2.4: Simultaneous Inference and Important Considerations

Dr. Bean - Stat 5100

Simultaneous inference is when we want to conduct multiple tests of significance at the same time.

1 Why Simultaneous Inference?

In handout 2.3, we conducted inference for parameters one at a time. We need to change our approach when looking at multiple parameters simultaneously.

(Groups) How and why do we need to change our approach when conducting simultaneous inference?
(check out **this comic** for help).

If we conduct several tests at the same level of significance, the probability of getting one false positive result (a type I) error becomes much higher than α.

As a result, we need to adjust the level of significance to account for a "multiplicity" of testing.

2 Bonferroni Adjustment

Multiplicity:

- Let A_j = event that an individual $(1-\alpha)100\%$ CI does not contain the true value of β_j.
- $P(A_0) = P(A_1) = \alpha \rightarrow$ Type I Error
  - $P(\text{NOT} A_j)$ = probability that an interval contains the true value of β_j.
- **Bonferroni Inequality:** $P(\text{NOT} A_0 \text{ AND NOT } A_1) \geq 1 - P(A_0) - P(A_1)$

This means that if we conduct g tests at a confidence level of $(1 - \frac{\alpha}{g})$, then we are guaranteed that overall level of confidence for all intervals *considered jointly* will be at least $(1-\alpha)$, we call this the **Bonferroni adjustment**.

- **Bonferroni Advantage:** Can be literally applied in *any* situation requires a multiplicity adjustment, including simultaneous intervals for $\hat{Y}$ at multiple X_h levels.
- **Bonferroni Disadvantage:** Can be overly conservative, producing inefficient (unnecessarily wide) intervals.

Comparison of Simultaneous Intervals for $\hat{Y}$

- Confidence intervals (mean response)
  - Bonferroni

$$\hat{Y} \pm t_{n-p}(1 - \frac{\alpha}{2g}) * s\{\hat{Y}_h\}$$

- Working-Hotelling (WH)

$$\hat{Y} \pm W * s\{\hat{Y}_h\} \qquad \left(W = \sqrt{pF_{p,n-p}(1-\alpha)}\right)$$

  Notice that the W-statistic does not consider g

  * WH provides a "confidence band" for the entire regression line (all possible X_h levels).
  * This means the WH interval at any individual X_h will be wider than the t-based confidence interval, but the WH intervals will eventually be narrower than Bonferroni confidence intervals if enough X_h are considered.

- Prediction intervals (new response)
  - Bonferroni

$$\hat{Y} \pm t_{n-p}(1 - \frac{\alpha}{2g}) * s\{\hat{Y}_{h(new)}\}$$

  - Scheffe (chef-eh)

$$\hat{Y} \pm S * s\{\hat{Y}_{h(new)}\} \qquad \left(S = \sqrt{gF_{g,n-p}(1-\alpha)}\right)$$

Rule of Thumb: Always pick the most efficient interval that guarantees your intended type I error (α).