2.4: Simultaneous Inference and Important Considerations

Dr. Bean - Stat 5100

Simultaneous inference is when we want to conduct multiple tests of significance at the same time.

1 Why Simultaneous Inference?

In handout 2.3, we conducted inference for parameters one at a time. We need to change our approach when looking at multiple parameters simultaneously.

How and why do we need to change our approach when conducting simultaneous inference?
(check out **this comic** for help).

If we conduct several tests at the same level of significance, the probability of getting one false positive result (a type I) error becomes much higher than α.

As a result, we need to adjust the level of significance to account for a "multiplicity" of testing.

2 Bonferroni Adjustment

Multiplicity:

- Let A_j = event that an individual $(1-\alpha)100\%$ CI does not contain the true value of β_j.
- $P(A_0) = P(A_1) = \alpha \to$ Type I Error
  - $P(\text{NOT} A_j)$ = probability that an interval contains the true value of β_j.
- **Bonferroni Inequality:** $P(\text{NOT} A_0 \text{ AND NOT } A_1) \geq 1 - P(A_0) - P(A_1)$

This means that if we conduct g tests at a confidence level of $(1-\frac{\alpha}{g})$, then we are guaranteed that overall level of confidence for all intervals *considered jointly* will be at least $(1-\alpha)$, we call this the **Bonferroni adjustment**.

- **Bonferroni Advantage:** Can be applied in *any* situation that requires a multiplicity adjustment, including simultaneous intervals for $\hat{Y}$ at multiple X_h levels.
- **Bonferroni Disadvantage:** Can be overly conservative, producing inefficient (unnecessarily wide) intervals.

Comparison of Simultaneous Intervals for $\hat{Y}$

- Confidence intervals (mean response)
  - Bonferroni

$$\hat{Y} \pm t_{n-p}(1-\frac{\alpha}{2g}) * s\{\hat{Y}_h\}$$

– Working-Hotelling (WH)

$$\hat{Y} \pm W * s\{\hat{Y}_h\} \qquad \left(W = \sqrt{pF_{p,n-p}(1-\alpha)}\right)$$

Notice that the W-statistic does not consider g

- WH provides a "confidence band" for the entire regression line (all possible X_h levels).
- This means the WH interval at any individual X_h will be wider than the t-based confidence interval, but the WH intervals will eventually be narrower than Bonferroni confidence intervals if enough X_h are considered.

- Prediction intervals (new response)

  – Bonferroni

$$\hat{Y} \pm t_{n-p}(1 - \frac{\alpha}{2g}) * s\{\hat{Y}_{h(new)}\}$$

  – Scheffe (chef-eh)

$$\hat{Y} \pm S * s\{\hat{Y}_{h(new)}\} \qquad \left(S = \sqrt{gF_{g,n-p}(1-\alpha)}\right)$$

Rule of Thumb: Always pick the most efficient interval that guarantees your intended type I error (α).

Table 1: Summary of Methods for Simultaneous Intervals

Simultaneous Interval on:	**Methods**
β's	Bonferroni
Population means of Y at multiple X_h	Bonferroni or Working-Hotelling
Predictions for Y at multiple X_h	Bonferroni of Scheffe

3 Inverse Prediction

Problem: What is the value of X_h necessary to achieve a specific value of $\hat{Y}$.

Solution: solve for X.

$$\begin{aligned} \hat{Y} &= b_0 + b_1 X_h \\ b_1 X_h &= \hat{Y} - b_0 \\ X_h &= \frac{\hat{Y} - b_0}{b_1} \end{aligned}$$

Problem: Use Y to predict values of X.

Solution: DO NOT solve for X.

Why?

- The least squares slope estimate of regression model that predicts Y using X: $\rho\frac{SD\{Y\}}{SD\{X\}}$.
- The least squares slope estimate of regression model that predicts X using Y: $\rho\frac{SD\{X\}}{SD\{Y\}}$.
- Notice that the slopes are NOT inverses of each other.

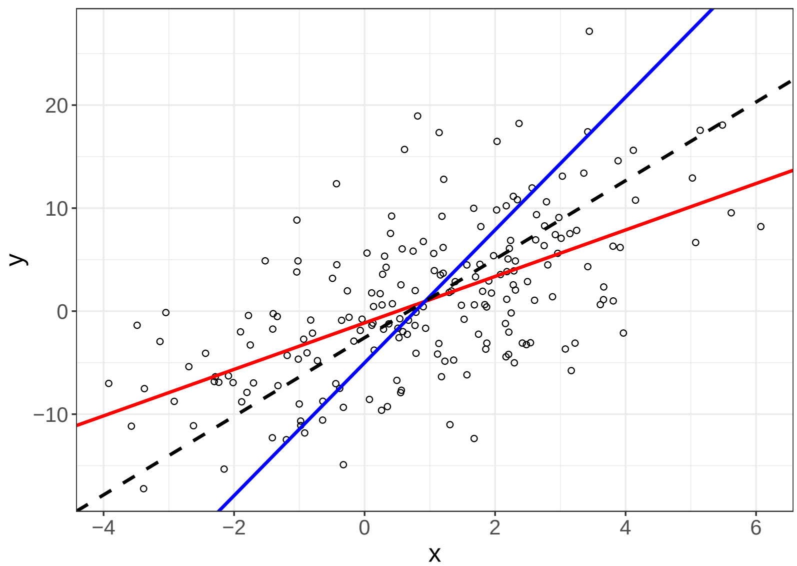

Figure 1: Scatterplot of points along with the regression line that uses X to predict Y (red), the regression line that uses Y to predict X (blue), the SD line (black).

4 Cautions for Linear Regression

- Remedial measures may not fix violations of assumptions
  - May need to abandon OLS regression altogether
- Interpretation: Sometimes the X vs Y relationship may look counterintuitive
  - May be the result of omitted predictors
- R^2 can be abused
  - Higher R^2 $\rightarrow$ not always better model
  - Lower R^2 $\rightarrow$ does not mean there is no linear relationship