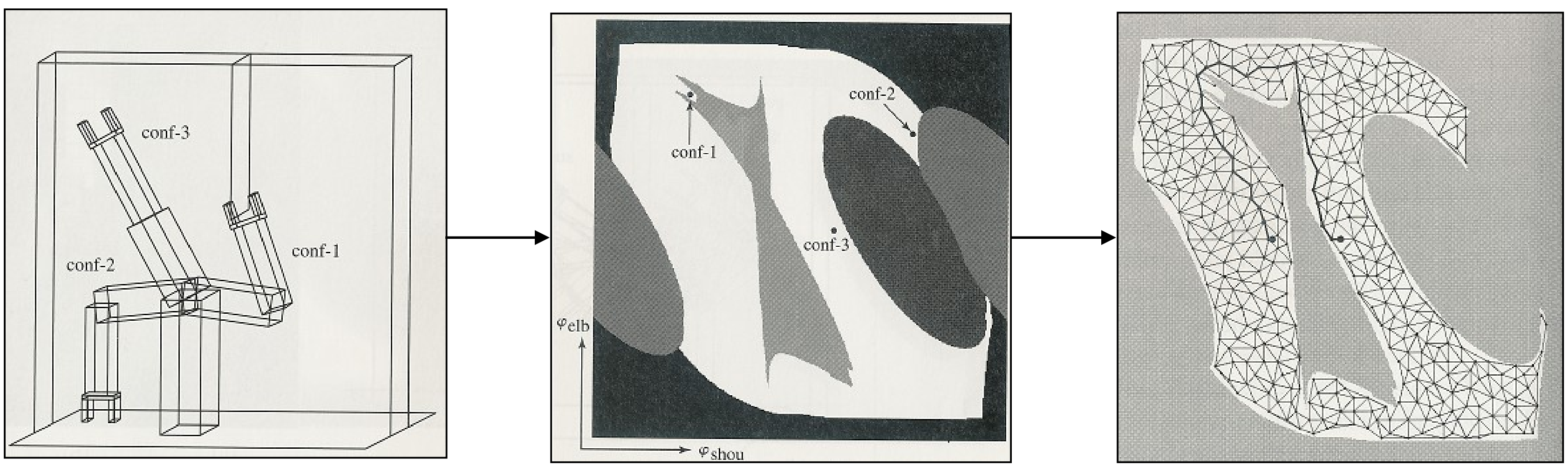
Resolution Complete vs. Sampling-based Planning

- Sampling-based planning:

Main observation:
The space is continuous and rather benign!



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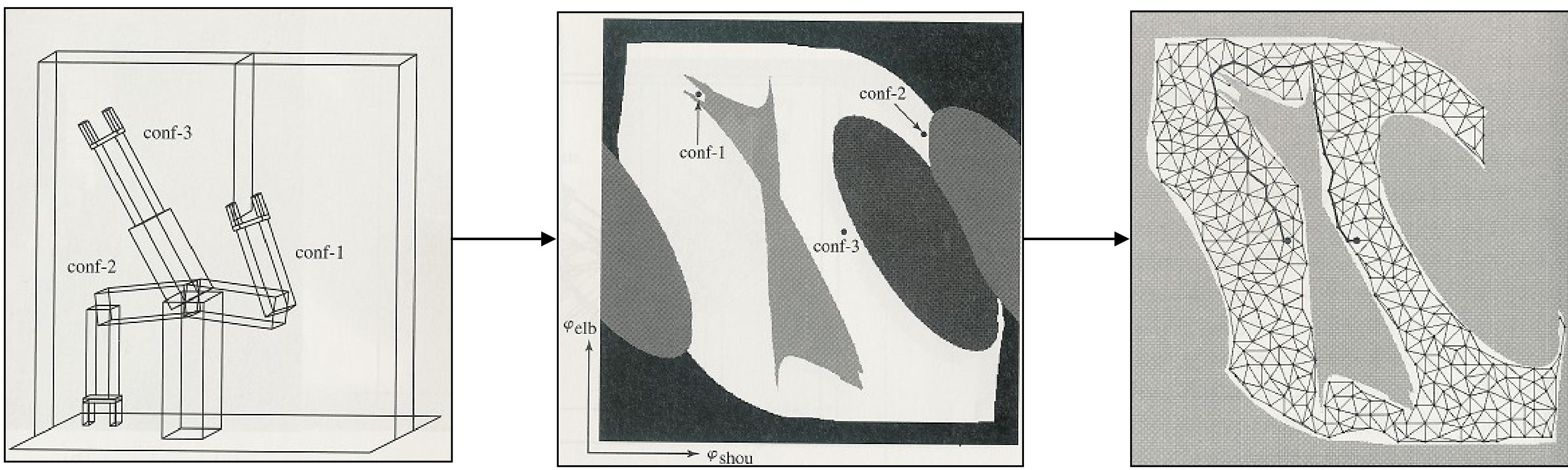
Resolution Complete vs. Sampling-based Planning

- Sampling-based planning:
  - generate a sparse (sample-based) representation (graph) of a free C-space (C_{free})
  - search the generated representation for a solution



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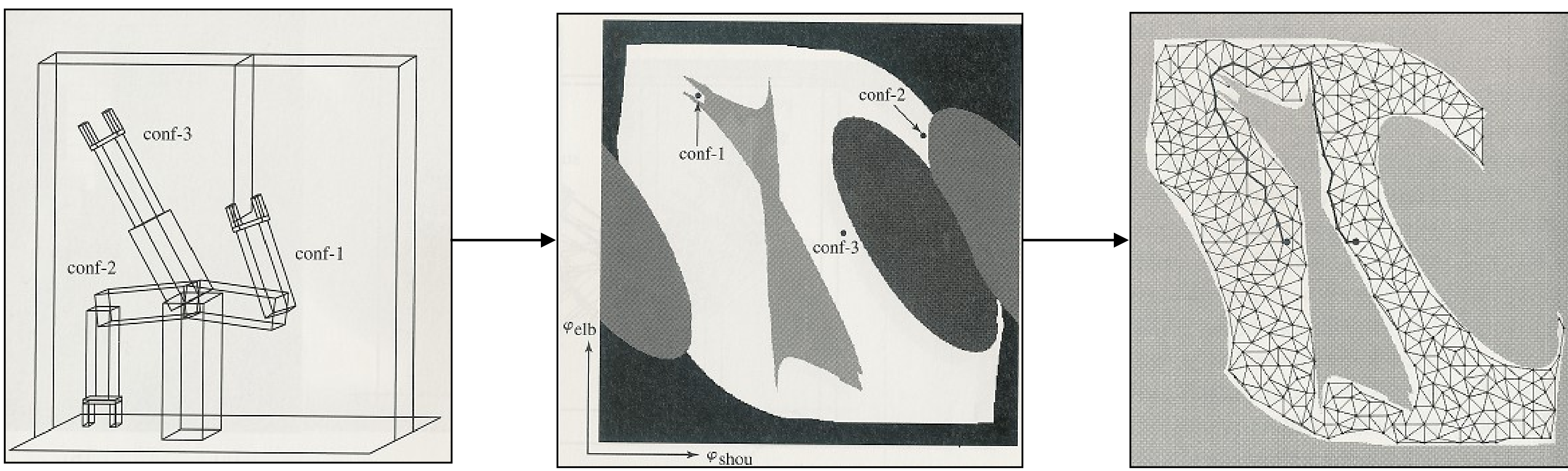
Resolution Complete vs. Sampling-based Planning

- Sampling-based planning:
  - provide **probabilistic** completeness guarantees
    - guaranteed to find a solution, if one exists, but only in the limit of the number of samples (that is, only as the number of samples approaches infinity)
  - well-suited for high-dimensional planning



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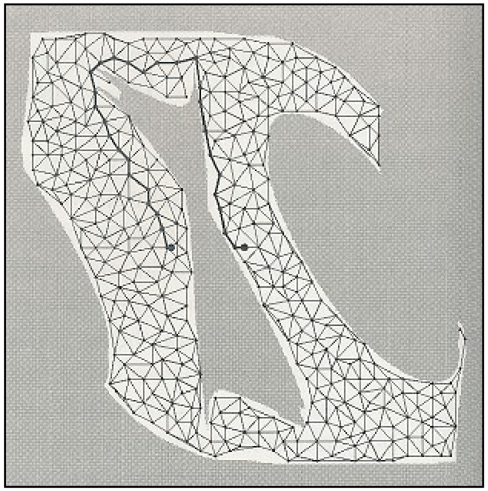
Main Questions in Sampling-based Planning

- How to select samples to construct a “good” graph
- How to search the graph
- Can we interleave these steps

Probabilistic Roadmaps (PRMs)

Step 1. Preprocessing Phase: Build a roadmap (graph) $\mathcal{G}$ which, hopefully, should be accessible from any point in C_{free}

Step 2. Query Phase: Given a start configuration q_I and goal configuration q_G, connect them to the roadmap $\mathcal{G}$ using a local planner, and then search the augmented roadmap for a shortest path from q_I to q_G



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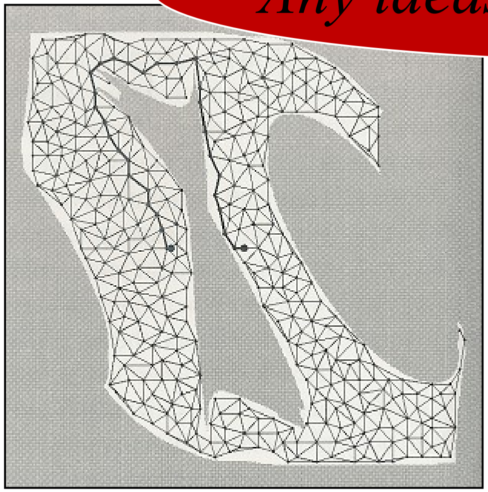
Any ideas for the local planner?



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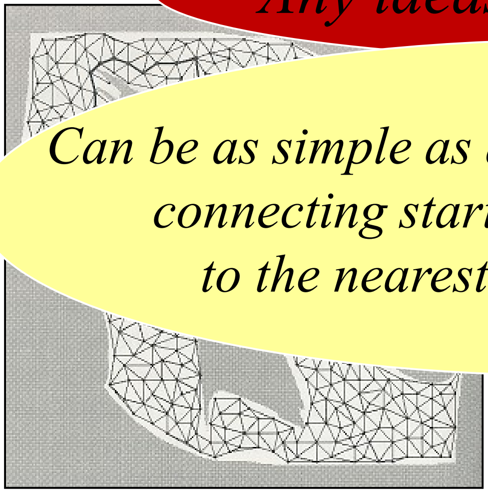


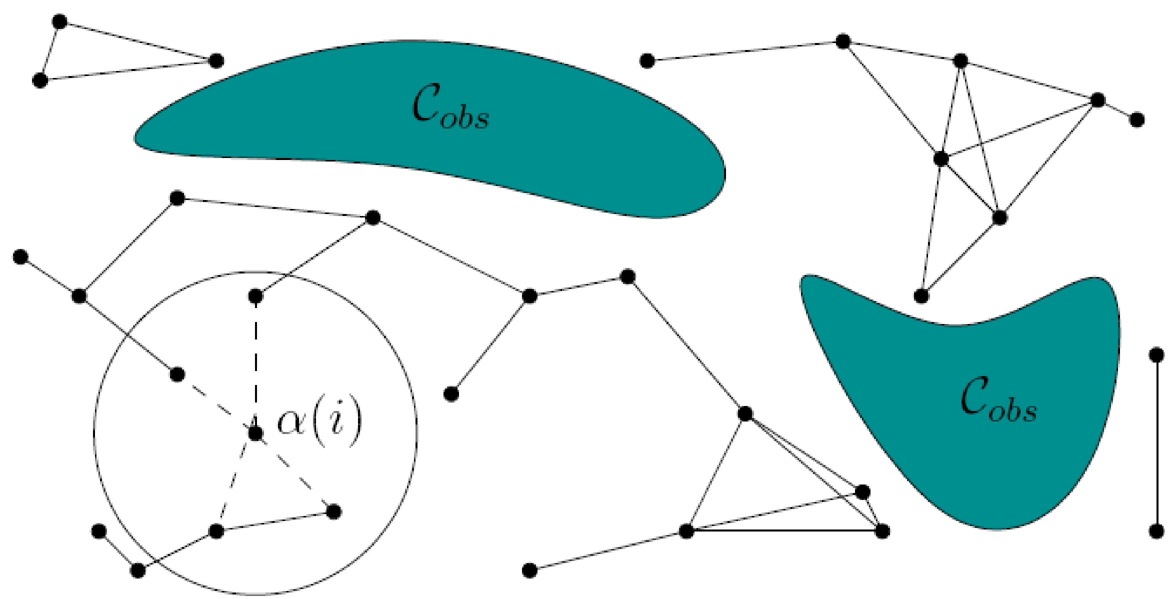
Can be as simple as a straight line (interpolation) connecting start (or goal) configuration to the nearest vertex in the roadmap

Probabilistic Roadmaps (PRMs)

Step 1: Preprocessing Phase.

BUILD_ROADMAP

$\mathcal{G}$.init(); $i \leftarrow 0$;
while $i < N$
  if $\alpha(i) \in \mathcal{C}_{free}$ **then**
    $\mathcal{G}$.add_vertex($\alpha(i)$); $i \leftarrow i + 1$;
    for each $q \in$ NEIGHBORHOOD($\alpha(i)$,$\mathcal{G}$)
      if ((**not** $\mathcal{G}$.same_component($\alpha(i), q$)) **and** CONNECT($\alpha(i), q$)) **then**
        $\mathcal{G}$.add_edge($\alpha(i), q$);



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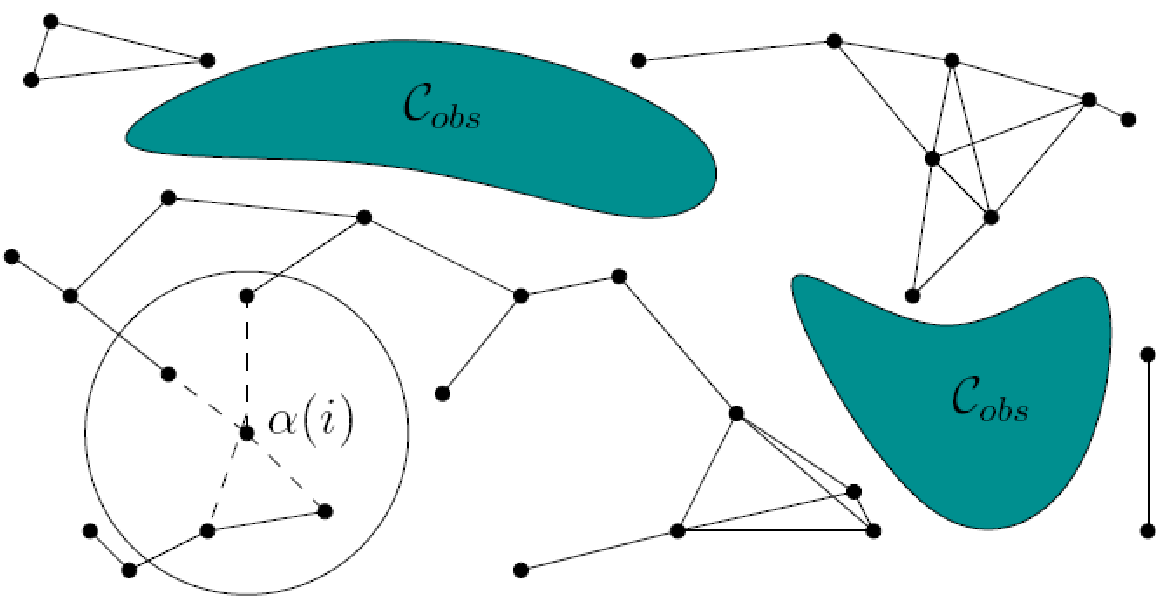
Probabilistic Roadmaps (PRMs)

Step 1: Preprocessing Phase.

new i^{th} configuration sample

```
BUILD_ROADMAP
  G.init(); i ← 0;
  while i < N
      if α(i) ∈ C_free then
          G.add_vertex(α(i))
          for each q ∈ NEIG
              if ((not G.same_com
                  G.add_edge(α(i), q);
```

α(i) is an ith sample in the configuration space. Each sample can be drawn uniformly (or more intelligently as described later)



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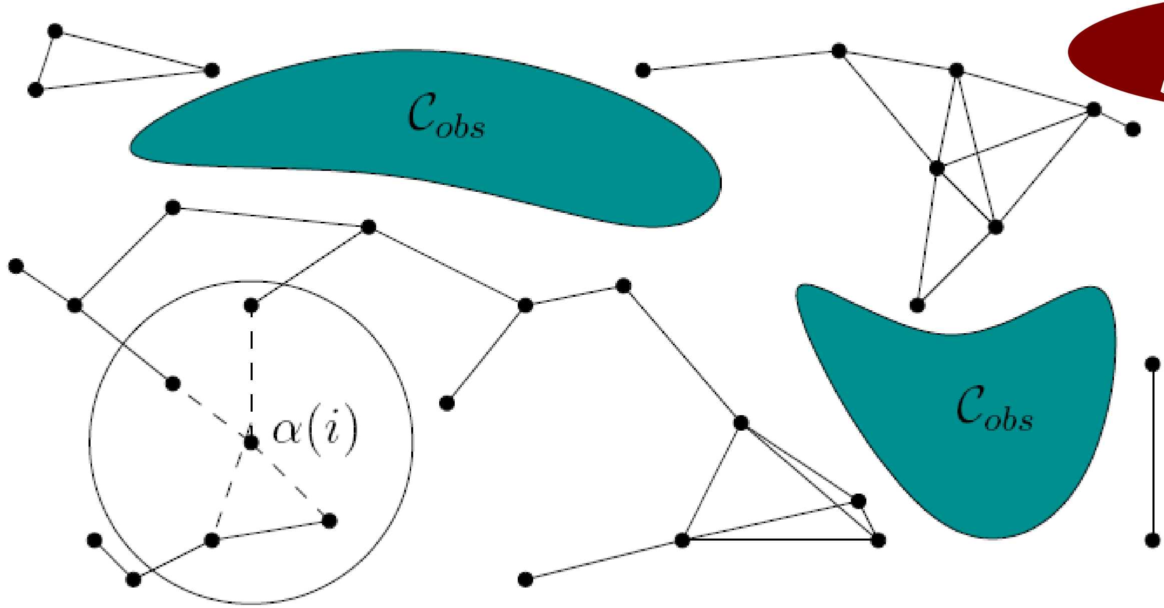
      if ((**not** $\mathcal{G}$.same_component($\alpha(i), q$)) **and** CONNECT($\alpha(i), q$)) **then**
        $\mathcal{G}$.add_edge($\alpha(i), q$);

some region around the configuration sample

can be connected by a local planner



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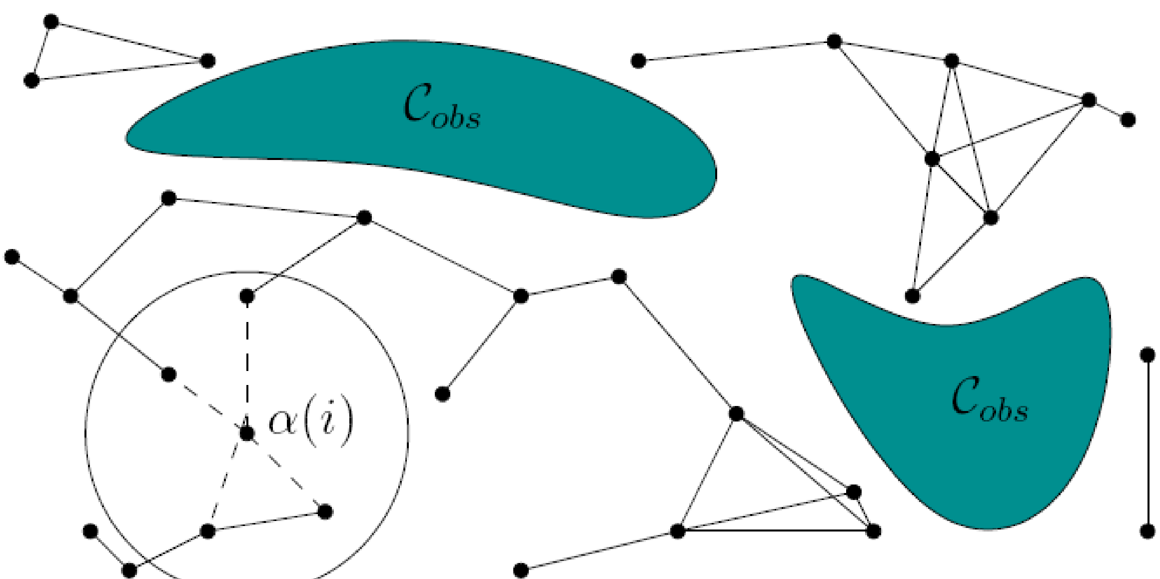
      if ((**not** $\mathcal{G}$.same_component($\alpha(i), q$)) **and** CONNECT($\alpha(i), q$)) **then**
        $\mathcal{G}$.add_edge($\alpha(i), q$);

can be replaced with:
"number of successors of q < K"



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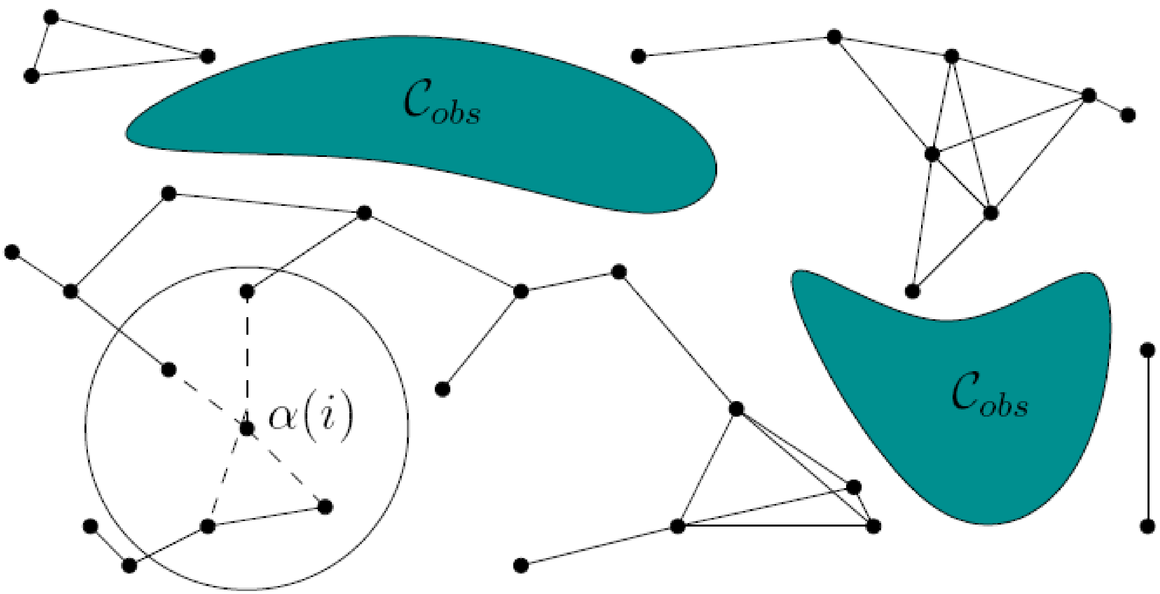
Probabilistic Roadmaps (PRMs)

Step 1: Preprocessing Phase.

Efficient implementation of $q \in$ NEIGHBORHOOD$(\alpha(i),\mathcal{G})$

- select K vertices closest to $\alpha(i)$
- select K (often just 1) closest points from each of the components in $\mathcal{G}$
- select all vertices within radius r from $\alpha(i)$



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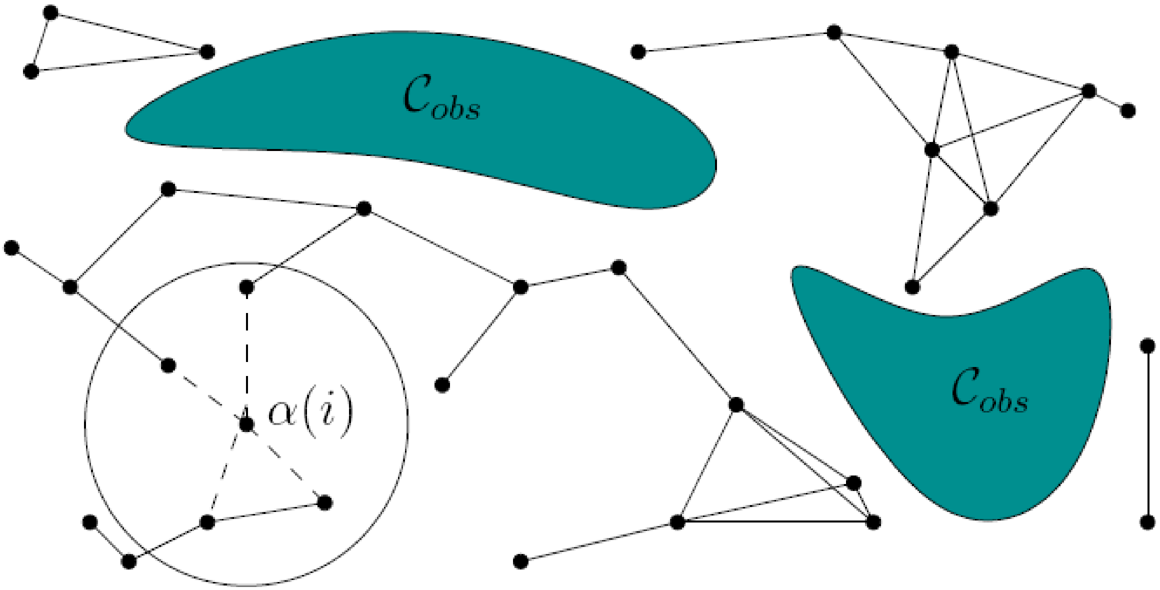
Probabilistic Roadmaps (PRMs)

Step 1: Preprocessing Phase.

Why do we need anything better than uniform sampling?

Sampling strategies

- sample uniformly from C_{free}



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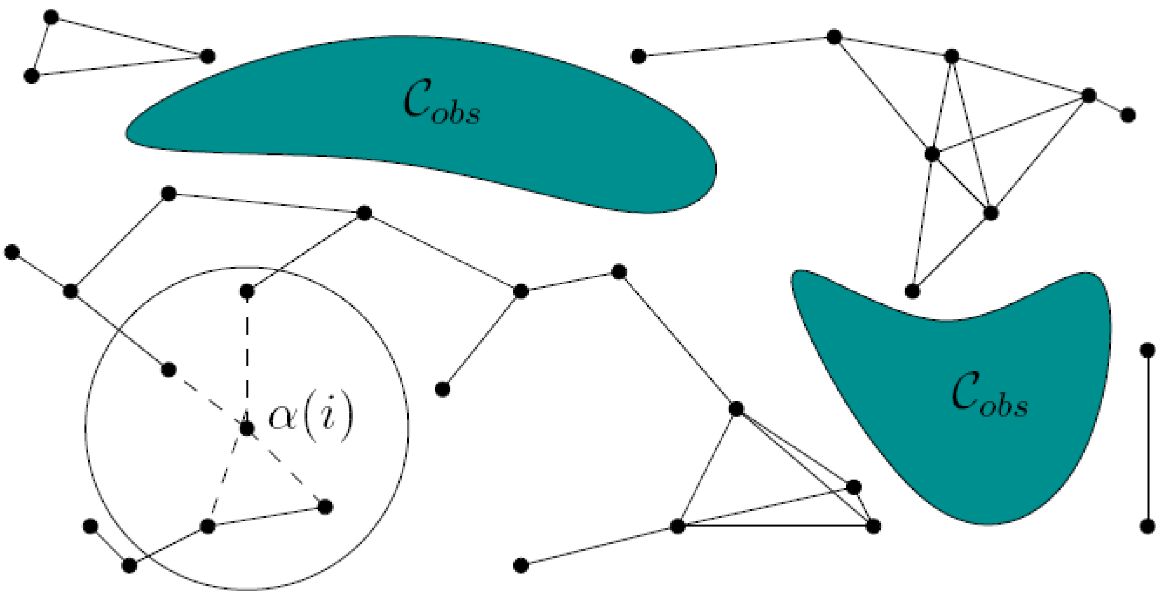
Probabilistic Roadmaps (PRMs)

Step 1: Preprocessing Phase.

Sampling strategies

- sample uniformly from C_{free}
- select at random an existing vertex with a probability distribution inversely proportional to how well-connected a vertex is, and then generate a random motion from it to get a sample $\alpha(i)$
- bias sampling towards obstacle boundaries



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Probabilistic Roadmaps (PRMs)

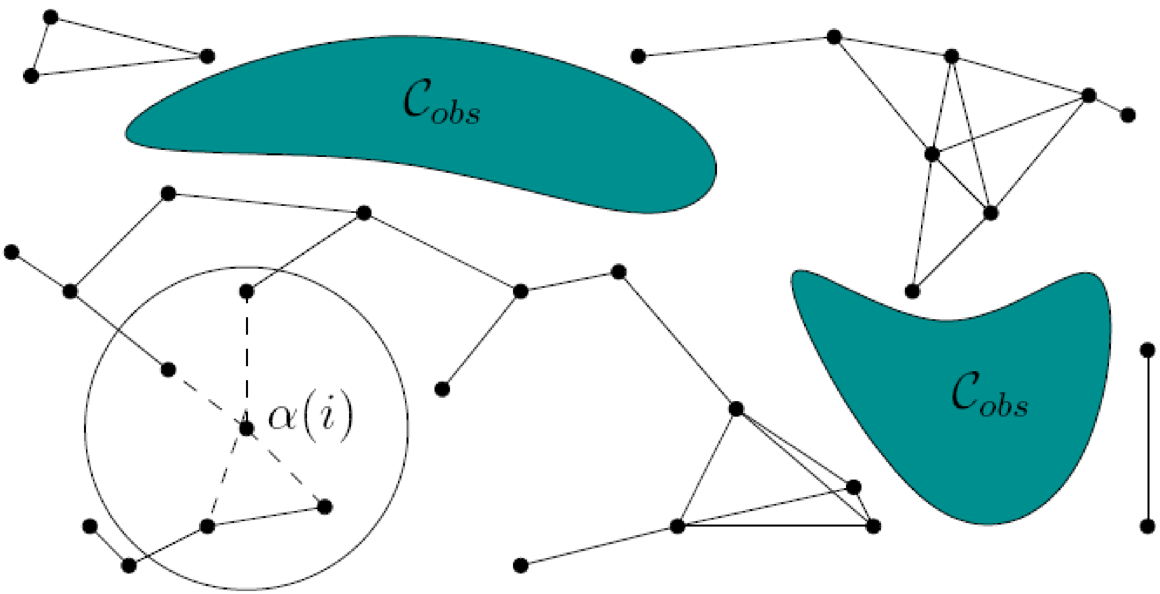
Step 1: Preprocessing Phase.

Sampling strategies

- sample q_1 and q_2 from Gaussian around q_1 and if either is in C_{obs}, then the other one is set as $\alpha(i)$

- sample q_1, q_2 , q_3 from Gaussian around q_2 and set q_2 as $\alpha(i)$ if q_2 is in C_{free}, and q_1 and q_3 are in C_{obs}

- bias sampling away from obstacles



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Summary

- Resolution-complete approaches (like Grid-based or Lattice-based graphs) tend to be used more for low-dimensional planning

- Sampling-based approaches (like PRMs) tend to be used more for high-dimensional continuous planning

- PRMs
  - typically fast but quite sub-optimal
  - mostly concerned with finding ANY solution (rather than close to optimal)
  - provide completeness guarantee in the limit of sample
  - can have difficulty for problems with "narrow passages"
  - very popular when planning for robotics arms