

6.1.2 - SAS: Time Series Case Study

Data: Weekly sales (in thousands of units) of Super Tech Videocassette Tapes over 161 weeks [see Bowerman & O'Connell "Forecasting and Time Series: An Applied Approach", 3rd Edition, Section 10.4 Case Study.

Goal: Want to forecast sales 25 weeks beyond end of data

```
data sales; input weekly @@; cards;
45.9 45.4 42.8 34.4 31.9 36.6 39.2 41.4 40.3 43.1 43.2
41.2 38.4 38.3 41.9 37.1 34.5 31.3 30.2 28.3 25.9 26.6
26.2 29 34.8 36.8 37.2 41.7 41.2 40.7 39.5 40.4 38
35.6 33.9 35.2 41.8 42.4 38.9 42.1 41.7 39.2 38.5 42.5
47.9 48.6 52 53.5 53.5 52.9 53.4 52.8 51.4 52.5 52.4
51.5 51.7 53.3 55.4 56.9 60 60.8 62.3 62.6 63.1 62.8
64.7 66.3 63 65.5 70.6 76 80.1 78.6 78.3 78.1 73.6
68.8 64.4 62.4 61.1 63.1 65.3 68.3 72.5 73.2 72.9 70.5
69.4 68.2 69.3 72.3 73.5 70.3 68.3 64.1 62.5 62.6 60.4
61.1 64.7 65.1 61.5 64.2 67.8 66.8 64.1 66.4 68 71
76.9 84.1 85.9 85.2 86.2 85.7 81.3 75.9 75 72.5 69.6
67.3 69.8 72.2 75.2 77.2 76.8 72.4 69.4 68.7 65.1 64.4
64.2 63.2 62.1 65.8 73.7 77.1 76 74.6 70.6 67.5 67.9
68.9 67.8 65.1 65 67.6 67.9 66.5 68.2 71.7 71.3 68.9
70 73.1 69.1 67.3 72.9 78.6 82.3
;
run;

/*****

/* Look at original data and check stationarity */
data sales; set sales;
    Time = _N_;
proc arima data=sales;
    identify var=weekly nlag=24;
    title1 'Look for Stationarity in SAC; Original Time
Series';
run;
```

Look for Stationarity in SAC; Original Time Series

The ARIMA Procedure

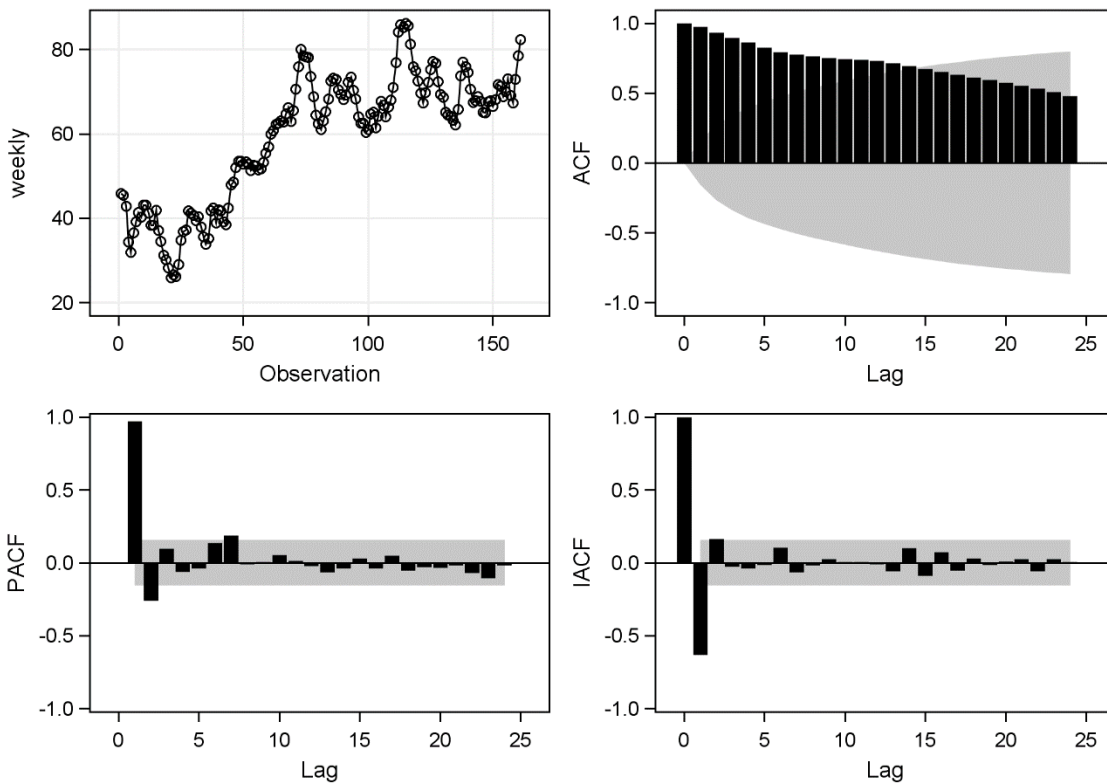
Name of Variable = weekly

Number of Observations 161

Autocorrelation Check for White Noise

To Lag	Chi-Square	DF	Pr > ChiSq	Autocorrelations					
6	779.33	6	<.0001	0.974	0.935	0.899	0.862	0.825	0.794
12	1366.70	12	<.0001	0.777	0.765	0.753	0.745	0.739	0.730
18	1845.25	18	<.0001	0.715	0.695	0.676	0.654	0.634	0.614
24	2177.35	24	<.0001	0.594	0.575	0.556	0.535	0.508	0.480

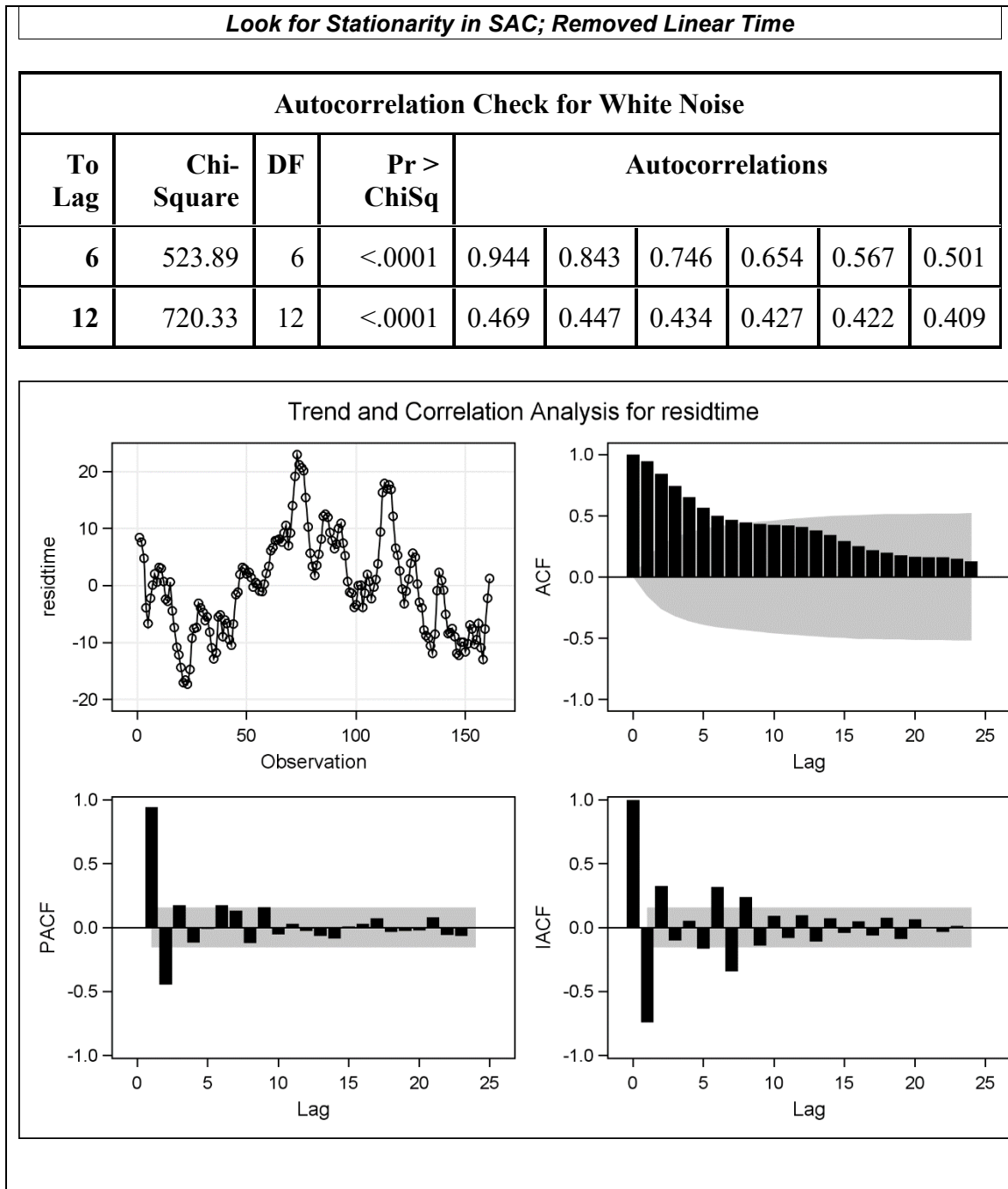
Trend and Correlation Analysis for weekly



```

/* Remove linear effect of time */
proc reg data=sales noprint;
  model weekly = time;
  output out=out1 r=residtime;
proc arima data=out1;
  identify var=residtime nlag=24;
  title1 'Look for Stationarity in SAC; Removed Linear
Time';
run;

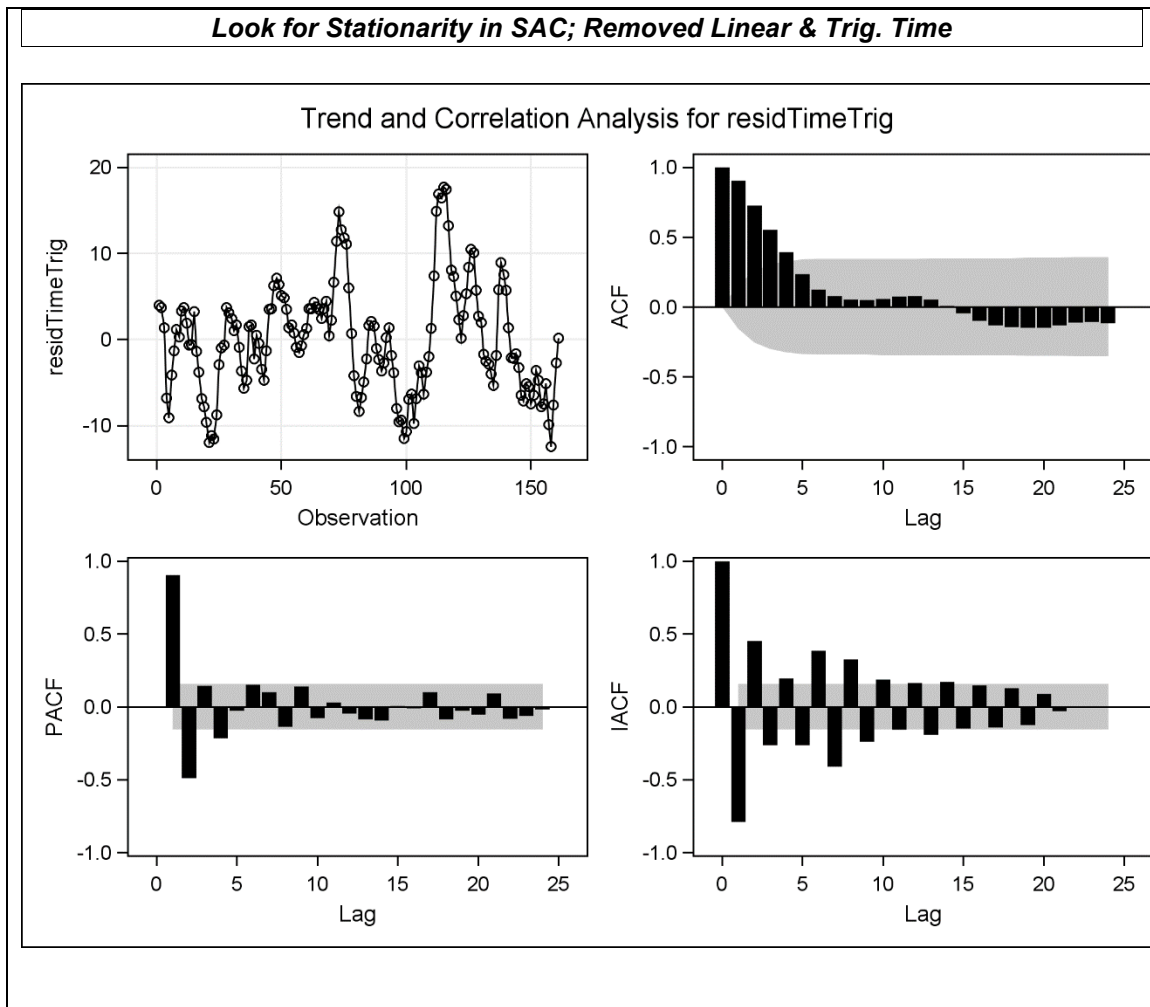
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/* Try to remove cyclic behavior
   -- there appears to be a 2-year cycle */
data sales; set sales;
    sin1 = sin(2*3.14*time/104);
    cos1 = cos(2*3.14*time/104);
proc reg data=sales noprint;
    model weekly = time sin1 cos1;
    output out=out2 r=residTimeTrig;
proc arima data=out2;
    identify var=residTimeTrig nlag=24;
    title1 'Look for Stationarity in SAC; Removed Linear &
Trig. Time';
run;

```

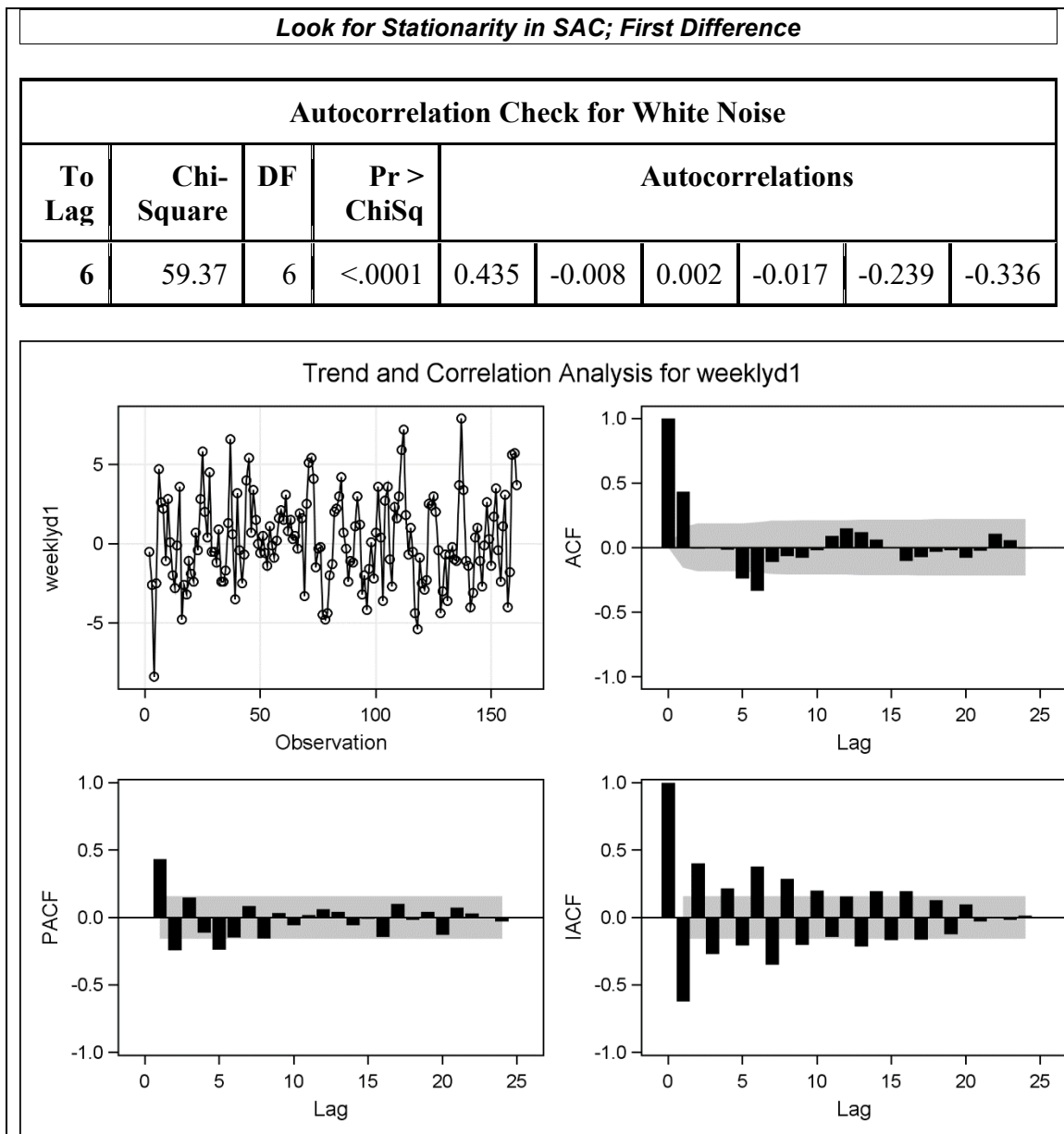


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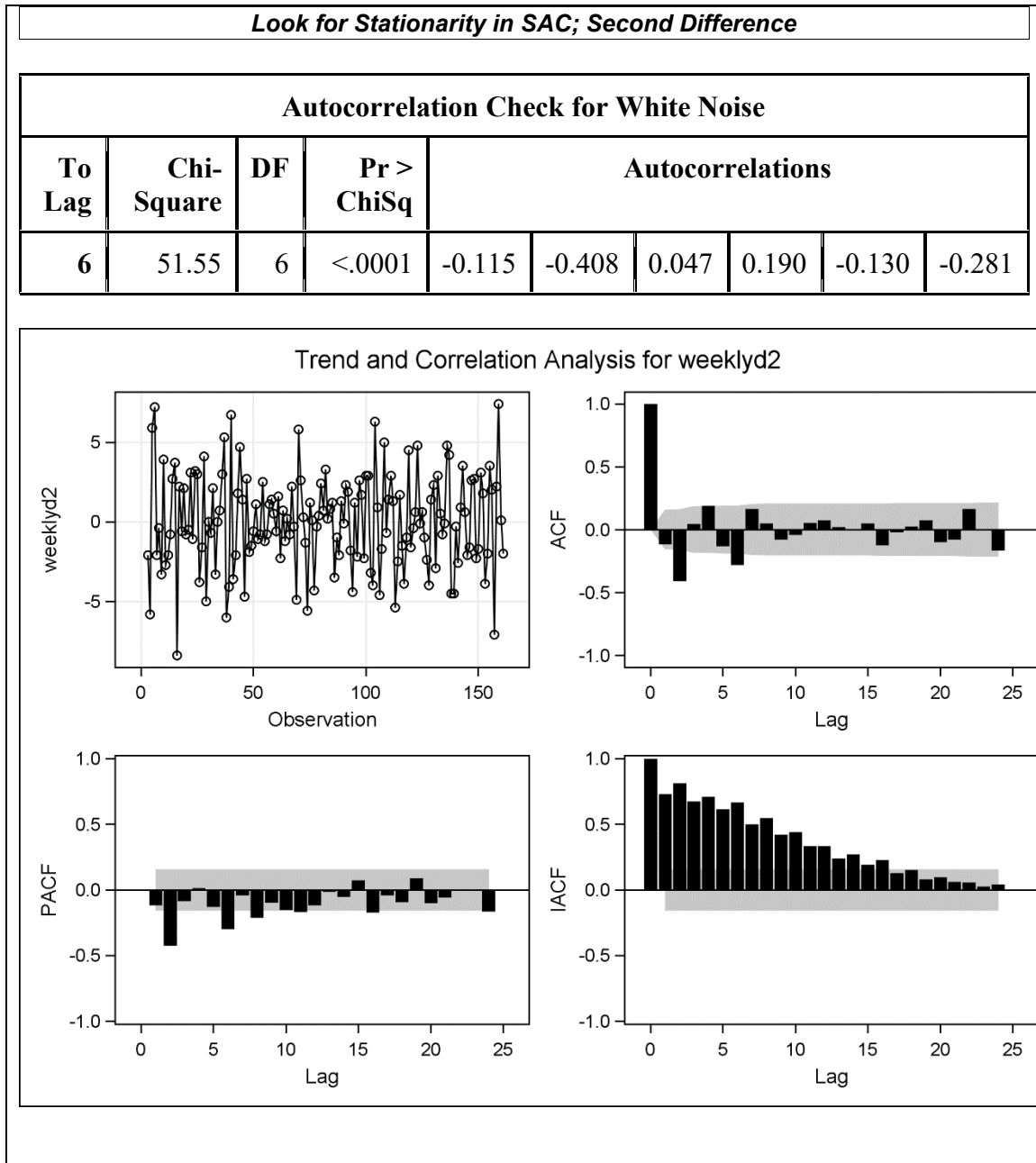
/* It doesn't look like this will work
   -- need to consider differencing */
data sales; set sales;
    weeklyd1 = weekly - lag(weekly);
    weeklyd2 = weeklyd1 - lag(weeklyd1);
run;

proc arima data=sales;
    identify var=weeklyd1 nlag=24;
    title1 'Look for Stationarity in SAC; First Difference';
run;

```



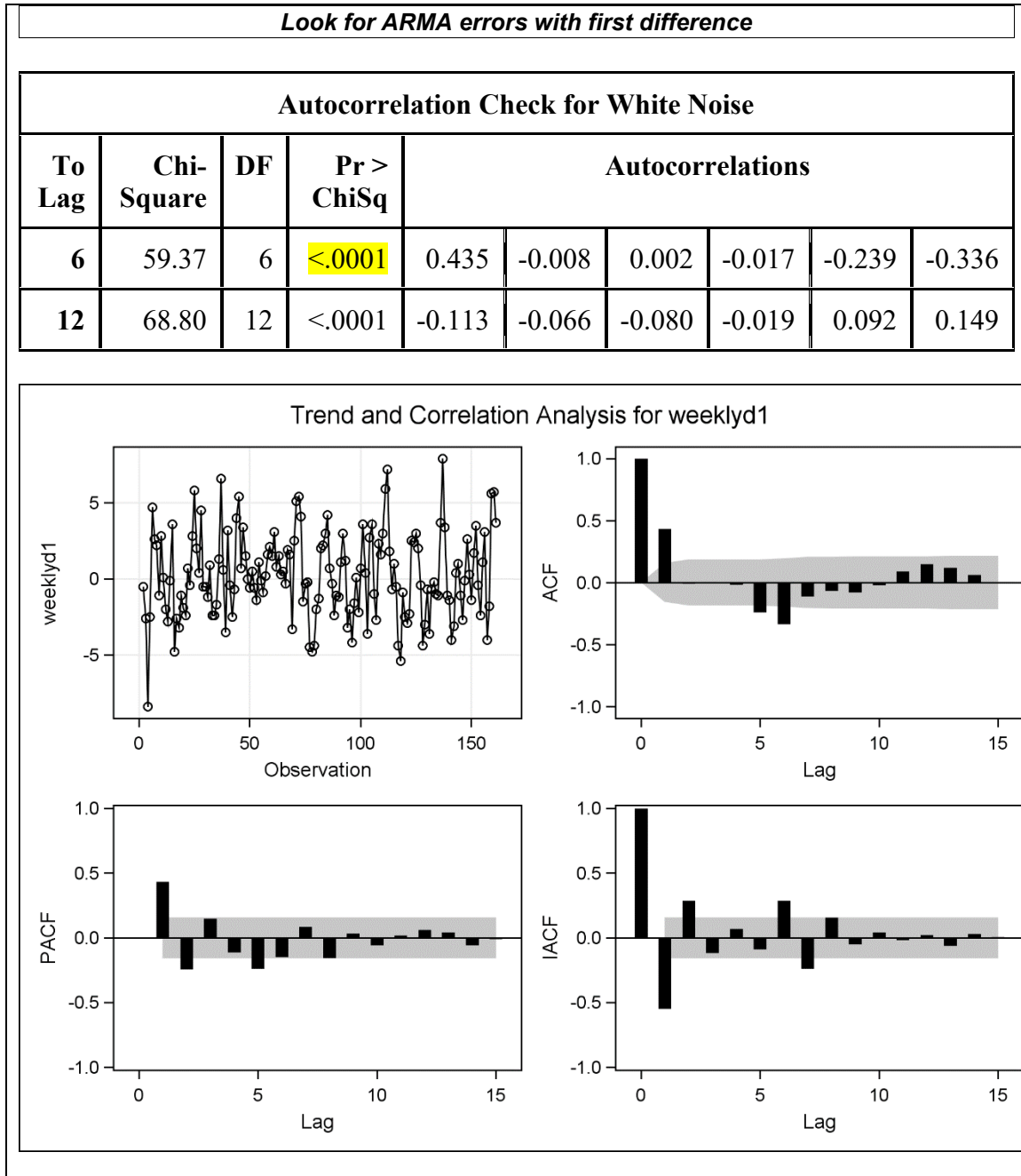
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proc arima data=sales;
  identify var=weeklyd2 nlag=24;
  title1 'Look for Stationarity in SAC; Second Difference';
run;
```



```

/* Now look for ARMA errors in first difference */
proc arima data=sales;
  identify var=weeklyd1 nlag=15;
  title1 'Look for ARMA errors with first difference';
run;

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/* Model 1: ARIMA(2,1,0), based on
   SAC's damped exponential / sine pattern,
   and SPAC spikes at 1 and 2 */
proc arima data=sales;
  identify var=weekly(1);
  estimate p=2 plot method=uls;
  forecast lead=25 alpha=0.05 noprint out=f1;
  title1 'Model 1: ARIMA(2,1,0)';
run;

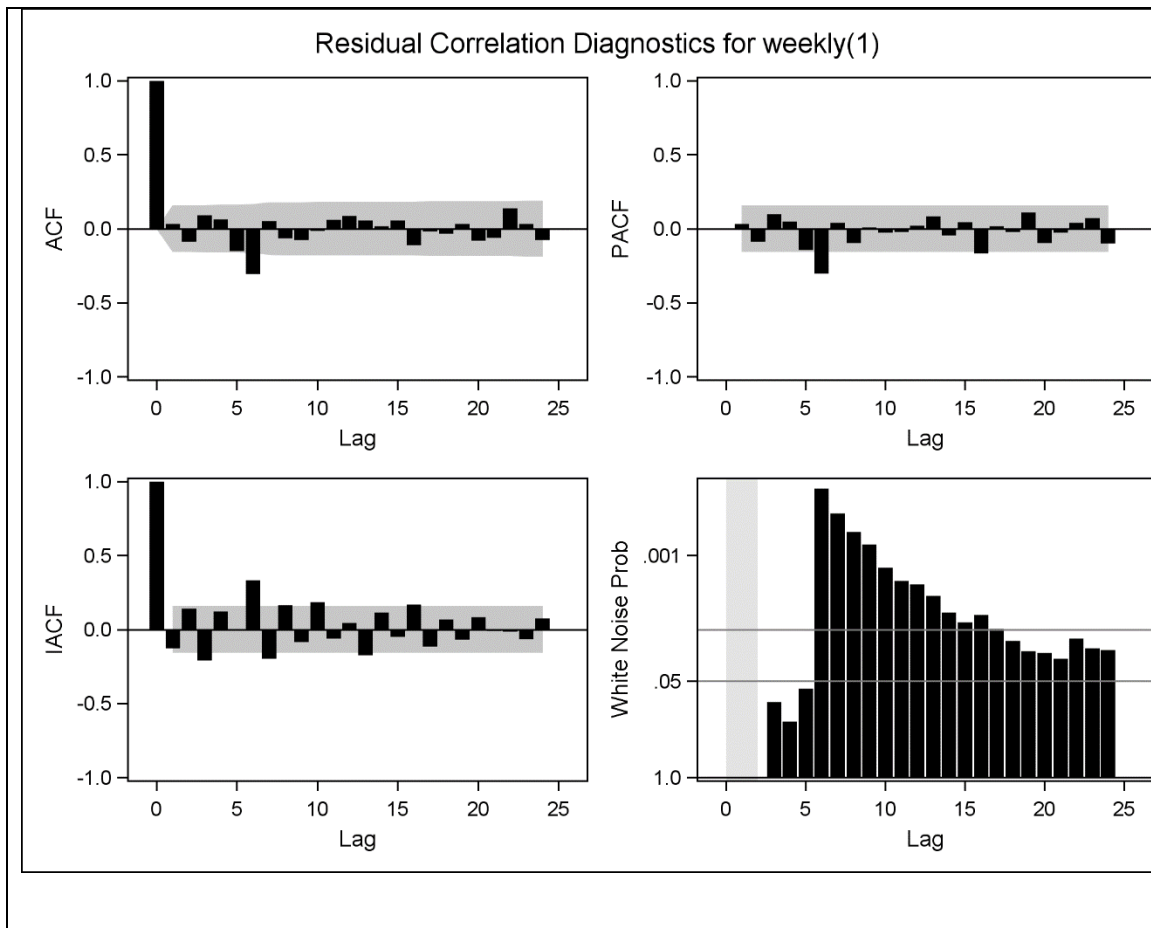
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Model 1: ARIMA(2,1,0)

Unconditional Least Squares Estimation					
Parameter	Estimate	Standard Error	t Value	Approx Pr > t	Lag
MU	0.22900	0.27904	0.82	0.4131	0
AR1,1	0.54277	0.07749	7.00	<.0001	1
AR1,2	-0.24502	0.07844	-3.12	0.0021	2

Constant Estimate	0.160818
Variance Estimate	6.147007
Std Error Estimate	2.479316

Autocorrelation Check of Residuals									
To Lag	Chi-Square	DF	Pr > ChiSq	Autocorrelations					
6	23.06	4	0.0001	0.035	-0.087	0.090	0.065	-0.151	-0.306
12	27.19	10	0.0024	0.053	-0.063	-0.074	-0.014	0.059	0.089
18	30.84	16	0.0141	0.057	0.017	0.056	-0.111	-0.019	-0.032
24	37.90	22	0.0188	0.035	-0.080	-0.058	0.140	0.034	-0.077
30	42.29	28	0.0407	0.107	0.092	-0.042	0.000	-0.030	-0.006



```

/* Model 2: ARIMA(2,1,(6)), based on
   RSAC/RSPAC spikes in Model 1 */
proc arima data=sales;
  identify var=weekly(1);
  estimate p=2 q=6 plot method=uls;
  forecast lead=25 alpha=0.05 noprint out=f2;
  title1 'Model 2: ARIMA(2,1,(6))';
run;

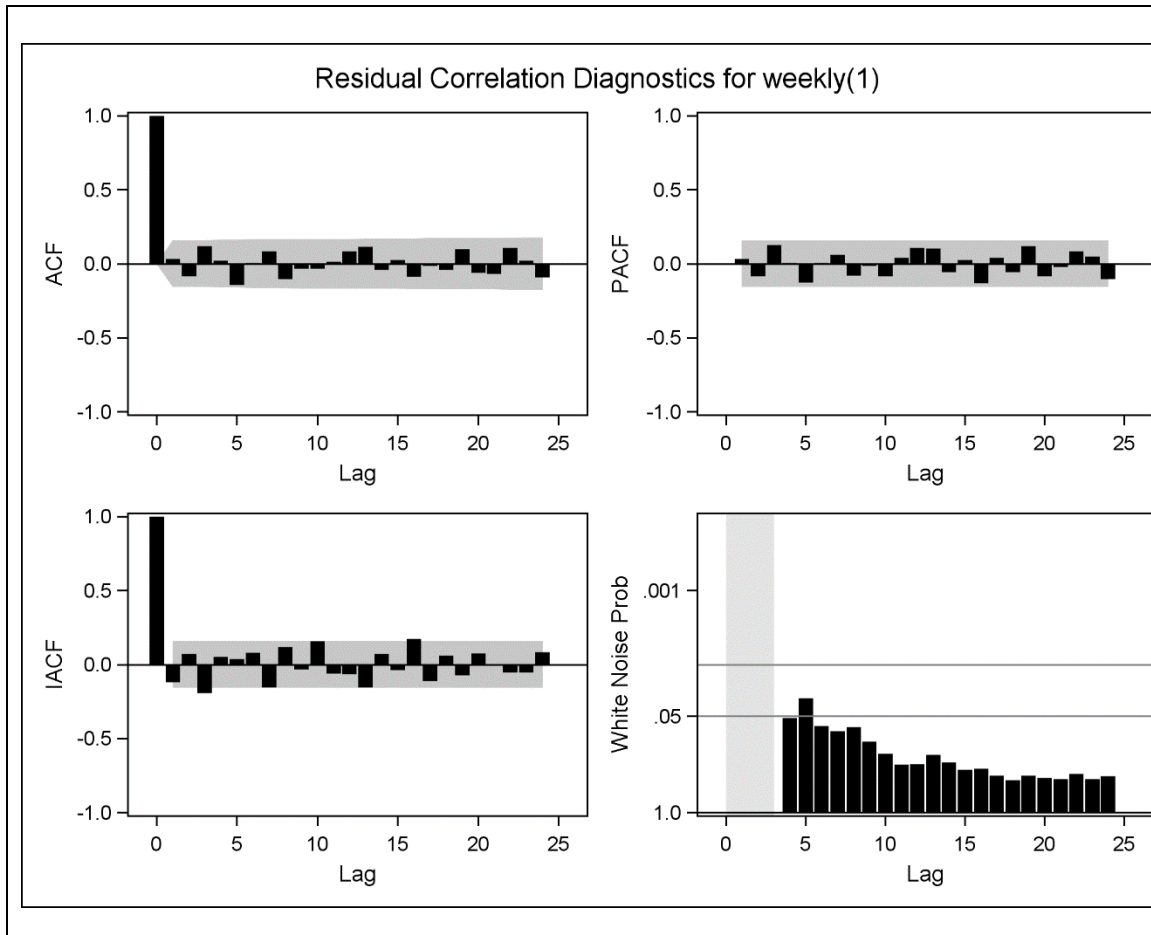
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Model 2: ARIMA(2,1,(6))

Unconditional Least Squares Estimation					
Parameter	Estimate	Standard Error	t Value	Approx Pr > t	Lag
MU	0.22841	0.16843	1.36	0.1770	0
MA1,1	0.34955	0.07858	4.45	<.0001	6
AR1,1	0.52984	0.07798	6.79	<.0001	1
AR1,2	-0.26407	0.07901	-3.34	0.0010	2

Constant Estimate	0.167703
Variance Estimate	5.54486
Std Error Estimate	2.354753

Autocorrelation Check of Residuals									
To Lag	Chi-Square	DF	Pr > ChiSq	Autocorrelations					
6	7.13	3	0.0679	0.032	-0.082	0.121	0.022	-0.142	0.007
12	11.84	9	0.2226	0.085	-0.103	-0.033	-0.034	0.015	0.084
18	16.27	15	0.3645	0.114	-0.040	0.028	-0.087	-0.013	-0.041
24	23.44	21	0.3210	0.101	-0.060	-0.067	0.105	0.023	-0.092
30	29.05	27	0.3584	0.110	0.078	-0.078	0.005	-0.046	-0.050



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/* Model 3: ARIMA(0,1,(1,6)), based on
   alternative reading of first diff. SAC */
proc arima data=sales;
  identify var=weekly(1);
  estimate p=0 q=(1,6) plot method=uls;
  forecast lead=25 alpha=0.05 noprint out=f3;
  title1 'Model 3: ARIMA(0,1,(1,6))';
run;

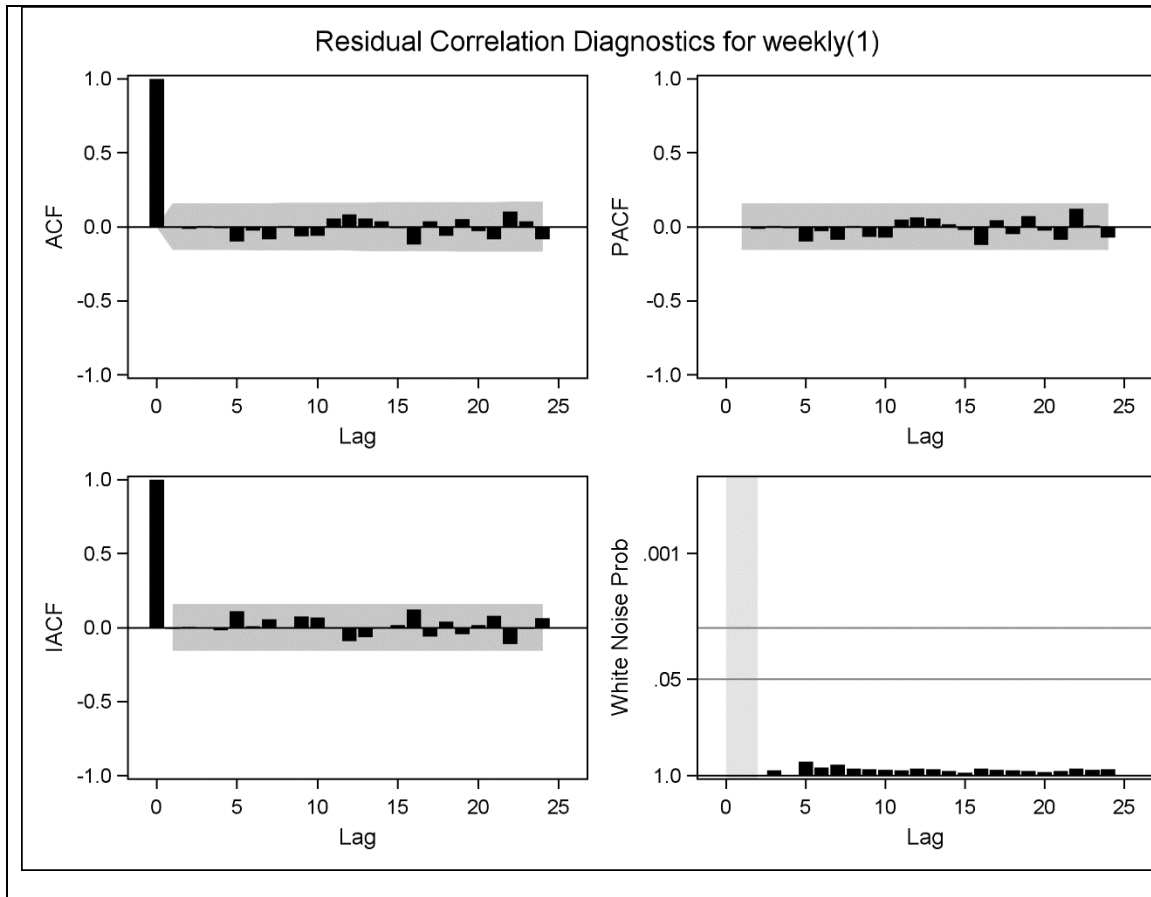
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Model 3: ARIMA(0,1,(1,6))

Unconditional Least Squares Estimation					
Parameter	Estimate	Standard Error	t Value	Approx Pr > t	Lag
MU	0.24618	0.22800	1.08	0.2819	0
MA1,1	-0.63823	0.09741	-6.55	<.0001	1
MA1,2	0.36176	0.07368	4.91	<.0001	6

Constant Estimate	0.246183
Variance Estimate	5.026094
Std Error Estimate	2.241895

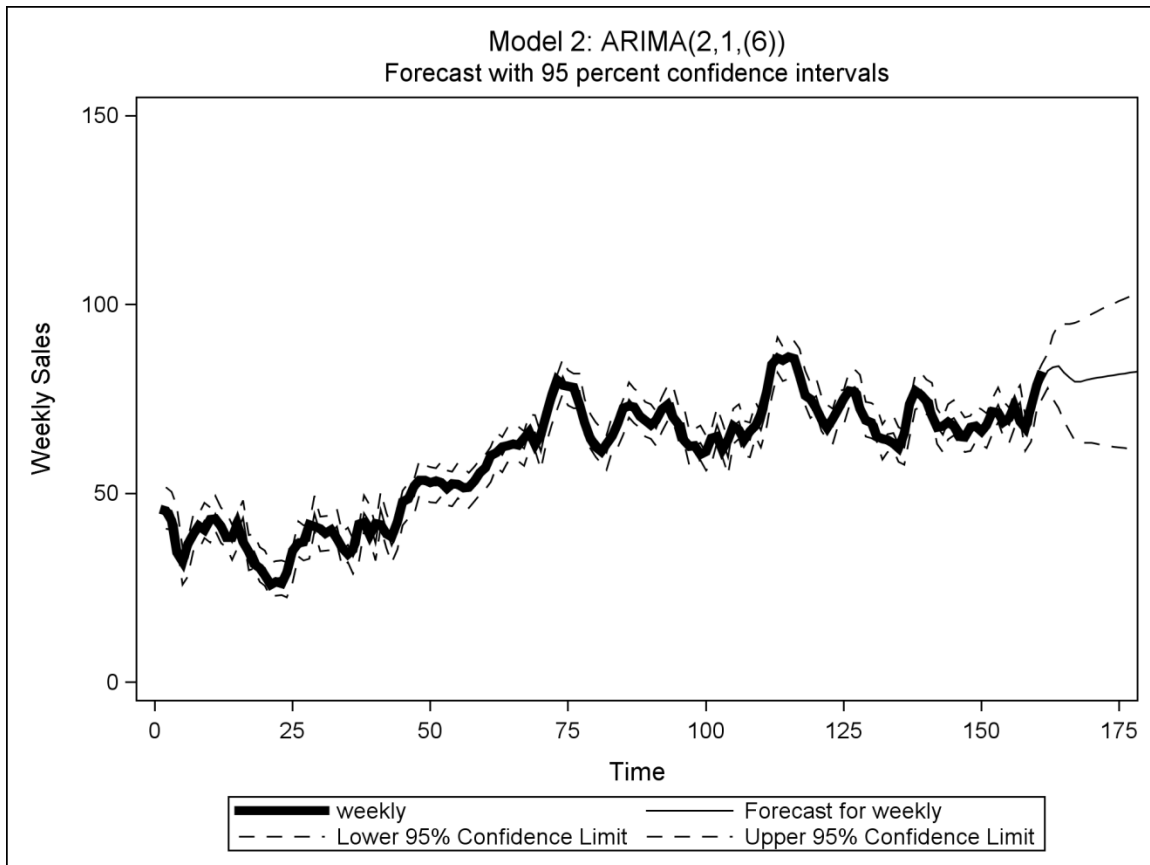
Autocorrelation Check of Residuals									
To Lag	Chi- Square	DF	Pr > ChiSq	Autocorrelations					
6	1.76	4	0.7793	-0.000	-0.014	0.007	-0.009	-0.098	-0.026
12	6.12	10	0.8055	-0.084	0.008	-0.063	-0.061	0.059	0.083
18	10.37	16	0.8464	0.056	0.036	-0.008	-0.118	0.039	-0.062
24	16.05	22	0.8135	0.055	-0.030	-0.084	0.105	0.037	-0.083
30	21.87	28	0.7873	0.120	0.066	-0.061	0.035	-0.076	-0.019



```

/* Forecasts from Model 2:  ARIMA(2,1,(6)) */
data f2; set f2;
  time = _n_;
proc sgplot data=f2;
  series x=time y=weekly / lineattrs=(pattern=solid
thickness=5);
  series x=time y=forecast / lineattrs=(pattern=solid);
  series x=time y=l95 / lineattrs=(pattern=dash);
  series x=time y=u95 / lineattrs=(pattern=dash);
  xaxis label='Time' values=(0 to 190 by 25);
  yaxis label='Weekly Sales' values=(0 to 190 by 50);
  title1 'Model 2: ARIMA(2,1,(6))';
  title2 'Forecast with 95 percent confidence intervals';
run;

```



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proc print data=f2;
  where time>159;
  var time weekly forecast l95 u95 std residual;
  title1 'Model 2: Forecasts from ARIMA(2,1,(6))';
run;

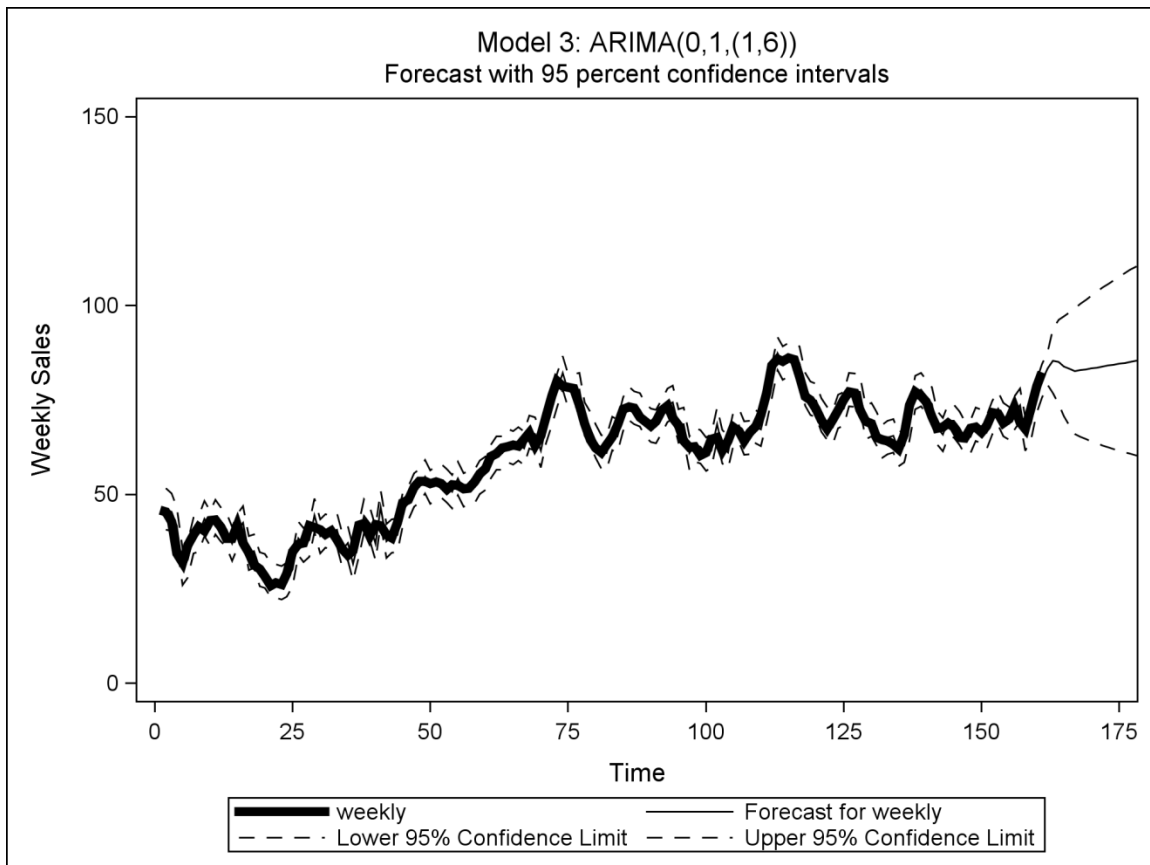
```

Model 2: Forecasts from ARIMA(2,1,(6))							
Obs	time	weekly	FORECAST	L95	U95	STD	RESIDUAL
160	160	78.6	76.8043	72.1891	81.420	2.3548	1.79569
161	161	82.3	79.6317	75.0165	84.247	2.3548	2.66829
162	162	.	82.4544	77.8392	87.070	2.3548	.
163	163	.	83.4632	75.0281	91.898	4.3037	.
164	164	.	83.6356	72.5860	94.685	5.6377	.
165	165	.	82.0292	69.1930	94.865	6.5492	.
166	166	.	80.6726	66.4211	94.924	7.2713	.
167	167	.	79.6130	64.0830	95.143	7.9236	.
168	168	.	79.5775	63.3702	95.785	8.2692	.
169	169	.	80.0062	63.3527	96.660	8.4968	.
...							
181	181	.	82.9144	61.1051	104.724	11.1274	.
182	182	.	83.1429	60.9536	105.332	11.3213	.
183	183	.	83.3713	60.8085	105.934	11.5119	.
184	184	.	83.5997	60.6694	106.530	11.6993	.
185	185	.	83.8281	60.5362	107.120	11.8839	.
186	186	.	84.0565	60.4085	107.705	12.0656	.

```

/* Forecasts from Model 3: ARIMA(0,1,(1,6)) */
data f3; set f3;
  time = _n_;
proc sgplot data=f3;
  series x=time y=weekly / lineattrs=(pattern=solid
thickness=5);
  series x=time y=forecast / lineattrs=(pattern=solid);
  series x=time y=l95 / lineattrs=(pattern=dash);
  series x=time y=u95 / lineattrs=(pattern=dash);
  xaxis label='Time' values=(0 to 190 by 25);
  yaxis label='Weekly Sales' values=(0 to 190 by 50);
  title1 'Model 3: ARIMA(0,1,(1,6))';
  title2 'Forecast with 95 percent confidence intervals';
run;

```




```

proc print data=f3;
  where time>159;
  var time weekly forecast l95 u95 std residual;
  title1 'Model 3: Forecasts from ARIMA(0,1,(1,6))';
run;

```

<i>Model 3: Forecasts from ARIMA(0,1,(1,6))</i>							
Obs	time	weekly	FORECAST	L95	U95	STD	RESIDUAL
160	160	78.6	76.0198	71.6123	80.427	2.2488	2.58023
161	161	82.3	80.0402	75.6328	84.448	2.2487	2.25981
162	162	.	83.4161	79.0221	87.810	2.2419	.
163	163	.	85.3235	76.8899	93.757	4.3029	.
164	164	.	85.0879	74.0000	96.176	5.6572	.
165	165	.	83.8918	70.6721	97.112	6.7449	.
166	166	.	83.2053	68.1528	98.258	7.6800	.
167	167	.	82.6389	65.9537	99.324	8.5130	.
168	168	.	82.8851	65.2824	100.488	8.9811	.
169	169	.	83.1313	64.6566	101.606	9.4260	.
...							
181	181	.	86.0855	59.2746	112.896	13.6793	.
182	182	.	86.3317	58.9403	113.723	13.9754	.
183	183	.	86.5779	58.6182	114.538	14.2654	.
184	184	.	86.8241	58.3073	115.341	14.5496	.
185	185	.	87.0702	58.0071	116.133	14.8284	.
186	186	.	87.3164	57.7170	116.916	15.1020	.

Rough script:

0. Introduce data and express desire to forecast 25 weeks.
1. See need for stationarity based on time plot and SAC (p. 2).
Try linear trend, see remaining ~2 year cycle (p. 3).
Try linear + trigonometric trends (p. 4).
-- But still see problems with 1st-order stationarity.
2. See stubbornness of time trends (p. 4), and need for differencing; first diff. appears sufficient (pp. 5-6).
3. See need for dependence structure after white noise check in first difference (p. 7).
4. Model 1: ARIMA(2,1,0), based on mixture of damped exp. decay and sine waves in SAC, and SPAC cuts off after lag 2 -- note may have additional spikes at lags 5 and 6 (pp. 8-9).
Goodness of fit checks: parameters significant, but model is inadequate (p. 8).
5. Model 2: ARIMA(2,1,(6)), based on spike in RSAC of Model 1 (pp. 10-11).
Goodness of fit checks: no evidence of model inadequacy (pp. 8-9) (? -- note Ljung-Box p-value).
6. Model 3: ARIMA(0,1,(1,6)), based on alternative reading of SAC and SPAC of first difference -- on page 7, SAC spikes at lags 1 and 6, SPAC dies down in oscillating fashion. (pp. 12-13).
Goodness of fit checks: no evidence of model inadequacy (pp. 12-13).
7. Compare forecasts from two 'adequate' models (pp 14-17):
Model 3 better only for short-term (2 week) forecasts, based on tighter confidence intervals (smaller STD for forecasts only for weeks 162-163).
Model 2 has tighter confidence intervals (smaller STD) for longer-term forecasts (weeks 164-186).

Model summaries:

Model 1: S=2.48, Q=23.06 (P=.0001),
RSAC & RSPAC have spike at lag 6
Model 2: S=2.35, Q=7.13 (P=.07),
RSAC & RSPAC have 'nothing'
Model 3: S=2.24, Q=1.76 (P=.78),
RSAC & RSPAC have 'less nothing'

Conclude: Model 1 inadequate,
Model 2 best for longer-term forecasts,
Model 3 best for short-term forecasts