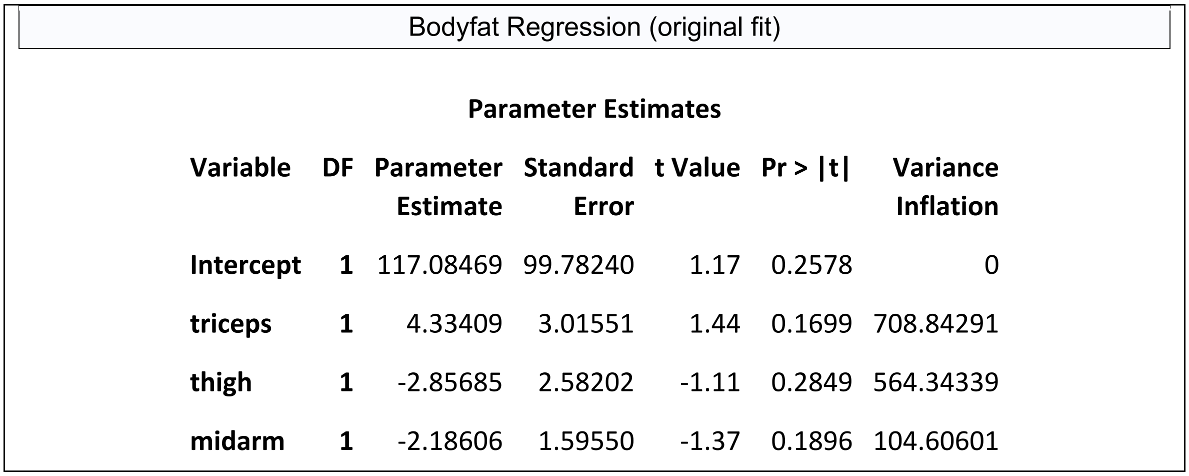
4.1.1: SAS - Penalized Regression Methods
(Ridge Regression, LASSO, and Elastic Net)

Example 1: (Ridge Regression; recall Handout 2.6.1 example) A study seeks to relate (in females) amount of body fat (Y) to triceps skinfold thickness (X_1), thigh circumference (X_2), and midarm circumference (X_3). Amount of body fat is expensive to measure, requiring immersion of person in water. This expense motivates the desire for a predictive model based on these inexpensive predictors.

```
/* Input data */
data bodyfat;
   input triceps thigh midarm body @@; cards;
  19.5  43.1  29.1  11.9       24.7  49.8  28.2  22.8
  30.7  51.9  37.0  18.7       29.8  54.3  31.1  20.1
  19.1  42.2  30.9  12.9       25.6  53.9  23.7  21.7
  31.4  58.5  27.6  27.1       27.9  52.1  30.6  25.4
  22.1  49.9  23.2  21.3       25.5  53.5  24.8  19.3
  31.1  56.6  30.0  25.4       30.4  56.7  28.3  27.2
  18.7  46.5  23.0  11.7       19.7  44.2  28.6  17.8
  14.6  42.7  21.3  12.8       29.5  54.4  30.1  23.9
  27.7  55.3  25.7  22.6       30.2  58.6  24.6  25.4
  22.7  48.2  27.1  14.8       25.2  51.0  27.5  21.1
;
run;

/* Look at original fit */
proc reg data=bodyfat;
  model body = triceps thigh midarm / vif;
  title1 'Bodyfat Regression (original fit)';
run;
```



Bodyfat Regression (original fit)

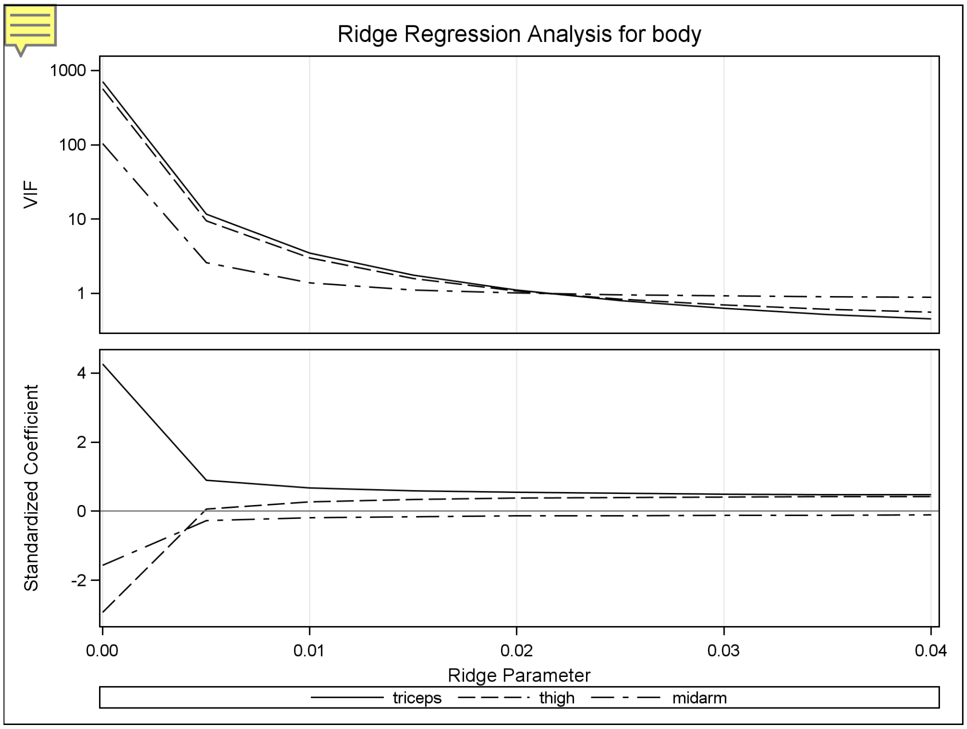
Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|	Variance Inflation
Intercept	**1**	117.08469	99.78240	1.17	0.2578	0
triceps	**1**	4.33409	3.01551	1.44	0.1699	708.84291
thigh	**1**	-2.85685	2.58202	-1.11	0.2849	564.34339
midarm	**1**	-2.18606	1.59550	-1.37	0.1896	104.60601

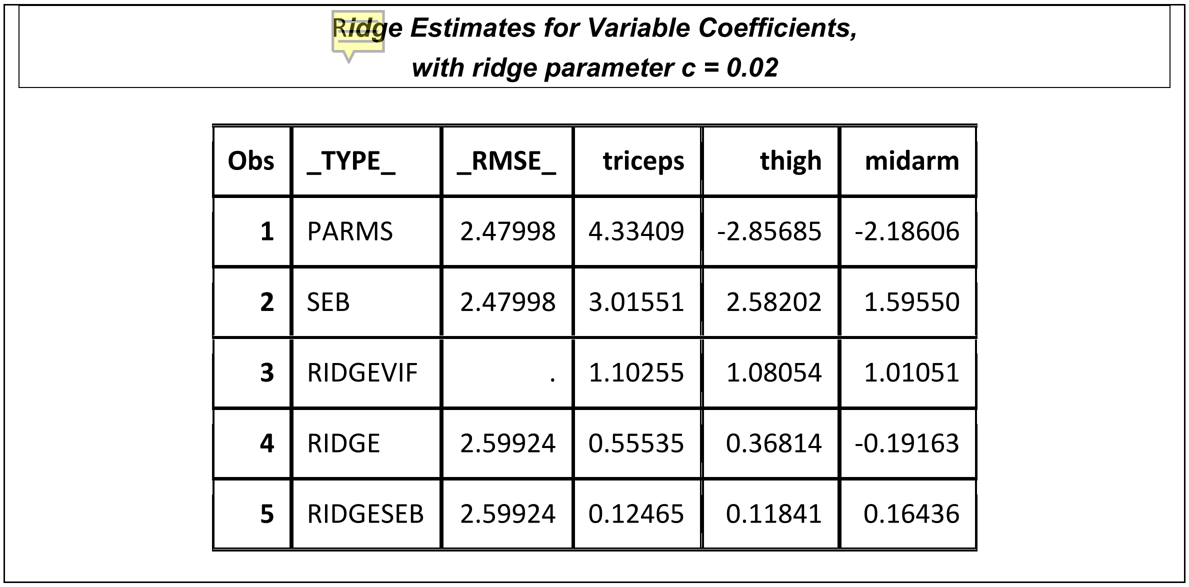
```
/* Try ridge regression as a remedial measure */
proc reg data=bodyfat ridge=0 to .04 by .005
     outvif outest=ridgests
     plots(only)=ridge(VIFaxis=log);
  model body = triceps thigh midarm / vif;
  title1 'Bodyfat Ridge Regression';
  title2 '(find adequate ridge parameter)';
run;
/* What these options do:

     ridge=0 to .04 by 0.005
        run a regression with each of these ridge parameter
       values

     outvif outest=ridgests
        ask for relevant output to be sent to a data set
       called ridgests (will include VIF and standardized
       coefficients for each ridge parameter)

     plots(only)=ridge(VIFaxis=log);
       make Ridge Trace and VIF plots only, with vertical axis
       in VIF plot on log scale
 */
```

Bodyfat Ridge Regression

(find adequate ridge parameter)



```
/* Now look at variable coeffs with ridge parameter 0.02 */
proc reg data=bodyfat outest=ridgenew outseb ridge=0.02
       outvif noprint;
    model body = triceps thigh midarm;
    title1 'Bodyfat Ridge Regression (c=.02)';
run;
proc print data=ridgenew;
  var _type_  _rmse_  triceps thigh midarm;
  title1 'Ridge Estimates for Variable Coefficients,';
  title2 'with ridge parameter c = 0.02';
run;
/* PARMS and SEB give the result of the regular OLS regression.
   RIDGE and RIDGESEB give the result of the ridge regression.
   -- Note no intercept is given; need to use textbook
      equation 7.46b to get intercept in ridge reg. (as below)
   Note substantial drop in SE for estimates in ridge reg.
   RIDGEVIF give the VIF after ridge regression.
 */
```



Ridge Estimates for Variable Coefficients,
with ridge parameter c = 0.02

Obs	_TYPE_	_RMSE_	triceps	thigh	midarm
1	PARMS	2.47998	4.33409	-2.85685	-2.18606
2	SEB	2.47998	3.01551	2.58202	1.59550
3	RIDGEVIF	.	1.10255	1.08054	1.01051
4	RIDGE	2.59924	0.55535	0.36814	-0.19163
5	RIDGESEB	2.59924	0.12465	0.11841	0.16436

```
/* Get intercept term in ridge regression */
proc means data=bodyfat mean;
  var body triceps thigh midarm;
  title1 'Summary Statistics';
run;
data temp;
 b0 = 20.195 - 0.55535*25.305 - 0.36814*51.17 + 0.19163*27.62;
proc print data=temp;
 var b0;
 title1 'Ridge Regression Intercept';
run;
```

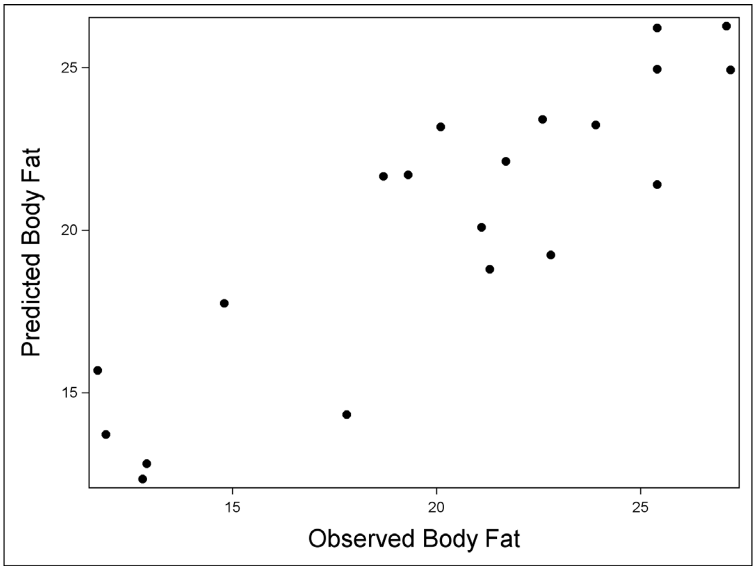
Summary Statistics

Variable	Mean
body	20.1950000
triceps	25.3050000
thigh	51.1700000
midarm	27.6200000

Ridge Regression Intercept

Obs	b0
1	-7.40303

```
/* Get predicted values in ridge regression */
data bodyfat; set bodyfat;
  predbody = -7.40303 + 0.55535*triceps
                 + 0.36814*thigh - 0.19163*midarm;
proc sgplot data=bodyfat;
  scatter x=body y=predbody / markerattrs=(symbol=CIRCLEFILLED);
  xaxis label='Observed Body Fat' labelattrs=(size=15pt);
  yaxis label='Predicted Body Fat' labelattrs=(size=15pt);
  title1;
run;
```



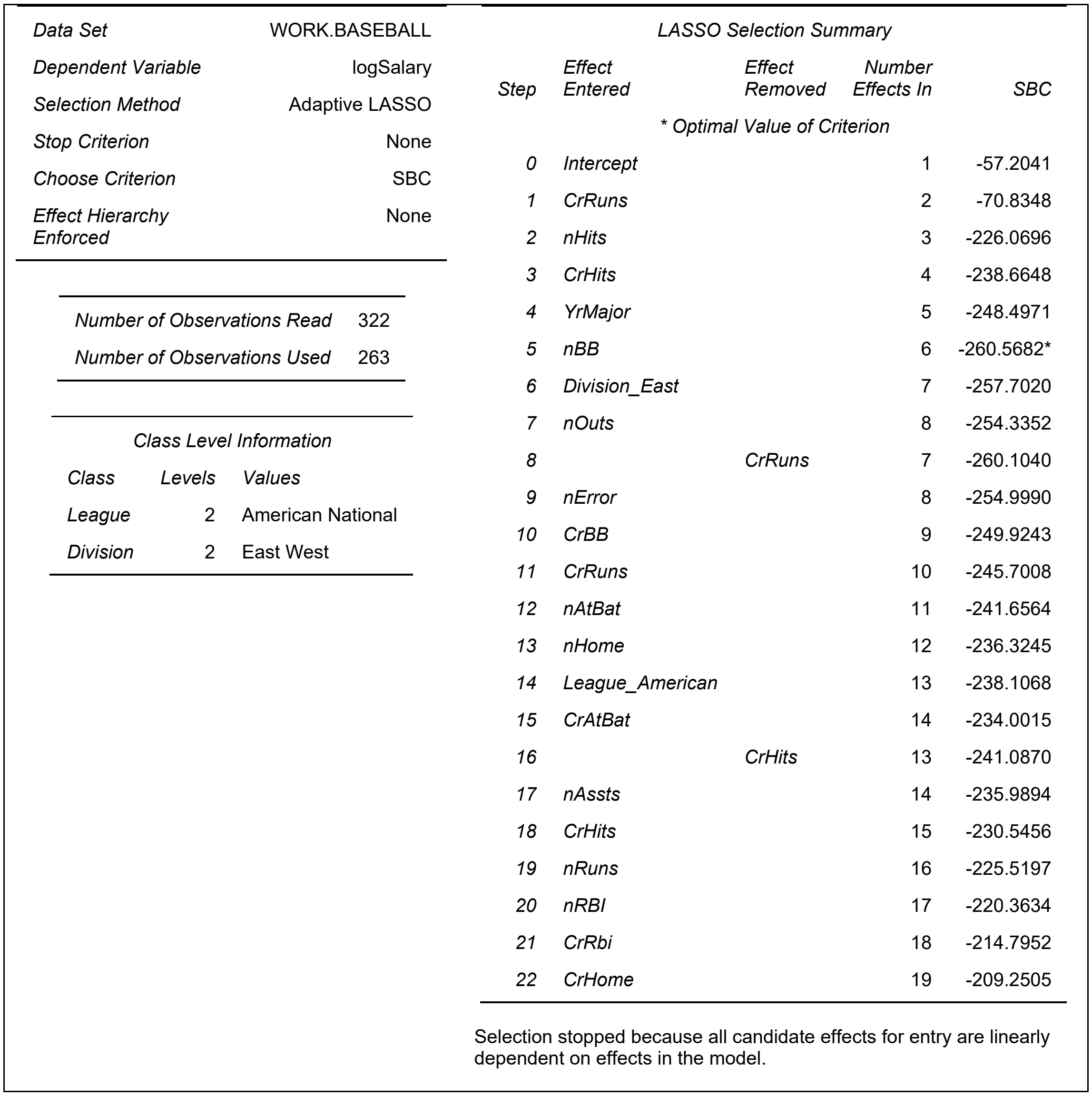
Example 2: (Baseball) This data set (from the SAS Help) contains salary (for 1987) and performance (1986 and some career) data for 322 MLB players who played at least one game in both 1986 and 1987 seasons, excluding pitchers. How can salary be predicted from performance?

```
data baseball; set sashelp.baseball;
proc contents varnum data=baseball;
   ods select position;
run;
```

Variables in Creation Order

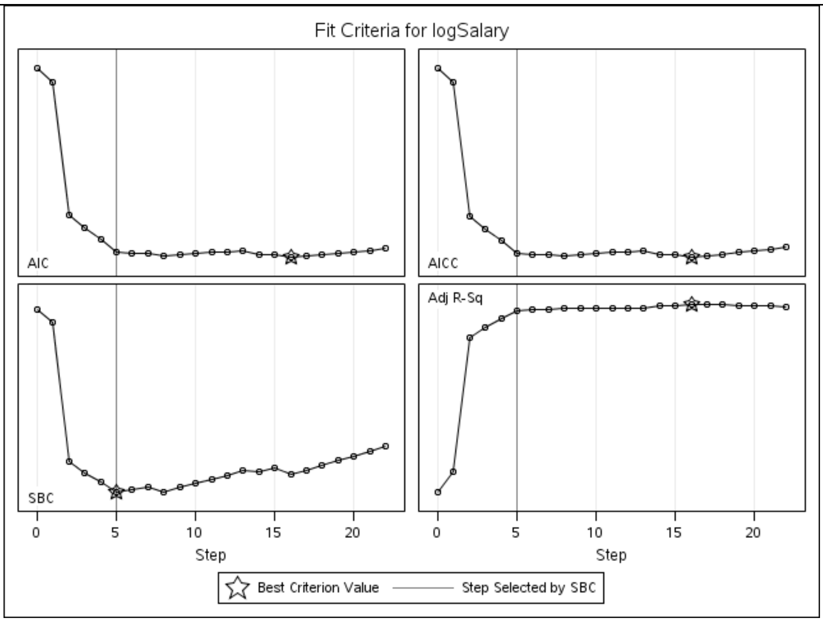
#	Variable	Type	Len	Label
1	Name	Char	18	Player's Name
2	Team	Char	14	Team at the End of 1986
3	nAtBat	Num	8	Times at Bat in 1986
4	nHits	Num	8	Hits in 1986
5	nHome	Num	8	Home Runs in 1986
6	nRuns	Num	8	Runs in 1986
7	nRBI	Num	8	RBIs in 1986
8	nBB	Num	8	Walks in 1986
9	YrMajor	Num	8	Years in the Major Leagues
10	CrAtBat	Num	8	Career Times at Bat
11	CrHits	Num	8	Career Hits
12	CrHome	Num	8	Career Home Runs
13	CrRuns	Num	8	Career Runs
14	CrRbi	Num	8	Career RBIs
15	CrBB	Num	8	Career Walks
16	League	Char	8	League at the End of 1986
17	Division	Char	8	Division at the End of 1986
18	Position	Char	8	Position(s) in 1986
19	nOuts	Num	8	Put Outs in 1986
20	nAssts	Num	8	Assists in 1986
21	nError	Num	8	Errors in 1986
22	Salary	Num	8	1987 Salary in $ Thousands
23	Div	Char	16	League and Division
24	logSalary	Num	8	Log Salary

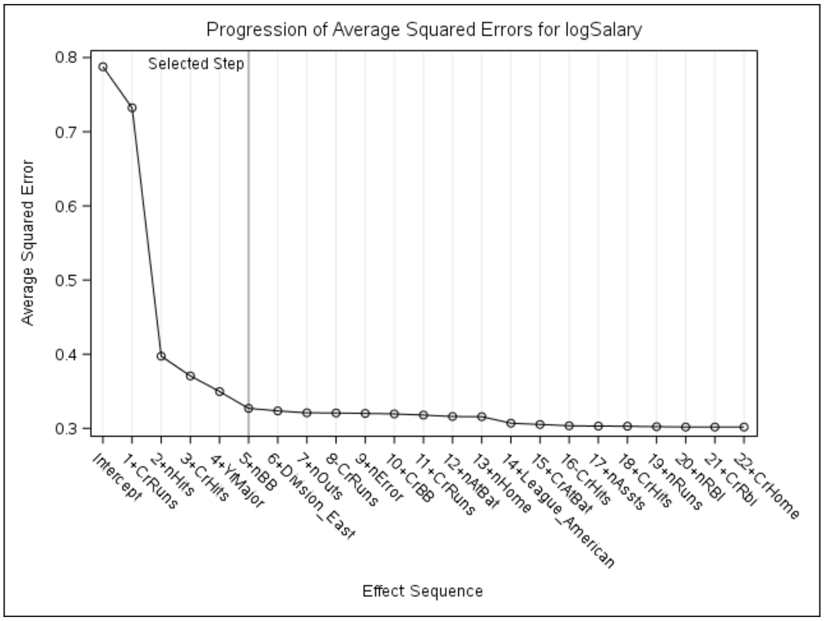
```
/* lasso */
proc glmselect data=baseball plots=(criterion ase);
 class league division;
 model logSalary = nAtBat nHits nHome nRuns nRBI nBB
                  yrMajor crAtBat crHits crHome crRuns crRbi
                  crBB league division nOuts nAssts nError
       / selection=lasso(adaptive choose=sbc stop=none);
 output out=out1 p=predlasso;
run;
```



```
Data Set                          WORK.BASEBALL
Dependent Variable                    logSalary
Selection Method                 Adaptive LASSO
Stop Criterion                             None
Choose Criterion                            SBC
Effect Hierarchy                           None
Enforced

   Number of Observations Read    322
   Number of Observations Used    263

       Class Level Information
  Class      Levels   Values
  League          2   American National
  Division        2   East West
```

```
                     LASSO Selection Summary
        Effect               Effect      Number
 Step   Entered              Removed     Effects In         SBC
                  * Optimal Value of Criterion
    0   Intercept                                 1    -57.2041
    1   CrRuns                                    2    -70.8348
    2   nHits                                     3   -226.0696
    3   CrHits                                    4   -238.6648
    4   YrMajor                                   5   -248.4971
    5   nBB                                       6  -260.5682*
    6   Division_East                             7   -257.7020
    7   nOuts                                     8   -254.3352
    8                        CrRuns               7   -260.1040
    9   nError                                    8   -254.9990
   10   CrBB                                      9   -249.9243
   11   CrRuns                                   10   -245.7008
   12   nAtBat                                   11   -241.6564
   13   nHome                                    12   -236.3245
   14   League_American                          13   -238.1068
   15   CrAtBat                                  14   -234.0015
   16                        CrHits              13   -241.0870
   17   nAssts                                   14   -235.9894
   18   CrHits                                   15   -230.5456
   19   nRuns                                    16   -225.5197
   20   nRBI                                     17   -220.3634
   21   CrRbi                                    18   -214.7952
   22   CrHome                                   19   -209.2505

Selection stopped because all candidate effects for entry are linearly
dependent on effects in the model.
```





Selected Model

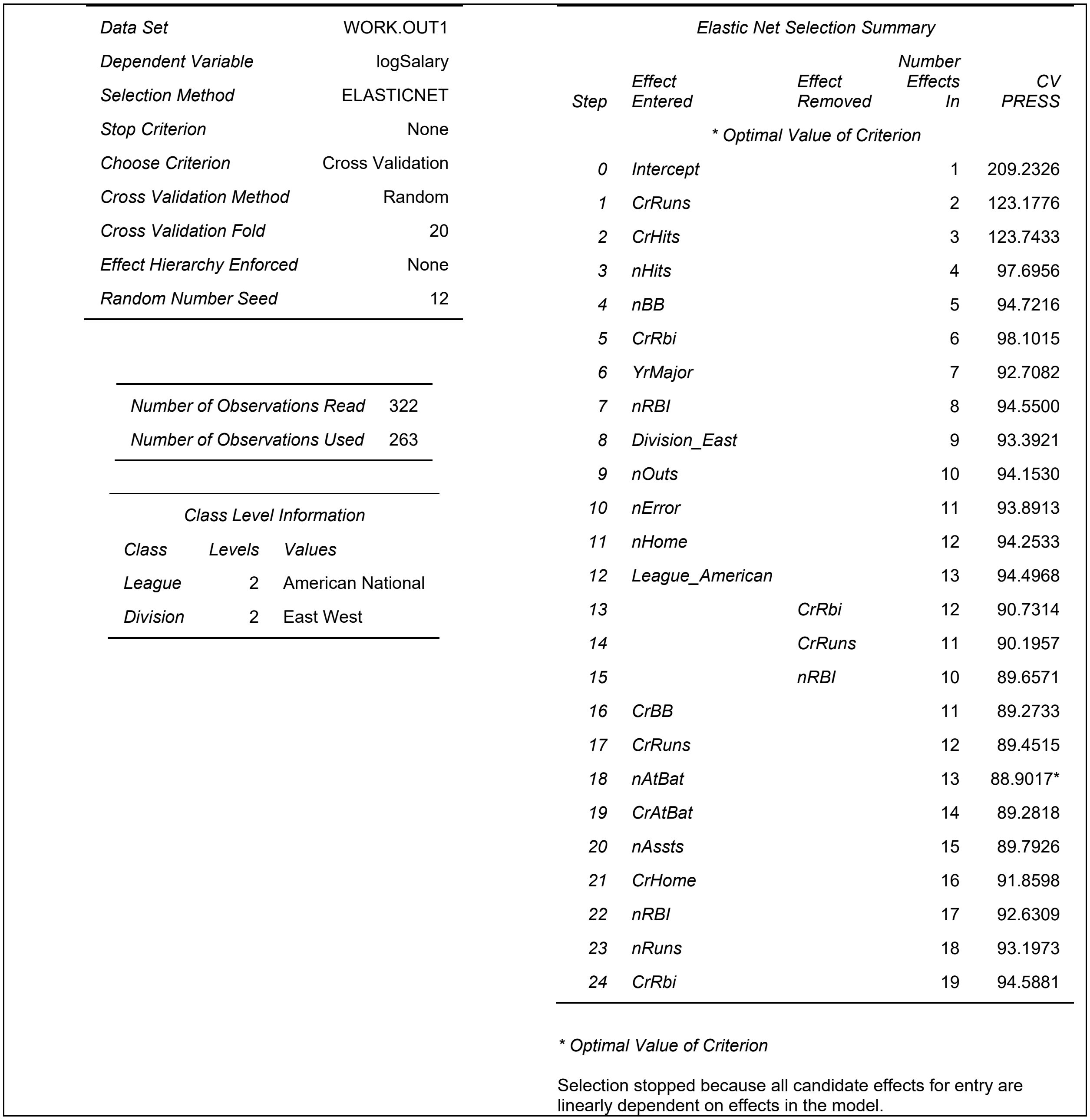
The selected model, based on SBC, is the model at Step 5.

Root MSE	0.57845
Dependent Mean	5.92722
R-Square	0.5849
Adj R-Sq	0.5768
AIC	-17.00115
AICC	-16.56194
SBC	-260.56823

Parameter Estimates

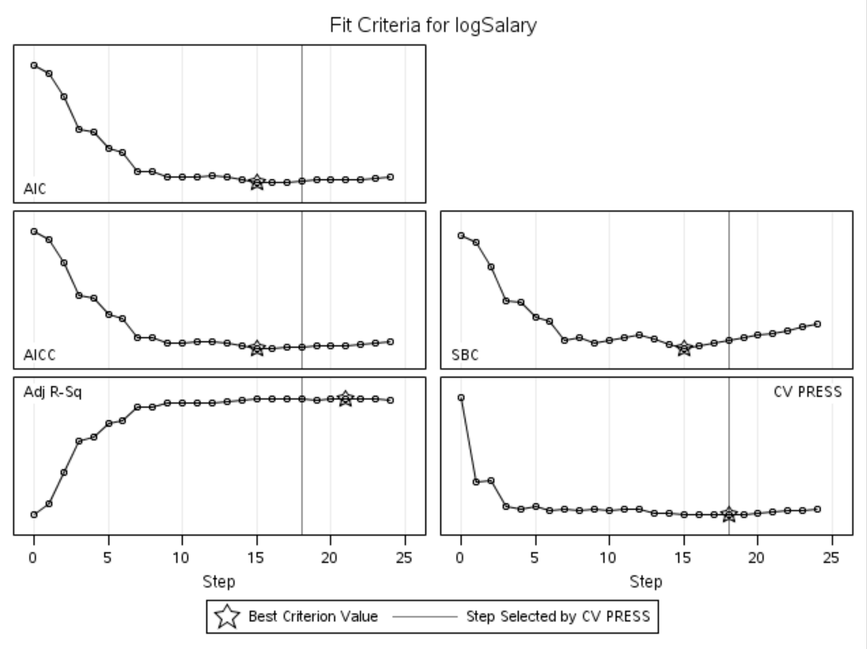
Parameter	*DF*	*Estimate*
Intercept	1	4.229778
nHits	1	0.007194
nBB	1	0.005629
YrMajor	1	0.062808
CrHits	1	0.000222
CrRuns	1	0.000136

```
/* elastic net */
proc glmselect data=out1 plots=(criterion ase) seed=12;
 class league division;
 model logSalary = nAtBat nHits nHome nRuns nRBI nBB
                  yrMajor crAtBat crHits crHome crRuns crRbi
                  crBB league division nOuts nAssts nError
      / selection=elasticnet(stop=none choose=cv)
        cvmethod=random(20);
 output out=out2 p=predelasticnet;
run;
```



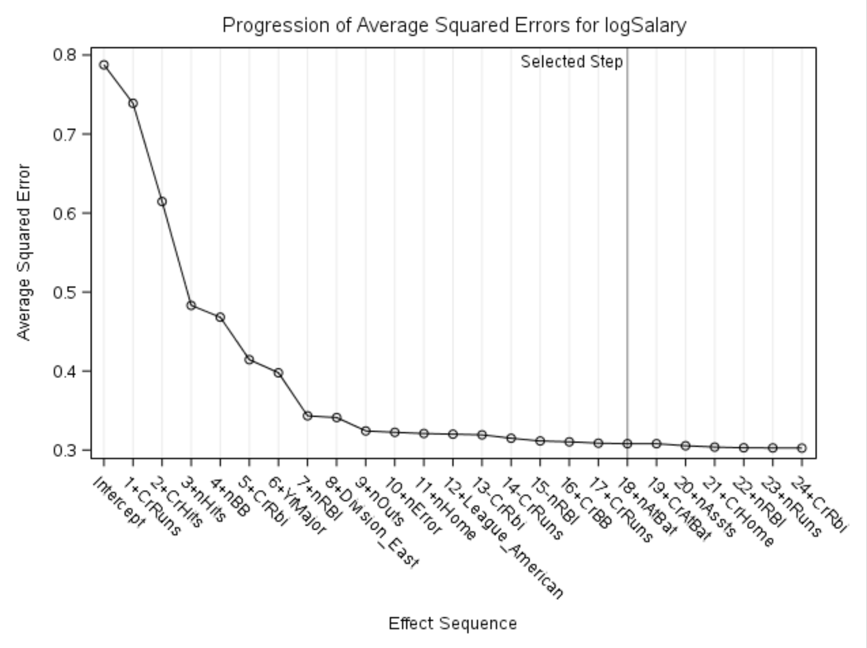
```
Data Set                          WORK.OUT1
Dependent Variable                logSalary
Selection Method                 ELASTICNET
Stop Criterion                         None
Choose Criterion           Cross Validation
Cross Validation Method              Random
Cross Validation Fold                    20
Effect Hierarchy Enforced              None
Random Number Seed                       12

Number of Observations Read    322
Number of Observations Used    263

        Class Level Information
Class       Levels   Values
League           2   American National
Division         2   East West
```

```
                   Elastic Net Selection Summary

                                                Number
       Effect                 Effect           Effects          CV
Step   Entered                Removed               In       PRESS

                   * Optimal Value of Criterion

   0   Intercept                                     1    209.2326
   1   CrRuns                                        2    123.1776
   2   CrHits                                        3    123.7433
   3   nHits                                         4     97.6956
   4   nBB                                           5     94.7216
   5   CrRbi                                         6     98.1015
   6   YrMajor                                       7     92.7082
   7   nRBI                                          8     94.5500
   8   Division_East                                 9     93.3921
   9   nOuts                                        10     94.1530
  10   nError                                       11     93.8913
  11   nHome                                        12     94.2533
  12   League_American                              13     94.4968
  13                          CrRbi                 12     90.7314
  14                          CrRuns                11     90.1957
  15                          nRBI                  10     89.6571
  16   CrBB                                         11     89.2733
  17   CrRuns                                       12     89.4515
  18   nAtBat                                       13     88.9017*
  19   CrAtBat                                      14     89.2818
  20   nAssts                                       15     89.7926
  21   CrHome                                       16     91.8598
  22   nRBI                                         17     92.6309
  23   nRuns                                        18     93.1973
  24   CrRbi                                        19     94.5881

* Optimal Value of Criterion

Selection stopped because all candidate effects for entry are
linearly dependent on effects in the model.
```



Selected Model
The selected model, based on Cross Validation, is the model at Step 18.

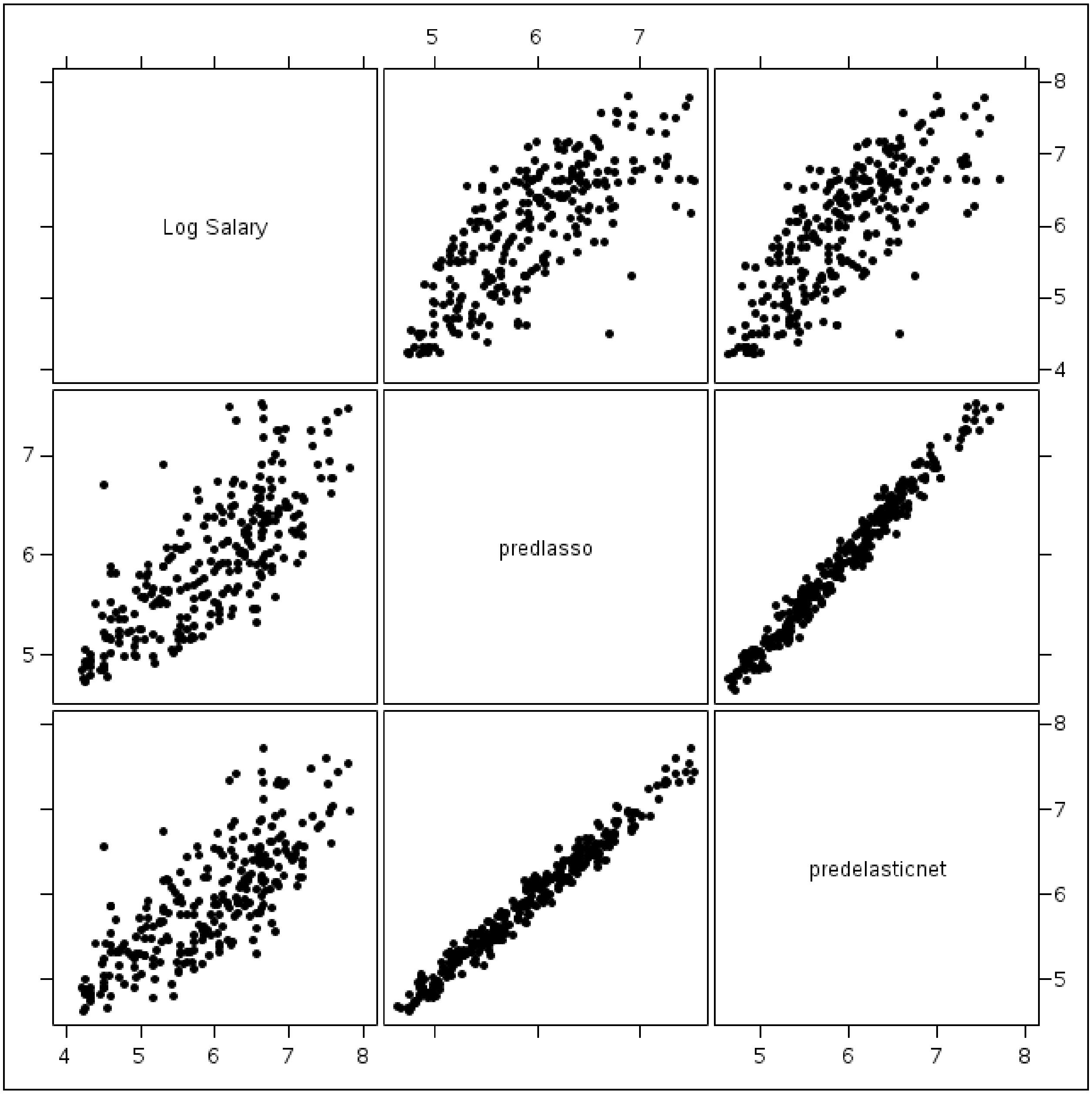
Root MSE	0.56923
Dependent Mean	5.92722
R-Square	0.6090
Adj R-Sq	0.5902
AIC	-18.72037
AICC	-17.02682
SBC	-237.28237
CV PRESS	88.90168



Parameter Estimates

Parameter	*DF*	*Estimate*
Intercept	1	4.195962
nAtBat	1	-0.000112
nHits	1	0.006807
nHome	1	0.003545
nBB	1	0.007082
YrMajor	1	0.070194
CrHits	1	0.000247
CrRuns	1	0.000212
CrBB	1	-0.000348
League_American	1	-0.092575
Division_East	1	0.144062
nOuts	1	0.000192
nError	1	-0.007767

```
proc sgscatter data=out2;
  matrix logSalary predlasso predelasticnet /
          markerattrs=(symbol=circlefilled size=6pt);
run;
```


Log Salary
predlasso
predelasticnet