LIST OF EQUATIONS

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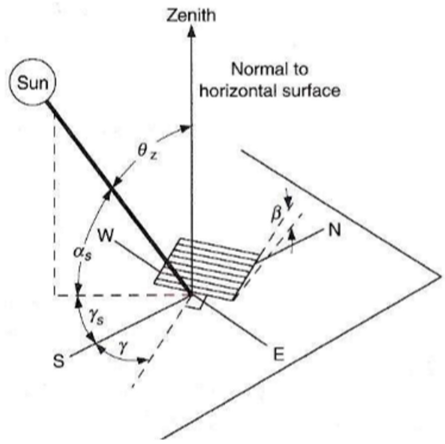
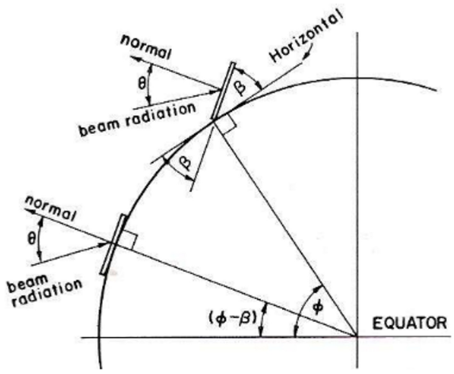
Energy System & Power Engineering

List of Equations

Principles of Solar Radiation





Solar irradiance: $$I = I_D + I_d$$

$$I_D = I_{DN}\cos\theta_z$$

Radiation intensity on titled surfaces: $$I_T = I_D R_T + I_d\left(\frac{1+\cos\beta}{2}\right) + I\rho_g\left(\frac{1-\cos\beta}{2}\right)$$

$$\text{where} \quad R_T = \frac{\text{direct radiation on a tilted surface}}{\text{direct radiation on a horizontal surface}} = \frac{\cos\theta}{\cos\theta_z}$$

Northern hemisphere for a south facing collector:

$$R_{T,\text{north}} = \frac{\cos(\Phi-\beta)\cos\delta_s\cos\omega + \sin(\Phi-\beta)\sin\delta_s}{\cos\Phi\cos\delta_s\cos\omega + \sin\Phi\sin\delta_s}$$

Southern hemisphere for a north facing collector:

$$R_{T,\text{south}} = \frac{\cos(\Phi+\beta)\cos\delta_s\cos\omega + \sin(\Phi+\beta)\sin\delta_s}{\cos\Phi\cos\delta_s\cos\omega + \sin\Phi\sin\delta_s}$$

Solar declination: $$\delta_s = 23.45° \sin\left(\frac{360°(284+n)}{365}\right)$$

Hour angle: $$\omega = 15° \text{ x } (\text{AST-12}) = \frac{\text{minutes from local solar noon}}{4 \text{ min/deg}}$$

Solar altitude angle $$\sin\alpha_s = \cos\theta_z = \sin\Phi\sin\delta_s + \cos\Phi\cos\omega\cos\delta_s$$

Solar azimuth angle: $$\sin\gamma_s = \frac{\cos\delta_s\sin\omega}{\cos\alpha_s}$$

Angle of incidence:

$$\begin{aligned}\cos\theta &= \cos\theta_z \cos\beta + \sin\theta_z \sin\beta\cos(\gamma_s - \gamma) \\ &= \sin\delta_s \sin\Phi\cos\beta - \sin\delta_s \cos\Phi\sin\beta\cos\gamma + \cos\delta_s \cos\Phi\cos\beta\cos\omega \\ &\quad + \cos\delta_s \sin\Phi\sin\beta\cos\omega\cos\gamma + \cos\delta_s \sin\beta\sin\omega\sin\gamma\end{aligned}$$

Apparent solar time:

$$\text{AST} = \text{LST} + \text{ET} + (l_{st} - l_{local}) \text{ x } 4 \text{ min}/\text{deg}; \quad \text{where } l_{st}, l_{local} \begin{cases} < 0 \text{ for East} \\ > 0 \text{ for West} \end{cases}$$

Equation of time:

$$\text{ET (minutes)} = 229.2\begin{bmatrix} 0.000075 + 0.001868\cos(\text{B}) - 0.032077\sin(\text{B}) - \\ 0.014615\cos(2\text{B}) - 0.04089\sin(2\text{B}) \end{bmatrix}; \quad \text{B (deg)} = \frac{360^{\circ}(n-1)}{365}$$

Sunset/Sunrise:

$$\text{Sunrise}/\text{Sunset} = 12{:}00 \text{ noon} \mp \left[\frac{\left(\cos^{-1}\left(-\tan\phi\cdot\tan\delta_s\right)\right)}{15 \text{ (deg/ hour)}}\right]$$

Heat Transfer

Dimensionless numbers:

$$\text{Nu} = \frac{hL}{k}, \quad \text{Re} = \frac{\rho v L_c}{\mu}, \quad \text{Pr} = \frac{\nu}{\alpha} = \frac{c_p \mu}{k}, \quad \text{Ra} = \frac{g\beta' \Delta T L^3}{\nu\alpha}$$

Conduction:

$$q'' = -k\frac{\partial T}{\partial n}$$

Convection:

$$q'' = h\left(T_s - T_\infty\right)$$

Radiation: Blackbody spectral/total emissive power (Planck's Law)

$$e_{\lambda b}(\lambda, T) = \frac{2\pi C_1}{\lambda^5(e^{C_2/(\lambda T)} - 1)}; \quad \begin{cases} C_1 = hc^2 = 0.59552197 \text{ x } 10^8 \text{ W}\cdot\mu\text{m}^4/(\text{m}^2\text{sr}) \\ C_2 = hc/k = 14{,}387.69 \ \mu\text{m}\cdot\text{K} \\ h = \text{Planck constant} = 6.6260755 \text{ x } 10^{-34} \text{ J}\cdot\text{s} \\ c = \text{speed of light} = 2.99792458 \text{ x } 10^8 \text{ m/s} \\ \sigma = \text{Stefan-Boltzmann constant} = 5.6704\cdot 10^{-8} \text{ W}\cdot\text{m}^{-2}\text{K}^{-4} \end{cases}$$

Blackbody total emissive power:

$$e_b(T) = \int_0^\infty e_{\lambda b}(\lambda, T)\, d\lambda = \sigma T^4$$

Wien's Displacement Law

$$\lambda_{max}\cdot T = C_3 = 2897.8 \ \mu\text{m}\cdot\text{K}$$

Blackbody fractional function:

$$F_{\lambda_1 - \lambda_2} = \frac{1}{\sigma T^4}\int_{\lambda_1}^{\lambda_2} e_{\lambda b}(\lambda, T)\, d\lambda$$

Infinite parallel plates

$$q_1 = -q_2 = \frac{\sigma(T_1^4 - T_2^4)}{1/\varepsilon_1 + 1/\varepsilon_2 - 1} = h_r\left(T_1 - T_2\right), \text{ where } h_r = \frac{\sigma(T_1 + T_2)(T_1^2 + T_2^2)}{1/\varepsilon_1 + 1/\varepsilon_2 - 1}$$

Solar Flat-Plate Collector

Solar irradiance absorbed: $$S = I_D R_T (\tau\alpha)_D + I_d (\tau\alpha)_d \left(\frac{1+\cos\beta}{2}\right) + I\rho_g (\tau\alpha)_g \left(\frac{1-\cos\beta}{2}\right) = I_T (\tau\alpha)_{av}$$

Net useful power absorbed:
$$\begin{aligned} Q_u &= A_c\left[S - U_L\left(T_{pm} - T_a\right)\right] \\ &= A_c F'\left[S - U_L\left(T_f - T_a\right)\right] \\ &= A_c F_R\left[S - U_L\left(T_{f,in} - T_a\right)\right] \\ &= \dot{m} c_p\left(T_{f,out} - T_{f,in}\right) \end{aligned}$$

Radiative heat transfer coefficients:
$$h_{r,p-w} = \frac{\sigma\left(T_{pm} + T_w\right)\left(T_{pm}^2 + T_w^2\right)}{1/\varepsilon_p + 1/\varepsilon_w - 1}$$
$$h_{r,w-s} = \frac{\sigma\varepsilon_w\left(T_w^4 - T_s^4\right)}{\left(T_w - T_a\right)}$$

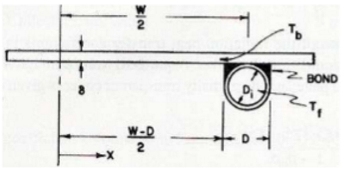
Overall heat transfer coefficient: $U_L = U_e + U_b + U_t$; $U_t = \left[1\Big/\left(h_{c,p-w} + h_{r,p-w}\right) + 1\Big/\left(h_{c,w-a} + h_{r,w-a}\right)\right]^{-1}$

Window transmittance/absorptance/reflectance: $T = \tau\left(\frac{1-\rho}{1+\rho}\right)$; $A = \frac{(1-\rho)(1-\tau)}{1-\rho\tau}$; $R = \rho\left[1 + \frac{(1-\rho)^2\tau^2}{1-\rho^2\tau^2}\right]$

"Tau-alpha" product: $$(\tau\alpha) = \left[\frac{T\alpha}{1-(1-\alpha)R}\right]$$

Incident angle modifier: $$K_{\tau\alpha} = \frac{(\tau\alpha)}{(\tau\alpha)_N}$$

Fin efficiency:



$$F = \frac{\tanh\left[m(W-D)/2\right]}{m(W-D)/2}, \quad m = \sqrt{U_L/k\delta}$$

Collector efficiency factor: $$F' = \left[U_L \cdot W\left(\frac{1}{U_L\left[D + (W-D)F\right]} + \frac{\delta_b}{k_b w_b} + \frac{1}{h_f \pi D_i}\right)\right]^{-1}$$

Collector heat removal factor: $$F_R = \frac{\dot{m}C_p}{A_c U_L}\left[1 - \exp\left(-\frac{A_c U_L F'}{\dot{m}C_p}\right)\right]$$

Efficiency / Hottel-Whillier equation:

$$\eta_c = \frac{Q_u}{A_c I_T} = \frac{F_R\left[S - U_L\left(T_{f,in} - T_a\right)\right]}{I_T} = F_R\left[(\tau\alpha)_{av} - U_L\,\frac{\left(T_{f,in} - T_a\right)}{I_T}\right]$$

Concentrated Solar Energy

Stagnation temperature: $$T_{\text{stagnation}} = \left(\frac{C \cdot I}{\sigma}\right)^{0.25}$$

Mean solar concentration ratio over aperture *A*: $$C = \frac{Q_{\text{solar}}}{A \cdot I}$$

Solar energy absorption efficiency: $$\eta_{\text{absoprtion}} = \frac{\alpha Q_{\text{solar}} - \varepsilon A \sigma T^4}{Q_{\text{solar}}} \overset{\alpha = \varepsilon = 1}{=} 1 - \frac{\sigma T^4}{IC}$$

Solar energy ideal exergy efficiency: $$\eta_{\text{exergy,ideal}} = \eta_{\text{absorption}} \cdot \eta_{\text{Carnot}} = \left(1 - \frac{\sigma T^4}{IC}\right) \cdot \left(1 - \frac{T_L}{T}\right)$$

Parabolic concentrator: $$C_{\text{peak,3D}} = \frac{\sin^2\phi}{\sin^2\theta}$$

$$C_{\text{mean,2D}} = \frac{\sin(2\phi)}{\sin(2\theta)} \qquad C_{\text{mean,3D}} = \frac{\sin^2(2\phi)}{\sin^2(2\theta)}$$

Compound parabolic concentrator (CPC): $$C_{\text{CPC,2D}} = \frac{r_{\text{in}}}{r_{\text{out}}} = \frac{1}{\sin\alpha},\ C_{\text{CPC,3D}} = \frac{r^2_{\text{in}}}{r^2_{\text{out}}} = \frac{1}{\sin^2\alpha}$$

$$L = \frac{r_{\text{in}} + r_{\text{out}}}{\tan\alpha}$$, where $\alpha = \phi + \theta$

2D-CPC: $$\begin{cases} r = \dfrac{2f\sin(\varphi-\alpha)}{1-\cos\varphi} - r_{\text{out}} \\ z = \dfrac{2f\cos(\varphi-\alpha)}{1-\cos\varphi} \end{cases},$$ where: $$\begin{cases} r_{\text{out}} = r_{\text{in}}\sin\alpha \\ f = r_{\text{out}}(1+\sin\alpha) \\ 2\alpha \le \varphi \le \dfrac{\pi}{2} + \alpha \end{cases}$$

2D-CPC with involutes:

$$\begin{cases} x = r\left[\sin\theta - M(\theta)\cos\theta\right] \\ z = r\left[-\cos\theta - M(\theta)\sin\theta\right] \end{cases},$$ where: $$M(\varphi)\begin{cases} \varphi & \text{for: } 0 \le \varphi \le \dfrac{\pi}{2} + \alpha \\ \dfrac{\pi/2 + \alpha + \varphi - \cos(\varphi-\alpha)}{1+\sin(\varphi-\alpha)} & \text{for: } \dfrac{\pi}{2} + \alpha \le \varphi \le \dfrac{3\pi}{2} - \alpha \end{cases}$$

Photovoltaics

Photon's energy: $$E = \frac{h \cdot c}{\lambda}$$

Blackbody spectral emissive power (Planck's law):

$$e_{\lambda b}\left(\lambda, T\right) = \frac{2\pi C_1}{\lambda^5 (e^{C_2/(\lambda T)} - 1)}; \quad \begin{cases} C_1 = hc^2 = 0.59552197 \times 10^8 \ \mathrm{W \cdot \mu m^4 / (m^2 sr)} \\ C_2 = hc / k = 14,387.69 \ \mathrm{\mu m \cdot K} \\ h = \text{Planck constant} = 6.6260755 \times 10^{-34} \ \mathrm{J \cdot s} \\ c = \text{speed of light} = 2.99792458 \times 10^8 \ \mathrm{m / s} \\ \sigma = \text{Stefan-Boltzmann constant} = 5.6704 \cdot 10^{-8} \ \mathrm{W \cdot m^{-2} K^{-4}} \end{cases}$$

Blackbody total emissive power: $$e_b\left(T\right) = \int_0^{\infty} e_{\lambda b}\left(\lambda, T\right) d\lambda = \sigma T^4$$

Wien's displacement law: $$\lambda_{max} T = 2897.8 \ \mathrm{\mu m \cdot K}$$

Blackbody fractional function: $$F_{\lambda_1 - \lambda_2} = \frac{1}{\sigma T^4} \int_{\lambda_1}^{\lambda_2} e_{\lambda b}\left(\lambda, T\right) d\lambda$$

Spectral efficiency: $$\eta = \sum_{\text{all intervals}} \frac{E_G}{E_\lambda} F_{\lambda_i - \lambda_{i+1}}$$