Stat 5100 Handout 3.3.1 – SAS: Influential Observations and Outliers

Example: Data collected on 50 countries relevant to a cross-sectional study of a life-cycle savings hypothesis, which states that the response variable

- SavRatio: aggregate personal saving divided by disposable income

can be explained by the following four predictor variables:

- AvIncome: per-capita disposable income, in USD (yearly average over decade)
- GrowRate: percentage growth rate in per-capita disposable income (over decade)
- PopU15: percentage of the population less than 15 years old (yearly average over decade)
- PopO75: percentage of the population over 75 years old (yearly average over decade)

The decade is 1960-1970. These data are published in section 2.2 of *Regression Diagnostics: Identifying Influential Data and Sources of Collinearity* (1980) by Belsley, Kuh, and Welsch (limited excerpt available through Google books).

```
/* Define options */
ods html image_dpi=300 style=journal;

/* Read in the data */
proc import out=work.savings dbms=csv replace
    datafile =
   "<file path here>"
  getnames=yes;
  datarow=2;
run;


/* Look at a regression model to predict SavRatio,
   with diagnostics for influential obs. and outliers */

proc reg data = savings
   plots(label)=(CooksD RStudentByLeverage DFFITS DFBETAS);
 id Country;
 model SavRatio = PopU15 PopO75 AvIncome GrowRate /
            partial partialdata;
 output out=out1 r=resid p=pred;
 title1 'Predict SavRatio';
run;
```

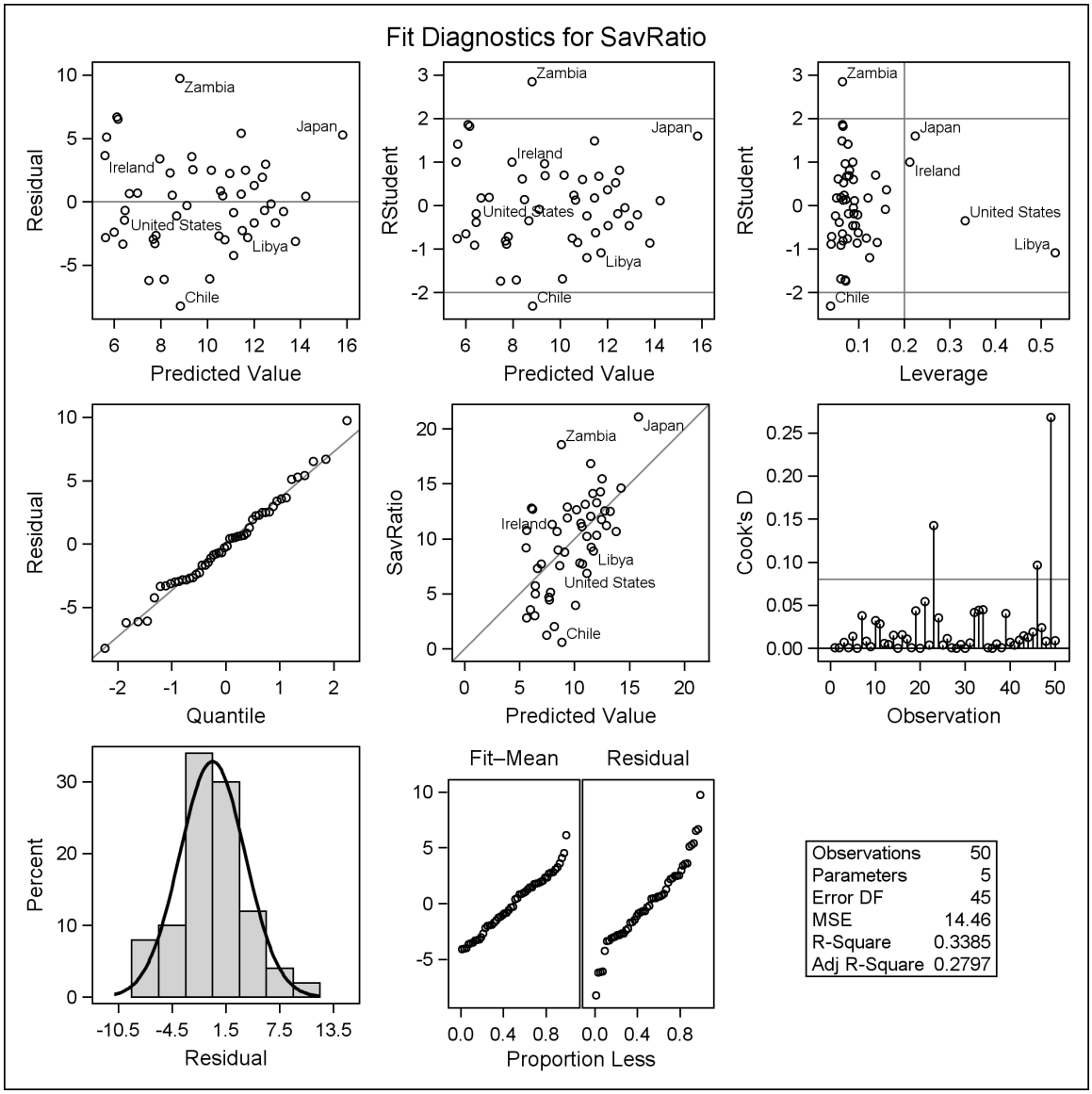
Predict SavRatio

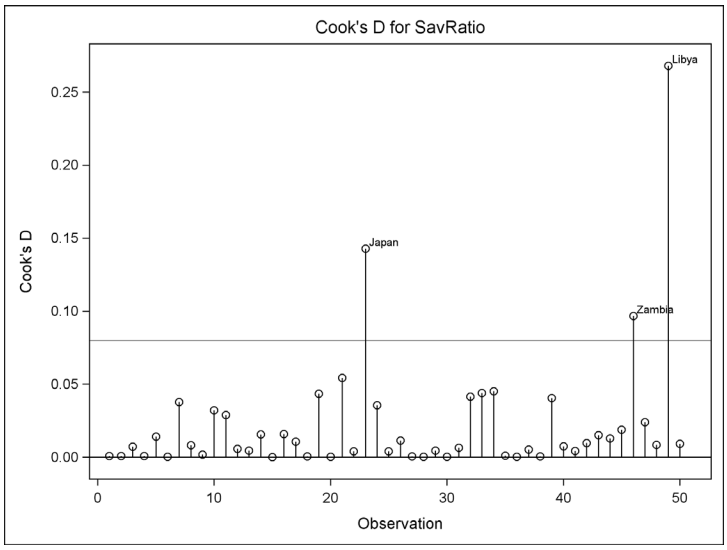
Number of Observations Read	50
Number of Observations Used	50

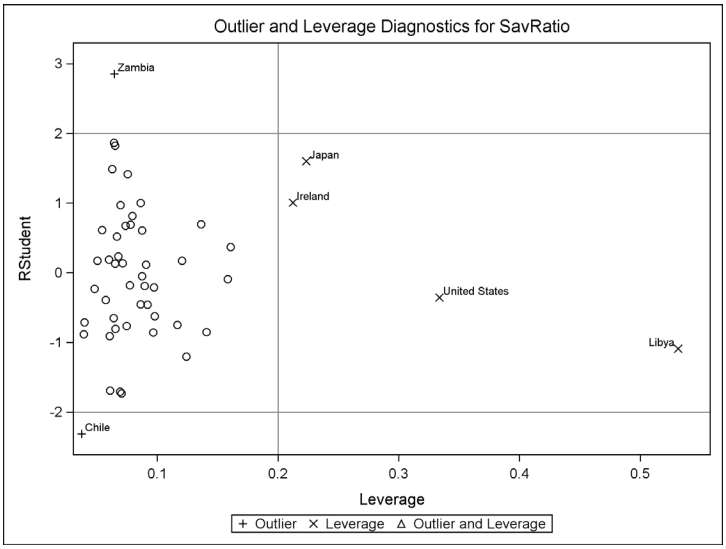
Analysis of Variance					
Source	**DF**	**Sum of Squares**	**Mean Square**	**F Value**	**Pr > F**
Model	4	332.91525	83.22881	5.76	0.0008
Error	45	650.71300	14.46029		
Corrected Total	49	983.62825			

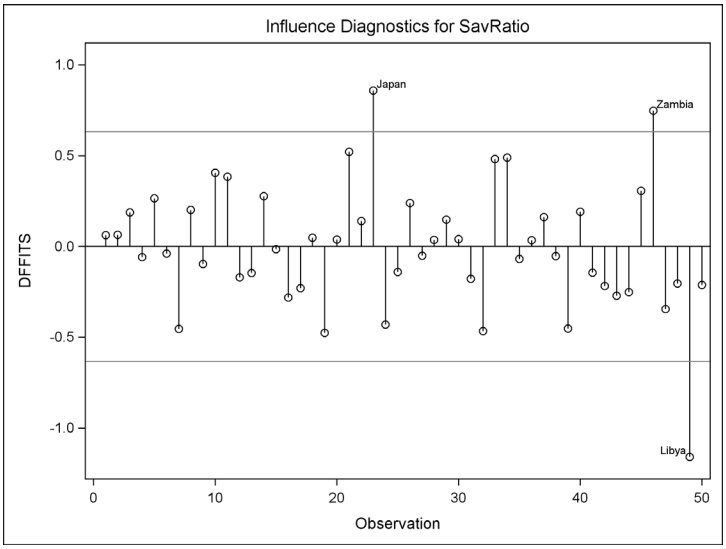
Root MSE	3.80267	**R-Square**	0.3385
Dependent Mean	9.67100	**Adj R-Sq**	0.2797
Coeff Var	39.32033		

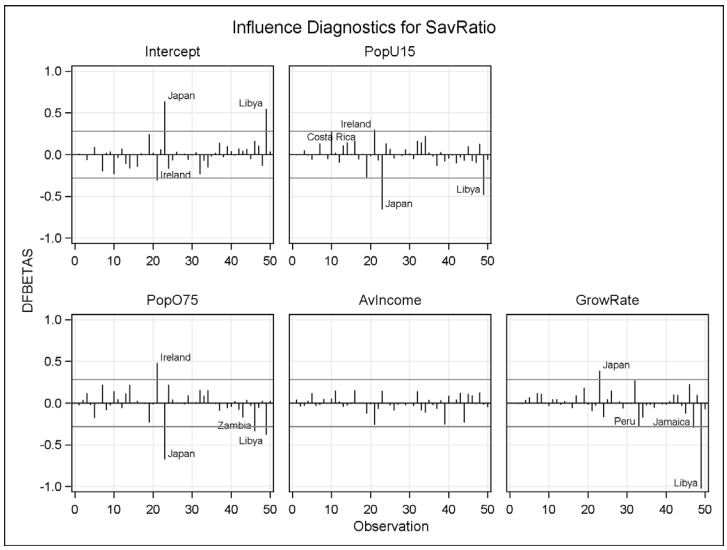
Parameter Estimates					
Variable	**DF**	**Parameter Estimate**	**Standard Error**	**t Value**	**Pr > |t|**
Intercept	**1**	28.56609	7.35452	3.88	0.0003
PopU15	**1**	-0.46119	0.14464	-3.19	0.0026
PopO75	**1**	-1.69150	1.08360	-1.56	0.1255
AvIncome	**1**	-0.00033690	0.00093111	-0.36	0.7192
GrowRate	**1**	0.40969	0.19620	2.09	0.0425

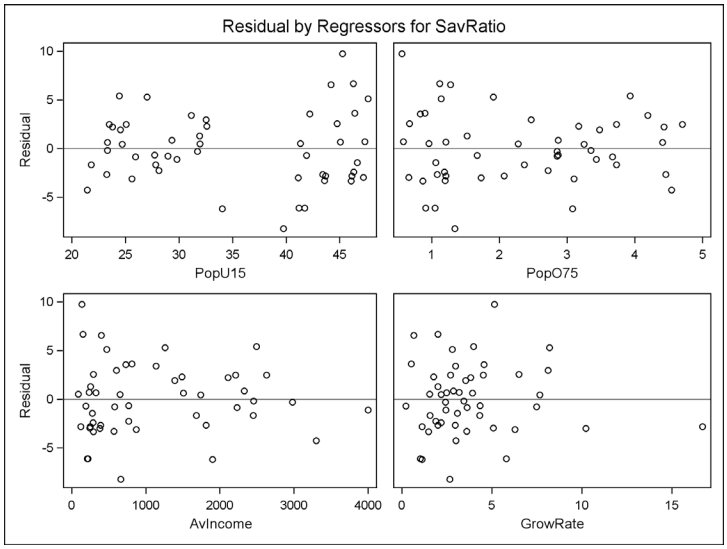

Fit Diagnostics for SavRatio
Residual
Predicted Value
Zambia
Japan
Ireland
United States
Libya
Chile
RStudent
Predicted Value
Zambia
Japan
Ireland
United States
Libya
Chile
RStudent
Leverage
Zambia
Japan
Ireland
United States
Libya
Chile
Residual
Quantile
SavRatio
Predicted Value
Japan
Zambia
Ireland
Libya
United States
Chile
Cook's D
Observation
Percent
Residual
Fit–Mean
Residual
Proportion Less
Observations 50
Parameters 5
Error DF 45
MSE 14.46
R-Square 0.3385
Adj R-Square 0.2797

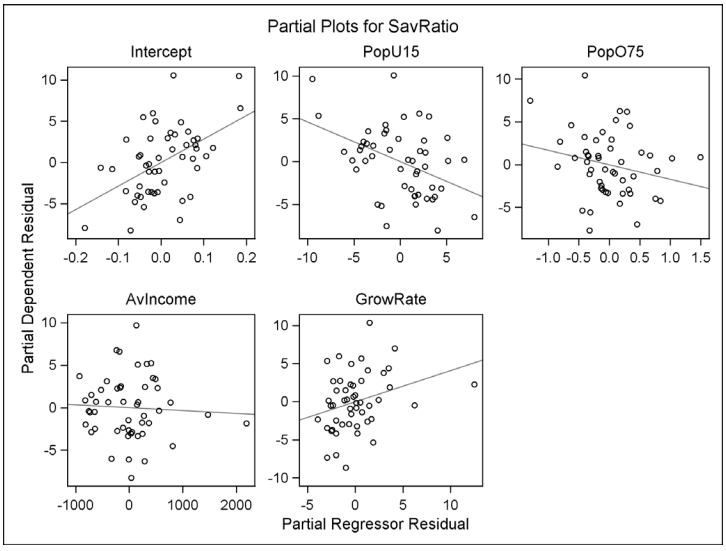

Cook's D for SavRatio
Cook's D
Observation
Libya
Japan
Zambia
0.25
0.20
0.15
0.10
0.05
0.00
0
10
20
30
40
50


Outlier and Leverage Diagnostics for SavRatio
RStudent
Leverage
Zambia
Japan
Ireland
United States
Libya
Chile
3
2
1
0
-1
-2
0.1
0.2
0.3
0.4
0.5
+ Outlier × Leverage Δ Outlier and Leverage


Influence Diagnostics for SavRatio
DFFITS
Observation
Japan
Zambia
Libya
1.0
0.5
0.0
-0.5
-1.0
0
10
20
30
40
50


Influence Diagnostics for SavRatio
Intercept
PopU15
PopO75
AvIncome
GrowRate
DFBETAS
Observation
Japan
Libya
Ireland
Costa Rica
Zambia
Peru
Jamaica


Residual by Regressors for SavRatio
Residual
PopU15
PopO75
AvIncome
GrowRate


Partial Plots for SavRatio
Intercept
PopU15
PopO75
AvIncome
GrowRate
Partial Dependent Residual
Partial Regressor Residual

Obs	Country	partial PopU15	SavRatio partial PopU15	partial PopO75	SavRatio partial PopO75	partial AvIncome	SavRatio partial AvIncome	partial GrowRate	SavRatio partial GrowRate
1	**Australia**	-1.13831	1.38856	-0.38628	1.51696	768.25943	0.60475	-0.01280	0.85833
…									
21	**Ireland**	6.87903	0.21857	1.49268	0.86626	-928.27477	3.70387	-1.59754	2.73663
22	**Italy**	-3.46588	3.52520	0.02001	1.89291	-530.90583	2.10562	-1.03136	1.50421
23	**Japan**	-9.48247	9.65473	-1.30002	7.48046	328.00826	5.17098	4.14032	6.97775
24	**Korea**	-2.03533	-5.16830	-0.44035	-5.36212	-11.96560	-6.10295	1.88266	-5.33567
…									
44	**United States**	4.41082	-3.14583	-0.30252	-0.59987	**2191.50614**	-1.84991	1.46020	-0.51335
45	**Venezuela**	2.53497	2.46341	-0.11300	3.82366	444.00821	3.48293	-2.30789	2.68698
46	**Zambia**	-0.70557	10.07632	-0.40325	10.43302	130.21746	9.70704	1.49950	10.36525
…									
49	**Libya**	7.98336	-6.51140	0.83739	-4.24597	49.71961	-2.84628	**12.47740**	2.28240
50	**Malaysia**	1.93033	-3.86112	-0.13636	-2.74022	243.12949	-3.05278	1.68312	-2.28130

```
/* Check other assumptions */
/* Define shortcut macro, using line copied from
Course Canvas Page */
%macro resid_num_diag(dataset, ...

/* Call shortcut macro */
%resid_num_diag(dataset=out1, datavar=resid,
   label='Residual', predvar=pred, predlabel='Predicted');
run;
```

P-value for Brown-Forsythe test of constant variance in Residual vs. Predicted Value

Obs	t_BF	BF_pvalue
1	2.40263	0.020193

Output for correlation test of normality of Residual (Check text Table B.6 for threshold)

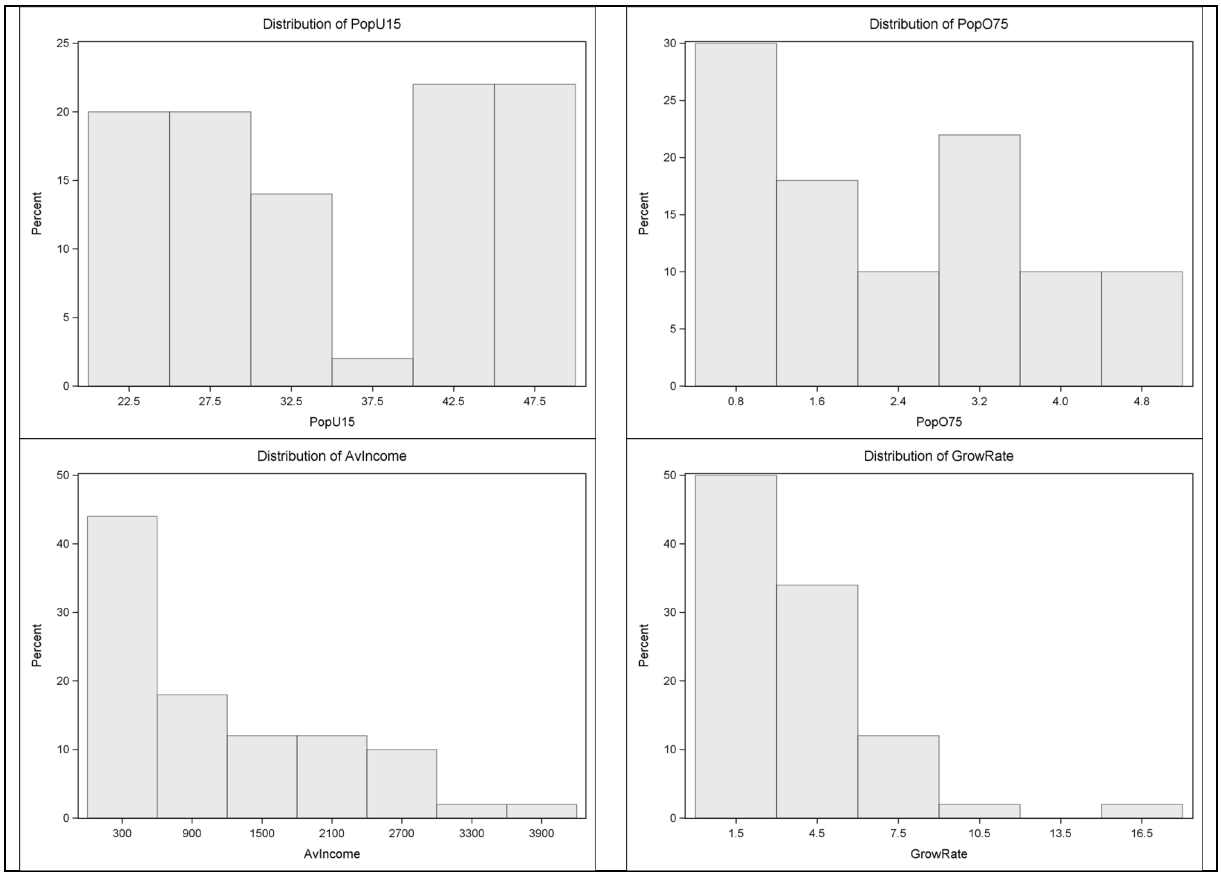
Pearson Correlation Coefficients, N = 50 Prob > |r| under H0: Rho=0		
	resid	expectNorm
resid Residual	1.00000	0.99252 <.0001
expectNorm	0.99252 <.0001	1.00000

```
/* Alternative thresholds for influential obs.
   and outlier diagnostics */
data temp;
  p=5;          /* p = # beta's (incl. intercept */
  n = 50;       /* n = sample size */
  CooksDsimple = 4/n;
  CooksD10 = finv(.10,p,n-p);
  CooksD20 = finv(.20,p,n-p);
  CooksD50 = finv(.50,p,n-p);
  RStudent95 = tinv((1-.05/2),(n-p));
  RStudent95Bonf = tinv((1-.05/2/n),(n-p));
  Leverage2 = 2*p/n;
  Leverage3 = 3*p/n;
  DFBETAS = 2/n**0.5; if (n <= 30) then DFBETAS = 1;
  DFFITS = 2*(p/n)**0.5; if (n <= 30) then DFFITS = 1;
  ;
proc print data=temp;
  var CooksDsimple CooksD10 CooksD20 CooksD50 RStudent95
      RStudent95Bonf Leverage2 Leverage3 DFBETAS DFFITS;
  title1 'Alternative thresholds';
run;
```

Alternative thresholds

Obs	CooksDsimple	CooksD10	CooksD20	CooksD50	RStudent95	RStudent95Bonf	Leverage2	Leverage3	DFBETAS	DFFITS
1	0.08	0.31729	0.46527	0.88349	2.01410	3.52025	0.2	0.3	0.28284	0.63246

```
/* Now look more closely at distribution of predictors,
   and suspect observations */
proc univariate data=savings noprint;
 var PopU15 PopO75 AvIncome GrowRate;
 histogram  PopU15 PopO75 AvIncome GrowRate;
 title1;
run;
```




```
proc print data=savings;
  where country = 'Ireland' | country = 'Japan'
     | country = 'United States' | country = 'Libya'
     | country = 'Zambia';
  var country SavRatio PopU15 PopO75 AvIncome GrowRate;
  title1 'Suspect observations';
run;
```

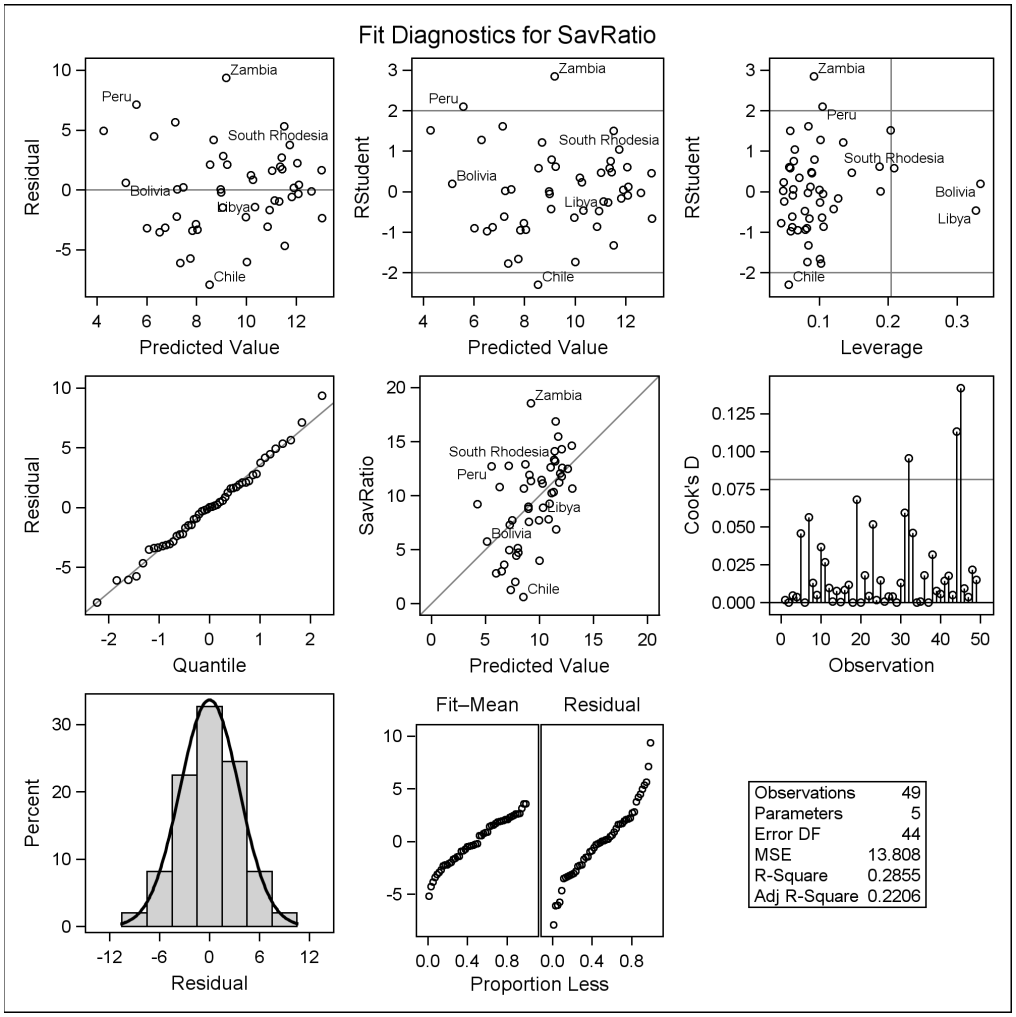
Suspect observations

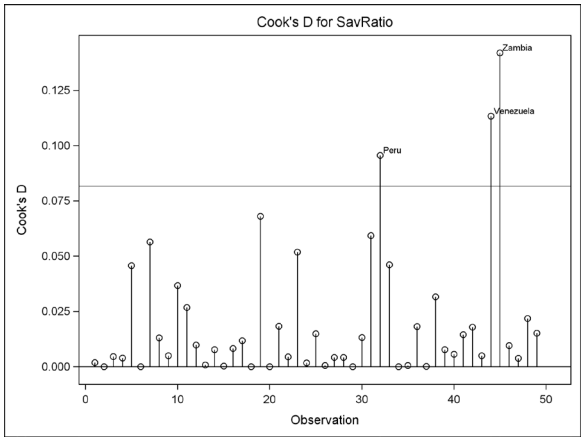
Obs	Country	SavRatio	PopU15	PopO75	AvIncome	GrowRate
21	Ireland	11.34	31.16	4.19	1139.95	2.99
23	Japan	21.1	27.01	1.91	1257.28	8.21
44	United States	7.56	29.81	3.43	4001.89	2.45
46	Zambia	18.56	45.25	0.56	138.33	5.14
49	Libya	8.89	43.69	2.07	123.58	16.71

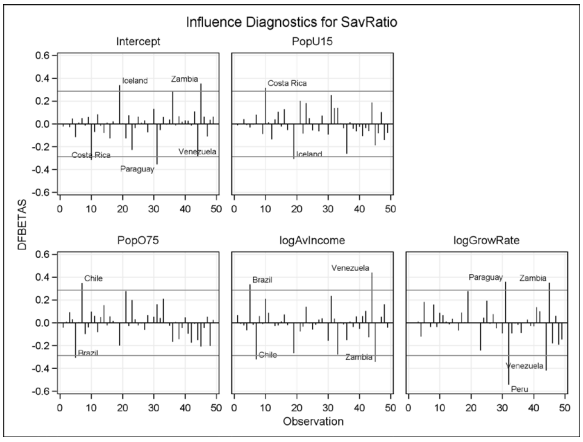
```
/*************************************************
  Possible Remedial Measures for these data:

    Drop Japan
       -- PopU15 and PopO75 don't match profile
          (influential but not outliers)

    Take log of AvIncome and log of GrowRate
       -- their distributions are skew right
       -- the extreme observation in each is a suspect obs.
            (United States for AvIncome,
             Libya for GrowRate)

 *************************************************/

/* Create new data set and fit regression model;
   check assumptions */
data newsavings; set savings;
   if country ne 'Japan';
   logAvIncome = log(AvIncome);
   logGrowRate = log(GrowRate);
run;

proc reg data = newsavings
 plots(label)=(CooksD RStudentByLeverage DFFITS DFBETAS);
 id Country;
 model SavRatio = PopU15 PopO75 logAvIncome logGrowRate
    / partial;
 output out=out2 r=resid p=pred;
 title1 'Predict SavRatio';
 title2 '(after remedial measures)';
run;
```

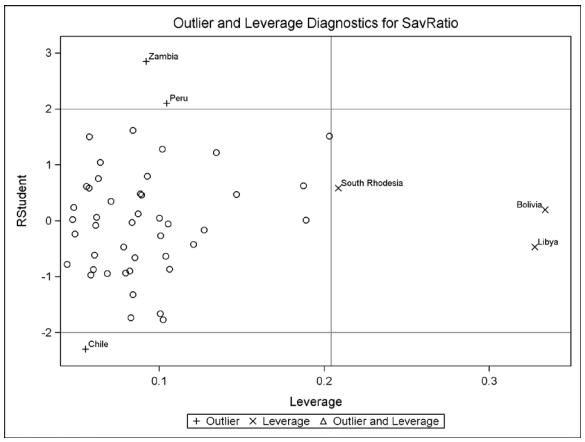
Predict SavRatio
(after remedial measures)

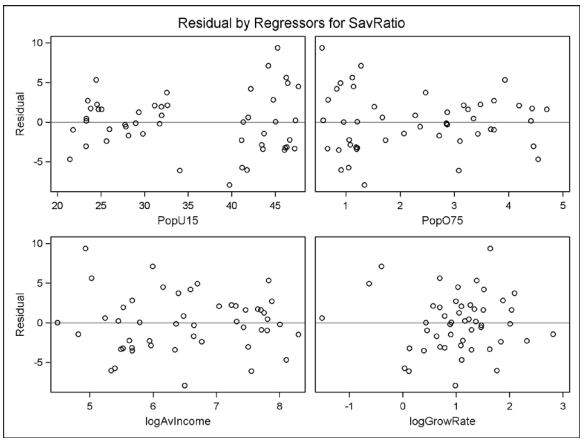
Parameter Estimates					
Variable	**DF**	**Parameter Estimate**	**Standard Error**	**t Value**	**Pr > |t|**
Intercept	**1**	26.25118	10.52632	2.49	0.0165
PopU15	**1**	-0.33837	0.15791	-2.14	0.0377
PopO75	**1**	-0.68558	1.13571	-0.60	0.5492
logAvIncome	**1**	-0.71860	0.97492	-0.74	0.4650
logGrowRate	**1**	1.33042	0.72528	1.83	0.0734

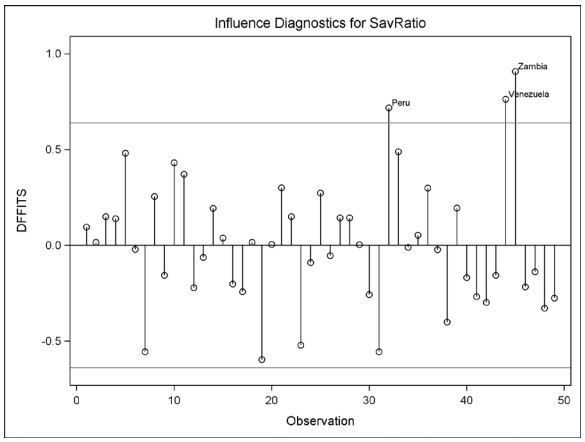


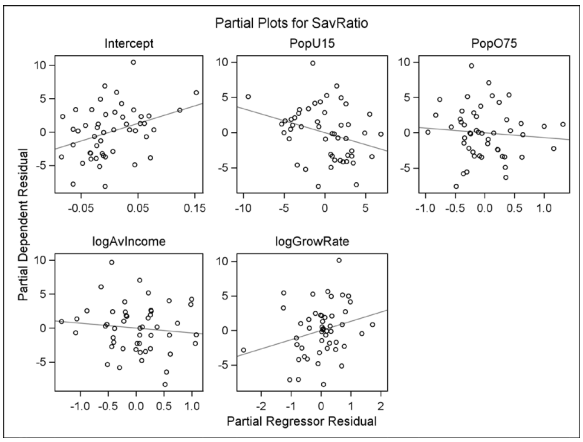













```
/* Check model assumptions */
%resid_num_diag(dataset=out2, datavar=resid,
  label='New Residual', predvar=pred,
  predlabel='New Predicted Value');
run;
```

P-value for Brown-Forsythe test of constant variance in New Residual vs. New Predicted Value

Obs	t_BF	BF_pvalue
1	2.43339	0.018815

Output for correlation test of normality of New Residual (Check text Table B.6 for threshold)

Pearson Correlation Coefficients, N = 49 Prob > |r| under H0: Rho=0		
	resid	expectNorm
resid New Residual	1.00000	0.99516 <.0001
expectNorm	0.99516 <.0001	1.00000

```
/* Look at final model */
proc reg data = newsavings;
     model SavRatio = PopU15 logGrowRate;
      title1 'Final Model';
run;
```

Parameter Estimates					
Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
Intercept	1	14.27955	2.40166	5.95	<.0001
PopU15	1	-0.18046	0.05915	-3.05	0.0038
logGrowRate	1	1.45209	0.71058	2.04	0.0468

```
/************************************************
  What if want to add your own reference lines?
 ************************************************/

proc reg data = savings ;
     id country;
     model SavRatio = PopU15 PopO75 AvIncome GrowRate /
             influence;
     ods output outputstatistics=out3;
run;

proc print data=out3;
run;
```

Observation	Country	Residual	RStudent	HatDiagonal	CovRatio	DFFITS	...	DFB_GrowRate
1	Australi	0.8636	0.2327	0.0677	1.1928	0.0627		-0.0002
2	Austria	0.6164	0.1710	0.1204	1.2678	0.0632		-0.0082
3	Belgium	2.2190	0.6066	0.0875	1.1762	0.1878		-0.0073
...								

```
proc sgplot data=out3;
 scatter x=HatDiagonal y=RStudent / markerchar=country;
 xaxis label='Leverage';
 yaxis label='Studentized Deleted Residual';
 refline 2.01 / axis=Y lineattrs=(pattern=2);
 refline 3.52 / axis=Y;
 refline .2 / axis=X lineattrs=(pattern=2);
 refline .3 / axis=X;
run;
```