Stat 5100 Handout #35 – SAS: Time Series Case Study (Unit 8)

Data: Weekly sales (in thousands of units) of Super Tech Videocassette Tapes over 161 weeks [see Bowerman & O'Connell "Forecasting and Time Series: An Applied Approach", 3rd Edition, Section 10.4 Case Study.

Goal: Want to forecast sales 25 weeks beyond end of data

```
data sales; input weekly @@; cards;
 45.9 45.4 42.8 34.4 31.9 36.6 39.2 41.4 40.3 43.1 43.2
 41.2 38.4 38.3 41.9 37.1 34.5 31.3 30.2 28.3 25.9
                                                  26.6
 26.2 29
           34.8 36.8 37.2 41.7 41.2 40.7 39.5 40.4 38
 35.6 33.9 35.2 41.8 42.4 38.9 42.1 41.7 39.2 38.5
                                                  42.5
 47.9 48.6 52
                53.5 53.5 52.9 53.4 52.8 51.4 52.5 52.4
 51.5 51.7 53.3 55.4 56.9 60
64.7 66.3 63 65.5 70.6 76
                              60.8 62.3
                                        62.6
                                             63.1
                              80.1 78.6 78.3
                                             78.1
                                                 73.6
 68.8 64.4 62.4 61.1 63.1 65.3 68.3 72.5 73.2
                                             72.9
                                                 70.5
 69.4 68.2 69.3 72.3 73.5 70.3 68.3 64.1
                                        62.5 62.6 60.4
 61.1 64.7 65.1 61.5 64.2 67.8 66.8 64.1
                                        66.4 68
                                                  71
 76.9 84.1 85.9 85.2 86.2 85.7 81.3 75.9 75
                                             72.5 69.6
 67.3 69.8 72.2 75.2 77.2 76.8 72.4 69.4
                                        68.7 65.1 64.4
 64.2 63.2 62.1 65.8 73.7 77.1 76
                                   74.6 70.6 67.5
                                                  67.9
 68.9 67.8 65.1 65
                     67.6 67.9 66.5 68.2 71.7 71.3 68.9
 70 73.1 69.1 67.3 72.9 78.6 82.3
run;
```

/*********************

```
/* Look at original data and check stationarity */
data sales; set sales;
    Time = _N_;
proc arima data=sales;
    identify var=weekly nlag=24;
    title1 'Look for Stationarity in SAC; Original Time Series';
run;
```

Look for Stationarity in SAC; Original Time Series

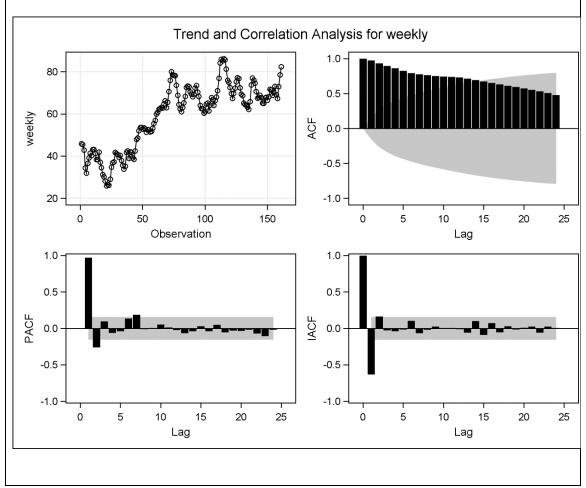
The ARIMA Procedure

Name of Variable = weekly

Number of Observations 1

161

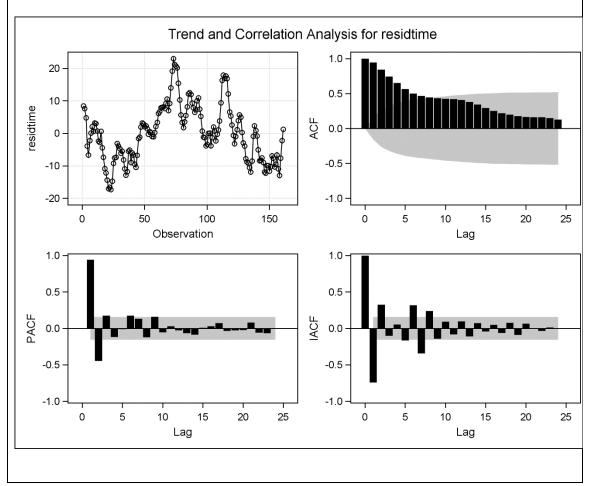
		Aut	ocorrelatio	n Check	for Wl	nite Nois	se		
To Lag	Chi- Square	DF	Pr > ChiSq	Autocorrelations					
6	779.33	6	<.0001	0.974	0.935	0.899	0.862	0.825	0.794
12	1366.70	12	<.0001	0.777	0.765	0.753	0.745	0.739	0.730
18	1845.25	18	<.0001	0.715	0.695	0.676	0.654	0.634	0.614
24	2177.35	24	<.0001	0.594	0.575	0.556	0.535	0.508	0.480



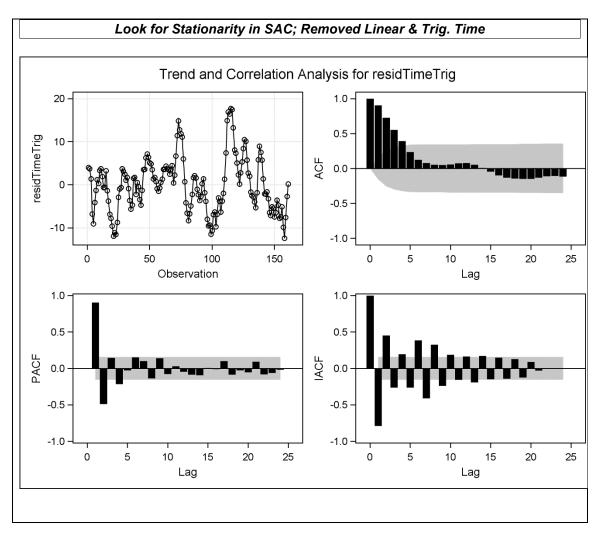
```
/* Remove linear effect of time */
proc reg data=sales noprint;
  model weekly = time;
  output out=out1 r=residtime;
proc arima data=out1;
  identify var=residtime nlag=24;
  title1 'Look for Stationarity in SAC; Removed Linear Time';
run;
```

Look for Stationarity in SAC; Removed Linear Time

	Autocorrelation Check for White Noise										
To Lag	Chi- Square	DF	Pr > ChiSq	Autocorrelations							
6	523.89	6	<.0001	0.944	0.843	0.746	0.654	0.567	0.501		
12	720.33	12	<.0001	0.469	0.447	0.434	0.427	0.422	0.409		



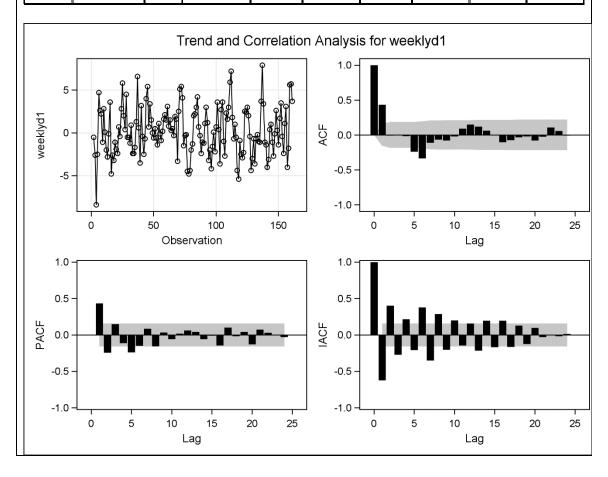
```
/* Try to remove cyclic behavior
    -- there appears to be a 2-year cycle */
data sales; set sales;
    sin1 = sin(2*3.14*time/104);
    cos1 = cos(2*3.14*time/104);
proc reg data=sales noprint;
    model weekly = time sin1 cos1;
    output out=out2 r=residTimeTrig;
proc arima data=out2;
    identify var=residTimeTrig nlag=24;
    title1 'Look for Stationarity in SAC; Removed Linear & Trig. Time';
run;
```



```
/* It doesn't look like this will work
    -- need to consider differencing */
data sales; set sales;
    weeklyd1 = weekly - lag(weekly);
    weeklyd2 = weeklyd1 - lag(weeklyd1);
run;

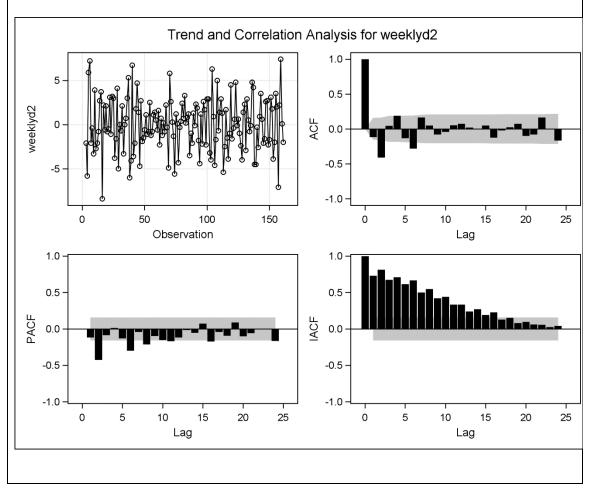
proc arima data=sales;
   identify var=weeklyd1 nlag=24;
   title1 'Look for Stationarity in SAC; First Difference';
run;
```

	Look for Stationarity in SAC; First Difference										
	Autocorrelation Check for White Noise										
To Lag	Chi- Square	DF	Pr > ChiSq	Autocorrelations							
6	59.37	6	<.0001	0.435	-0.008	0.002	-0.017	-0.239	-0.336		



```
proc arima data=sales;
  identify var=weeklyd2 nlag=24;
  title1 'Look for Stationarity in SAC; Second Difference';
run;
```

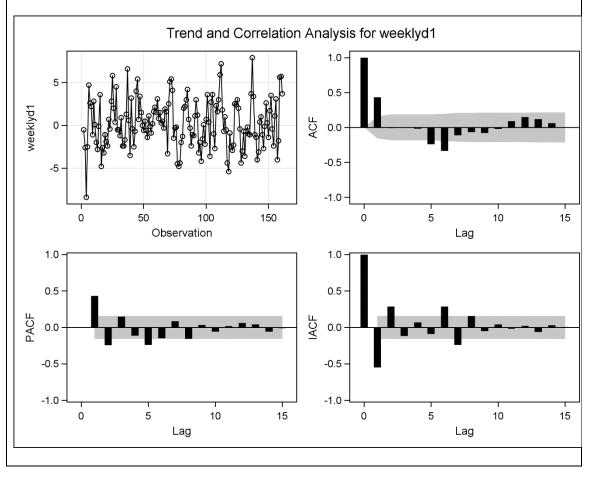
	Look for Stationarity in SAC; Second Difference									
	Autocorrelation Check for White Noise									
To Lag	Chi- Square	DF	Pr > ChiSq	Autocorrelations						
6	51.55	6	<.0001	-0.115	-0.408	0.047	0.190	-0.130	-0.281	



```
/* Now look for ARMA errors in first difference */
proc arima data=sales;
   identify var=weeklyd1 nlag=15;
   title1 'Look for ARMA errors with first difference';
run;
```

Look for AR	RMA errors with	first difference

Autocorrelation Check for White Noise									
To Lag	Chi- Square	DF	Pr > ChiSq	Autocorrelations					
6	59.37	6	<.0001	0.435	-0.008	0.002	-0.017	-0.239	-0.336
12	68.80	12	<.0001	-0.113	-0.066	-0.080	-0.019	0.092	0.149



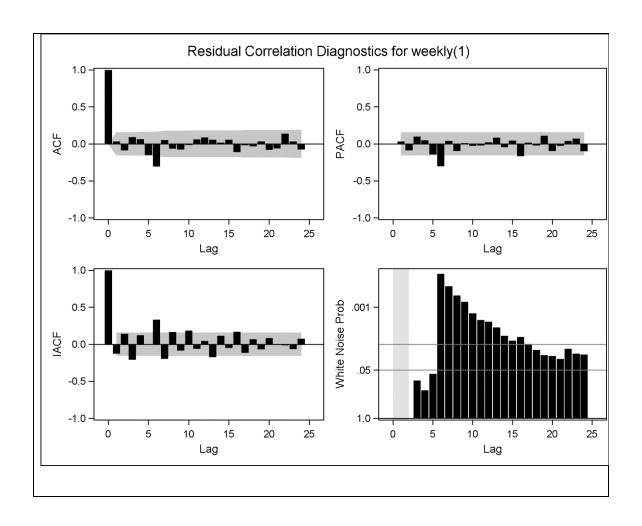
```
/* Model 1: ARIMA(2,1,0), based on
    SAC's damped exponential / sine pattern,
    and SPAC spikes at 1 and 2 */
proc arima data=sales;
    identify var=weekly(1);
    estimate p=2 plot method=uls;
    forecast lead=25 alpha=0.05 noprint out=f1;
    title1 'Model 1: ARIMA(2,1,0)';
run;
```

Model 1: ARIMA(2,1,0)

Unconditional Least Squares Estimation										
Parameter	Estimate	Standard Error	t Value	Approx Pr > t	Lag					
MU	0.22900	0.27904	0.82	0.4131	0					
AR1,1	0.54277	0.07749	7.00	<.0001	1					
AR1,2	-0.24502	0.07844	-3.12	0.0021	2					

Constant Estimate	0.160818
Variance Estimate	6.147007
Std Error Estimate	2.479316

			Autocorr	elation (Check of	f Residua	als		
To Lag	Chi- Square	DF	Pr > ChiSq	Autocorrelations					
6	23.06	4	0.0001	0.035	-0.087	0.090	0.065	-0.151	-0.306
12	27.19	10	0.0024	0.053	-0.063	-0.074	-0.014	0.059	0.089
18	30.84	16	0.0141	0.057	0.017	0.056	-0.111	-0.019	-0.032
24	37.90	22	0.0188	0.035	-0.080	-0.058	0.140	0.034	-0.077
30	42.29	28	0.0407	0.107	0.092	-0.042	0.000	-0.030	-0.006



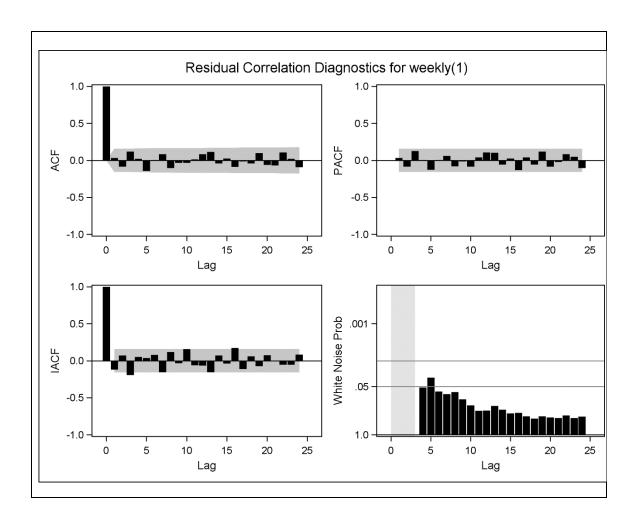
```
/* Model 2: ARIMA(2,1,(6)), based on
   RSAC/RSPAC spikes in Model 1 */
proc arima data=sales;
   identify var=weekly(1);
   estimate p=2 q=(6) plot method=uls;
   forecast lead=25 alpha=0.05 noprint out=f2;
   title1 'Model 2: ARIMA(2,1,(6))';
run;
```

Model 2: ARIMA(2,1,(6))

Unconditional Least Squares Estimation										
Parameter	Estimate	Standard Error	t Value	Approx Pr > t	Lag					
MU	0.22841	0.16843	1.36	0.1770	0					
MA1,1	0.34955	0.07858	4.45	<.0001	6					
AR1,1	0.52984	0.07798	6.79	<.0001	1					
AR1,2	-0.26407	0.07901	-3.34	0.0010	2					

Constant Estimate	0.167703
Variance Estimate	5.54486
Std Error Estimate	2.354753

			Autocorr	elation (Check of	f Residua	als		
To Lag	Chi- Square	DF	Pr > ChiSq	Autocorrelations					
6	7.13	3	0.0679	0.032	-0.082	0.121	0.022	-0.142	0.007
12	11.84	9	0.2226	0.085	-0.103	-0.033	-0.034	0.015	0.084
18	16.27	15	0.3645	0.114	-0.040	0.028	-0.087	-0.013	-0.041
24	23.44	21	0.3210	0.101	-0.060	-0.067	0.105	0.023	-0.092
30	29.05	27	0.3584	0.110	0.078	-0.078	0.005	-0.046	-0.050



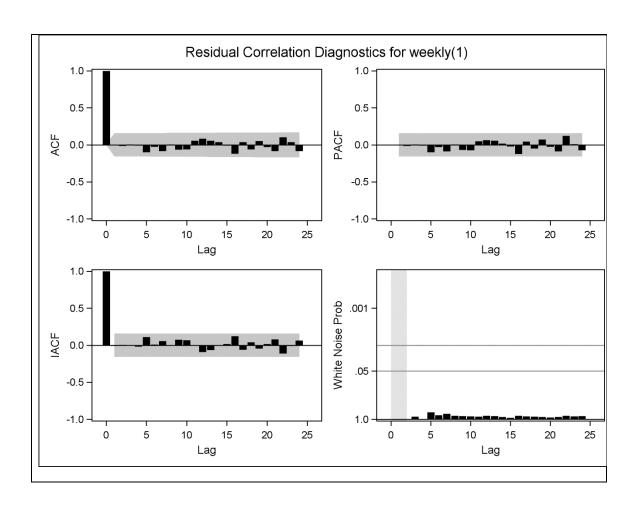
```
/* Model 3: ARIMA(0,1,(1,6)), based on
   alternative reading of first diff. SAC */
proc arima data=sales;
   identify var=weekly(1);
    estimate p=0 q=(1,6) plot method=uls;
   forecast lead=25 alpha=0.05 noprint out=f3;
   title1 'Model 3: ARIMA(0,1,(1,6))';
run;
```

Model 3: ARIMA(0,1,(1,6))

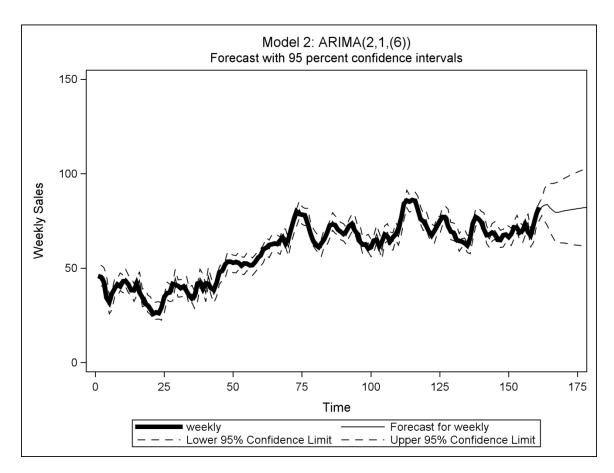
Unconditional Least Squares Estimation									
Parameter	Estimate	Standard Error	t Value	Approx Pr > t	Lag				
MU	0.24618	0.22800	1.08	0.2819	0				
MA1,1	-0.63823	0.09741	-6.55	<.0001	1				
MA1,2	0.36176	0.07368	4.91	<.0001	6				

Constant Estimate	0.246183
Variance Estimate	5.026094
Std Error Estimate	2.241895

Autocorrelation Check of Residuals									
To Lag	Chi- Square	DF	Pr > ChiSq	Autocorrelations					
6	1.76	4	0.7793	-0.000					
12	6.12	10	0.8055	-0.084	0.008	-0.063	-0.061	0.059	0.083
18	10.37	16	0.8464	0.056	0.036	-0.008	-0.118	0.039	-0.062
24	16.05	22	0.8135	0.055	-0.030	-0.084	0.105	0.037	-0.083
30	21.87	28	0.7873	0.120	0.066	-0.061	0.035	-0.076	-0.019



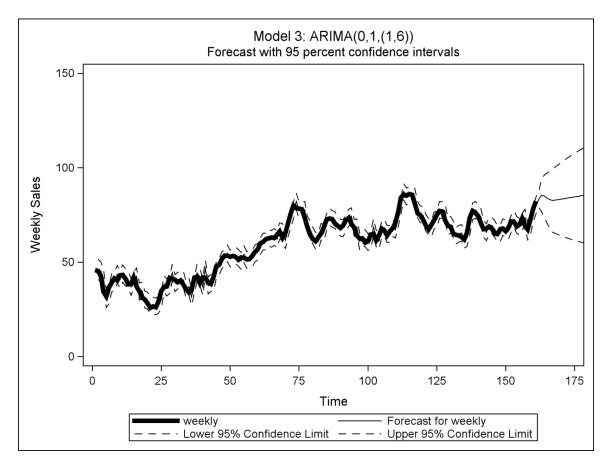
```
/* Forecasts from Model 2: ARIMA(2,1,(6)) */
data f2; set f2;
  time = _n_;
proc sgplot data=f2;
  series x=time y=weekly / lineattrs=(pattern=solid
thickness=5);
  series x=time y=forecast / lineattrs=(pattern=solid);
  series x=time y=195 / lineattrs=(pattern=dash);
  series x=time y=u95 / lineattrs=(pattern=dash);
  series x=time y=u95 / lineattrs=(pattern=dash);
  xaxis label='Time' values=(0 to 190 by 25);
  yaxis label='Weekly Sales' values=(0 to 190 by 50);
  title1 'Model 2: ARIMA(2,1,(6))';
  title2 'Forecast with 95 percent confidence intervals';
run;
```



```
proc print data=f2;
  where time>159;
  var time weekly forecast 195 u95 std residual;
  title1 'Model 2: Forecasts from ARIMA(2,1,(6))';
run;
```

Model 2: Forecasts from ARIMA(2,1,(6))								
Obs	time	weekly	FORECAST	L95	U95	STD	RESIDUAL	
160	160	78.6	76.8043	72.1891	81.420	2.3548	1.79569	
161	161	82.3	79.6317	75.0165	84.247	2.3548	2.66829	
162	162		82.4544	77.8392	87.070	2.3548		
163	163		83.4632	75.0281	91.898	4.3037		
164	164		83.6356	72.5860	94.685	5.6377		
165	165		82.0292	69.1930	94.865	6.5492		
166	166		80.6726	66.4211	94.924	7.2713		
167	167		79.6130	64.0830	95.143	7.9236		
168	168		79.5775	63.3702	95.785	8.2692		
169	169		80.0062	63.3527	96.660	8.4968		
•••								
181	181		82.9144	61.1051	104.724	11.1274		
182	182		83.1429	60.9536	105.332	11.3213		
183	183		83.3713	60.8085	105.934	11.5119		
184	184		83.5997	60.6694	106.530	11.6993		
185	185		83.8281	60.5362	107.120	11.8839		
186	186		84.0565	60.4085	107.705	12.0656		

```
/* Forecasts from Model 3: ARIMA(0,1,(1,6)) */
data f3; set f3;
  time = _n_;
proc sgplot data=f3;
  series x=time y=weekly / lineattrs=(pattern=solid
thickness=5);
  series x=time y=forecast / lineattrs=(pattern=solid);
  series x=time y=195 / lineattrs=(pattern=dash);
  series x=time y=u95 / lineattrs=(pattern=dash);
  series x=time y=u95 / lineattrs=(pattern=dash);
  xaxis label='Time' values=(0 to 190 by 25);
  yaxis label='Weekly Sales' values=(0 to 190 by 50);
  title1 'Model 3: ARIMA(0,1,(1,6))';
  title2 'Forecast with 95 percent confidence intervals';
run;
```



```
proc print data=f3;
  where time>159;
  var time weekly forecast 195 u95 std residual;
  title1 'Model 3: Forecasts from ARIMA(0,1,(1,6))';
run;
```

Model 3: Forecasts from ARIMA(0,1,(1,6))								
Obs	time	weekly	FORECAST	L95	U95	STD	RESIDUAL	
160	160	78.6	76.0198	71.6123	80.427	2.2488	2.58023	
161	161	82.3	80.0402	75.6328	84.448	2.2487	2.25981	
162	162		83.4161	79.0221	87.810	2.2419		
163	163		85.3235	76.8899	93.757	4.3029		
164	164		85.0879	74.0000	96.176	5.6572		
165	165		83.8918	70.6721	97.112	<mark>6.7449</mark>		
166	166		83.2053	68.1528	98.258	<mark>7.6800</mark>		
167	167	•	82.6389	65.9537	99.324	8.5130		
168	168	•	82.8851	65.2824	100.488	8.9811		
169	169	•	83.1313	64.6566	101.606	9.4260		
•••								
181	181	•	86.0855	59.2746	112.896	13.6793		
182	182		86.3317	58.9403	113.723	13.9754		
183	183		86.5779	58.6182	114.538	14.2654		
184	184		86.8241	58.3073	115.341	14.5496		
185	185		87.0702	58.0071	116.133	14.8284		
186	186		87.3164	57.7170	116.916	15.1020		

Rough script:

- 0. Introduce data and express desire to forecast 25 weeks.
- See need for stationarity based on time plot and SAC (p. 2).

Try linear trend, see remaining ~2 year cycle (p. 3). Try linear + trigonometric trends (p. 4).

- -- But still see problems with 1st-order stationarity.
- See stubbornness of time trends (p. 4), and need for differencing; first diff. appears sufficient (pp. 5-6).
- 3. See need for dependence structure after white noise check in first difference (p. 7).
- 4. Model 1: ARIMA(2,1,0), based on mixture of damped exp. decay and sine waves in SAC, and SPAC cuts off after lag 2 -- note may have additional spikes at lags 5 and 6 (pp. 8-9).

Goodness of fit checks: parameters significant, but model is inadequate (p. 8).

5. Model 2: ARIMA(2,1,(6)), based on spike in RSAC of Model 1 (pp. 10-11).

Goodness of fit checks: no evidence of model inadequacy (pp. 8-9) (? -- note Ljung-Box p-value).

6. Model 3: ARIMA(0,1,(1,6)), based on alternative reading of SAC and SPAC of first difference -- on page 7, SAC spikes at lags 1 and 6, SPAC dies down in oscillating fashion. (pp. 12-13).

Goodness of fit checks: no evidence of model inadequacy (pp. 12-13).

7. Compare forecasts from two 'adequate' models (pp 14-17):

Model 3 better only for short-term (2 week) forecasts,
based on tighter confidence intervals (smaller STD for
forecasts only for weeks 162-163).

Model 2 has tighter confidence intervals (smaller STD) for longer-term forecasts (weeks 164-186).

Model summaries:

Model 1: S=2.48, Q=23.06 (P=.0001),

RSAC & RSPAC have spike at lag 6

Model 2: S=2.35, Q=7.13 (P=.07),

RSAC & RSPAC have 'nothing'

Model 3: S=2.24, Q=1.76 (P=.78),

RSAC & RSPAC have 'less nothing'

Conclude: Model 1 inadequate,

Model 2 best for longer-term forecasts,

Model 3 best for short-term forecasts