

16-350

Planning Techniques for Robotics

***Planning Representations/Search Algorithms:
Rapidly Exploring Random Trees (RRT)***

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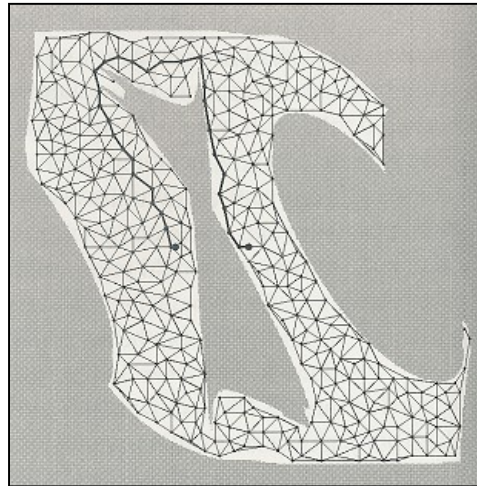
Probabilistic Roadmaps (PRMs)

*Great for problems where a planner
has to plan many times for different start/goal pairs
(step 1 needs to be done only once)*

Not so great for single shot planning

Step 1. Preprocessing Phase: Build a roadmap (graph) \mathcal{G} which, hopefully, should be accessible from any point in C_{free}

Step 2. Query Phase: Given a start configuration q_I and goal configuration q_G , connect them to the roadmap \mathcal{G} using a local planner, and then search the augmented roadmap for a shortest path from q_I to q_G



Rapidly Exploring Random Trees (RRTs)

No preprocessing step: starting with the initial configuration q_I build the graph (actually, tree) until the goal configuration g_G is part of it

Very effective for single shot planning

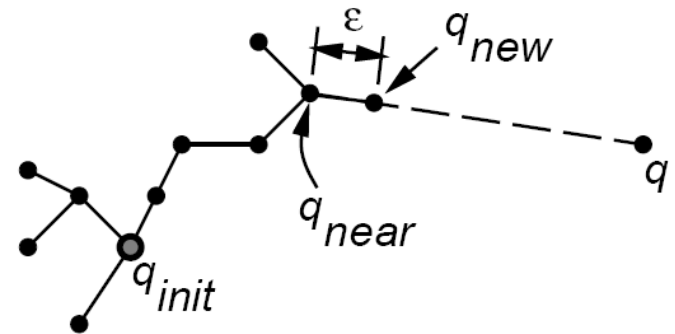
Rapidly Exploring Random Trees (RRTs)

BUILD_RRT(q_{init})

```
1   $\mathcal{T}.\text{init}(q_{init});$ 
2  for  $k = 1$  to  $K$  do
3       $q_{rand} \leftarrow \text{RANDOM\_CONFIG}();$ 
4       $\text{EXTEND}(\mathcal{T}, q_{rand});$ 
5  Return  $\mathcal{T}$ 
```

EXTEND(\mathcal{T}, q)

```
1   $q_{near} \leftarrow \text{NEAREST\_NEIGHBOR}(q, \mathcal{T});$ 
2  if  $\text{NEW\_CONFIG}(q, q_{near}, q_{new})$  then
3       $\mathcal{T}.\text{add\_vertex}(q_{new});$ 
4       $\mathcal{T}.\text{add\_edge}(q_{near}, q_{new});$ 
5      if  $q_{new} = q$  then
6          Return Reached;
7      else
8          Return Advanced;
9  Return Trapped;
```



EXTEND operation

borrowed from “RRT-Connect: An Efficient Approach to Single-Query Path Planning” paper by J. Kuffner & S. LaValle

Rapidly Exploring Random Trees (RRTs)

*Path to the goal is a path in the tree
from q_{init} to the vertex closest to goal*

BUILD_RRT(q_{init})

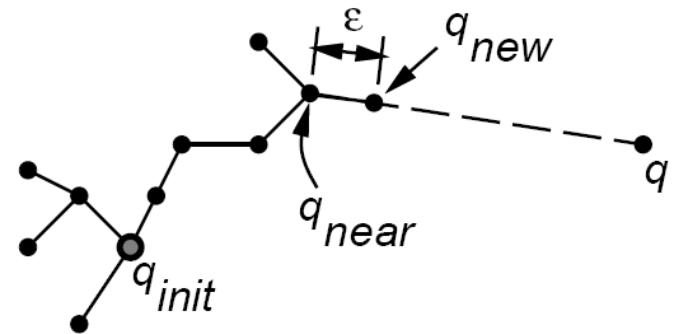
```
1   $\mathcal{T}.\text{init}(q_{init});$ 
2  for  $k = 1$  to  $K$  do
3       $q_{rand} \leftarrow \text{RANDOM\_CONFIG}();$ 
4      EXTEND( $\mathcal{T}, q_{rand}$ );
5  Return  $\mathcal{T}$ 
```

selects closest vertex in the tree

EXTEND(\mathcal{T}, q)

```
1   $q_{near} \leftarrow \text{NEAREST\_NEIGHBOR}(q, \mathcal{T});$ 
2  if NEW_CONFIG( $q, q_{near}, q_{new}$ ) then
3       $\mathcal{T}.\text{add\_vertex}(q_{new});$ 
4       $\mathcal{T}.\text{add\_edge}(q_{near}, q_{new});$ 
5      if  $q_{new} = q$  then
6          Return Reached;
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```

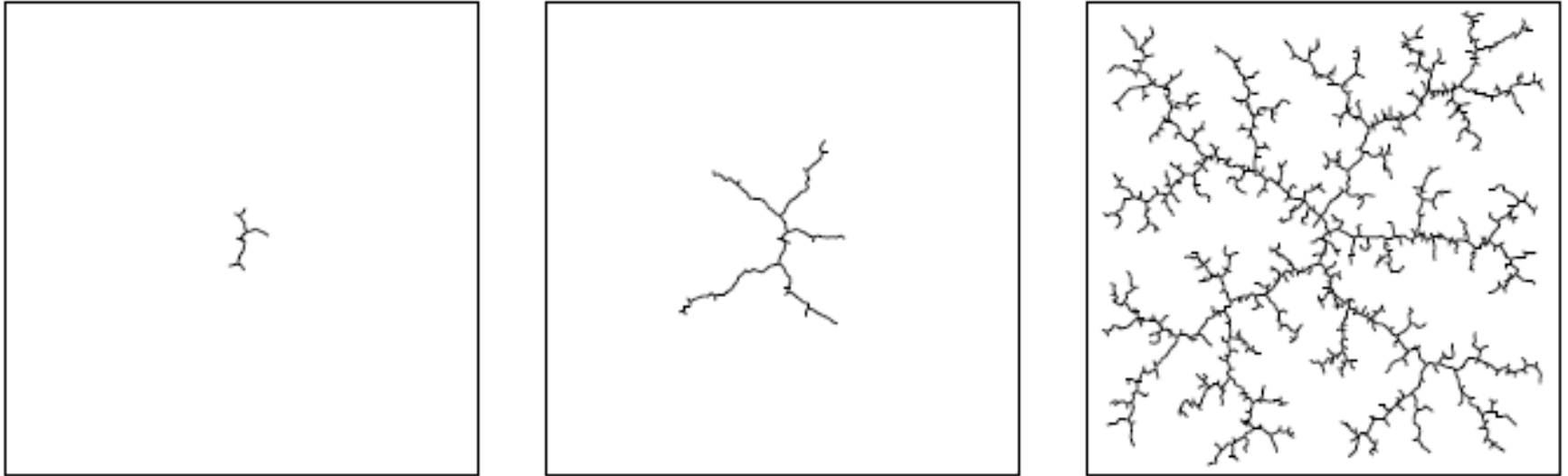
*moves by at most ϵ
from q_{near} towards q*



EXTEND operation

borrowed from “RRT-Connect: An Efficient Approach to Single-Query Path Planning” paper by J. Kuffner & S. LaValle

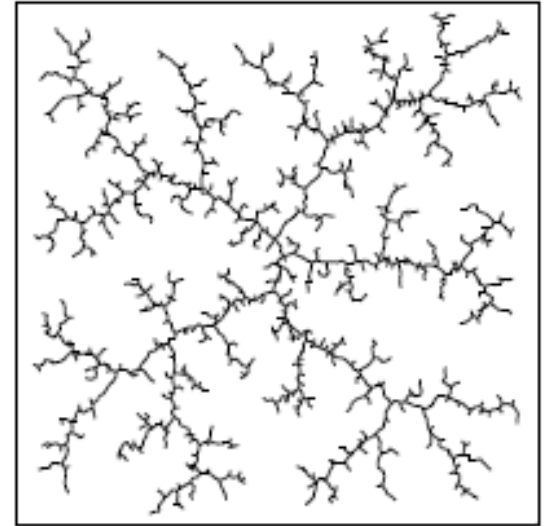
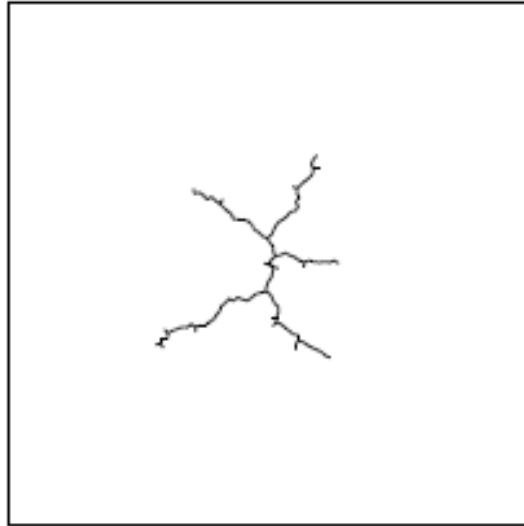
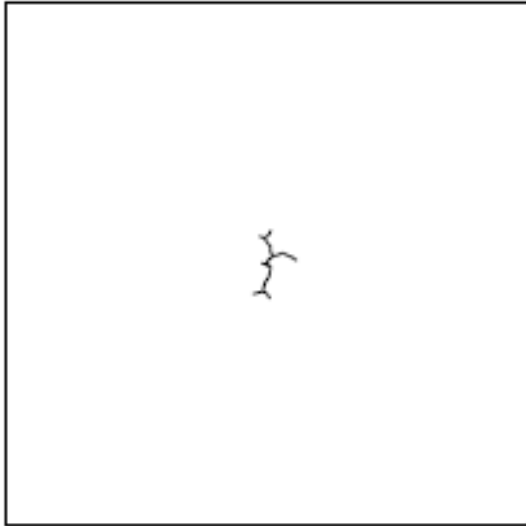
Rapidly Exploring Random Trees (RRTs)



- RRT provides uniform coverage of space

borrowed from “RRT-Connect: An Efficient Approach to Single-Query Path Planning” paper by J. Kuffner & S. LaValle

Rapidly Exploring Random Trees (RRTs)

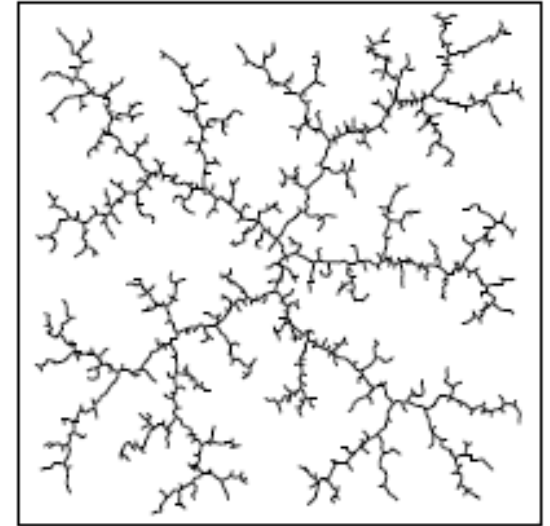
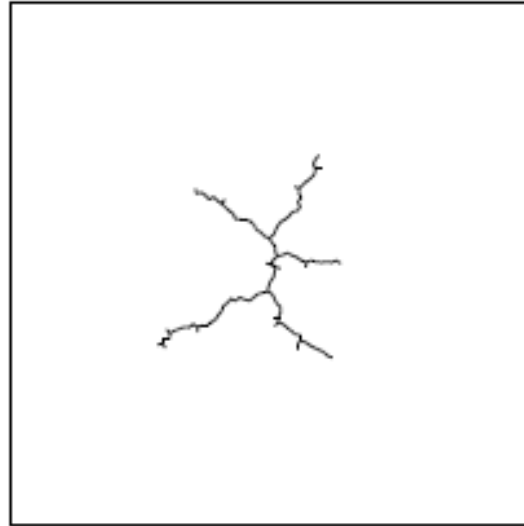
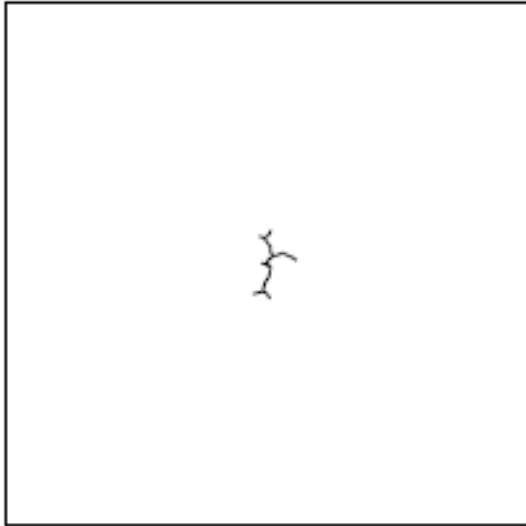


- RRT provides uniform coverage of space

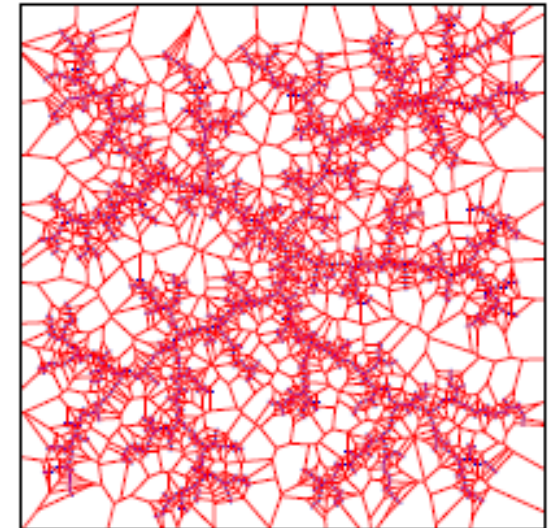
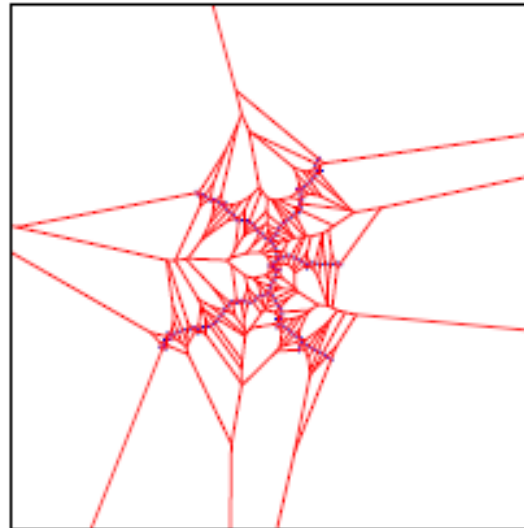
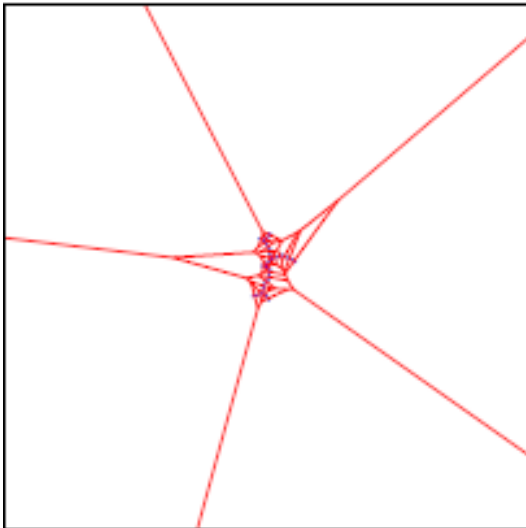
Pros/cons?

borrowed from “RRT-Connect: An Efficient Approach to Single-Query Path Planning” paper by J. Kuffner & S. LaValle

Rapidly Exploring Random Trees (RRTs)

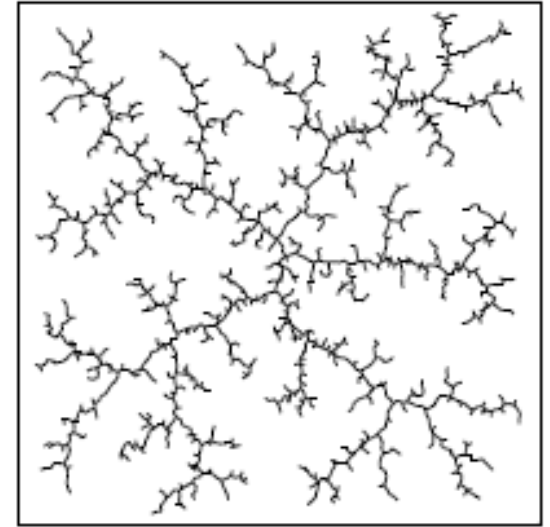
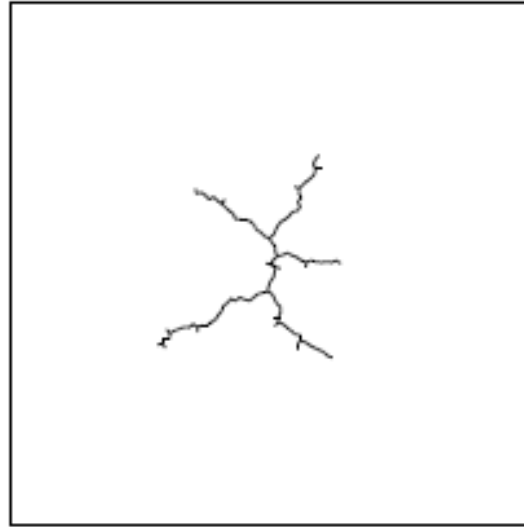
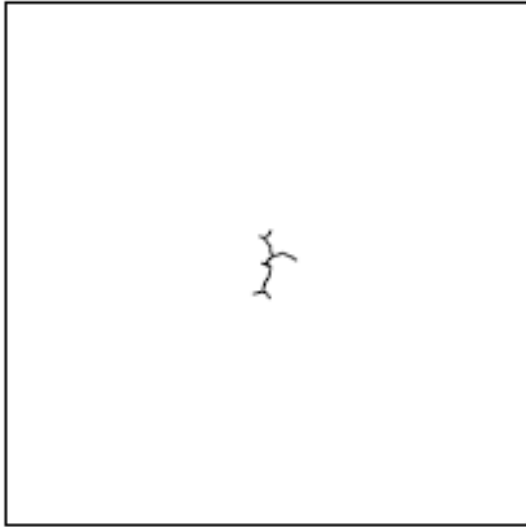


- Alternatively, the growth is always biased by the largest unexplored region

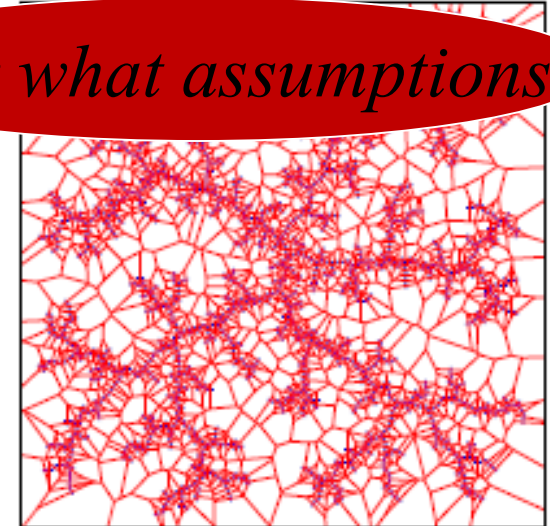
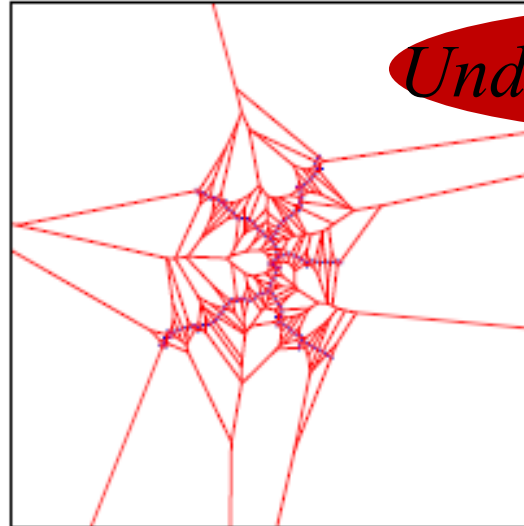
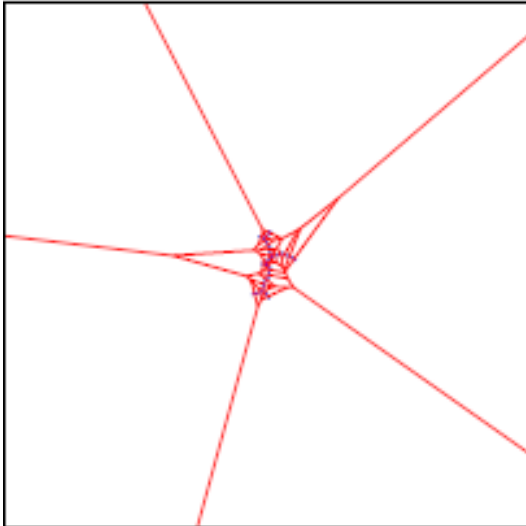


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Rapidly Exploring Random Trees (RRTs)



- Alternatively, the growth is always biased by the largest unexplored region



Under what assumptions?

borrowed from “RRT-Connect: An Efficient Approach to Single-Query Path Planning” paper by J. Kuffner & S. LaValle

RRT-Connect

Bi-directional growth of the tree

+

relax the ε constraint on the growth of the tree

RRT-Connect

RRT_CONNECT_PLANNER(q_{init}, q_{goal})

```
1   $\mathcal{T}_a$ .init( $q_{init}$ );  $\mathcal{T}_b$ .init( $q_{goal}$ );
2  for  $k = 1$  to  $K$  do
3       $q_{rand} \leftarrow \text{RANDOM\_CONFIG}()$ ;
4      if not (EXTEND( $\mathcal{T}_a, q_{rand}$ ) = Trapped) then
5          if (CONNECT( $\mathcal{T}_b, q_{new}$ ) = Reached) then
6              Return PATH( $\mathcal{T}_a, \mathcal{T}_b$ );
7      SWAP( $\mathcal{T}_a, \mathcal{T}_b$ );
8  Return Failure
```

CONNECT(\mathcal{T}, q)

```
1  repeat
2       $S \leftarrow \text{EXTEND}(\mathcal{T}, q)$ ;
3  until not ( $S = \text{Advanced}$ )
4  Return  $S$ ;
```

borrowed from “RRT-Connect: An Efficient Approach to Single-Query Path Planning” paper by J. Kuffner & S. LaValle

RRT-Connect

RRT_CONNECT_PLANNER(q_{init}, q_{goal})

```
1   $\mathcal{T}_a.\text{init}(q_{init}); \mathcal{T}_b.\text{init}(q_{goal});$ 
2  for  $k = 1$  to  $K$  do
3       $q_{rand} \leftarrow \text{RANDOM\_CONFIG}();$ 
4      if not ( $\text{EXTEND}(\mathcal{T}_a, q_{rand}) = \text{Trapped}$ ) then
5          if ( $\text{CONNECT}(\mathcal{T}_b, q_{new}) = \text{Reached}$ ) then
6              Return  $\text{PATH}(\mathcal{T}_a, \mathcal{T}_b);$ 
7       $\text{SWAP}(\mathcal{T}_a, \mathcal{T}_b);$ 
8  Return Failure
```

*tries to grow T_b to q_{new}
that was just added to T_a*

Why swap the trees?

CONNECT(\mathcal{T}, q)

```
1  repeat
2       $S \leftarrow \text{EXTEND}(\mathcal{T}, q);$ 
3  until not ( $S = \text{Advanced}$ )
4  Return  $S;$ 
```

*CONNECT function grows the tree
by more than just one ϵ*

borrowed from “RRT-Connect: An Efficient Approach to Single-Query Path Planning” paper by J. Kuffner & S. LaValle

RRT-Connect

- For any $q \in C_{free}$, $\lim_{k \rightarrow \infty} P[d(q) < \varepsilon] = 1$, where $d(q)$ is a distance from configuration q to the closest vertex in the tree, and assuming C_{free} is connected, bounded and open
- RRT-Connect is probabilistically complete: *as # of samples approaches infinity, the algorithm is guaranteed to find a solution if one exists*

Sampling-based approaches

Typical setup:

- Run PRM/RRT/RRT-Connect/...
- Post-process the generated solution to make it more optimal



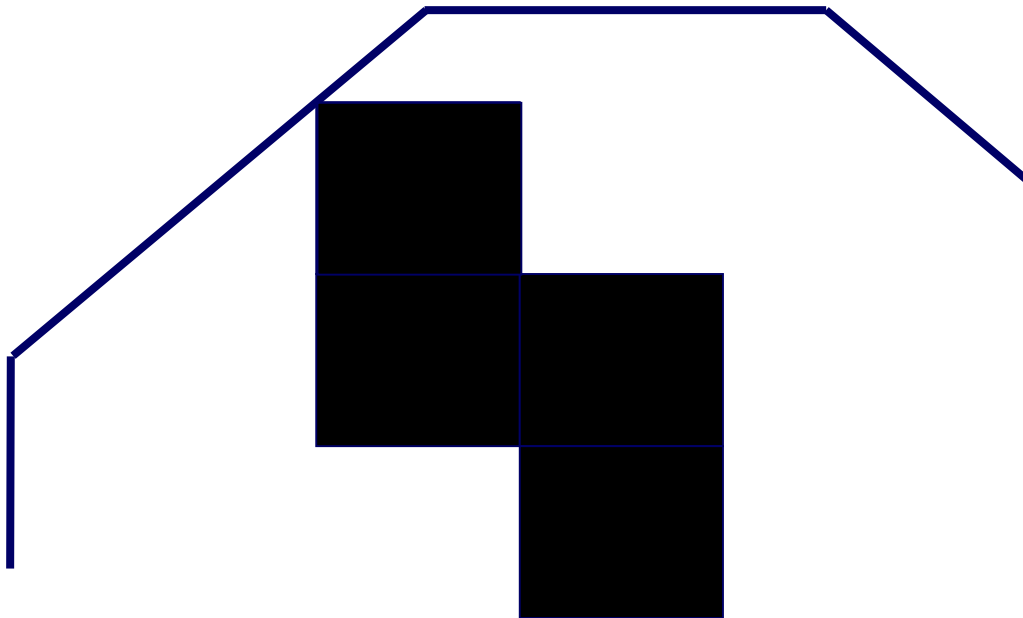
*An important but
often time-consuming step*

Could also be highly non-trivial

Post-processing

Any ideas how to post-process it?

Consider this path generated by RRT or PRM or A^ on a grid-based graph:*



Simple Post-processing via Short-cutting

- Short-cutting a path consisting of a series of points

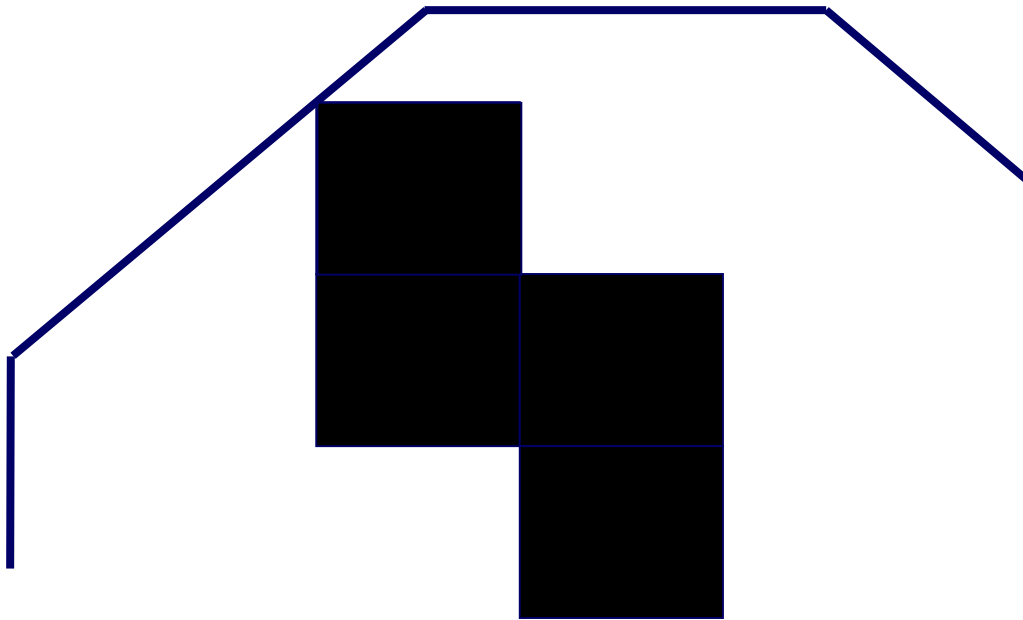
NewPath=[]; P=start point, P1 = point P+1 along the path

while P != goal point

while line segment [P,P1+1] is obstacle-free AND P1+1 < goal point

P1 = point P1+1 along the path;

NewPath+= [P,P1]; P = P1; P1 = point P+1 along the path;



Simple Post-processing via Short-cutting

- Short-cutting a path consisting of a series of points

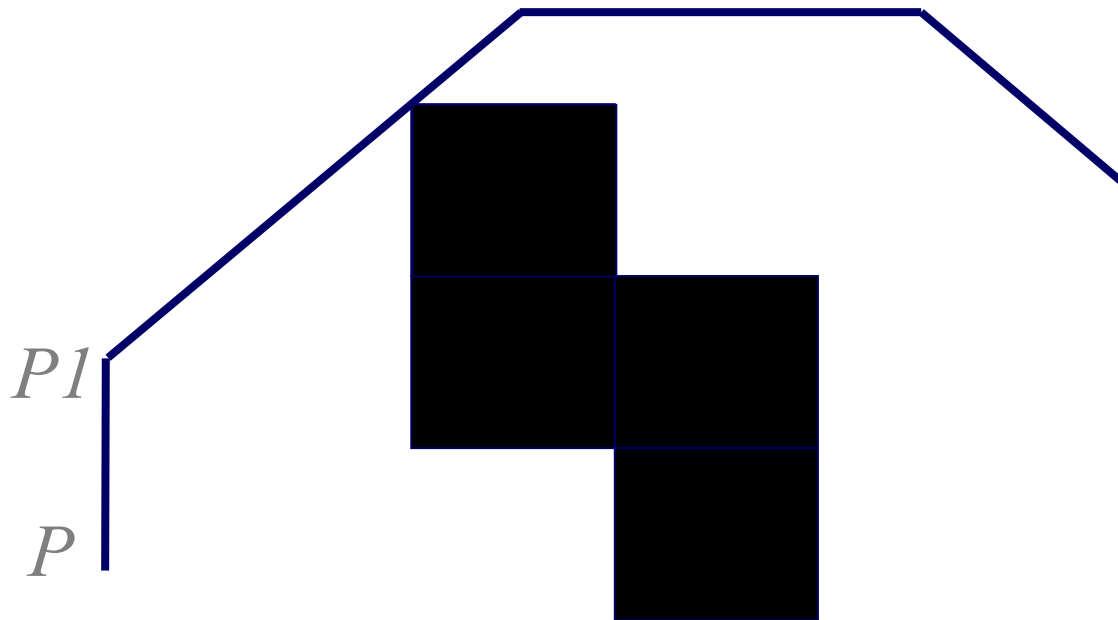
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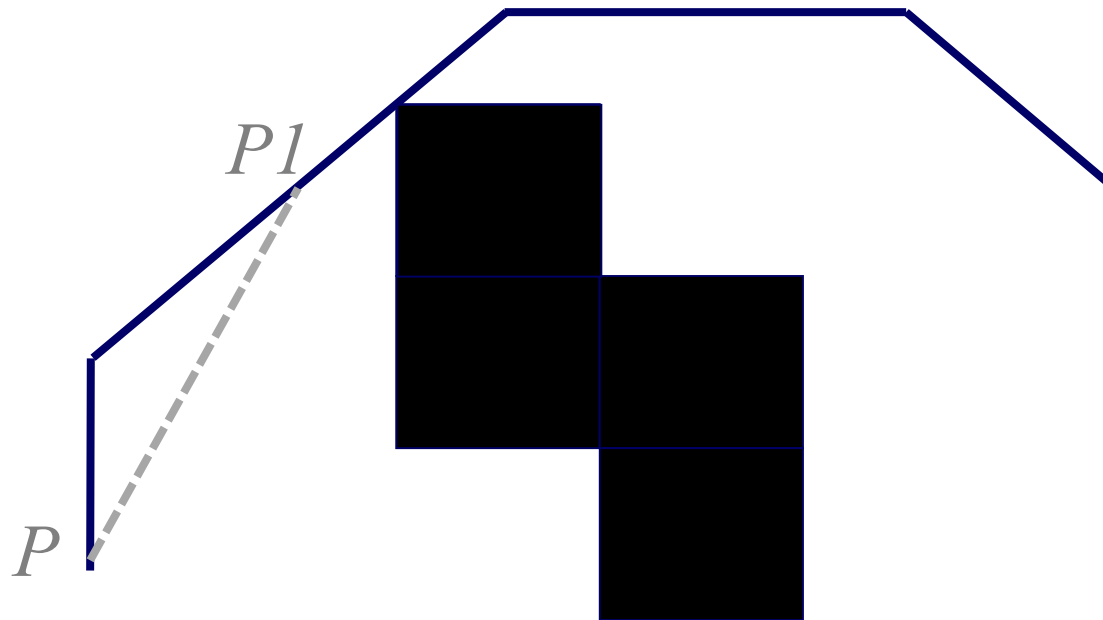
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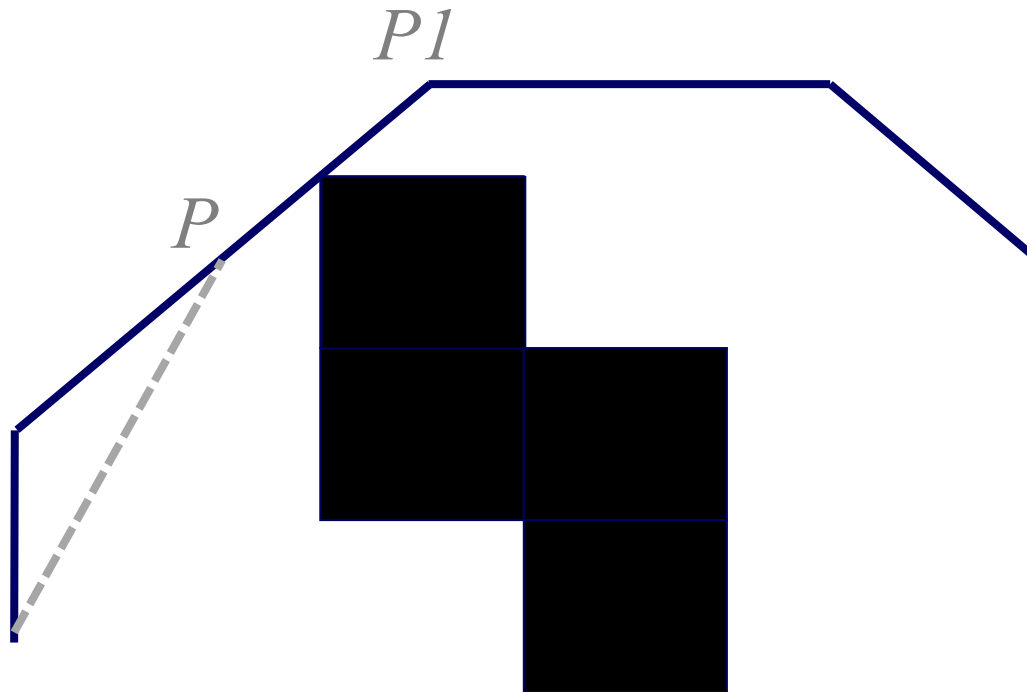
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Simple Post-processing via Short-cutting

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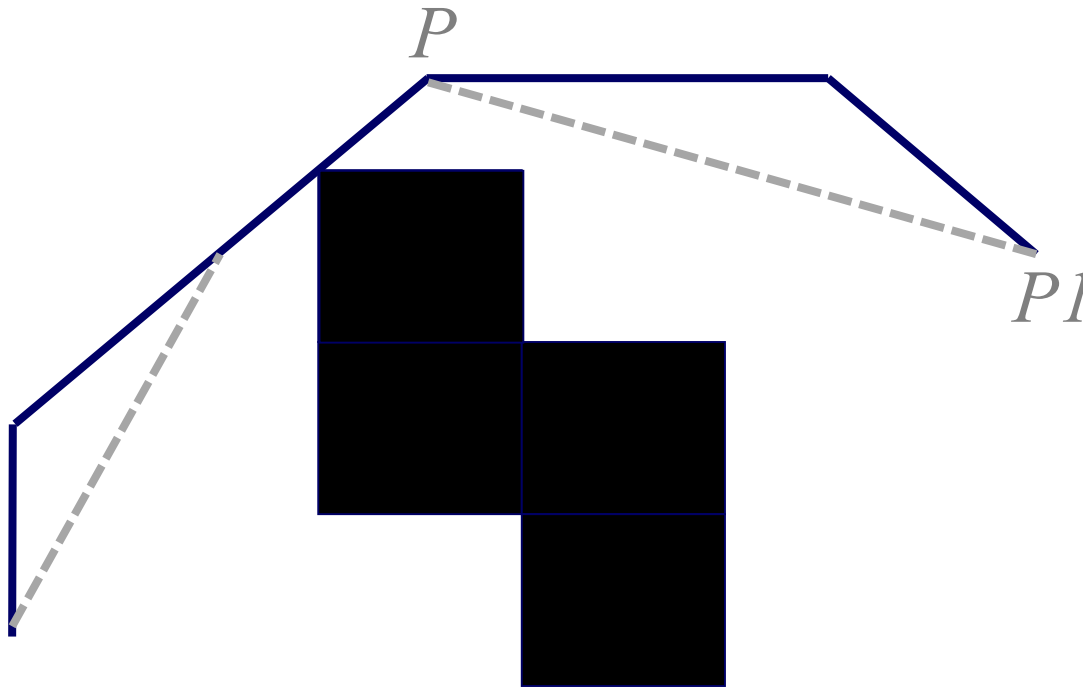
NewPath=[]; P=start point, P1 = point P+1 along the path

while $P \neq \text{goal point}$

while line segment $[P, P1+1]$ is obstacle-free AND $P1+1 < \text{goal point}$

$P1 = \text{point } P1+1 \text{ along the path;}$

NewPath += [P,P1]; P = P1; P1 = point P+1 along the path;



Simple Post-processing via Short-cutting

- Short-cutting a path consisting of a series of points

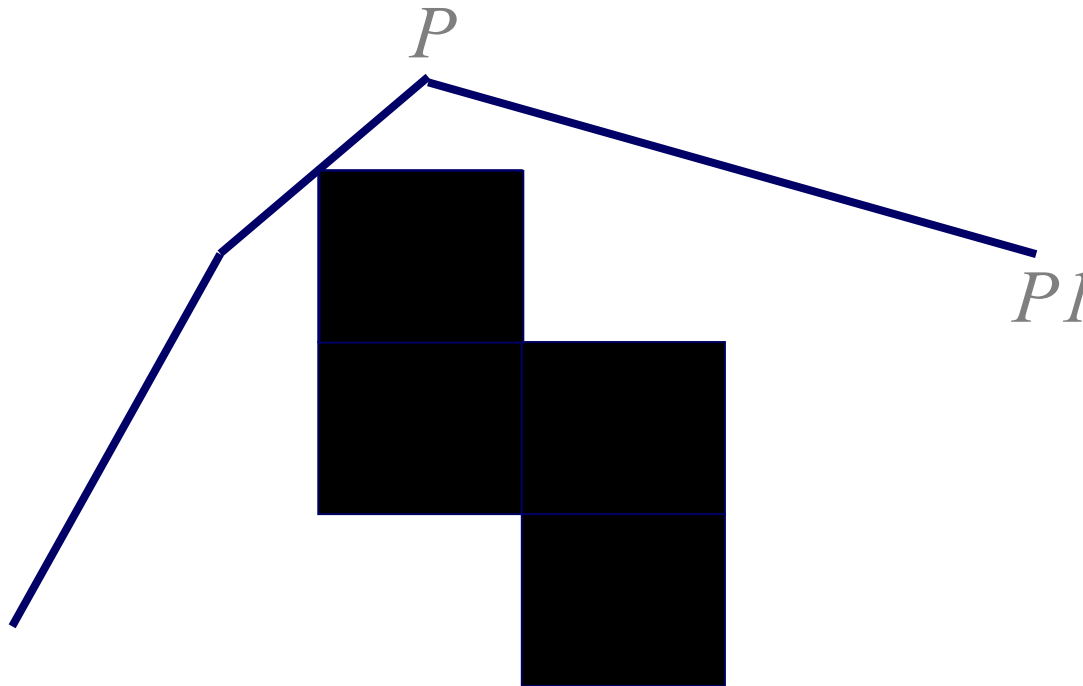
NewPath=[]; P=start point, P1 = point P+1 along the path

while P != goal point

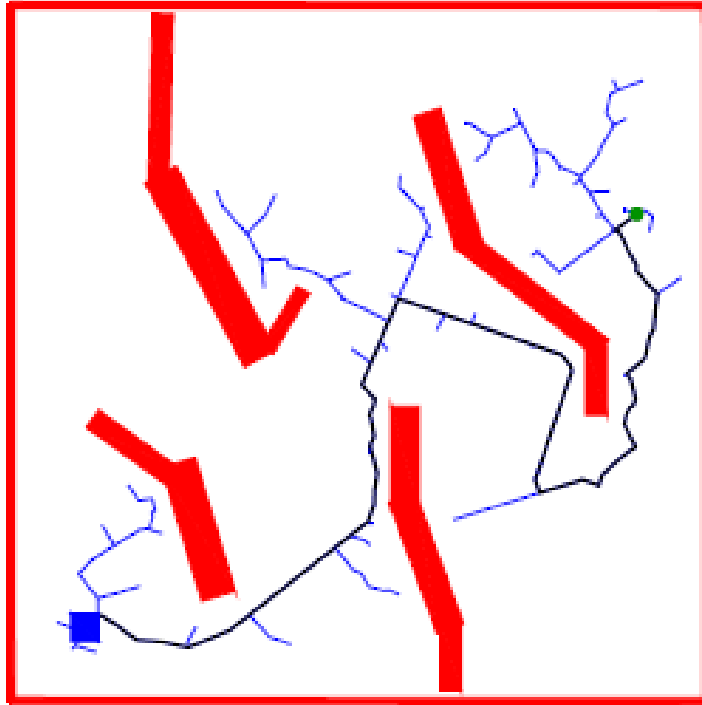
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P1 = point P1+1 along the path;

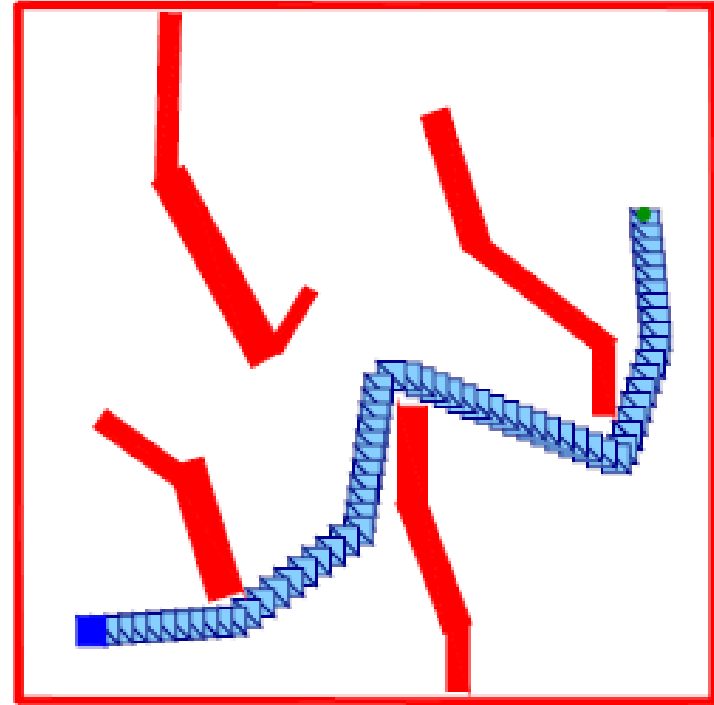
NewPath+= [P,P1]; P = P1; P1 = point P+1 along the path;



Examples of RRT in action



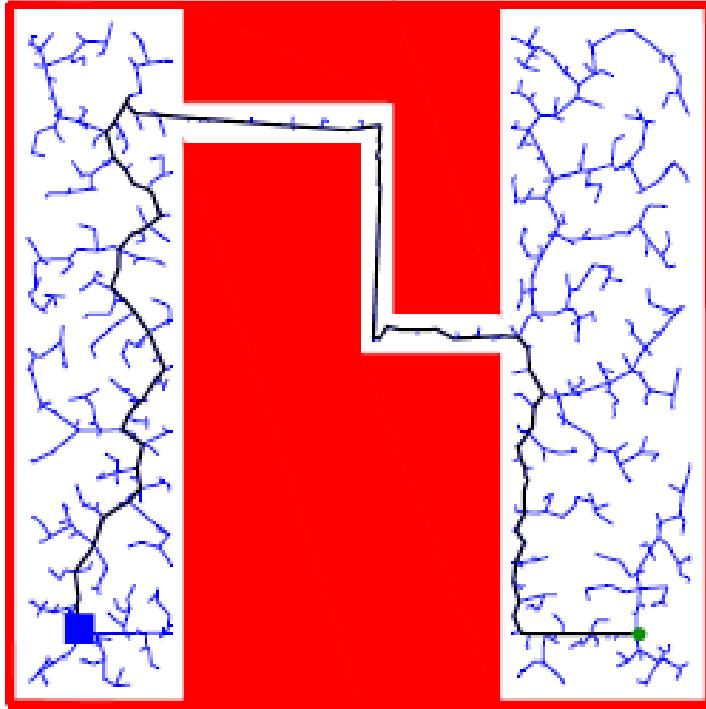
RRT-connect



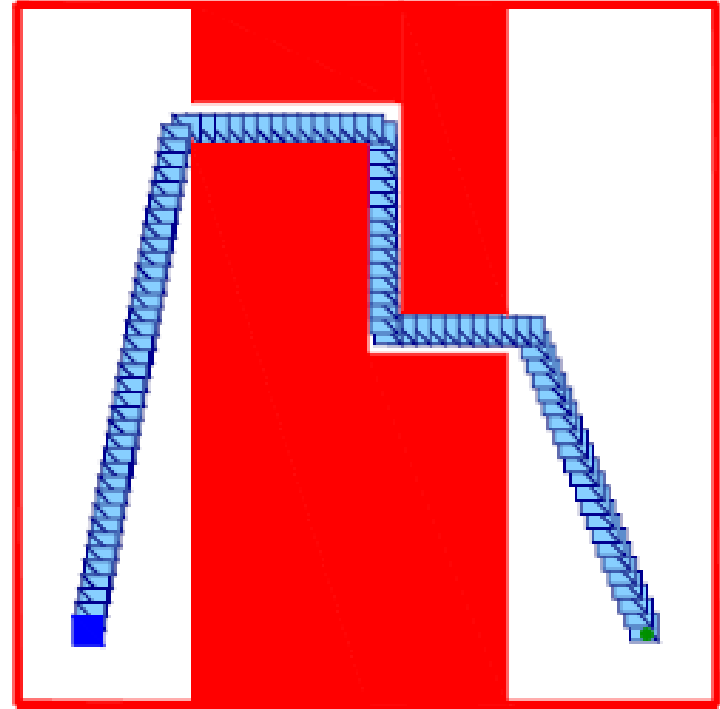
path after postprocessing

borrowed from “RRT-Connect: An Efficient Approach to Single-Query Path Planning” paper by J. Kuffner & S. LaValle

Examples of RRT in action



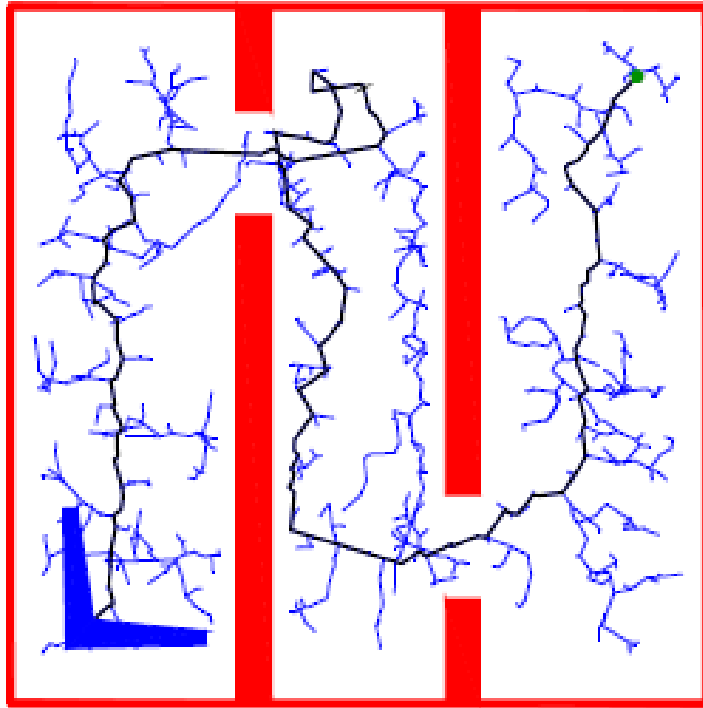
RRT-connect



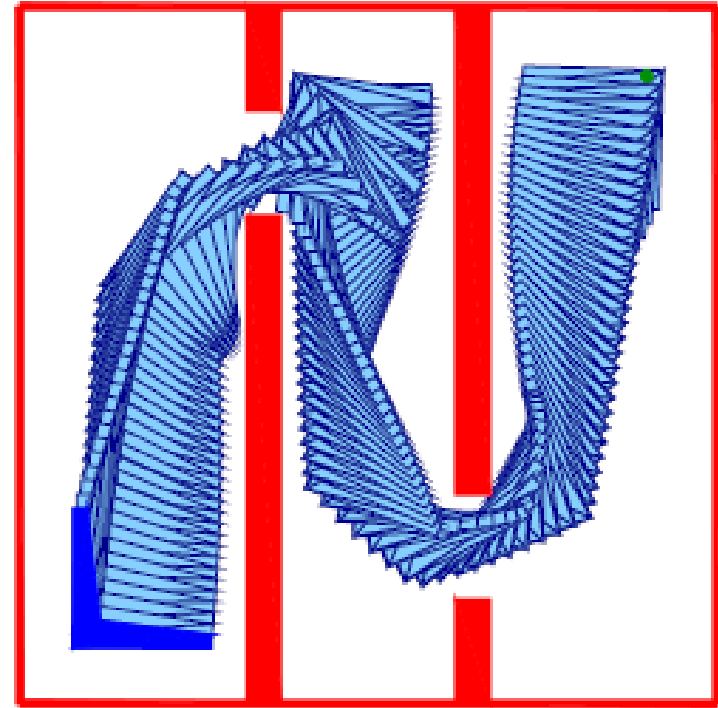
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Examples of RRT in action



RRT-connect



path after postprocessing

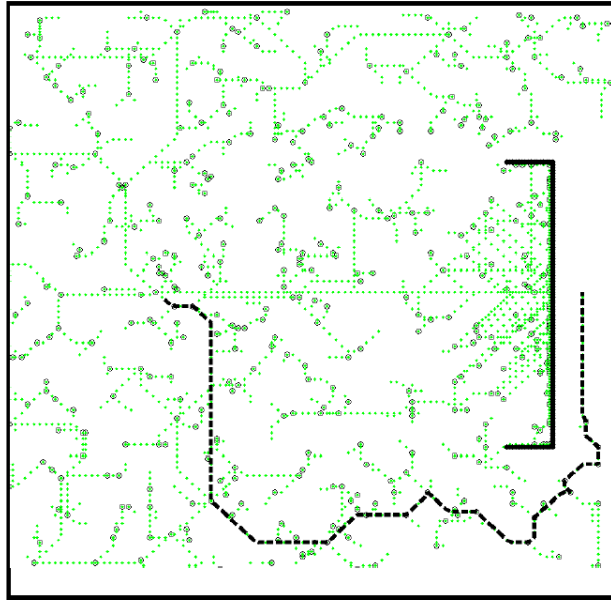
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PRMs vs. RRTs

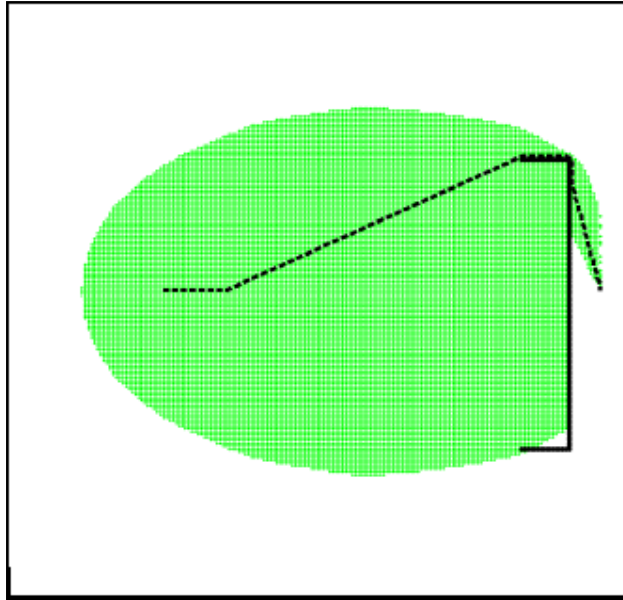
- PRMs construct a roadmap and then searches it for a solution whenever q_I, g_G are given
 - well-suited for repeated planning in between different pairs of q_I, g_G (*multiple queries*)
- RRTs construct a tree for a given q_I, q_G until the tree has a solution
 - well-suited for single-shot planning in between a single pair of q_I, g_G (*single query*)
 - There exist extensions of RRTs that try to reuse a previously constructed tree when replanning in response to map updates

RRTs vs A*-based planning

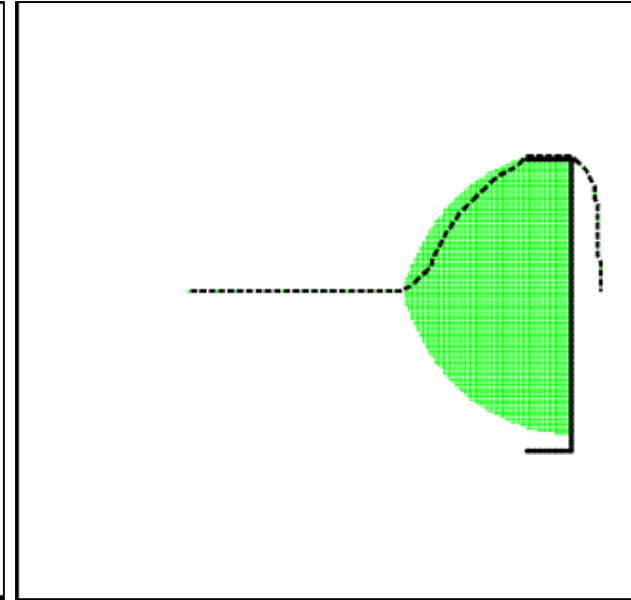
RRT



*A**



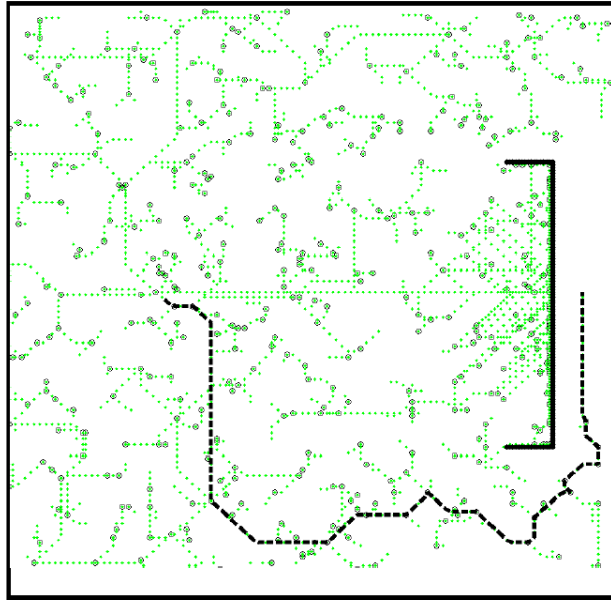
wA with $\varepsilon = 3$*



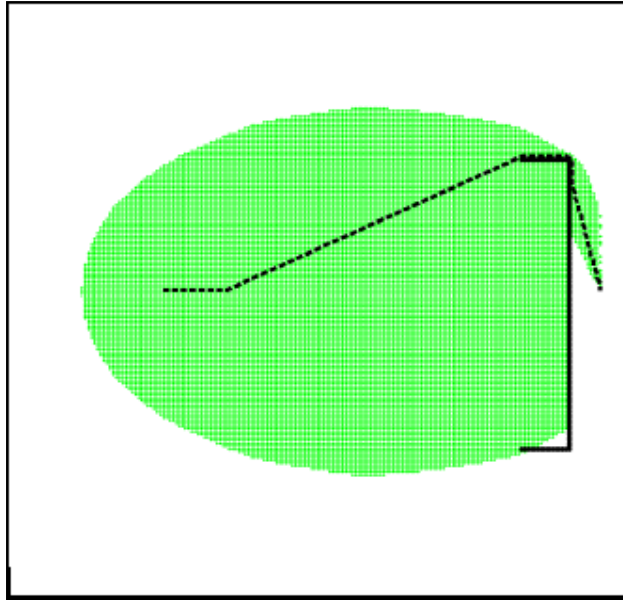
- RRTs:
 - sparse exploration, usually little memory and computations required, works well in high-D
 - solutions can be highly sub-optimal, requires post-processing, which in some cases can be very hard to do, the solution is still restricted to the same homotopic class

RRTs vs A*-based planning

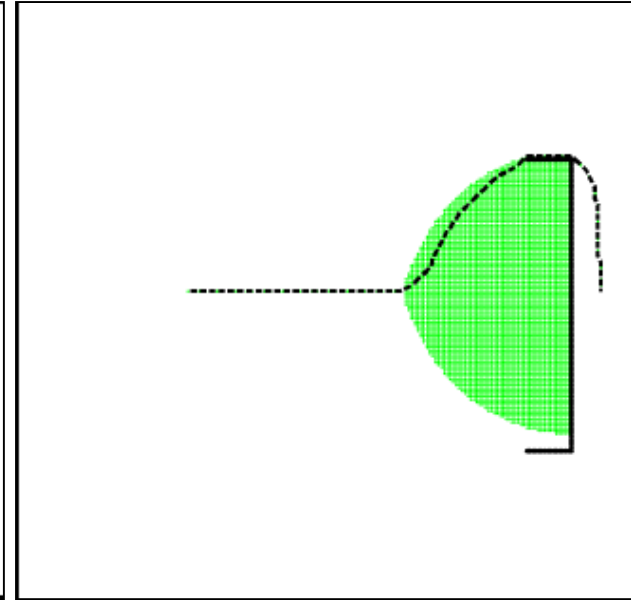
RRT



*A**



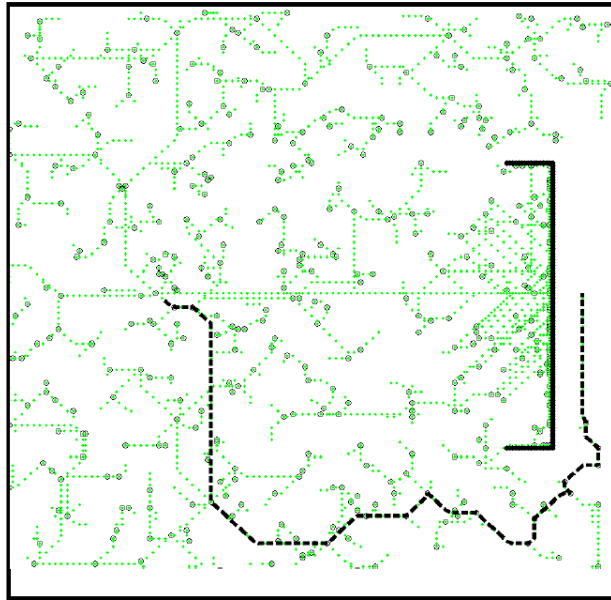
wA with $\varepsilon = 3$*



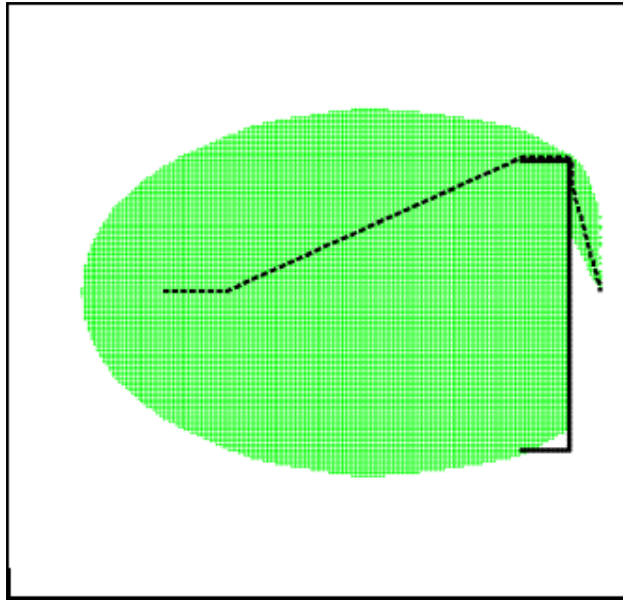
- RRTs:
 - does not incorporate a (potentially complex) cost function
 - there exist versions (e.g., RRT*) that try to incorporate the cost function and converge to a provably least-cost solution in the limit of samples (but typically computationally more expensive than RRT)

RRTs vs A*-based planning

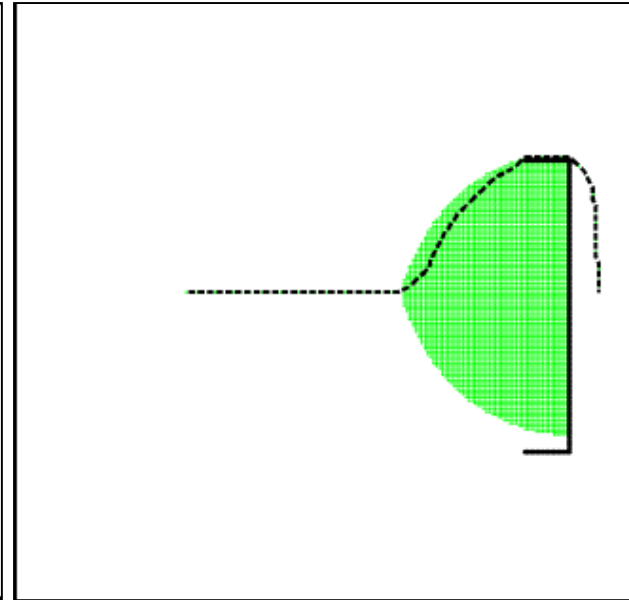
RRT



*A**



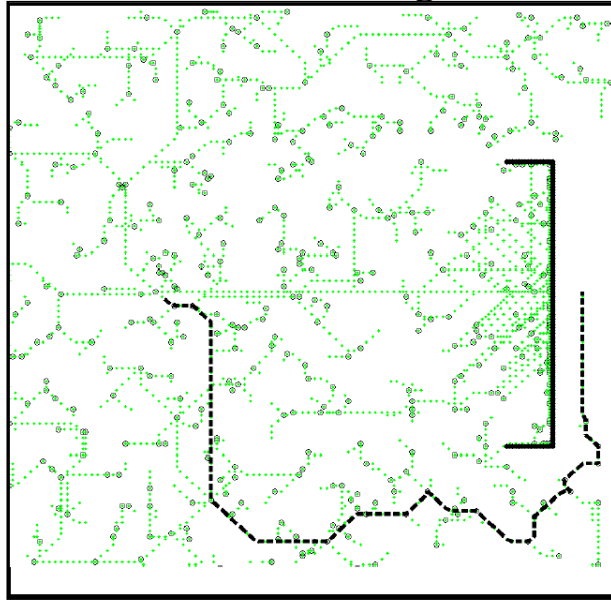
*wA** with $\varepsilon = 3$



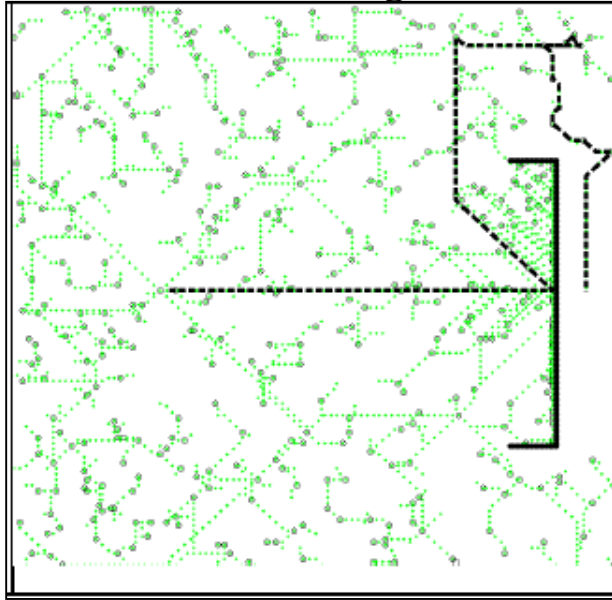
- A* and weighted A* (wA*):
 - returns a solution with optimality (or sub-optimality) guarantees with respect to the discretization used
 - explicitly minimizes a cost function
 - requires a thorough exploration of the state-space resulting in high memory and computational requirements

Sampling in RRTs

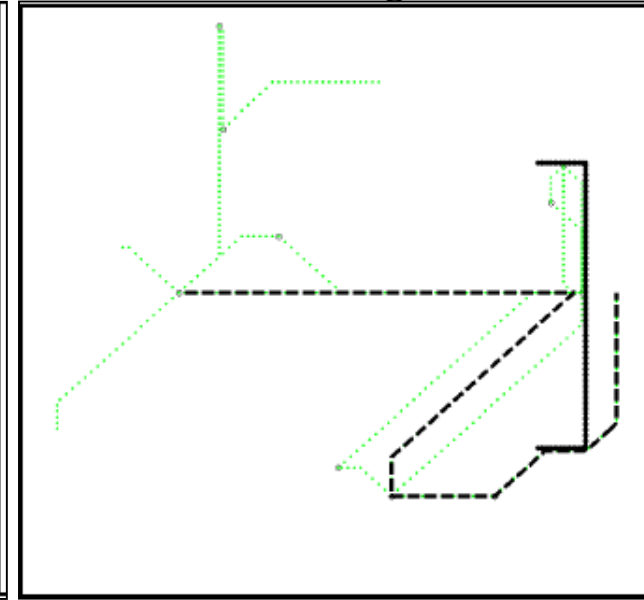
$RRT, P_g=0$



$RRT, P_g=0.1$



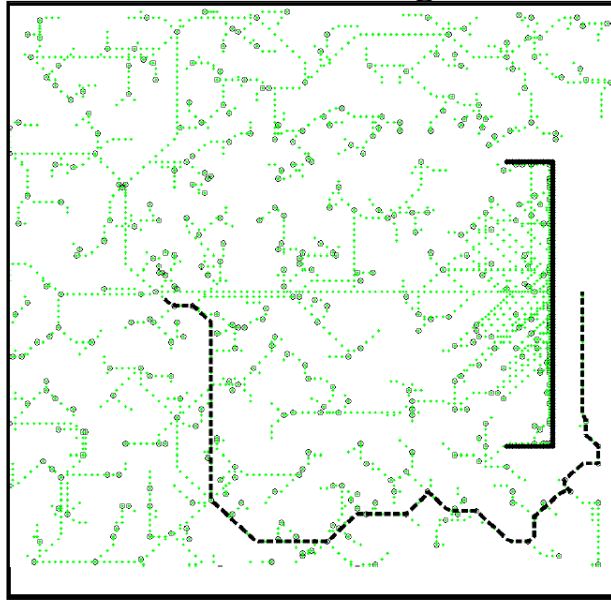
$RRT, P_g=0.5$



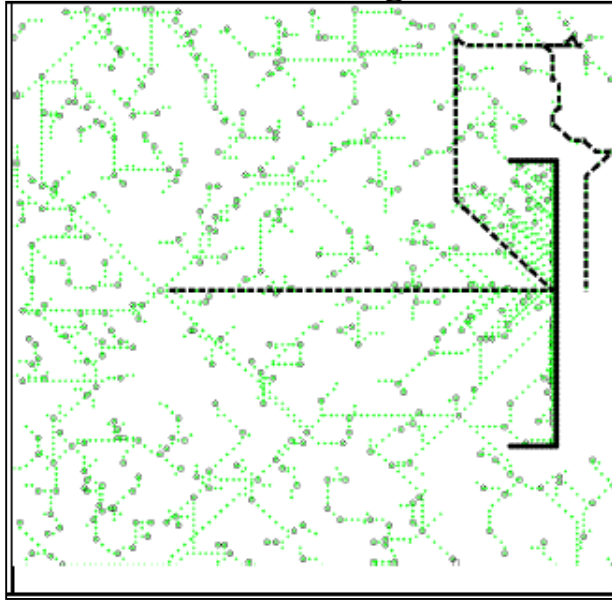
- Uniform: q_{rand} is a random sample in C_{free}
- Goal-biased: with a probability $(1-P_g)$, q_{rand} is chosen as a random sample in C_{free} , with probability P_g , q_{rand} is set to g_G

Sampling in RRTs

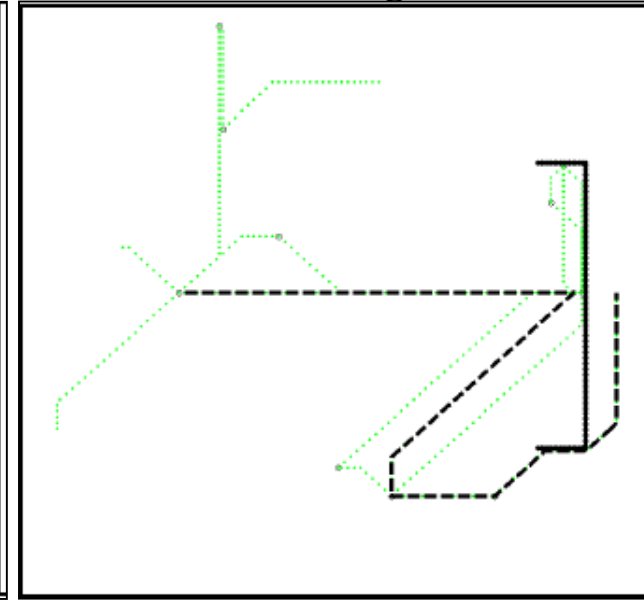
$RRT, P_g=0$



$RRT, P_g=0.1$



$RRT, P_g=0.5$



- Uniform: q_{rand} is a random sample in C_{free}
- Goal-biased: with a probability $(1-P_g)$, q_{rand} is chosen as a random sample in C_{free} , with probability P_g , q_{rand} is set to g_G

Very useful!

Summary

- RRT
 - interleaves tree construction and search
 - great for high-dimensional planning in continuous spaces
 - RRT-Connect typically much faster than RRT
 - Good post-processing is very important
- Sampling-based approaches are heavily used in planning for articulated robots (e.g., arms, humanoids, etc.)
- Provide probabilistic guarantees (i.e., in the limit of the number of samples)